Reconciling the FOPT and CIPT Predictions for the Hadronic Tau Decay Rate

André H. Hoang

University of Vienna

Based on arXiv:2008.00578 and arXiv:2105.11222 (with Christoph Regner)
and work with Miguel Benitez-Rathgeb, Diogo Boito and Matthias Jamin

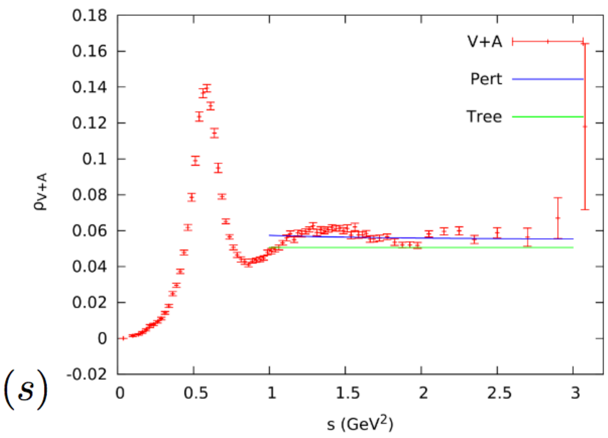
Doktoratskolleg Particles and Interactions

universität wien

FWF Der Wissenschaftsfonds.

Hadronic τ Spectral Function Moments

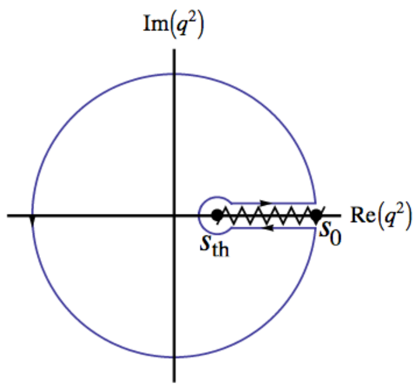
ALEPH: τ hadronic width (HFLAV 2019)



$$R_\tau \equiv \frac{\Gamma[\tau^- \to \text{hadrons}\,\nu_\tau(\gamma)]}{\Gamma[\tau^- \to e^-\overline{\nu}_e\nu_\tau(\gamma)]} = 3.6355 \pm 0.0081$$

$$A^\omega_{V/A}(s_0) \equiv \int_{s_{\rm th}}^{s_0} \frac{ds}{s_0}\,\omega(s)\,\mathrm{Im}\,\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0}\frac{ds}{s_0}\,\omega(s)\,\Pi_{V/A}(s)$$

$$(p^\mu p^\nu - g^{\mu\nu}p^2)\,\Pi(p^2) \equiv i\int dx\, e^{ipx}\,\langle\Omega|\,T\{j^\mu_{v/av,jk}(x)\,j^\nu_{v/av,jk}(0)^\dagger\}\Omega\rangle$$



Theory: Operator product expansion

$$A_{W_i}(s_0) = \frac{N_c}{2}|V_{ud}|^2\left[\delta^{\rm tree}_{W_i} + \delta^{(0)}_{W_i}(s_0) + \sum_{d\geq 2}\delta^{(d)}_{W_i}(s_0) + \delta^{\rm DV}_{W_i}(s_0)\right]$$

$$W_i(x) = \sum_{n=0}^{m} a_n\, x^n$$

pQCD — OPE — Duality violation

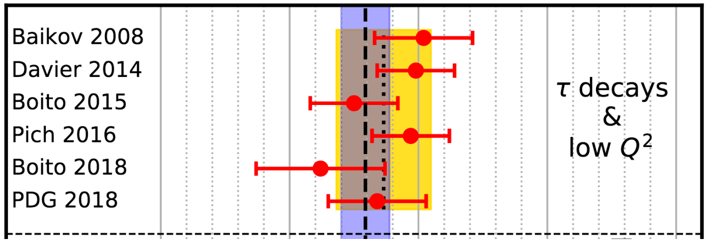
$$\delta^{(0)}_{W_i}(s_0) = \frac{1}{2\pi i}\oint_{|s|=s_0}\frac{\mathrm{d}s}{s}\,W_i\!\left(\tfrac{s}{s_0}\right)\hat{D}(s)$$

Adler function: $$\frac{1}{4\pi^2}\left(1+\hat{D}(s)\right) \equiv -\,s\,\frac{\mathrm{d}\Pi(s)}{\mathrm{d}s}$$

