On the use of the Operator Product Expansion in finite-energy sum rules for light-quark correlators

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Strong coupling determinations:

- From the FLAG-19 review:

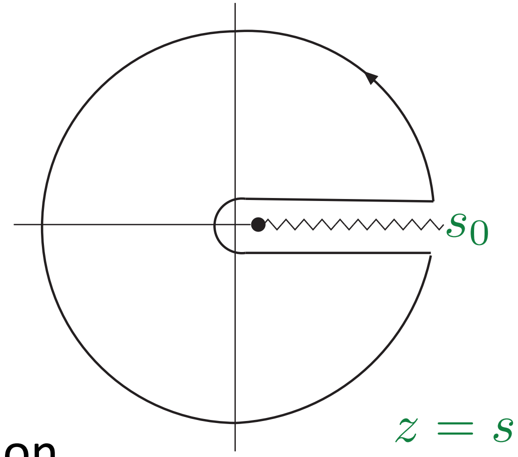
  "Since the size of the nonperturbative effects is very hard to estimate one should try to avoid such regions [i.e., below the tau mass] of the coupling."

- Strong coupling from tau decays: no such luxury!
  1) high precision (if non-perturbative effects can be controlled)
  2) direct test of QCD-running based on experimental data

- Need to face the need to control non-perturbative effects:

  Operator Product Expansion and quark-hadron duality violations: Test assumptions!

Finite Energy Sum Rules



$z = s = q^2$

Linear combinations of $\int_0^{s_0} ds\, s^n\, \rho(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, z^n\, \Pi(z)$

$\Pi(z) = \Pi_{\text{pert.th.}}(z) + \Pi_{\text{OPE}}(z) + \Pi_{\text{DV}}(z)$ is V+A or EM vacuum polarization

↑ α_s ↑ ↑ resonances

$$\Pi_{\text{OPE}}(q^2) = \frac{C_4}{q^4} - \frac{C_6}{q^6} + \frac{C_8}{q^8} - \dots$$

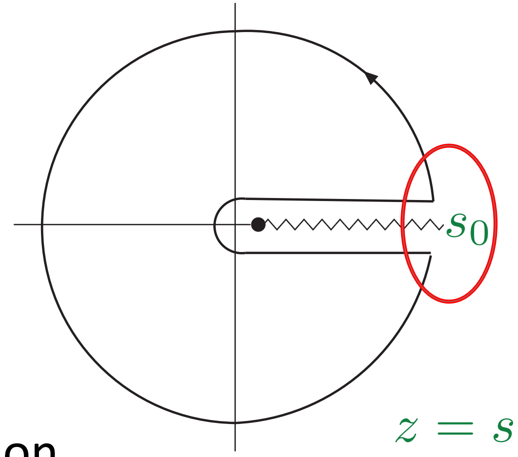
- OPE does not converge – (at best) asymptotic series; z^n sum rule picks out $1/q^{2(n+1)}$ (residue thrm)
- Resonances correspond to cut on positive axis, effect decreases exponentially with q^2, but, $s_0 \leq m_\tau^2$!

Finite Energy Sum Rules

experiment: $I_w^{\rm exp}(s_0)$ theory: $I_w^{\rm th}(s_0)$

$\Downarrow$ $\Downarrow$

Linear combinations of $$\int_0^{s_0} ds\, s^n\, \rho(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, z^n\, \Pi(z)$$



$\Pi(z) = \Pi_{\rm pert.th.}(z) + \Pi_{\rm OPE}(z) + \Pi_{\rm DV}(z)$ is V+A or EM vacuum polarization

$\uparrow$ α_s $\uparrow$ $\uparrow$ resonances

$$\Pi_{\rm OPE}(q^2) = \frac{C_4}{q^4} - \frac{C_6}{q^6} + \frac{C_8}{q^8} - \dots$$

