The strong coupling from an improved tau vector isovector spectral function

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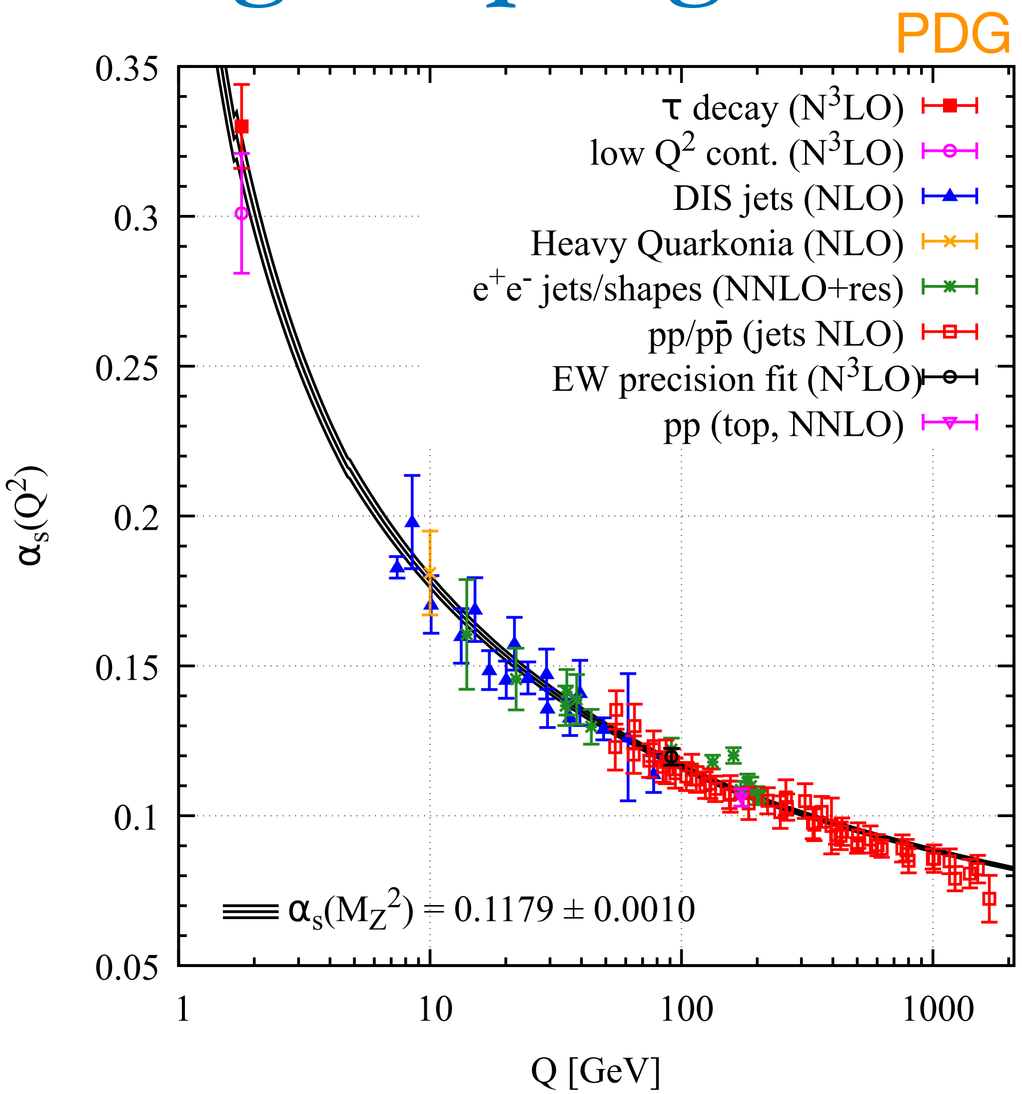
with Maarten Golterman, Kim Maltman, Santi Peris, Marcus Rodrigues and Wilder Schaaf

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, arXiv:2012.10440, PRD 103 (2021)

strong coupling from tau decays



Lower energies

Larger coupling, more sensitivity to QCD corrections.

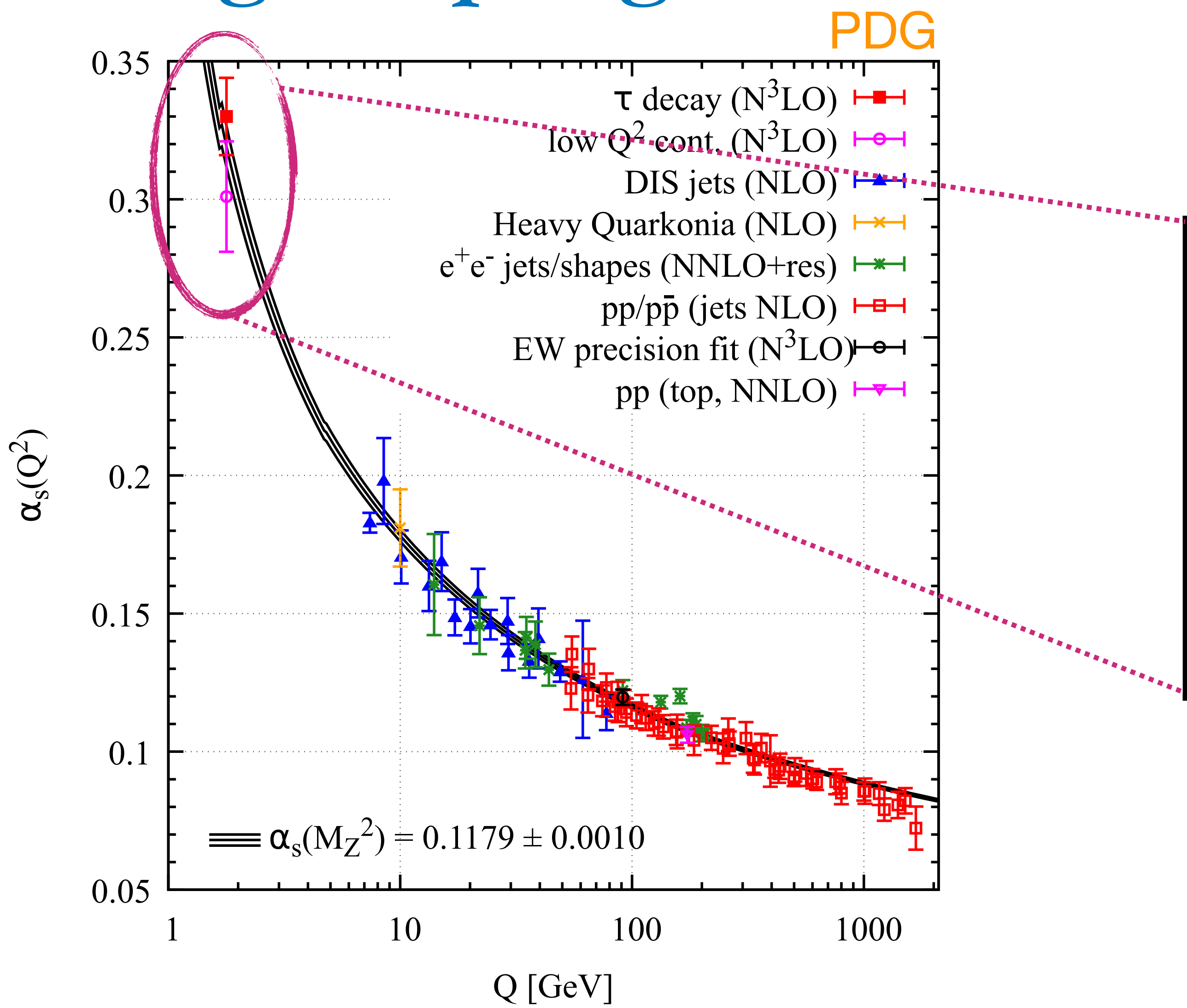
Larger non-perturbative physics (OPE, DVs),
Problems with pt. theory (renormalons,...).

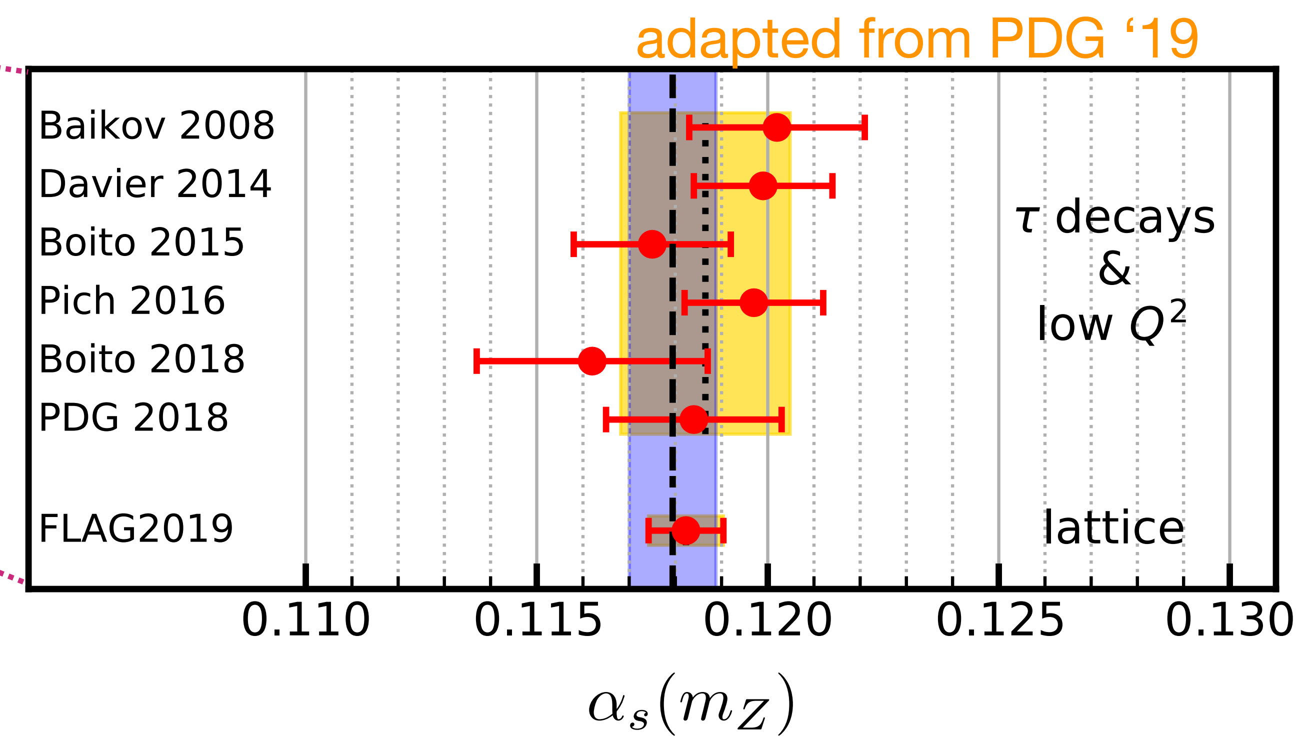
Higher energies

Smaller coupling, less sensitive to QCD corrections, more precision required from exp.