Strong coupling from τ decays

$$\hat{D}(s) = \sum_{n=1}^{\infty} c_{n,1} \left(\frac{\alpha_s(-s)}{\pi}\right)^n ,$$
$$= \sum_{n=1}^{\infty} \left(\frac{\alpha_s(s_0)}{\pi}\right)^n \sum_{k=1}^{n+1} k\, c_{n,k} \ln^{k-1}\left(\frac{-s}{s_0}\right)$$

FOPT CIPT



$$c_{0,1} = c_{1,1} = 1 , \quad c_{2,1} = 1.640$$
$$c_{3.1} = 6.371$$
$$c_{4,1} = 49.076$$

4-loop: Gorishni etal., Surguladze etal. 1991

5-loop: Baikov etal. 2008

Contour-improved perturbation theory (CIPT):

$$\delta_{W_i}^{(0),\mathrm{CIPT}}(s_0) = \frac{1}{2\pi i} \sum_{n=1}^{\infty} c_{n,1} \oint_{|x|=1} \frac{\mathrm{d}x}{x} W_i(x) \left(\frac{\alpha_s(-x s_0)}{\pi}\right)^n$$

- CIPT leads to smaller moments than FOPT
- OPE and DV corrections assumed to be universal
- Strong coupling from CIPT larger than from FOPT

Fixed-order perturbation theory (FOPT):

$$\delta_{W_i}^{(0),\mathrm{FOPT}}(s_0) = \frac{1}{2\pi i} \sum_{n=1}^{\infty} \left(\frac{\alpha_s(s_0)}{\pi}\right)^n \sum_{k=1}^{n+1} k\, c_{n,k} \oint_{|x|=1} \frac{\mathrm{d}x}{x} W_i(x) \ln^{k-1}(-x)$$

Outline

Starting point of the work: We consider the possibility that the discrepancy is related to the asymptotic character of the perturbative CIPT and FOPT spectral function moment series.

A lot of previous work: Beneke, Jamin 0806.3156 — Jamin hep-ph/0509001 — Beneke, Jamin, Boito 1210.8038 — Caprini, Fischer 0906.5211 — Descotes-Genon, Malaescu 1002.2968

- Asymptotic series and renormalons & previous studies
- FOPT and CIPT Borel representation: **Asymptotic Separation**
- Properties and computation of the asymptotic separation
- Reconciling CIPT and FOPT
- Conclusions

$$\alpha_s(m_\tau) = 0.34$$

$$a(x) \equiv \frac{\beta_0\,\alpha_s(s)}{4\pi} = \frac{\beta_0\,\alpha_s(x s_0)}{4\pi}$$

$$a_0 \equiv \frac{\beta_0\,\alpha_s(s_0)}{4\pi}$$

$$s_0 \equiv |s|$$

Renormalon Calculus: Euclidean Adler Function

Perturbative series in QCD are not convergent, but asymptotic. $\rightarrow$ $\hat{D}(-s_0) \sim \sum_{n=1}^{\infty} n! \left(\frac{\alpha_s(s_0)}{\pi}\right)^n$

(consequence of using dim. reg. and MSbar)

Reminder of renormalon calculus for the Euclidean Adler Function: \`t Hooft; David; Müller; ... Beneke; ...

$$\hat{D}^{\rm CIPT}(-s_0) = \sum_{n=1}^{\infty} c_{n,1} \left(\frac{\alpha_s(s_0)}{\pi}\right)^n \Longrightarrow B[\hat{D}](u) \sim \sum_{n=1}^{\infty} \frac{\bar{c}_{n,1}}{\Gamma(n)} u^{n-1} \Longrightarrow B[\hat{D}](u) \sim \frac{1}{(p-u)^{\gamma}} + \frac{1}{(\tilde{p}+u)^{\tilde{\gamma}}}$$

Original series (asymptotic)

Borel function series (convergent)

Borel function

Borel transform

Sumation +Analytic continuation

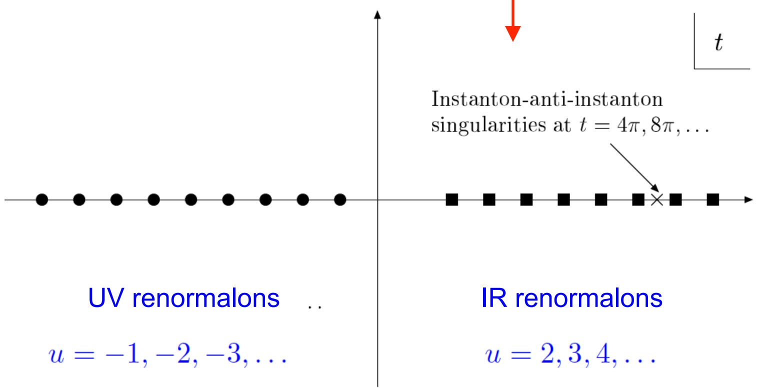
Borel representation and Borel sum:

(inverse Borel transform)

$$\hat{D}_{\rm Borel}(-s_0) = {\rm PV} \int_0^{\infty} {\rm d}u \; B[\hat{D}](u) \, e^{-\frac{4\pi u}{\beta_0 \alpha_s(s_0)}}$$

Some regularization needed: PV prescription

(IR cutoff)

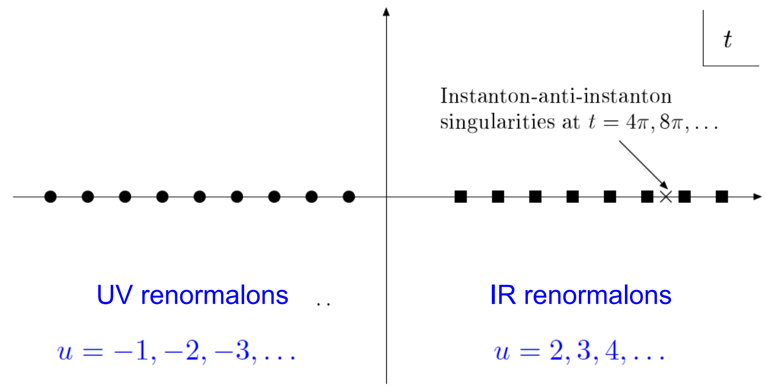


Beneke „Renormalons“

Renormalon Calculus: Euclidean Adler Function

What do the renormalon poles mean?

`t Hooft; David; Müller 1980's

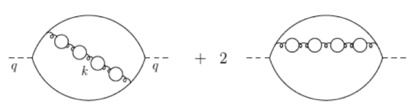


Correspondence: IR renormalon poles ⟺ (standard) OPE Corrections

$$\hat{D}^{\rm OPE}(-s_0) = \frac{\langle \alpha_s G^2 \rangle}{s_0^2} + \sum_{p=3}^{\infty} \frac{1}{s_0^p} \Big[C_0(\alpha_s(s_0)\langle \mathcal{O}_{2p,0}\rangle + C_1(\alpha_s(s_0)\langle \mathcal{O}_{2p,1}\rangle + \ldots \Big]$$

Large-β_0 approximation:

(cut in full QCD)



$$B(u) = \frac{1}{(2-u)} \qquad \Longleftrightarrow \qquad \langle \alpha_s G^{\mu\nu} G_{\mu\nu} \rangle$$

- Exact form of B(u) unknown in full QCD $\rightarrow$ models
- Connection to the asymptotic character relevant in practice only if the the gluon condensate renormalon dominates the series already at low orders (true for large-β_0)

FOPT vs. CIPT Borel Representation

Contour-improved perturbation theory (CIPT):

$$\delta^{(0),\mathrm{CIPT}}_{W_i}(s_0) \;=\; \frac{1}{2\pi i}\sum_{n=1}^{\infty} c_{n,1} \oint\limits_{|x|=1} \frac{\mathrm{d}x}{x}\, W_i(x)\, \Big(\tfrac{\alpha_s(-xs_0)}{\pi}\Big)^n$$

Fixed-order perturbation theory (FOPT):

$$\delta^{(0),\mathrm{FOPT}}_{W_i}(s_0) \;=\; \frac{1}{2\pi i}\sum_{n=1}^{\infty} \Big(\tfrac{\alpha_s(s_0)}{\pi}\Big)^n \sum_{k=1}^{n+1} k\, c_{n,k} \oint\limits_{|x|=1} \frac{\mathrm{d}x}{x}\, W_i(x)\, \ln^{k-1}(-x)$$

- It is impossible to switch between the CIPT and FOPT moment series through a change of scheme of the strong coupling and a reexpansion of the series due to the **contour integration** !!

- The CIPT and FOPT series are intrinsically different and their Borel representations have to be considered to be different to start with.

  (Equivalence cannot be assumed from the start!)