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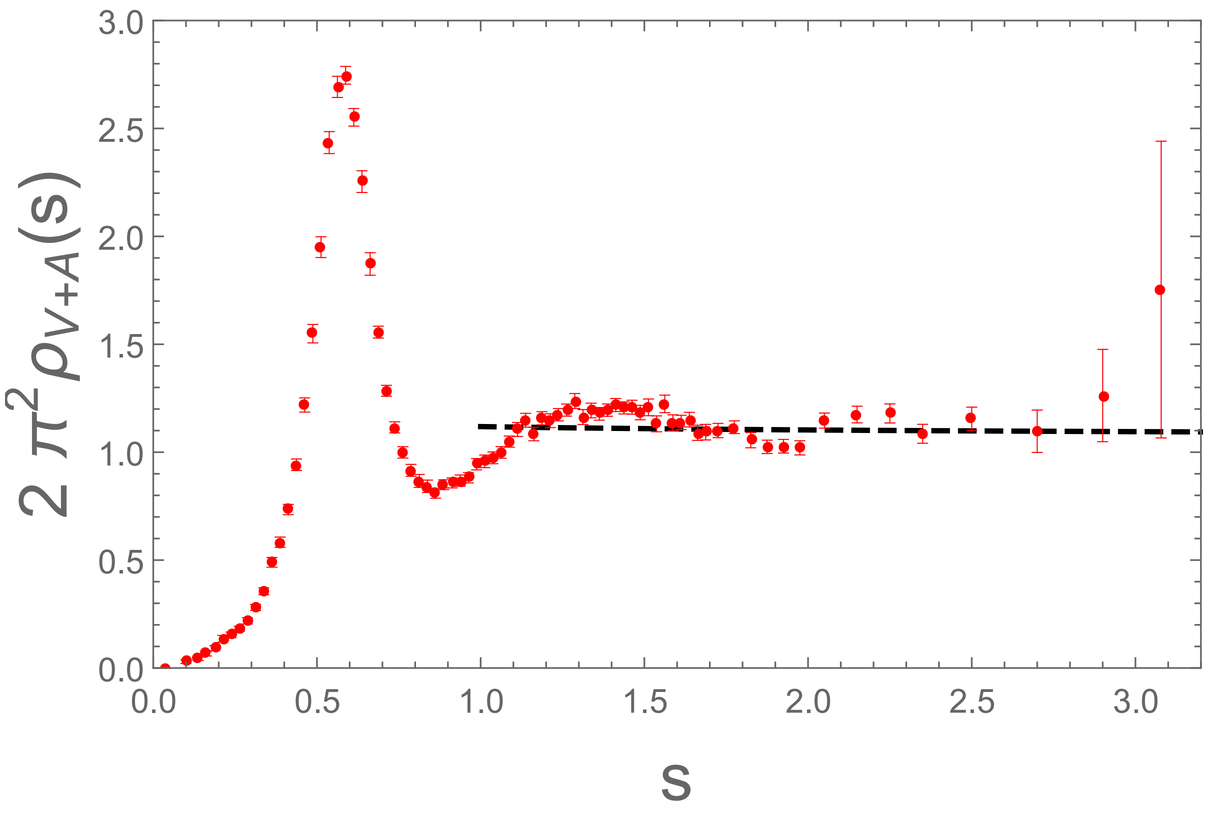