Small contamination from non-perturbative physics,
pt. series is "almost" convergent

strong coupling from tau decays





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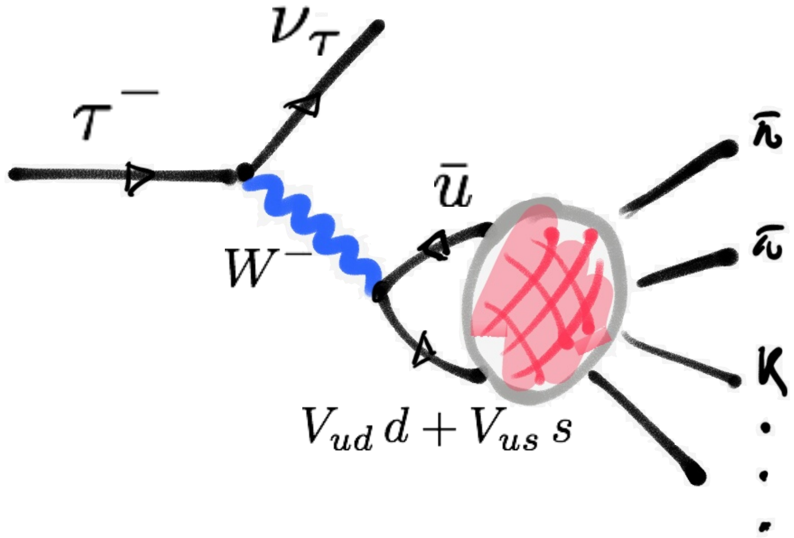
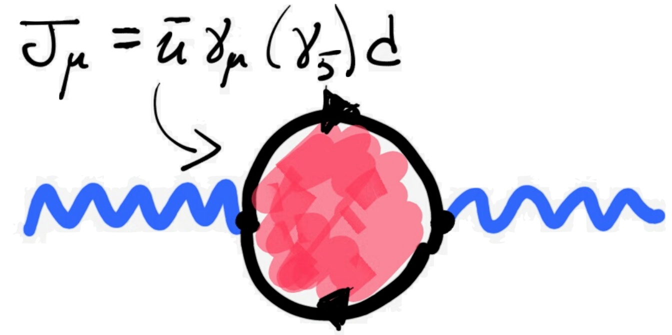


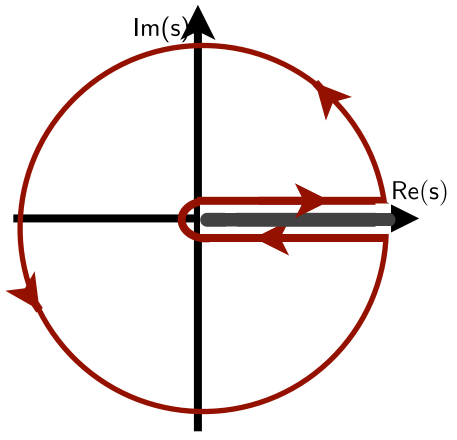


$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|\rangle$$

Massless (V&A) correlators

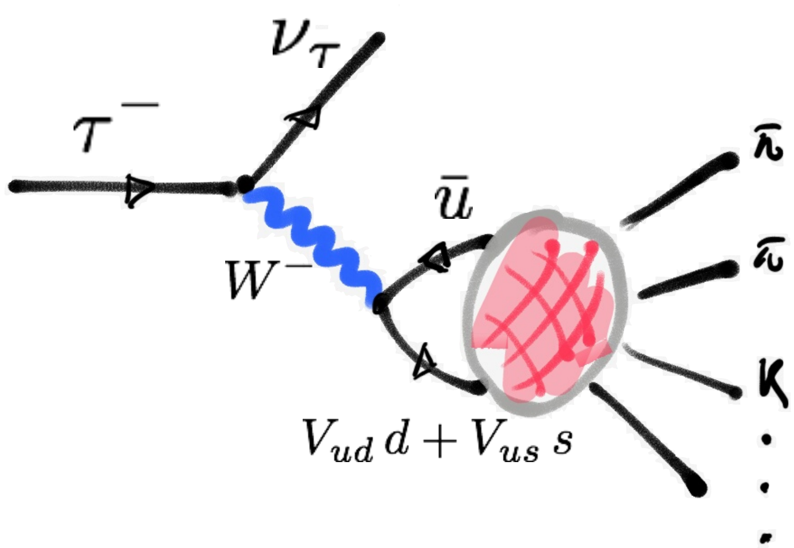
Braaten, Narison, and Pich '92

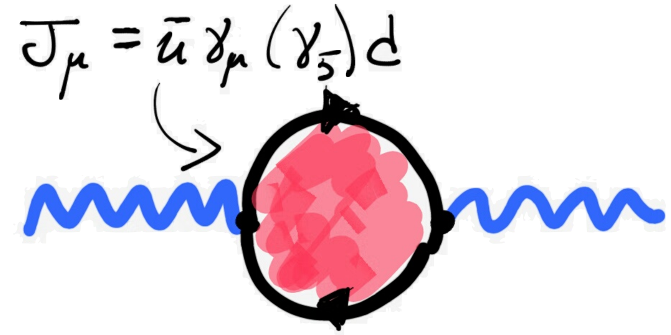
Sum rules (using Cauchy's theorem)



$$\frac{1}{s_0}\int_0^{s_0} ds w(s) \frac{1}{\pi} \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0} \oint_{|z|=s_0} dz w(z) \Pi(z)$$

strong coupling from tau decays





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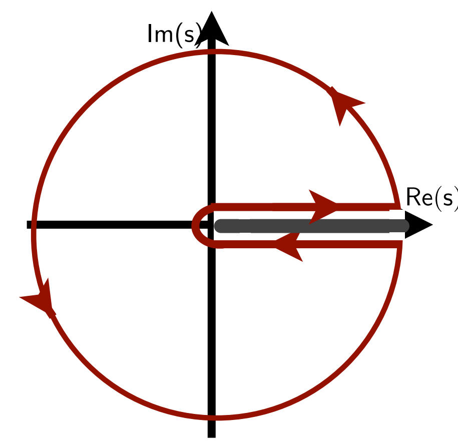

experiment

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s) \frac{1}{\pi} \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0} \oint_{|z|=s_0} dz\, w(z) \Pi(z)$$

theory

theory overview

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\,\Pi(z) \approx S_{\rm EW}\, N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Perturbation theory (OPE) ⟶

$$\sum_{n=0}^{4}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{n+1} c_{n,k} \log^k\left(\frac{-s}{\mu^2}\right) + \frac{C_4}{Q^4} + \frac{C_6}{Q^6} + \frac{C_8}{Q^8} + \cdots$$

see Pich's talk

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

α_s^1 α_s^2 α_s^3 α_s^4

pt. correction is ~20%

$$\delta^{(0)}_{\rm FO} = 0.1012 + 0.0533 + 0.0273 + 0.0133 = 0.1952$$

(fixed order perturbation theory)

theory overview

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theory

Perturbation theory (OPE) →

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Duality Violations →

$$\rho_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha + \beta s)$$

Ansatz based on widely accepted assumptions about QCD: Regge behaviour and large-N_c. Main expected corrections: logarithmic and powers of *1/s*.

DB, Caprini, Golterman, Maltman, Peris, PRD '18

theory: FOPT vs CIPT

Fixed Order (FO) or Contour Improved (CI) lead to different α_s values

theoretical uncertainty?

see Hoang's talk

$$\qquad \alpha_s^1 \qquad\qquad \alpha_s^2 \qquad\qquad \alpha_s^3 \qquad\qquad \alpha_s^4$$

$$\delta^{(0)}_{\rm FO} = 0.1012 + 0.0533 + 0.0273 + 0.0133 \ = \ 0.1952$$

$$\delta^{(0)}_{\rm CI} = 0.1375 + 0.0262 + 0.0104 + 0.0072 = 0.1814$$

theory: FOPT vs CIPT

Fixed Order (FO) or Contour Improved (CI) lead to different α_s values

theoretical uncertainty?

see Hoang's talk

$$\begin{array}{cccccc} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 & \\ \delta^{(0)}_{\rm FO} = & 0.1012 + & 0.0533 + & 0.0273 + & 0.0133 & = 0.1952 \\ \delta^{(0)}_{\rm CI} = & 0.1375 + & 0.0262 + & 0.0104 + & 0.0072 & = 0.1814 \end{array}$$

Discrepancy between FOPT and CIPT
linked to an incoherent treatment of the OPE
(previously assumed to be the same for both prescriptions)

Hoang and Regner '20.'21

We will not quote results from CIPT in this talk

analysis strategy

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) \approx S_{\rm EW}\, N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Desired properties from the choice of weights

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

analysis strategy

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) \approx S_{\rm EW} N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Desired properties from the choice of weights

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

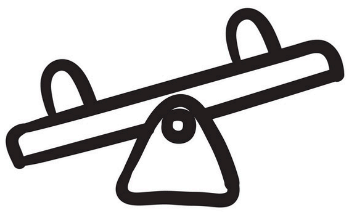
Choice of weights

$w_0(y) = 1$	Tiny condensate contributions, sensitive to DVs
$w_2(y) = 1 - y^2$	Only D=6
$w_3(y) = (1-y)^2(1+2y)$	Only D=6 and 8 Tau kinematical Moment (R_τ)
$w_4(y) = (1-y^2)^2$	Only D=6 and 10

DB, Cata, Golterman, Jamin, Maltman '11, Beneke, DB, Jamin '12, DB, M. Golterman, K. Maltman, S. Peris '16
DB F Oliani '20

analysis strategy

Suppression of DVs comes with the price of additional (unknown) higher dim. contributions from the OPE.

