A CONTINUUM ChPT DETERMINATION OF THE STRONG ISOSPIN BREAKING CONTRIBUTION TO $a_\mu^{LO,HVP}$

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CONTEXT: ERRORS ON a_μ, $a_\mu^{LO,HVP}$

a_μ EXPERIMENTAL ERROR

- BNL E821: 6.3×10^{-10}
- FNAL E989 Run 1: 5.4×10^{-10}
- **Combined BNL/FNAL: 4.1×10^{-10}**
- **FNAL E989 final target: 1.6×10^{-10}**

- *BaBar/KLOE ππ discrepancy:*
  - *0.3→1.94 GeV difference: 9.8×10^{-10}*
  - *averages excluding one of KLOE, BaBar differ by 5.5×10^{-10}*

$a_\mu^{LO,HVP}$ THEORY

- $a_\mu^{LO,HVP}$ (dispersive) $\times 10^{10}$
  - DHMZ 2020: 694.0(4.0)
  - KNT 2019: 692.8(2.4)

- $a_\mu^{LO,HVP}$ (lattice) $\times 10^{10}$
  - RBC/UKQCD2018: 717.4(18.7)
  - ETM 2019: 692.1(16.3)
  - FHM 2019: 699(15)
  - Mainz 2019: 720.0(15.9)
  - PACS 2019: 737(20)
  - **BMW 2020: 707.5(5.5)**

- Lattice approaching dispersive-level accuracy
- Sub-% goal for lattice $a_\mu^{LO,HVP}$ ⇒ EM, strong IB (SIB) mandatory
- **This talk: SIB**

- Quark-line connected and -disconnected contributions
  - calculated separately on lattice
  - isospin limit: *ud* disconnected ~-2% *ud* connected
  - Lehner-Meyer (LM20): PQChPT for SIB: exact cancellation at NLO of ππ connected and disconnected contributions
  - ⇒expect strong cancellation for SIB

Context for continuum determination

- EM, strong IB (SIB) mandatory
- Strong connected-disconnected cancellation in SIB
- Lattice SIB results
  - FHM17/19: PRL 120, 152001; PRD 101, 034512
  - RBC/UKQCD 18: PRL 121, 022003
  - ETM19: PRD 99, 114502
  - LM20: PRD 101, 074515
  - BWM20: Nature 593, 51

[Caution: FV in separate connected, disconnected]

Contributions to $[a_\mu^{SIB}] \times 10^{10}$

Connected	Disconnected	Collaboration
9.5(4.5)	---	FHM17/20
10.6(8.0)	---	RBC/UKQCD18
6.0(2.3)	---	ETM19
9.0(1.4)	-6.9(3.5)*	LM20
6.6(0.8)	-4.7(9)	BMW20

- * PQChPT estimate, not lattice
- BMW sum: enhanced relative error

THE EUCLIDEAN Q^2 INTEGRAL REPRESENTATION OF $a_\mu^{LO,HVP}$, a_μ^{SIB}

$$\Pi^{ab}_{\mu\nu}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{ab}(Q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\{V^a_\mu(x)V^b_\nu(0)\}|0\rangle$$

$$V^a_\mu = \bar{q}\frac{\lambda^a}{2}\gamma_\mu q, \qquad J^{EM}_\mu = V^3_\mu + \frac{1}{\sqrt{3}}V^8_\mu$$

$$\Pi_{EM}(Q^2) = \Pi^{33}(Q^2) + \frac{2}{\sqrt{3}}\Pi^{38}(Q^2) + \frac{1}{3}\Pi^{88}(Q^2)$$