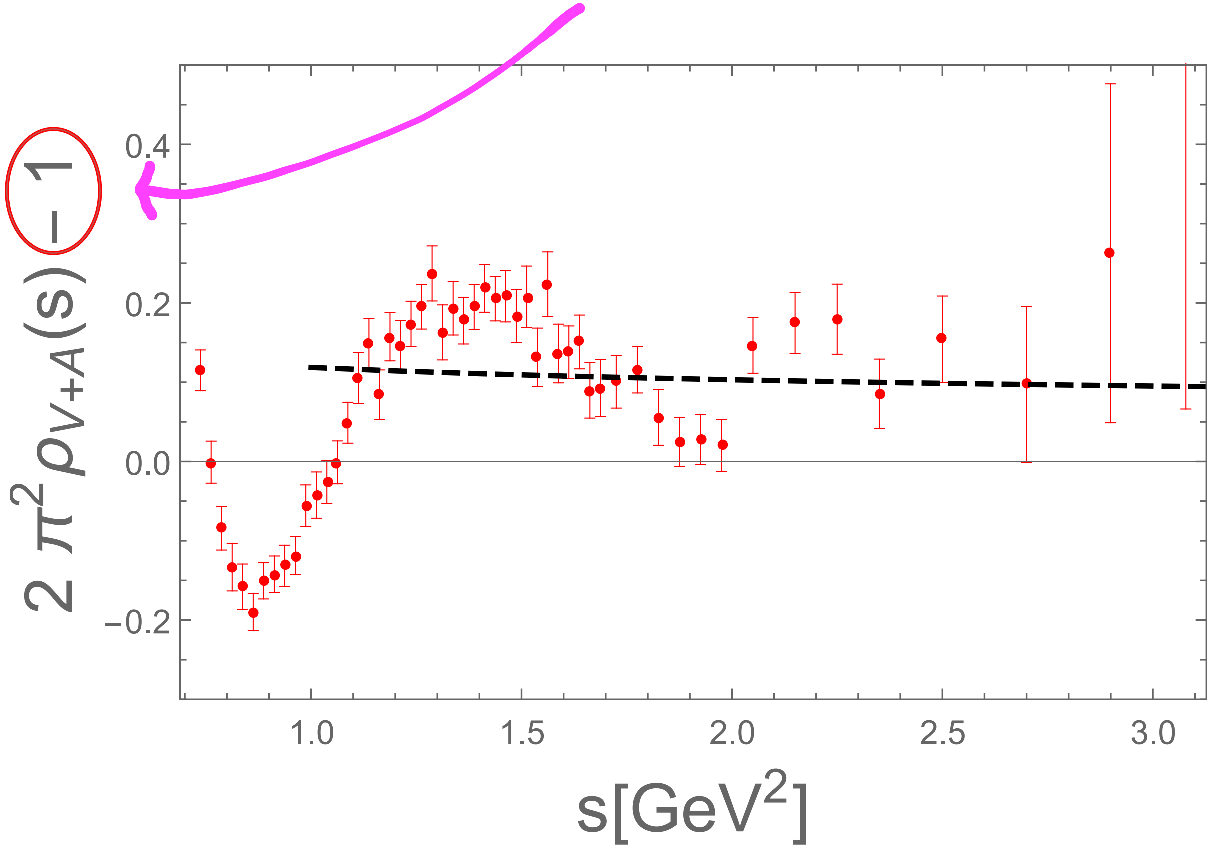
Non-strange spectral functions from hadronic tau decays: data (ALEPH, OPAL, ...)