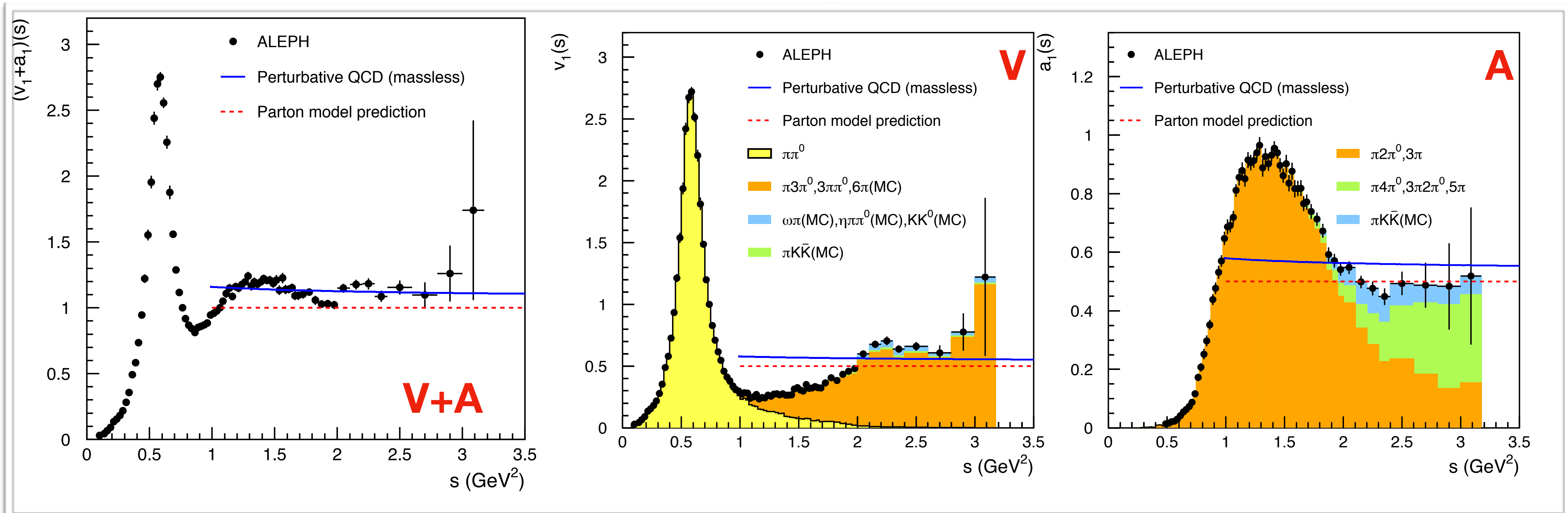


DV strategy	Truncated OPE strategy
DB, M. Golterman, K. Maltman, S. Peris, M. V. Rodrigues and W. Schaaf, 2012.10440	A Pich, A. Rodriguez-Sanchez 1605.06830 Davier, Höcker, Malaescu, Yuan, Zhang 1312.1501 see Golterman's talk
– Accept some DVs, strongly suppress contamination on the OPE side.	– Suppress DVs but need to ignore the higher order contributions on the OPE side (too many parameters).
	(Serious issues with the truncation of the OPE) DB, M. Golterman, K. Maltman, S. Peris '16 '19

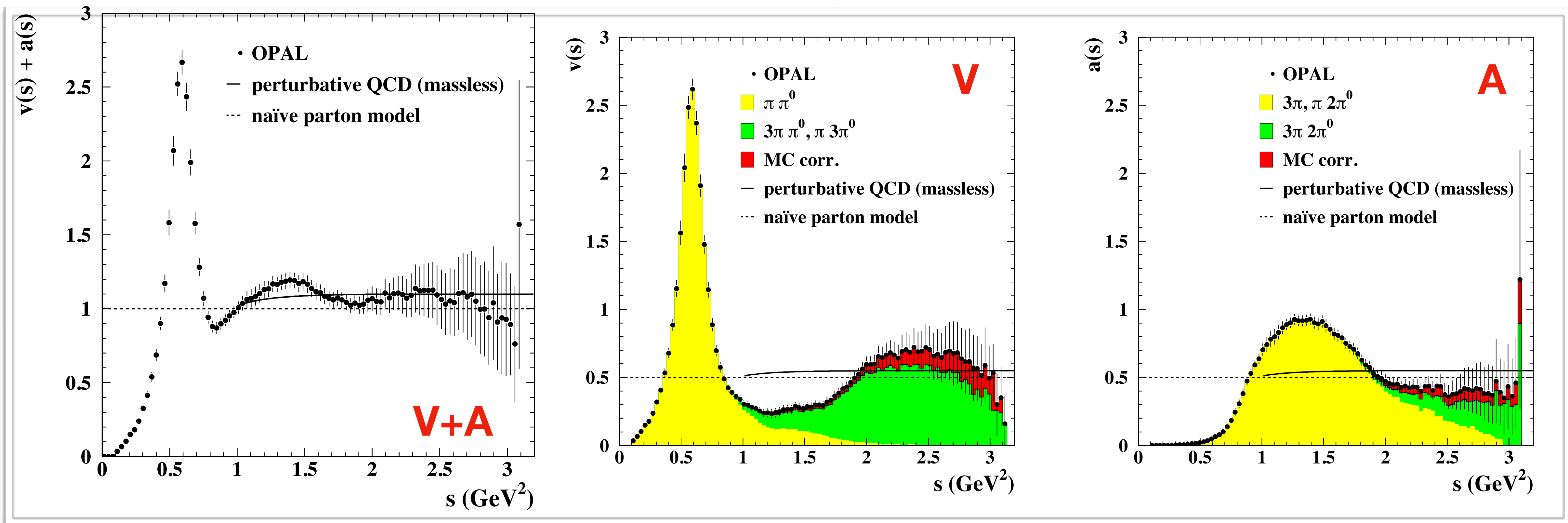
Data

inclusive hadronic tau decay data

Davier et al [ALEPH] '14



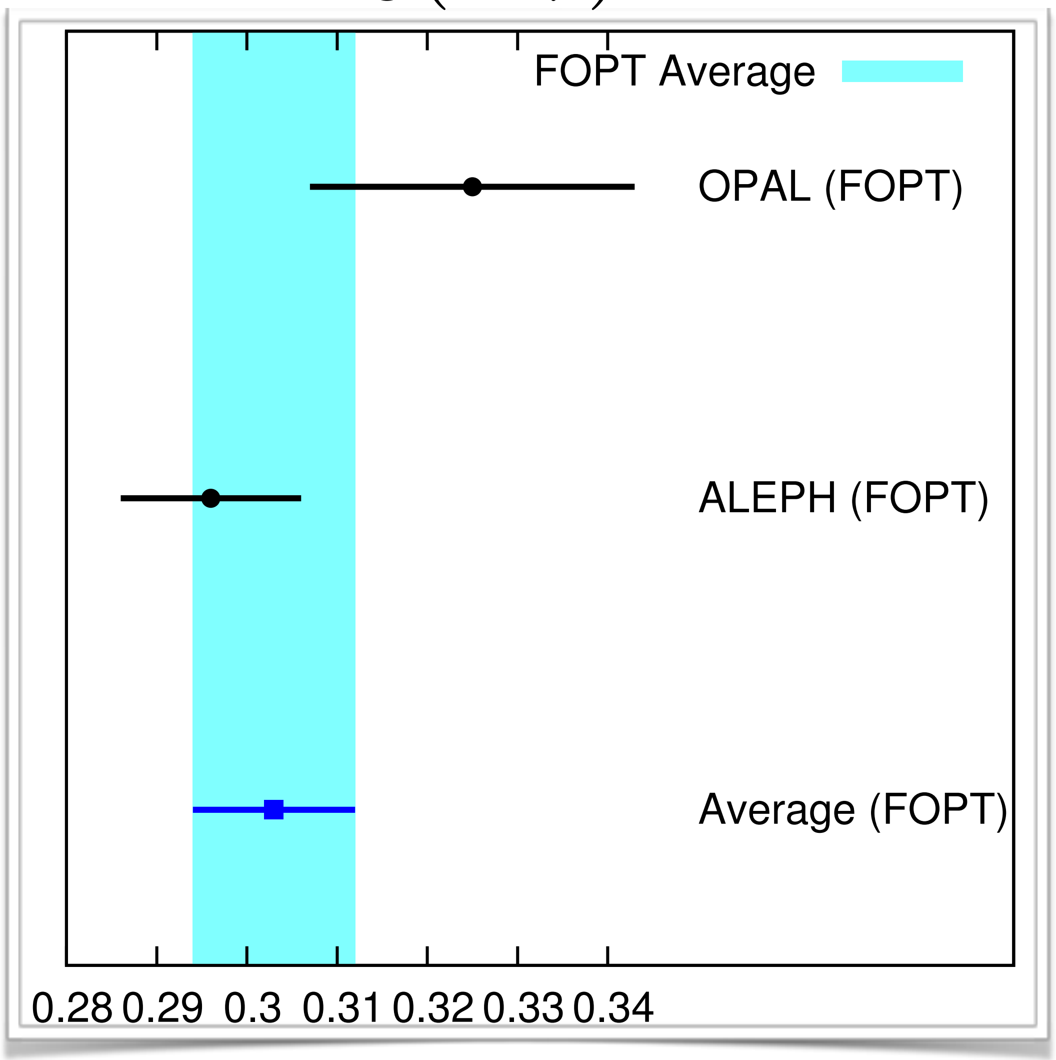
Ackerstaff et al [OPAL] '98 updated for recent values of branching fractions Boito et al '11, '21