FOPT vs. CIPT Borel Representation

Renormalon calculus: (Euclidean Adler function)

$$\hat{D}^{\rm CIPT}(-s_0) = \sum_{n=1}^{\infty} c_{n,1} \left(\frac{\alpha_s(s_0)}{\pi}\right)^n \quad \Longrightarrow \quad B[\hat{D}](u) \sim \sum_{n=1}^{\infty} \frac{\bar{c}_{n,1}}{\Gamma(n)}\, u^{n-1} \quad \Longrightarrow \quad B[\hat{D}](u) \sim \frac{1}{(p-u)^{\gamma}} + \frac{1}{(\tilde{p}+u)^{\tilde{\gamma}}}$$

$$\Longrightarrow \quad \hat{D}_{\rm Borel}(-s_0) = {\rm Reg.} \int_0^{\infty} {\rm d}u \; B[\hat{D}](u)\, e^{-\frac{4\pi u}{\beta_0 \alpha_s(s_0)}}$$

FOPT approach (large-β_0): (more complicated in full QCD, outcome the same)

$$\delta^{(0),\rm FOPT}_{W_i}(s_0) = \frac{1}{2\pi i} \sum_{n=1}^{\infty} \left(\frac{\alpha_s(s_0)}{\pi}\right)^n \underbrace{\sum_{k=1}^{n+1} k\, c_{n,k} \oint_{|x|=1} \frac{{\rm d}x}{x}\, W_i(x)\, \ln^{k-1}(-x)}_{\text{coefficient}}$$

$$\underbrace{B[\hat{D}](u)\, e^{-u\ln(-x)}}_{\text{Summed u-Taylor series}}\; \overbrace{e^{-\frac{4\pi u}{\beta_0 \alpha_s(s_0)}}}^{e^{-\frac{4\pi u}{\beta_0 \alpha_s(-x s_0)}}} = B[\hat{D}](u)\, e^{-\frac{4\pi u}{\beta_0 \alpha_s(-x s_0)}}$$

coefficient → Summed u-Taylor series

→ FOPT Borel representation = previously known Borel representation

$$\delta^{(0),\rm FOPT}_{W_i,\rm Borel}(s_0) = {\rm PV} \int_0^{\infty} {\rm d}u \; \frac{1}{2\pi i} \oint_{|x|=1} \frac{{\rm d}x}{x}\, W_i(x)\, B[\hat{D}](u)\, e^{-\frac{4\pi u}{\beta_0 \alpha_s(-x s_0)}}$$

FOPT vs. CIPT Borel Representation

Renormalon calculus: (Euclidean Adler function)

$$\hat{D}^{\rm CIPT}(-s_0) = \sum_{n=1}^{\infty} c_{n,1}\left(\frac{\alpha_s(s_0)}{\pi}\right)^n \quad\Longrightarrow\quad B[\hat{D}](u) \sim \sum_{n=1}^{\infty} \frac{\bar{c}_{n,1}}{\Gamma(n)}\, u^{n-1} \quad\Longrightarrow\quad B[\hat{D}](u) \sim \frac{1}{(p-u)^{\gamma}} + \frac{1}{(\tilde{p}+u)^{\tilde{\gamma}}}$$

$$\Longrightarrow\quad \hat{D}_{\rm Borel}(-s_0) = \text{Reg.} \int_0^{\infty} \mathrm{d}u\; B[\hat{D}](u)\, e^{-\frac{4\pi u}{\beta_0 \alpha_s(s_0)}}$$

CIPT approach:

Complex-valued coupling is not the expansion parameter

$$\delta^{(0),\rm CIPT}_{W_i}(s_0) = \frac{1}{2\pi i}\sum_{n=1}^{\infty} c_{n,1} \oint\limits_{|x|=1} \frac{\mathrm{d}x}{x}\, W_i(x)\left(\frac{\alpha_s(-x s_0)}{\pi}\right)^n = \frac{1}{2\pi i}\sum_{n=1}^{\infty}\left(\frac{\alpha_s(s_0)}{\pi}\right)^n \underbrace{c_{n,1}\oint\limits_{|x|=1}\frac{\mathrm{d}x}{x}\, W_i(x)\left(\frac{\alpha_s(-x s_0)}{\alpha_s(s_0)}\right)^n}_{\text{coefficient}},$$

➡ CIPT Borel representation: NEW !

$$\delta^{(0),\rm CIPT}_{W_i,\rm Borel}(s_0) = \int_0^{\infty} \mathrm{d}\bar{u}\, \frac{1}{2\pi i} \oint\limits_{\mathcal{C}_x} \frac{\mathrm{d}x}{x}\, W_i(x) \left(\frac{\alpha_s(-x s_0)}{\alpha_s(s_0)}\right) B[\hat{D}]\left(\frac{\alpha_s(-x s_0)}{\alpha_s(s_0)}\,\bar{u}\right) e^{-\frac{4\pi \bar{u}}{\beta_0 \alpha_s(s_0)}}$$

!