V+A spectral function (Davier *et al.*, '14, ALEPH)



← perturbation theory, no DVs

Blow up of large-s region (subtracting α_s-independent parton-model contribution):

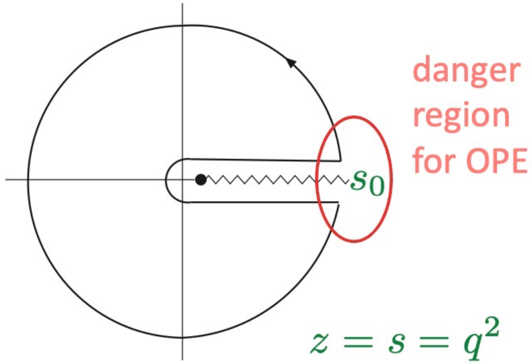


QCD perturbative part dependent on α_s

Quark-hadron duality violations – resonance effects – are **not** small!

$\Longrightarrow$ suppress duality violations (DVs), or take into account in fits

Two strategies to non-perturbative "contamination"



- ALEPH (Davier *et al.*), OPAL, Pich *et al.* ("Truncated-OPE strategy"):

  Ignore Duality Violations, but attempt to suppress dangerous region by "pinching":

  use polynomials with multiple zeroes at $s = s_0 = m_\tau^2$, up to degree 7

  Fit $\alpha_s(m_\tau^2)$ and $C_4\,,\ C_6\,,\ C_8\ (C_{10})$, set higher orders in OPE and DVs to zero by hand

  Difficulty: inconsistent treatment of the OPE THIS TALK

- Boito *et al.* ("DV-model strategy"):

  Treat OPE consistently, use low-degree weights (up to degree 3 or 4)

  Keep and model DVs with *ansatz* based on theory (Boito *et al.*, *Phys. Rev.* D97 (2018) 5, 054007)

  Vary s_0 over a range of values below m_τ^2

  Fit $\alpha_s(m_\tau^2)$ and OPE/DV parameters

  Difficulty: need to model DVs D. Boito's talk

Truncated OPE strategy: example

$$\int_0^{s_0} ds\, w(s)\, \rho(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z)$$

- Choose weights, e.g., the Pich & Rodríguez-Sánchez ('16) "optimal" set:

$$\begin{aligned} w^{(2,1)} &= 1 - 3x^2 + 2x^3 \\ w^{(2,2)} &= 1 - 4x^3 + 3x^4 \\ w^{(2,3)} &= 1 - 5x^4 + 4x^5 \\ w^{(2,4)} &= 1 - 6x^5 + 5x^6 \\ w^{(2,5)} &= 1 - 7x^6 + 6x^7 \end{aligned} \qquad x = s/s_0$$

all doubly pinched (double zero @ $s = s_0$)

In principle OPE terms up to dimension 16 (and suppresses $C_4; C_2 \approx 0$ for non-strange case)

- Set $C_{12} = C_{14} = C_{16} = 0$ by hand, $s_0 = m_\tau^2$: 5 data points, 4 parameters, $\alpha_s,\ C_6,\ C_8,\ C_{10}$

- Argue DV and OPE-truncation effects less severe in V+A $\Longrightarrow$ consider this case

Compare two choices with different *D* = 12, 14, 16 assumed input:

(1) $C_{12} = C_{14} = C_{16} = 0$ (Pich & Rodríguez-Sánchez ‘16, Davier *et al*. ‘14)

(2) $C_{12} = 0.161\ \mathrm{GeV}^{12},\ C_{14} = -0.17\ \mathrm{GeV}^{14},\ C_{16} = -0.55\ \mathrm{GeV}^{16}$ equally arbitrary, but reasonable

FOPT fits of free parameters to ALEPH V+A non-strange spectral data:

	α_s	C_6	C_8	C_{10}	χ^2/dof
choice 1	0.317(3)	0.0014(4)	-0.0010(5)	0.0004(3)	1.26/1
choice 2	0.295(4)	-0.0130(4)	0.0356(5)	-0.0836(3)	1.09/1

Errors statistical only, C_D in GeV^D; similar results for CIPT

OPE coefficients all reasonable; grow with order, but consistent with asymptotic expansion

Huge effect on $\alpha_s(m_\tau^2)$: 7% shift of central value – double the total P&R-S error

Test of the Truncated OPE strategy on data for $e^+e^- \to \mathrm{hadrons}$

- R-ratio data not limited by tau mass, use this to test Truncated OPE approach:
  If the truncated OPE approach works at $s_0 = m_\tau^2$, **it should work** at $s_0 > m_\tau^2$!

- Differences with tau isospin-1 ud V+A analysis:
  (1) V only: Davier *et al.,* Pich *et al*. find V fits consistent with and as good as V+A (*p*-value).
  (2) Additional isospin-0 component (SU(3)-flavor partner of isospin-1 V).