tau decay data and the strong coupling

The two data sets lead to compatible values for $\alpha_s(m_\tau)$: average

$\alpha_s(m_\tau)$



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, '12

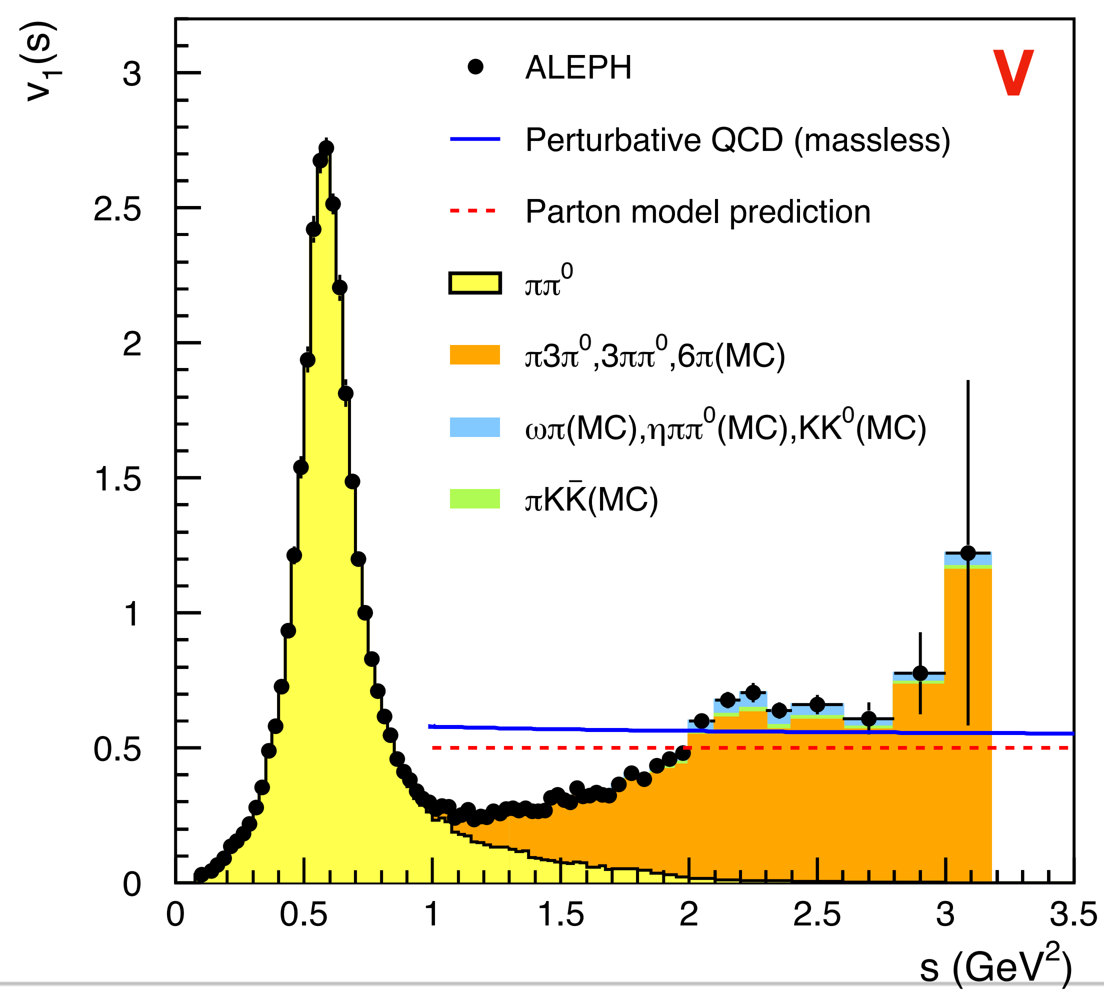
DB, Golterman, Maltman, Osborne, Peris, '15

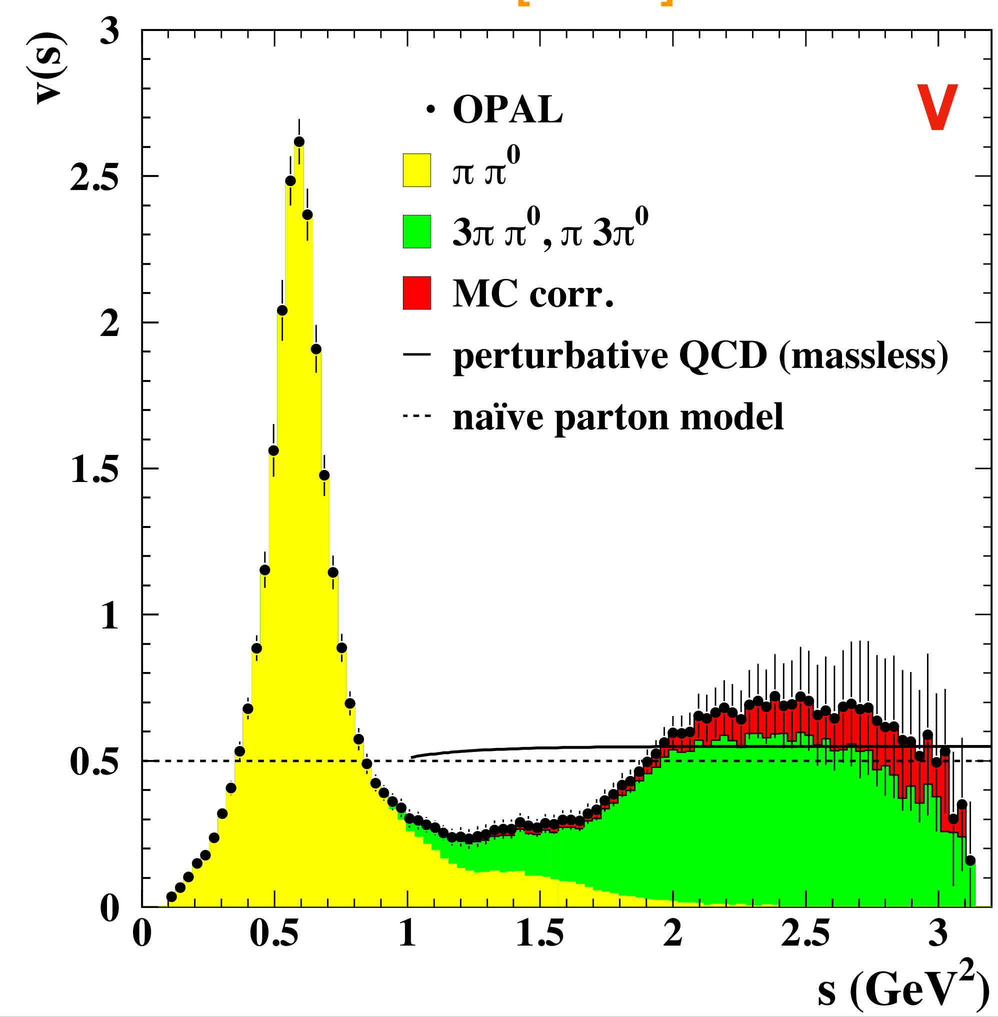
Are the data sets compatible? Can we combine them?

Can the inclusive spectral functions be improved with recent data?

anatomy of the data sets

- V channel dominated by $\tau \to 2\pi + \nu_\tau$ and $\tau \to 4\pi + \nu_\tau$
- "Residual" channels subdominant (but important for α_s!)
- Monte Carlo (MC) inputs for several channels





Recently measured channels in e^+e^- can be used to improve the vector channel

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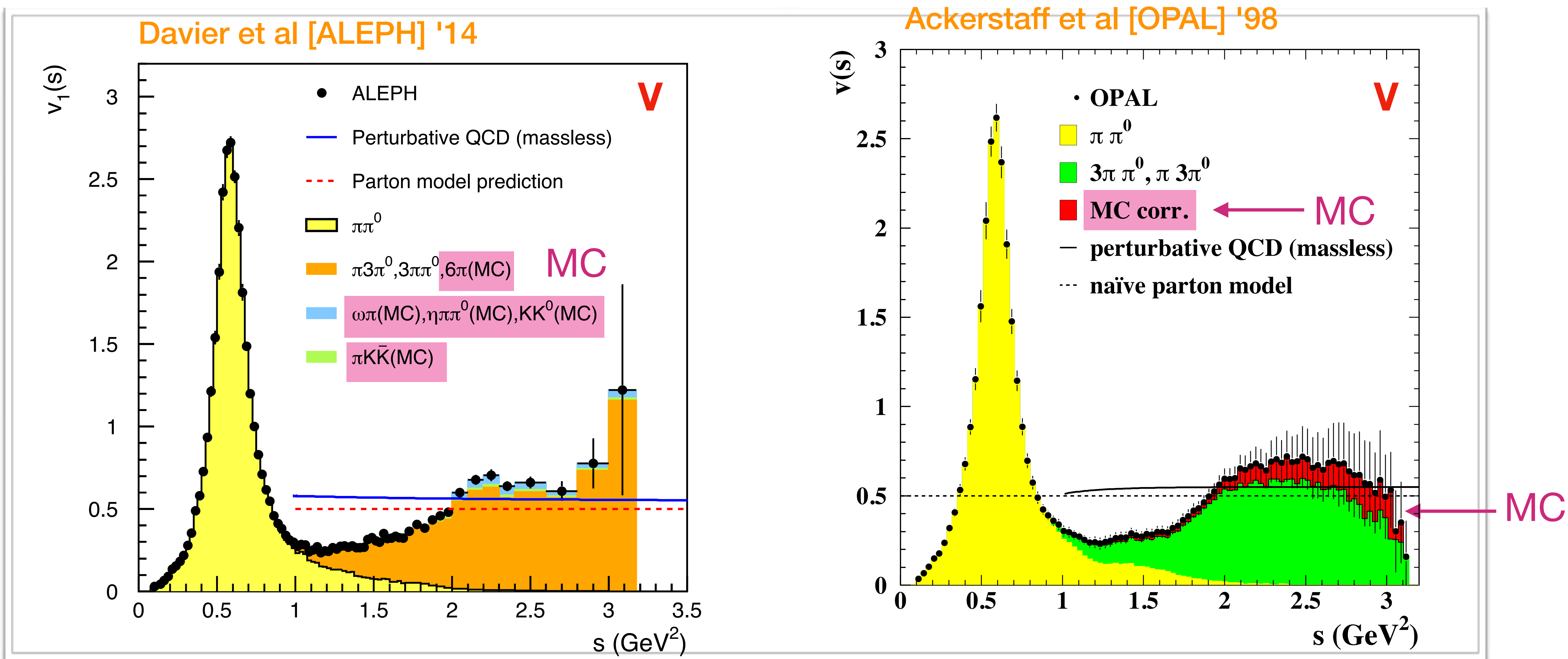


Recently measured channels in e^+e^- can be used to improve the vector channel

new vector isovector spectral function

- Combined data for 2π and 4π channels from ALEPH & OPAL

Data combination: same algorithm used in R-data combination for muon g-2.