(1) Character of the Asymptotic Separation

FOPT Borel representation

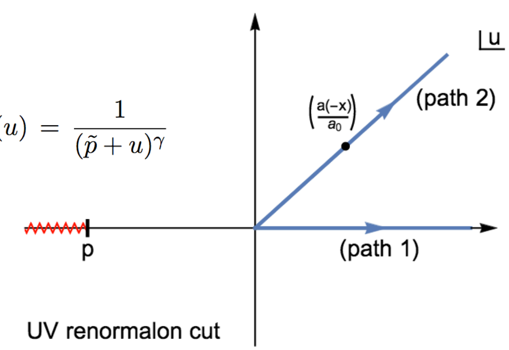
$$\delta^{(0),\mathrm{FOPT}}_{W_i,\mathrm{Borel}}(s_0) = \mathrm{PV}\int_0^\infty \mathrm{d}u\,\frac{1}{2\pi i}\oint_{|x|=1}\frac{\mathrm{d}x}{x}\,W_i(x)\,B[\hat{D}](u)\,e^{-\frac{4\pi u}{\beta_0\alpha_s(-xs_0)}}$$

CIPT Borel representation

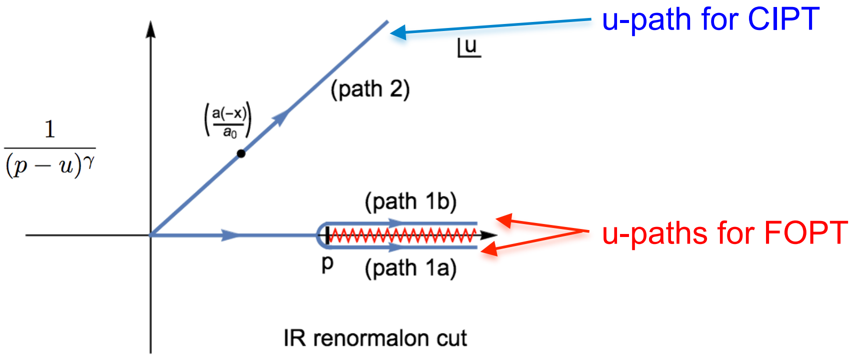
$$\delta^{(0),\mathrm{CIPT}}_{W_i,\mathrm{Borel}}(s_0) = \int_0^\infty \mathrm{d}\bar{u}\,\frac{1}{2\pi i}\oint_{\mathcal{C}_x}\frac{\mathrm{d}x}{x}\,W_i(x)\left(\frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\right)B[\hat{D}]\left(\frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\bar{u}\right)e^{-\frac{4\pi\bar{u}}{\beta_0\alpha_s(s_0)}}$$

- Related through complex-valued change of variables $u = \frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\bar{u}$
- Equivalent in perturbation theory (u-Taylor series)
- Agree at Euclidean point x = -1
- Difference in presence of IR renormalon cuts

$$B^{\mathrm{UV}}_{\hat{D},-\tilde{p},\gamma}(u) = \frac{1}{(\tilde{p}+u)^\gamma}$$



$$B^{\mathrm{IR}}_{\hat{D},p,\gamma}(u) = \frac{1}{(p-u)^\gamma}$$



UV renormalons:

FOPT and CIPT Borel representations equivalent because closing up paths 1 and 2 does not contain cuts

IR renormalons: finite difference !

FOPT and CIPT Borel representations inequivalent

- FOPT: PV prescription needs to be imposed
- CIPT: automatically well-defined by complex-valued α_s
- Difference because closing paths 1a/1b and 2 always contains cuts

(2) CIPT Borel Sum Contour Integration

The contour integration for the CIPT Borel representation must be deformed away from |x| = 1.

(Leaves FOPT Borel sum unchanged!)

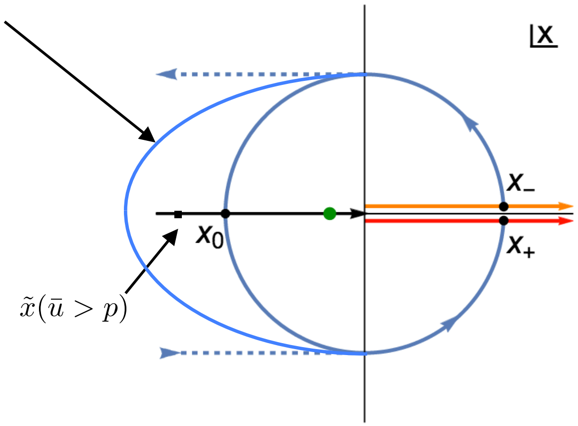
Do the contour-integral first:

$$\delta^{(0),\mathrm{CIPT}}_{\{(-x)^m,p,\gamma\},\mathrm{Borel}}(s_0) = \int_0^\infty \mathrm{d}\bar{u}\,\frac{1}{2\pi i}\oint_{\mathcal{C}_x}\frac{\mathrm{d}x}{x}\,(-x)^m\left(\frac{a(-x)}{a_0}\right)\frac{e^{-\frac{\bar{u}}{a_0}}}{\left(p-\frac{a(-x)}{a_0}\bar{u}\right)^\gamma}$$

$$= \int_0^\infty \mathrm{d}\bar{u}\,e^{-\frac{\bar{u}}{a_0}}\,\tilde{C}(p,\gamma,m,s_0;\bar{u})\,.$$

pole in x-plane at (arge-β_0)

Contour must always cross real axis for $x < \tilde{x}(\bar{u})$



$$\tilde{x}(\bar{u}) = -e^{(\bar{u}-p)/pa_0} = -\left(\frac{\Lambda^2_{\mathrm{QCD}}}{s_0}\right)^{\frac{p-\bar{u}}{p}}$$

$$< -1 \quad \text{for} \quad \bar{u} > p$$

$$\tilde{x}(0) = -\left(\frac{\Lambda^2_{\mathrm{QCD}}}{s_0}\right) \quad \text{(Landau pole)}$$

$$\tilde{x}(\bar{u}\to\infty) \to -\infty$$

(2) CIPT Borel Sum Contour Integration

The contour integration for the CIPT Borel representation must be deformed away from |x| = 1.

(Leaves FOPT Borel sum unchanged!)

Do the Borel-u-integral first:

$$\Delta(m,p,\gamma,s_0) \equiv \delta^{(0),\mathrm{CIPT}}_{\{(-x)^m,p,\gamma\},\mathrm{Borel}}(s_0) - \delta^{(0),\mathrm{FOPT}}_{\{(-x)^m,p,\gamma\},\mathrm{Borel}}(s_0)$$

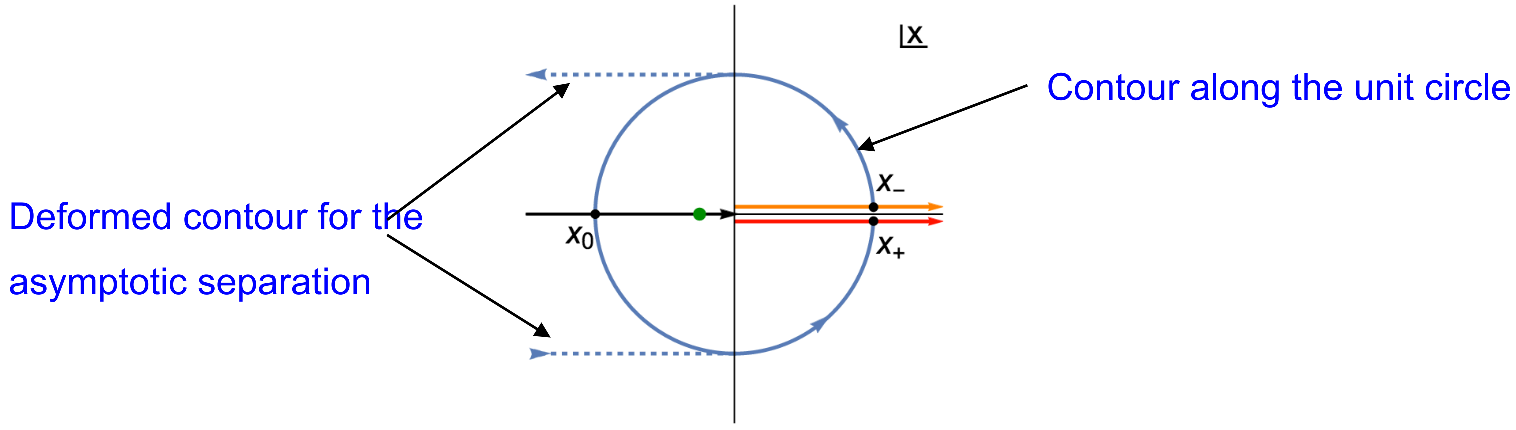
„Asymptotic Separation"

$$= \frac{1}{2\Gamma(\gamma)} \oint_{\mathcal{C}_x} \frac{\mathrm{d}x}{x} (-x)^m \,\mathrm{sig}[\mathrm{Im}[a(-x)]]\, (a(-x))^{1-\gamma}\, e^{-\frac{p}{a(-x)}} \,.$$

Cut along the negative real s-axis!