$$\Pi^{SIB}(Q^2) = \frac{2}{\sqrt{3}}\Pi^{38}_{QCD}(Q^2)$$

$$\hat{\Pi}_{EM}(Q^2) = \Pi_{EM}(Q^2) - \Pi_{EM}(0)$$

$$\hat{\Pi}^{SIB}(Q^2) = \Pi^{SIB}(Q^2) - \Pi^{SIB}(0)$$

$$a^{LO,HVP}_\mu = -4\alpha^2\int_0^\infty dQ^2 f(Q^2)\hat{\Pi}_{EM}(Q^2)$$

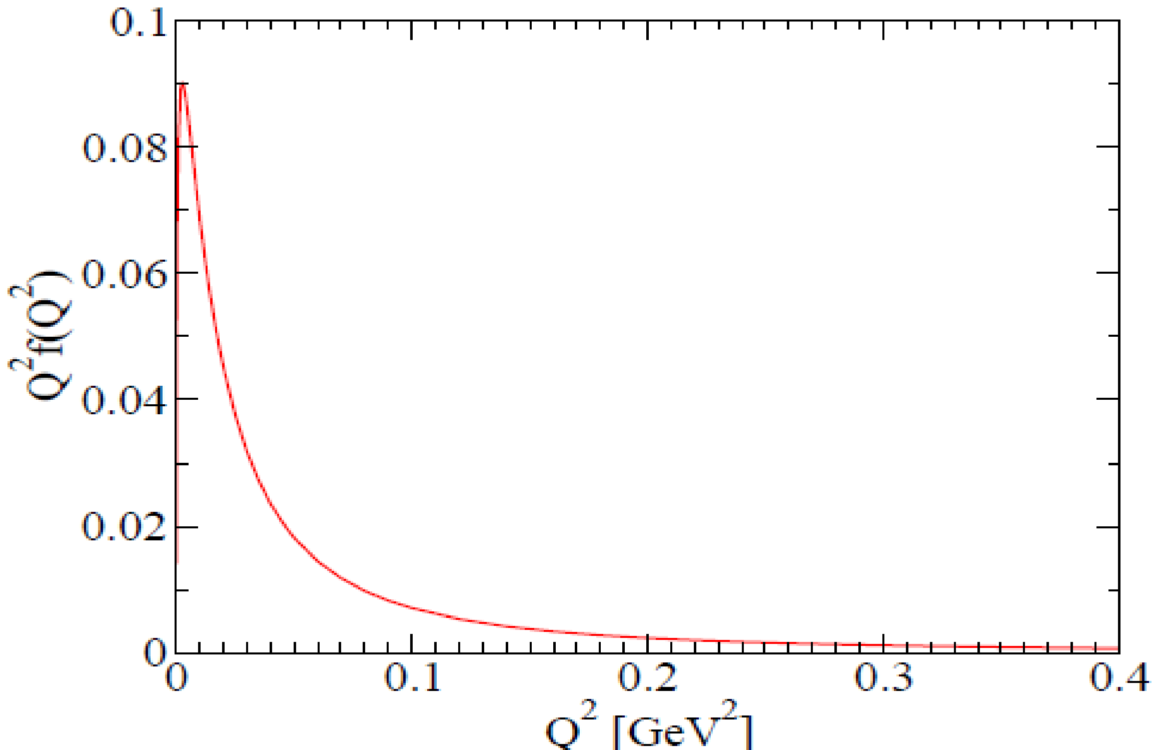
$$a^{SIB}_\mu = -4\alpha^2\int_0^\infty dQ^2 f(Q^2)\hat{\Pi}^{SIB}(Q^2)$$

$$f(Q^2) = \left[m_\mu^2 Q^2 Z^3\ (1 - Q^2 Z)\right] / \left[1 + m_\mu^2 Q^2 Z^2\right]$$

$$Z = \left[\sqrt{Q^4 + 4m_\mu^2 Q^2} - Q^2\right] / 2m_\mu^2 Q^2$$

WHY CHPT?