Keshavarzi, Nomura, Teubner '18

- Exp. data only: 7 residual channels from e^+e^- using CVC (conserved vector current) and BaBar data for $\tau \to K K_S \nu_\tau$

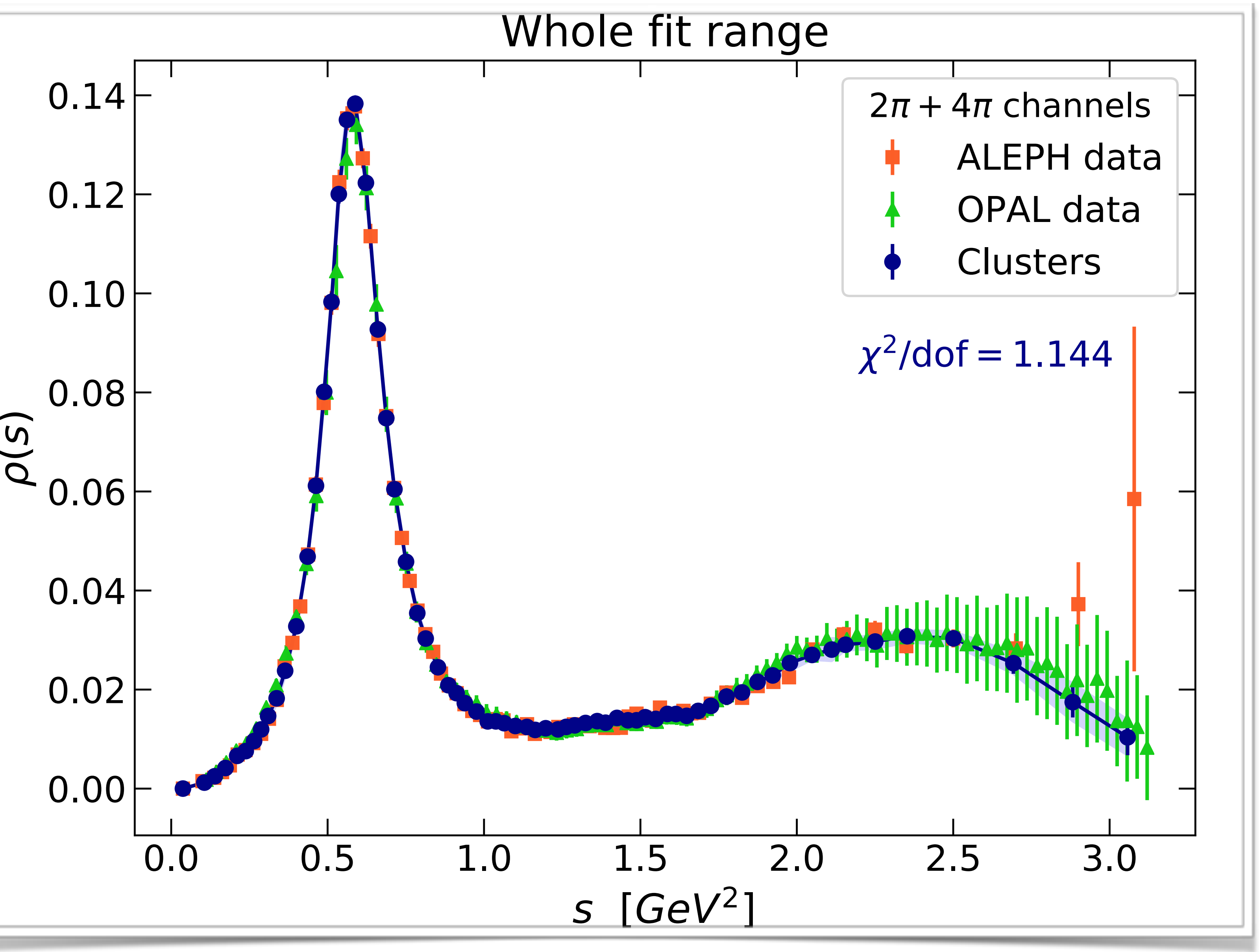
No Monte Carlo inputs; IB corrections to CVC negligible

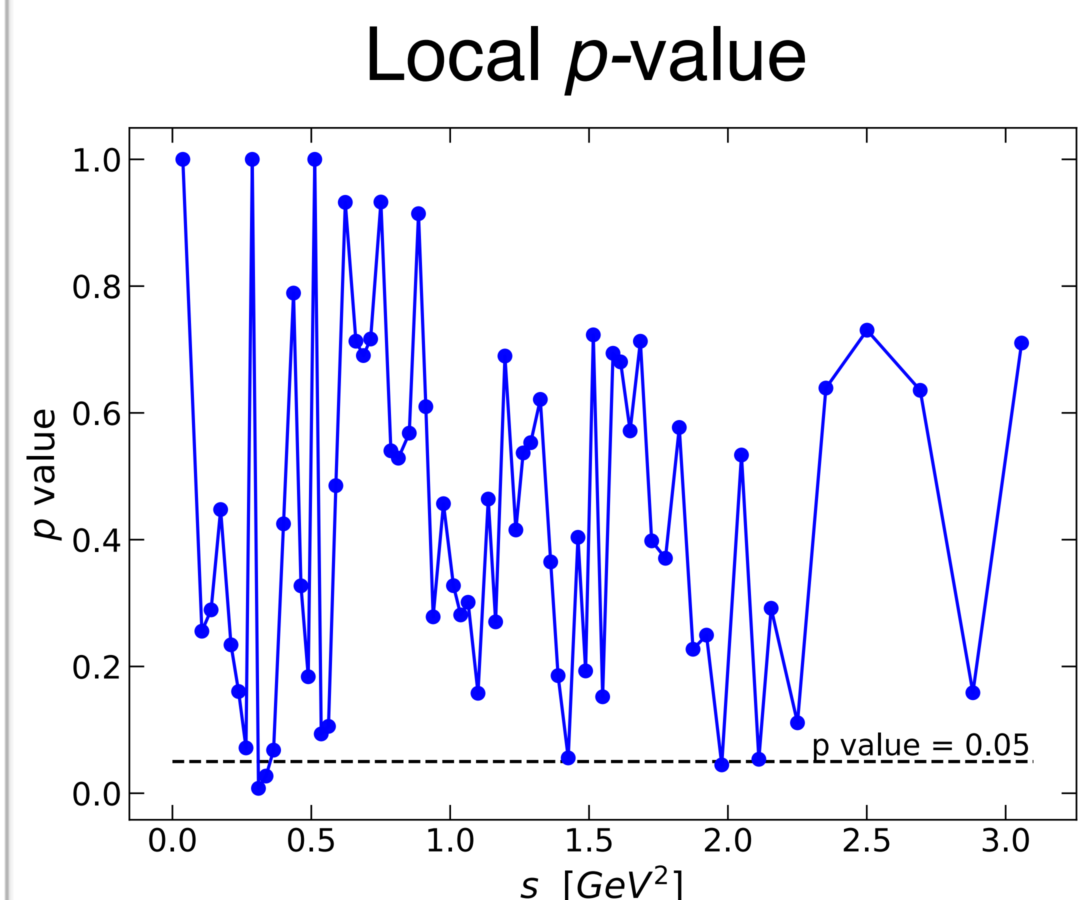
- Results updated for recent branching ratio measurements