Power-suppressed $\sim \left(\frac{\Lambda^2_{\mathrm{QCD}}}{s}\right)^p$

Remaining contour integration must be deformed (to negative real infinity in the x-plane)



(3) Computation of the CIPT Borel Sum

An analytic continuation is mandatory to compute the CIPT Borel sum for m>p

$$W(x) \sim x^m$$

$$B(u) \sim \frac{1}{(p-u)^\gamma}$$

$$\Delta(m,p,\gamma,s_0) \equiv \delta^{(0),\mathrm{CIPT}}_{\{(-x)^m,p,\gamma\},\mathrm{Borel}}(s_0) - \delta^{(0),\mathrm{FOPT}}_{\{(-x)^m,p,\gamma\},\mathrm{Borel}}(s_0)$$

$$= \frac{1}{2\Gamma(\gamma)} \oint_{\mathcal{C}_x} \frac{\mathrm{d}x}{x} \, (-x)^m \, \mathrm{sig}[\mathrm{Im}[a(-x)]] \, (a(-x))^{1-\gamma} \, e^{-\frac{p}{a(-x)}} \, .$$

$$\sim x^{-p}$$

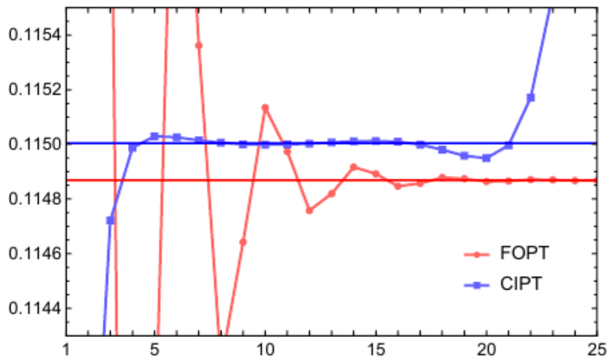
Properties of the asymptotic separation:

- Renormalization scheme invariant
- **Much larger than canonical FOPT Borel sum ambiguity estimate if the Borel function has a sizeable gluon condensate cut**
- Fully analytic results
- **Existence of the asymptotic separation implies that OPE corrections differ between CIPT and FOPT**

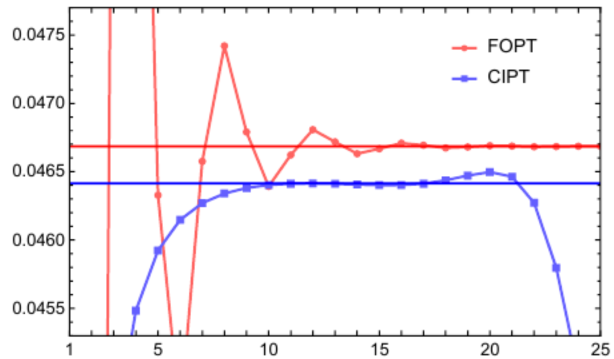
Numerical Analysis

Single renormalon model (large-b0):

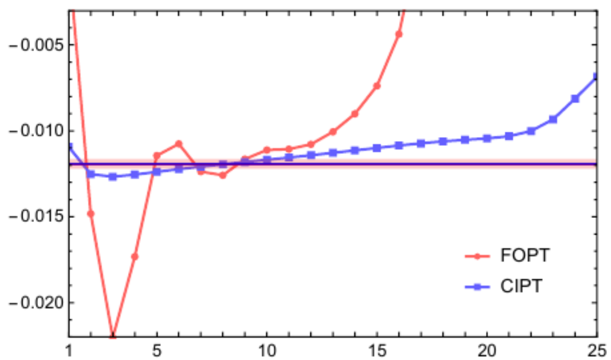
$$B(u) = \frac{1}{(2-u)} \qquad \Longleftrightarrow \qquad \langle \alpha_s G^{\mu\nu} G_{\mu\nu} \rangle$$