- Use R-ratio data from Keshavarzi *et al.* '18 for $m_\tau^2 \le s_0 \le 4\ \mathrm{GeV}^2$ (exclusive region)

- "Diagonal" fits: only diagonal errors in fit, *include full data covariance matrix in fit errors*

Test of the Truncated OPE strategy – R-ratio with optimal weights: sample fits

- First, $s_0 = m_\tau^2$:

  χ^2 fit: $\alpha_s(m_\tau^2) = 0.308(4)$ $p\text{—value} = 2 \times 10^{-15}$

  diagonal fit: $\alpha_s(m_\tau^2) = 0.245(10)$

  This is a disaster.

- Try larger s_0 : $s_0 = 3.6\ \mathrm{GeV}^2$ (this gets *p*-value above 10%)

  χ^2 fit: $\alpha_s(m_\tau^2) = 0.264(5)$ $p\text{—value} = 0.41$

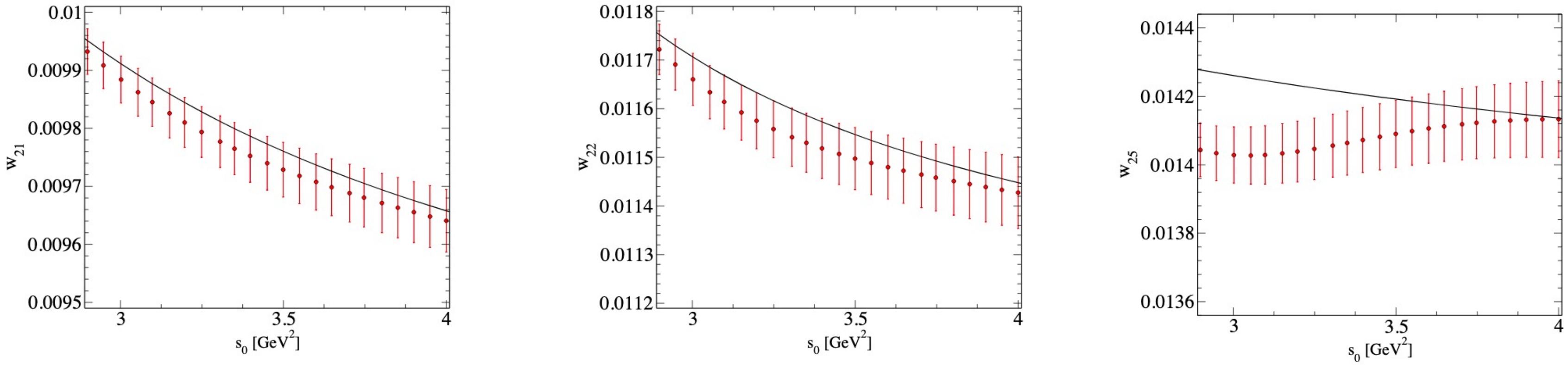
  diagonal fit: $\alpha_s(m_\tau^2) = 0.256(12)$

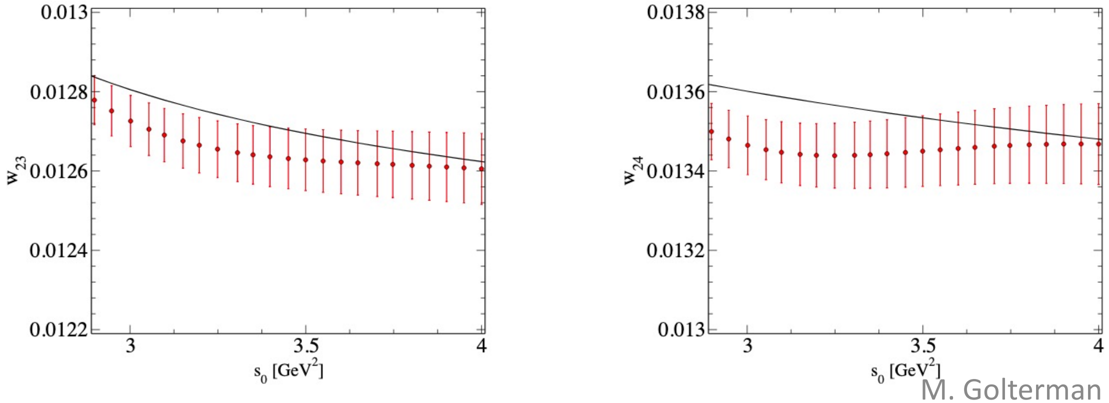
  Good fit, with consistent, but extremely low values for $\alpha_s(m_\tau^2)$: $\alpha_s(m_Z^2) = 0.110$!