new vector isovector spectral function

Combination of 2π $+4\pi$ channels

Good χ^2 both locally and globally, no χ^2 inflation needed





DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

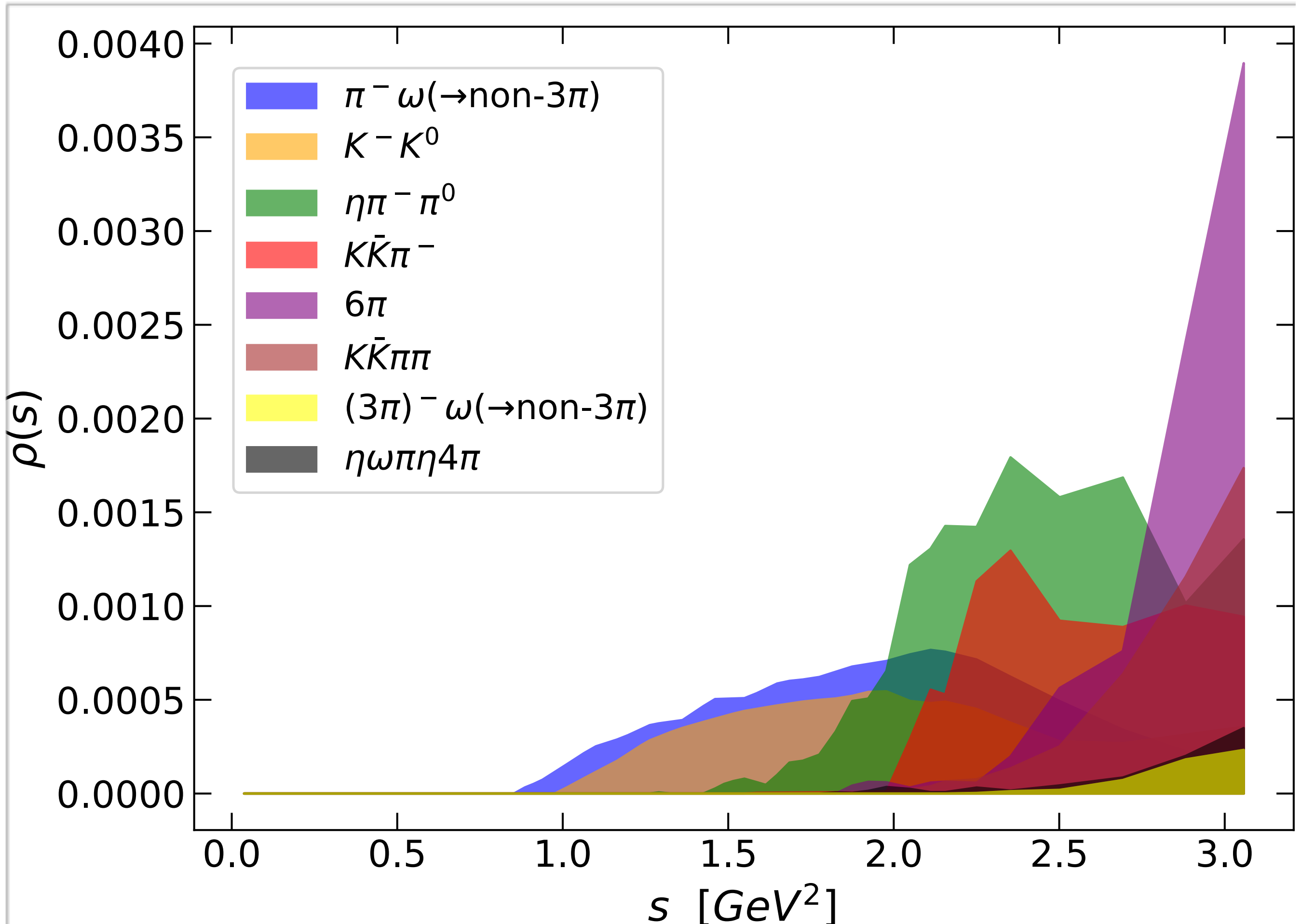
new vector isovector spectral function

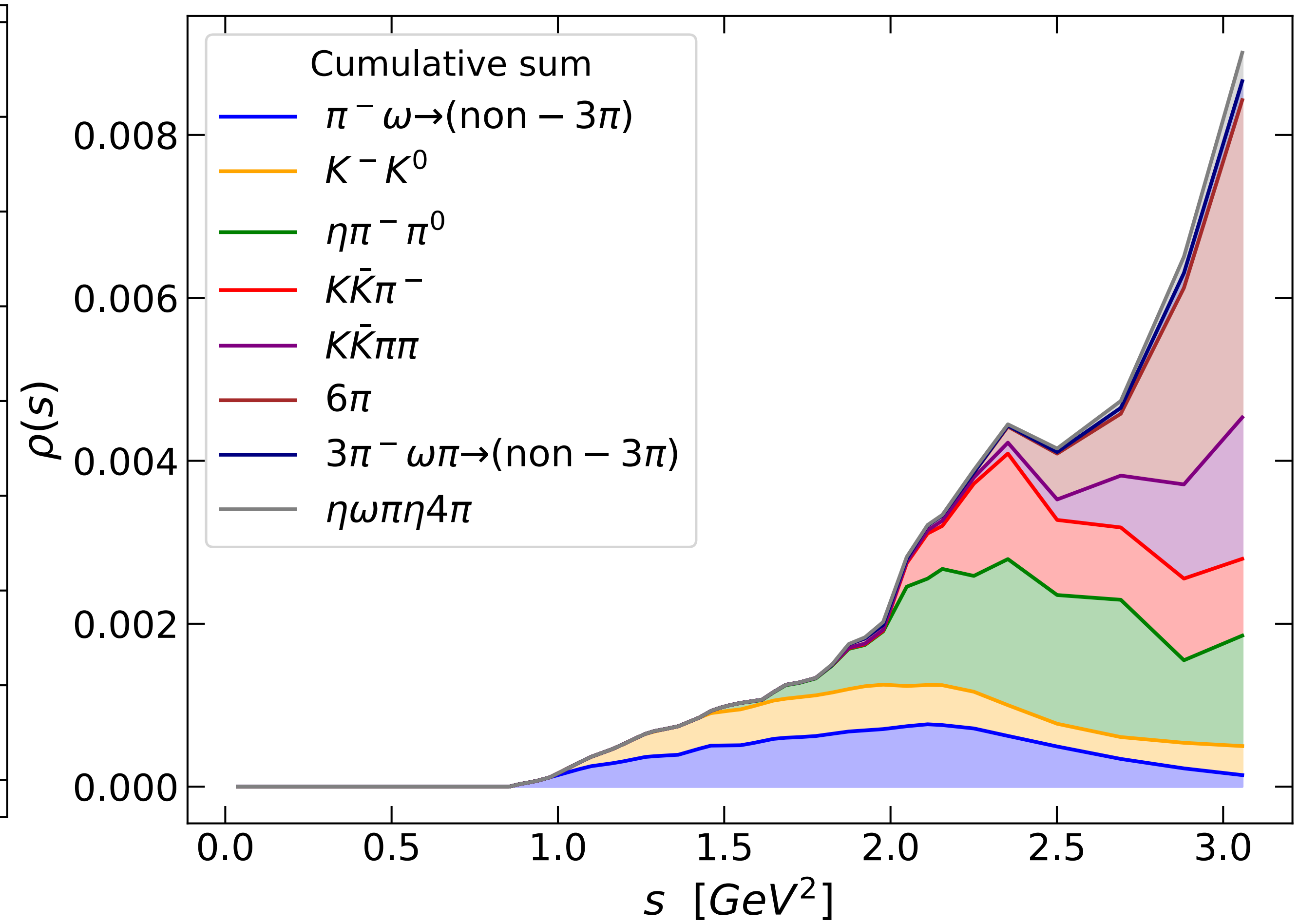
7 residual channels extracted from e^+e^- data + BaBar data for $\tau \to K K_S \nu_\tau$

Dramatic improvement in errors for higher multiplicity modes (near end point)

No Monte Carlo input

Original data sets from: BABAR, CMD-3 and SND (results from 16 papers)