- $f(Q^2)$ diverges as $1/(m_\mu Q)$ as $Q^2 \to 0$, rapid fall-off with increasing Q^2
- result is $a_\mu^{LO,HVP}$ integrand peaked at very low $Q^2 \sim m_\mu^2/4$ **(region of linear behavior of subtracted EM polarization)**
- $\Rightarrow a_\mu^{SIB}$ integrand will also peak in linear region, at $Q^2 \sim m_\mu^2/4$



I=1 (ab=33) analogue, τ input, dispersive representation:

- ~82% from $Q^2<0.1$ GeV^2
- ~92% from $Q^2<0.2$ GeV^2
- >94% from $Q^2<0.25$ GeV^2

THE ChPT REPRESENTATION OF a_μ^{SIB}

- $a_\mu^{SIB}(Q^2_{max})$, $Q^2_{max}=0.25\ \mathrm{GeV}^2 \simeq m_K^2$, with ChPT representation of subtracted polarization, as approximation to full a_μ^{SIB}
- Estimate error from I=1 analogue case
  - Advantage: I=1 subtracted polarization (and associated a_μ^{33}) from dispersive representation with experimental input
  - Need chiral representation to NNLO to incorporate resonance region (ρ) effects (in NNLO LEC C_{93})
  - $a_\mu^{33}(0.25\ \mathrm{GeV}^2)$ approximation accurate to 1.5% (combination of ~ -6% from truncation of the integral at $Q^2_{max}=0.25\ \mathrm{GeV}^2$, ~ +5% from effect of missing NNNLO and higher curvature in the NNLO representation)
  - Cancellation between integral-truncation underestimate, missing higher order curvature overestimate also expected for SIB case

ChPT ESTIMATE(S) FOR a_μ^{SIB}

- The subtracted SIB polarization to NNLO [KM PRD53 (1996) 2573]

$$\hat{\Pi}^{38}(Q^2) = \frac{\sqrt{3}}{4}(m_{K^0}^2 - m_{K^+}^2)_{QCD}\left[\frac{2i\bar{B}(\bar{m}_K^2, Q^2)}{Q^2} - \frac{1}{48\pi^2\bar{m}_K^2} + \frac{8i\bar{B}(\bar{m}_K^2, Q^2)}{f_\pi^2}\left(\frac{i}{2}\bar{B}_{21}(m_\pi^2, Q^2) + i\bar{B}_{21}(\bar{m}_K^2, Q^2) + \frac{\log\left(m_\pi^2\bar{m}_K^4/\mu^6\right)}{384\pi^2} - L_9^r(\mu)\right)\right]$$

- *N.B.: no tree-level contact term ⇒ FB resonance-region (ρ-ω, ρ'-ω' interference etc.) contributions not yet encoded at NNLO (first appear at NNNLO: CQ^2 from operator with 4 derivatives, 1 power of the quark mass matrix)*

ChPT ESTIMATE(S) FOR a_μ^{SIB} (2)

- More re the NNLO form
  - $O(\alpha_{EM}\,[m_d+m_u])$ EM/SIB separation ambiguity entirely in $[\Delta m_K^2]_{QCD}$ factor (FLAG result for violation of Dashen's theorem as input)
  - Absence of NLO π loop contribution (LM20: connected-disconnected cancellation of NLO contributions from ππ intermediate states)
  - No exact connected-disconnected ππ cancellation at NNLO
  - ππ cancellation makes NLO small; NNLO larger, but still small

$$\left[a_\mu^{SIB}(0.25\ GeV^2)\right]_{NLO} = 0.073 \times 10^{-10}$$
$$\left[a_\mu^{SIB}(0.25\ GeV^2)\right]_{NNLO} = 0.552(37) \times 10^{-10}$$

  - NLO+NNLO total, $0.625(37) \times 10^{-10}$: small c.f. *few* x 10^{-10} ρ-ω region contributions from fits to ππ cross sections in interference region

ChPT ESTIMATE(S) FOR a_μ^{SIB} (3)

➢ Beyond NNLO (encoding leading resonance-region contributions)

- ❖ Contributions from states integrated out in forming low-energy effective Lagrangian appear first at NNNLO in subtracted FB/IB V current polarizations
- ❖ Resulting leading chiral order tree-level contributions to subtracted FB/IB V current polarizations $\propto C_{FB/IB}\, Q^2$
- ❖ Only one NNNLO operator producing such contributions for external vector sources only:

$$L_{eff}^{NNNLO} = 8B_0 Q^2 \delta C_{93}^{(1)}\, Tr\left[M v^\mu v^\nu\right]\left(q_\mu q_\nu - g_{\mu\nu} q^2\right)$$