- Other sets of weights used in Truncated OPE strategy: very similar results.

Test of the Truncated OPE strategy – R-ratio with optimal weights: s_0 dependence

If the truncated OPE provides a valid strategy above the tau mass, there should be a good match between theory and experiment for all $s_0 \geq m_\tau^2$, using R-ratio data.





Fits using optimal weights with $s_0^* = 3.6\ \mathrm{GeV}^2$

points: $I_w^{\mathrm{exp}}(s_0)$

curves: $I_w^{\mathrm{th}}(s_0)$

Test of the Truncated OPE strategy – R-ratio with optimal weights: s_0 dependence

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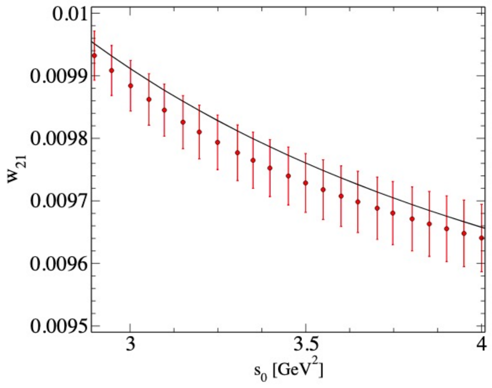
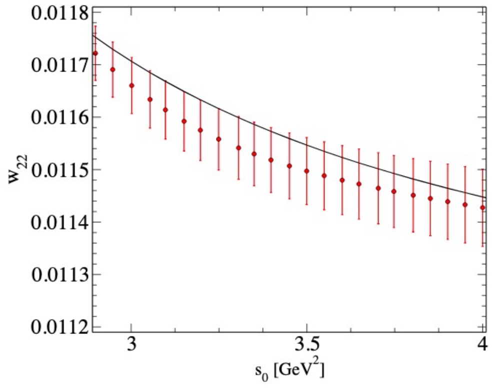
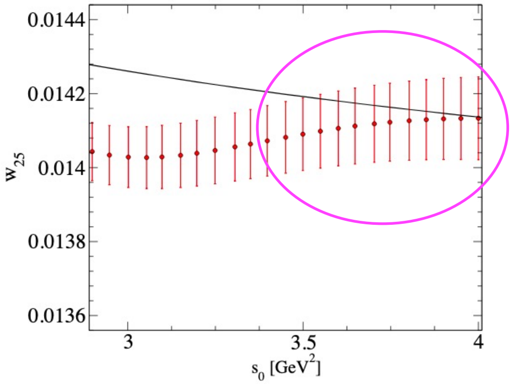
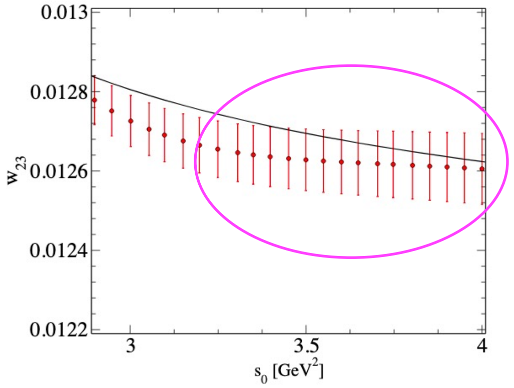
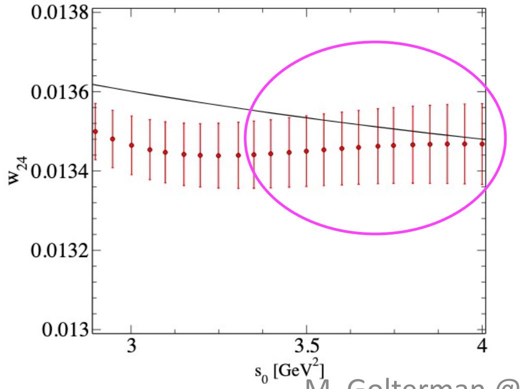