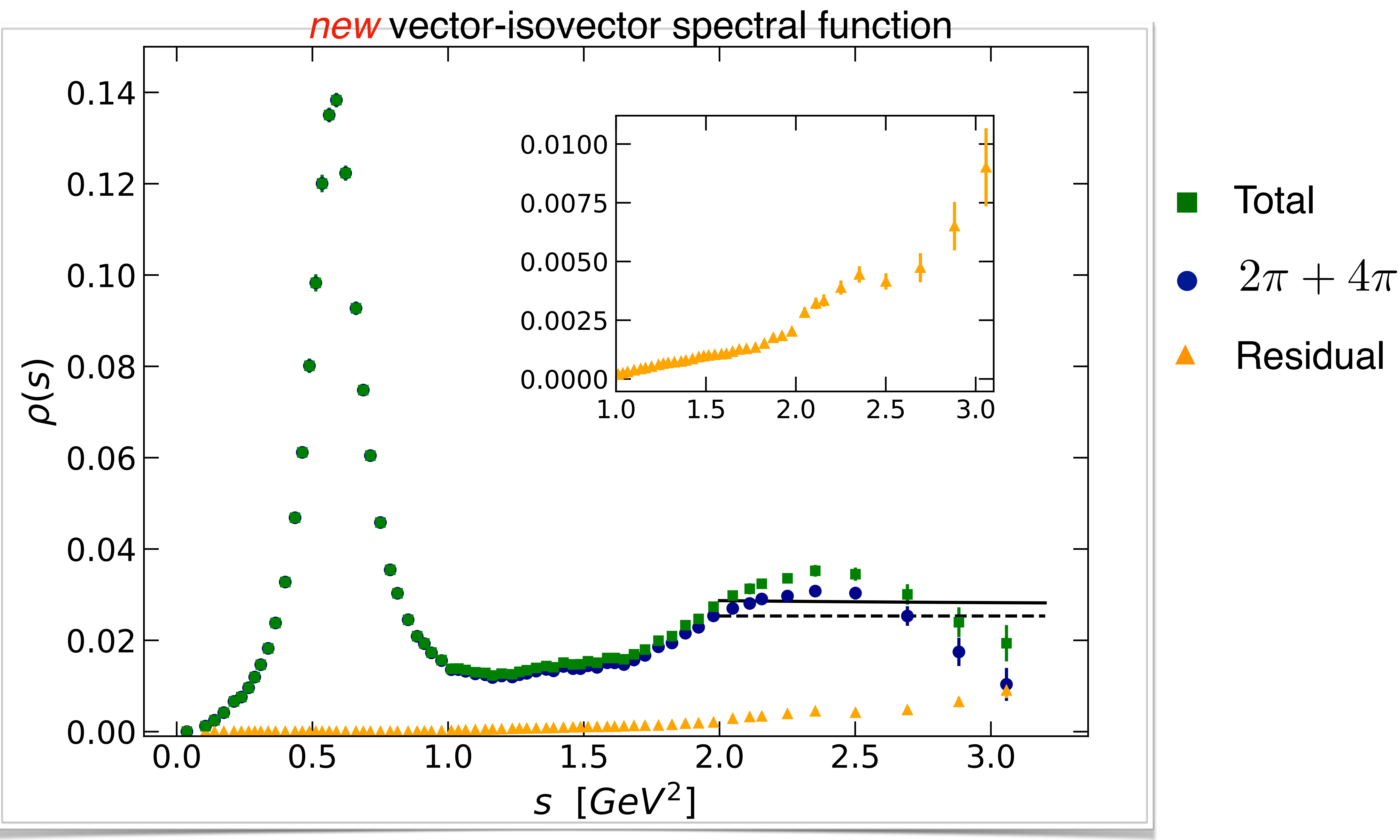


DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

new vector isovector spectral function

Combined $2\pi + 4\pi$ (ALEPH and OPAL) + residual channels from data

99.95% of the Branching Fraction covered



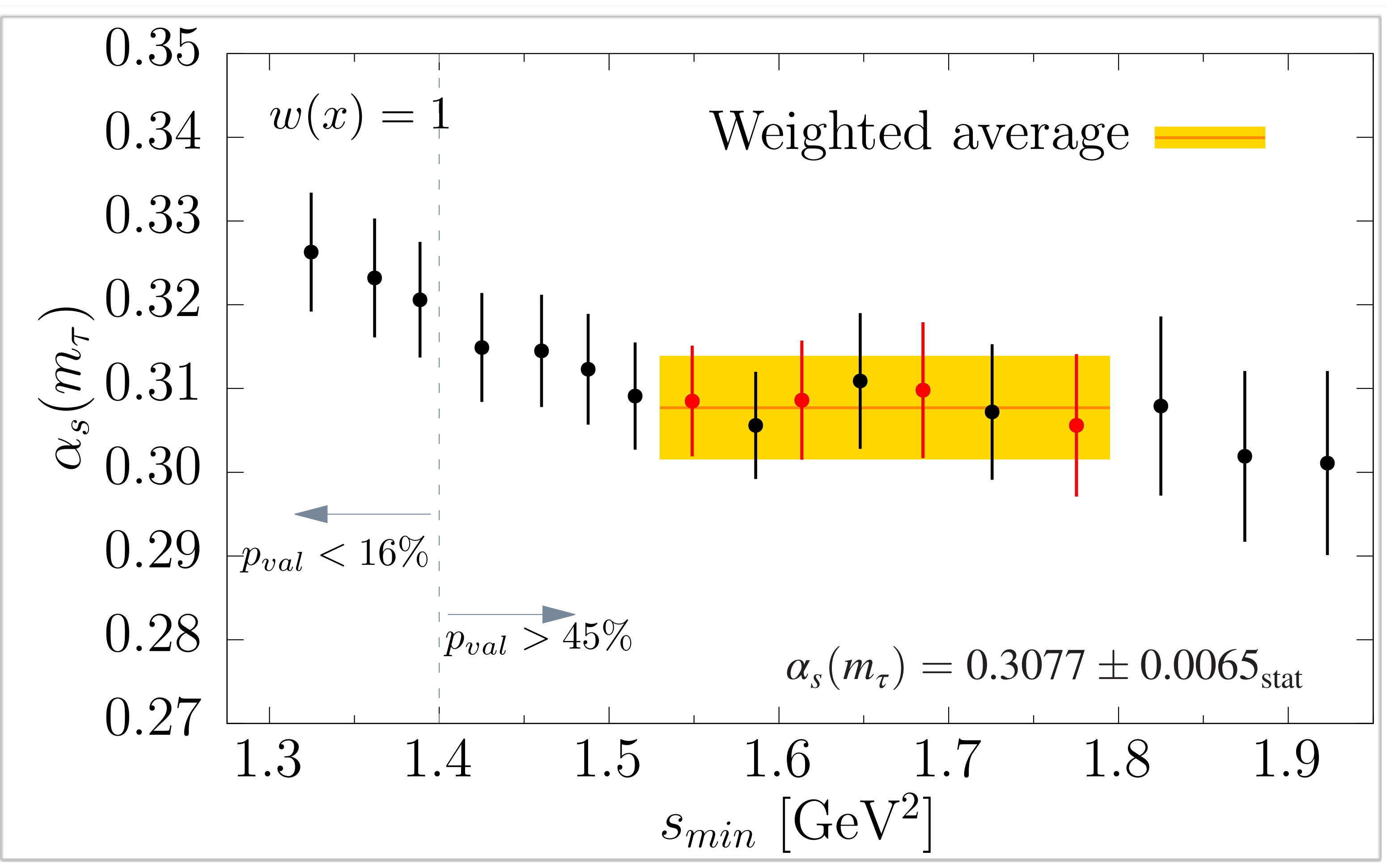
Results

strong coupling from the new spectral function

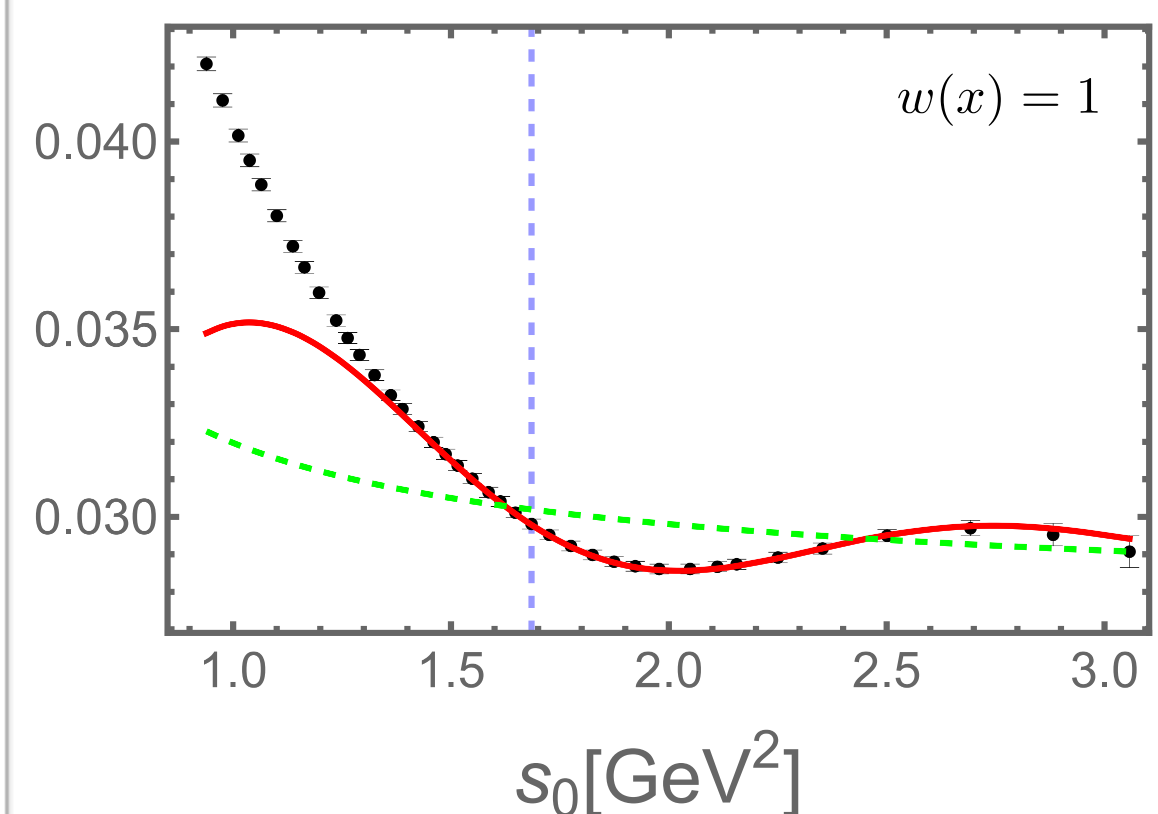
Several fits, single moments or in combination

Many fit windows: $[s_{\min}, m_\tau^2]$

Consistency between different fits (α_s , condensates, DV params.)



$$\frac{1}{s_0}\int_0^{s_0} ds\,\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i\, s_0}\oint_{|z|=s_0} dz\,\Pi(z)$$



$$w_0(y) = 1$$
$$w_2(y) = 1 - y^2$$
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Consistency between different fits

mom.	α_s	$c_6[\mathrm{GeV}^6]$
w_0	$0.3077(65)$	$--$
$w_0\&w_2$	$0.3091(69)$	$-0.0059(13)$
$w_0\&w_3$	$0.3080(70)$	$-0.0070(12)$
$w_0\&w_4$	$0.3079(70)$	$-0.0068(12)$

strong coupling from the new spectral function

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Final value

pt. series truncation, scale variation

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\mathrm{stat}} \pm 0.0038_{\mathrm{pert}}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \mathrm{FOPT})$$