- ❖ Associated contribution to subtracted SIB polarization:

$$\left[\hat{\Pi}^{SIB}(Q^2)\right]_{NNNLO,LEC} = -\frac{8}{3} Q^2 \left(m_{K^0}^2 - m_{K^+}^2\right)_{QCD} \delta C_{93}^{(1)}$$

ChPT ESTIMATE(S) FOR a_μ^{SIB} (4)

- Beyond NNLO (encoding leading resonance-region contributions)
  - The NNNLO LEC $\delta C_{93}^{(1)}$ also encodes leading resonance-region contributions to the FB flavor *ud-us* vector current polarization

$$\left[\hat{\Pi}_{ud-us;V}(Q^2)\right]_{NNNLO,LEC} = -8Q^2(m_K^2 - m_\pi^2)\,\delta C_{93}^{(1)}$$

  - $\Rightarrow \delta C_{93}^{(1)}$ measurable from slope wrt Q^2 at $Q^2=0$
  - Slope measurable from FB IMFESR [GMP17, PRD96, 045027]

$$\left.\frac{d\hat{\Pi}_{ud-us;V}(Q^2)}{dQ^2}\right|_{Q^2=0} = -\int_{4m_\pi^2}^{s_0} ds\, w_\tau(s/s_0)\,\frac{\rho_{ud;V}(s)-\rho_{us;V}(s)}{s^2}$$

$$-\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, w_\tau(s/s_0)\,\frac{\hat{\Pi}_{ud-us;V}(Q^2=-s)}{s^2}$$

- 1^{st} term RHS: non-strange, strange hadronic τ decay distributions
- 2^{nd} term RHS (very small): OPE
- $w_\tau(x)=1-3x^2+2x^3$ factor a technical convenience

FINAL ChPT ESTIMATE FOR a_μ^{SIB}

- Update GMP17 $\delta C_{93}^{(1)}$ result [main impact: HFLAV 2019 strange BF input]

$$\frac{d\hat{\Pi}_{ud-us;V}(Q^2)}{dQ^2}\Big|_{Q^2=0} = -0.0862(24)\ GeV^{-2}$$

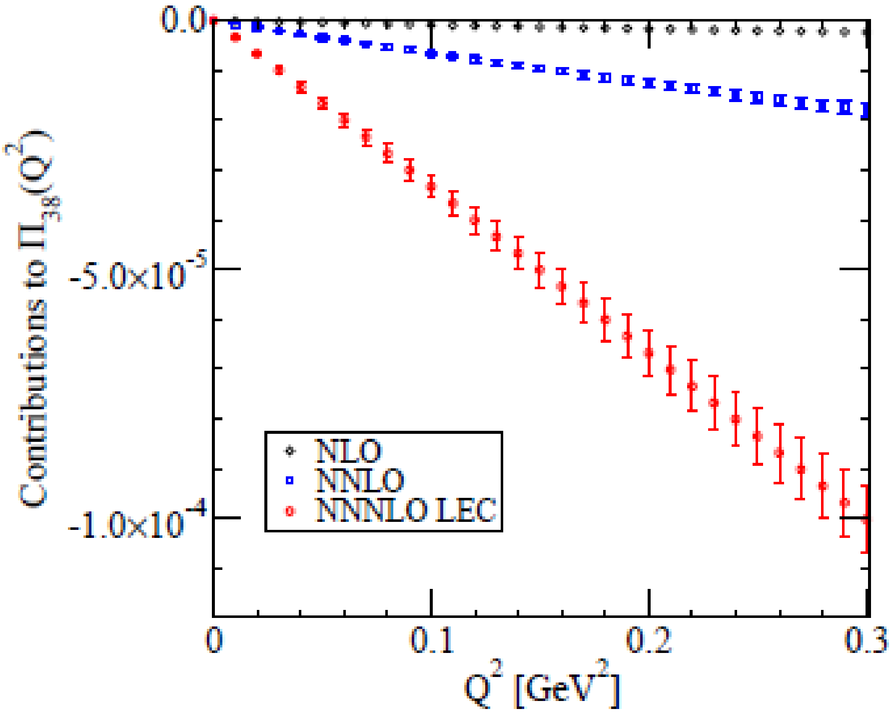
$$\delta C_{93}^{(1)}\ (m_K^2 - m_\pi^2) = 0.00534(37)\ GeV^{-2}$$