theory and exp.
slopes very different;
strong correlations!





Fits using optimal weights with $s_0^* = 3.6\ \mathrm{GeV}^2$

points: $I_w^{\mathrm{exp}}(s_0)$

curves: $I_w^{\mathrm{th}}(s_0)$

- Assessment of agreement between experimental and theory moments is hard because of strong correlations between theory and experiment and between different s_0 values.

- Resolve using double differences: for fit at $s_0 = s_0^*$, consider

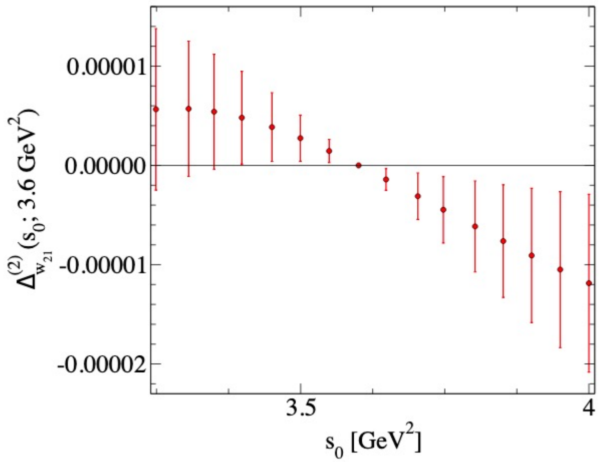
$$\Delta_w^{(2)}(s_0; s_0^*) = \left[I_w^{\rm th}(s_0) - I_w^{\rm exp}(s_0)\right] - \left[I_w^{\rm th}(s_0^*) - I_w^{\rm exp}(s_0^*)\right]$$

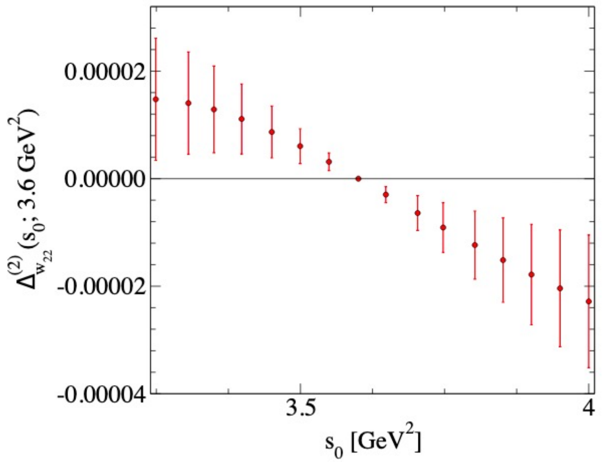
$I_w^{\rm exp/th}(s_0)$ is exp/theory side of FESR with weight w.