Final result

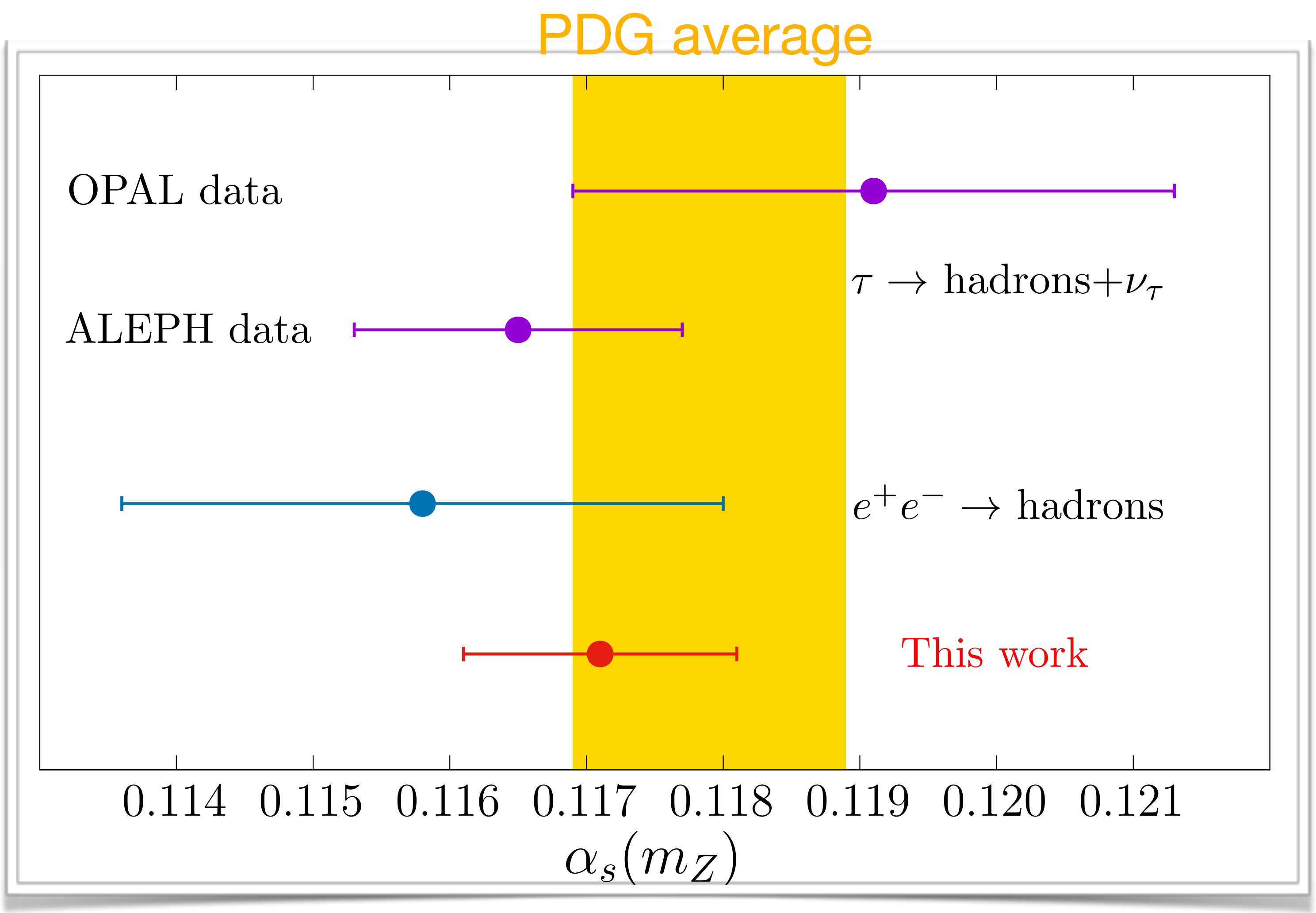
Results at m_τ

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\rm stat} \pm 0.0038_{\rm pert}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \text{FOPT})$$

Results evolved to m_Z

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

$(\overline{\text{MS}}, N_f = 5)$



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, ‘12

DB, Golterman, Maltman, Osborne, Peris, ‘15

DB, Golterman, Keshavarzi, Maltman, Nomura, Peris, Teubner ‘18

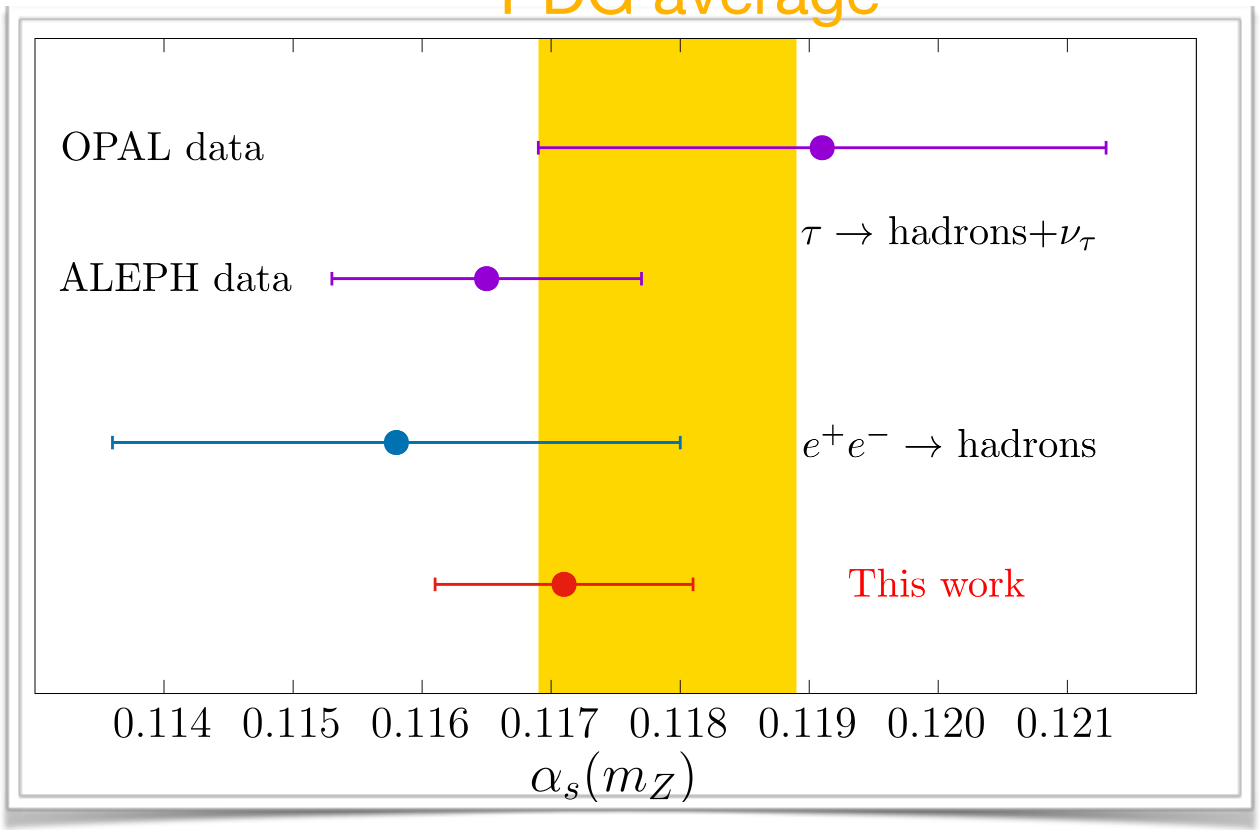
DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, ’21

conclusions

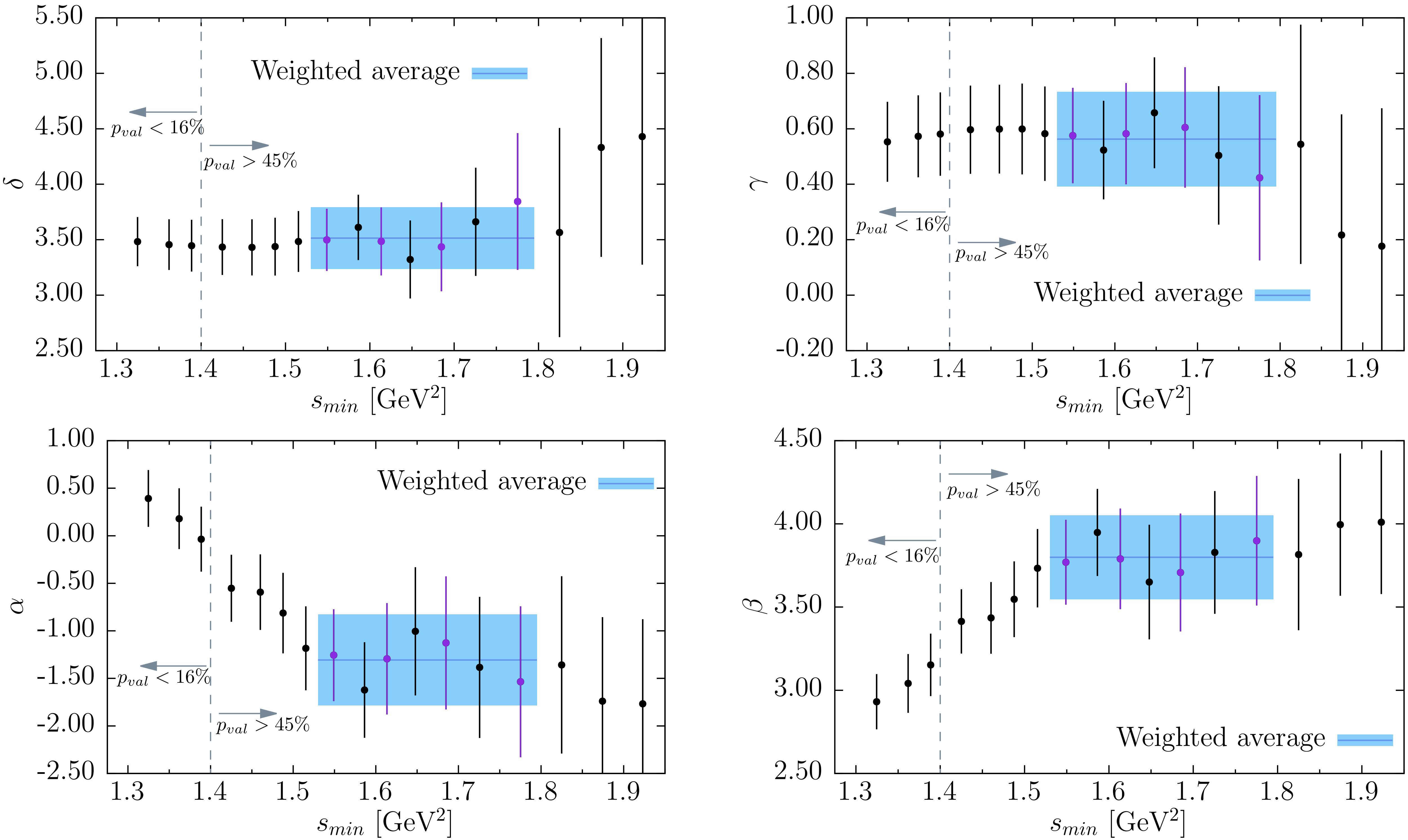
- Vector channel is special: CVC allows improvement near tau kin. end point.
- New vector isovector spectral function purely based on data, no MC input.
- Analysis can be improved with new data for the $2\pi + 4\pi$ channels only!
- Improvements of this type not possible for the axial channel (no axial photon).
- Final result from the new vector spectral function is competitive.

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

PDG average



Extra



Weighted average
$p_{val} < 16\%$
$p_{val} > 45\%$
δ
γ
α
β
s_{min} [GeV^2]
5.50
5.00
4.50
4.00
3.50
3.00
2.50
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
0.50
-0.50
-1.00
-1.50
-2.00
-2.50
1.3
1.4
1.5
1.6
1.7
1.8
1.9