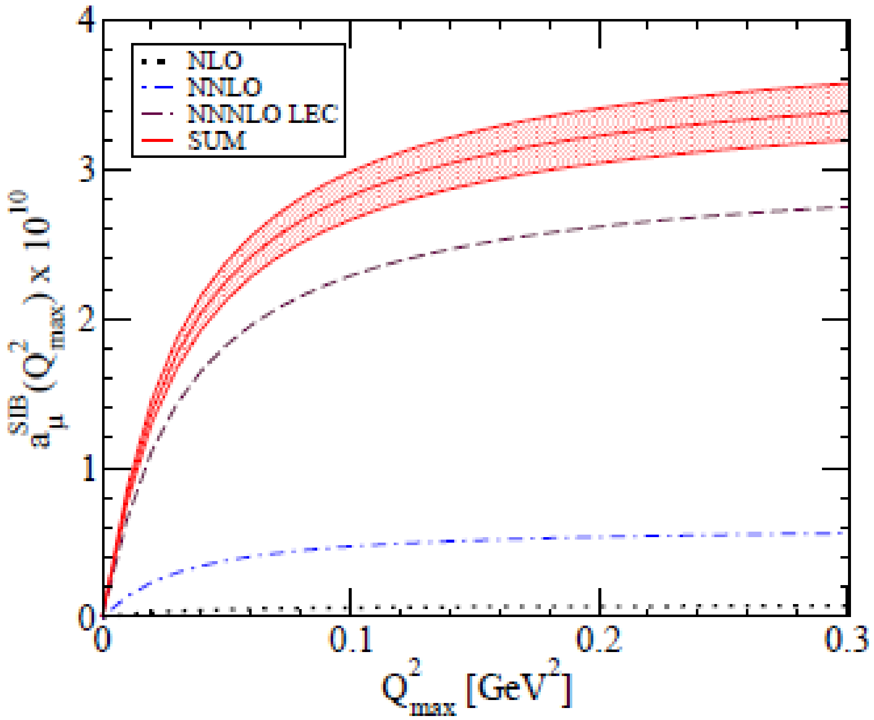
- $\Rightarrow \left[a_\mu^{SIB}(0.25\ GeV^2)\right]_{NNNLO} = 2.69(18)_{IMFESR\ slope}(81)_{HO\ FB\ IMFESR} \times 10^{-10}$

- Final NLO+NNLO+NNNLO result [c.f. BMW20 lattice: 1.93(83)(87) x 10^{-10}]

$$a_\mu^{SIB} = 3.32(4)_{L_9}(19)_{IMFESR\ slope}(33)_{Q^2,curv\ trunc}(81)_{HO\ FB\ IMFESR} \times 10^{-10}$$

NLO, NNLO, NNNLO LEC SIB contributions vs Q^2 and Q^2_{max}





SUMMARY/CONCLUSIONS

- $a_\mu^{SIB} = 3.32(90) \times 10^{-10}$
- error dominated by 30% estimate of possible higher order FB contributions in FB IMFESR determination of $\delta C_{93}^{(1)}$
- compatible with BMW20 lattice result within errors
- dominance by resonance-region dominated tree-level NNNLO LEC contribution $\Rightarrow$ small (numerically negligible) FV effects in connected+disconnected sum (NOT true of individual terms)
- Of interest for lattice groups to quote slope wrt Q^2 at $Q^2=0$ of connected+disconnected SIB polarization sum (from Euclidean t^4 moment of zero-spatial-momentum SIB 2-point function sum)