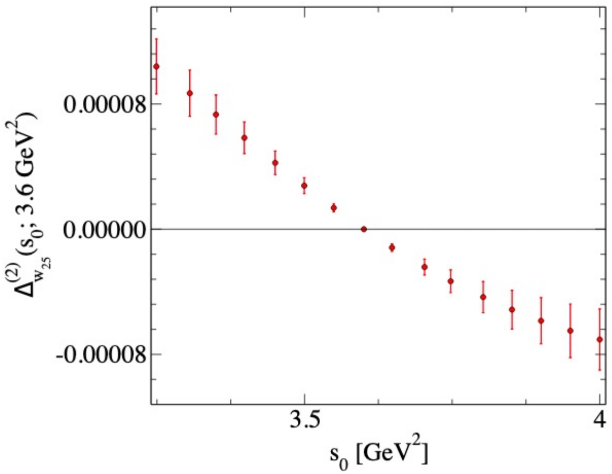
- This compares theory with experiment, as a function of s_0, relative to a reference value s_0^*. *Take all correlations into account, including those between data and fitted parameter values!*

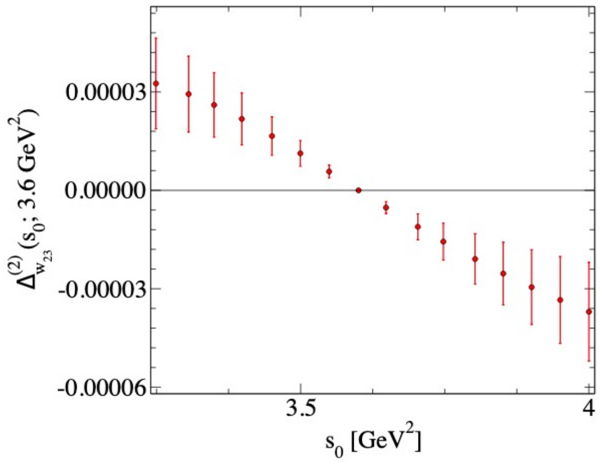
- This double difference should be consistent with zero for the Truncated OPE strategy to be valid.

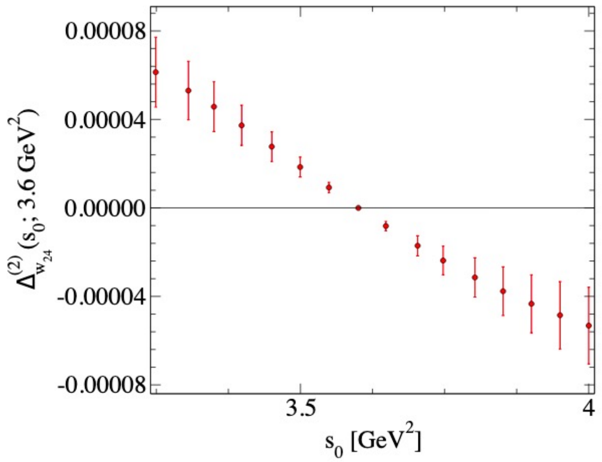
Sum rule for optimal weights with $s_0^* = 3.6\ \mathrm{GeV}^2$: double differences











- All correlations taken into account
- Diffs should vanish for *all* weights

- *Fits based on Truncated OPE strategy clearly fail!*

Conclusions

- Assumptions for dealing with non-perturbative effects in hadronic tau decays are needed; here we considered the Truncated OPE strategy, a model in which higher-order terms in the OPE are neglected. Setting $(C_{10} =)C_{12} = C_{14} = C_{16} = 0$ is arbitrary. Is this dangerous given the asymptotic nature of the OPE?

- YES: Truncated OPE strategy does not pass EM based self-consistency tests.

- YES: Truncated OPE strategy does not pass tau non-strange V+A based self-consistency tests.

- Truncated OPE strategy fails if the goal is to obtain $\alpha_s(m_\tau^2)$ with competitive accuracy; its value depends strongly on arbitrary assumptions made about the OPE in this approach.

BACK-UP

Why does the truncated-OPE approach get it wrong?

- Rely on uncontrolled assumption about the OPE in higher orders.
- Assume that duality violations (resonance effects) can be neglected, at least in V+A, *without testing this*.

- Potentially large effect at $s_0 = m_\tau^2$! **Not excluded by data.**