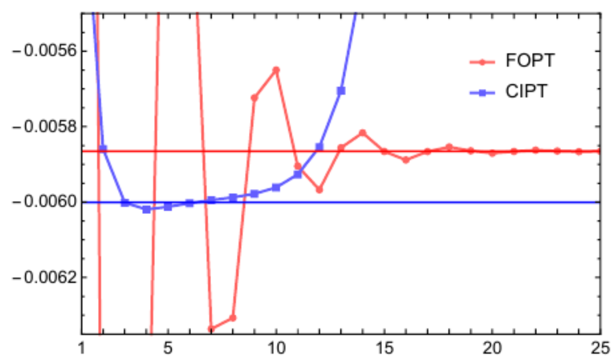
(a) Simple pole, p=2, $W(x) = 1$, large-β_0



(b) Simple pole, p=2, $W(x) = (-x)$, large-β_0



(c) Simple pole, p=2, $W(x) = (-x)^2$, large-β_0



(d) Simple pole, p=2, $W(x) = (-x)^4$, large-β_0

Excellent description of the asymptotic behavior of the CIPT series using the CIPT Borel representation.

Convergence behavior strongly depending on the power of the weight function.

Intriguing observation:

For moments with $W(x) = x^{m \neq 2}$

FOPT convergent series!

Standard Gluon cond. OPE correction vanishes!

CIPT series divergent!

(Apparently unnoticed in the literature)

→ CIPT expansion not compatible with standard OPE

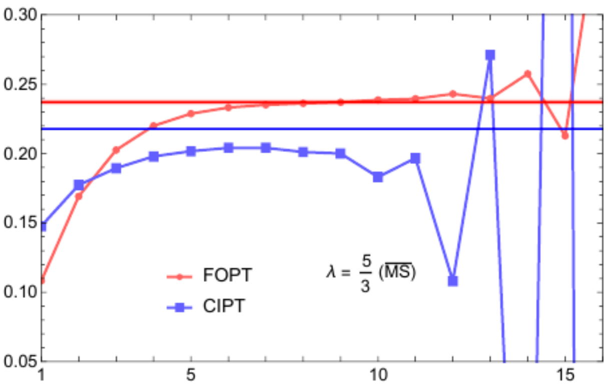
Numerical Analysis

Full QCD: Tau decay rate R_τ

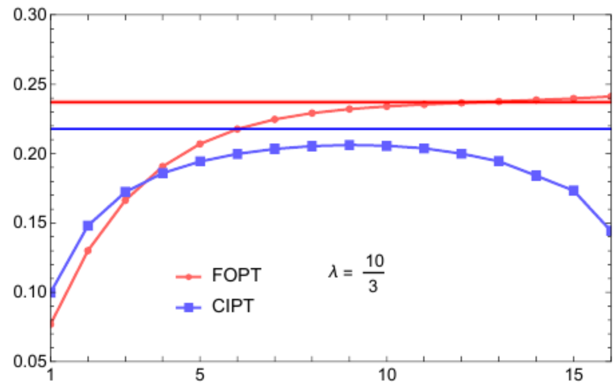
(Beneke/Jamin Borel Model, with gluon cond. cut)

$$W_\tau(x) = (1-x)^3(1+x) = 1 - 2x + 2x^3 - x^4$$

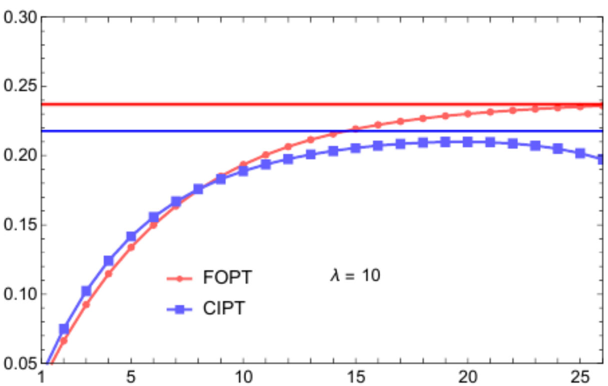
- Updated to 5-loop precision



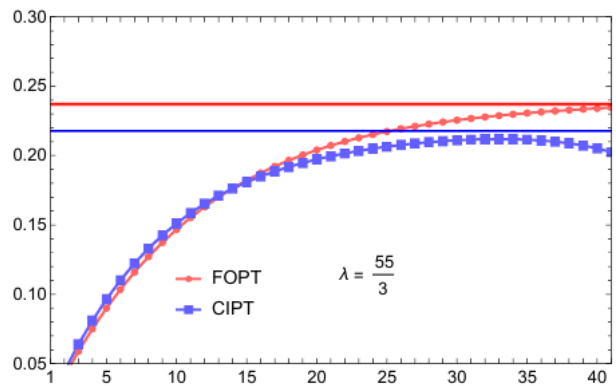
(a) $\delta^{(0)}_{W_\tau}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{\overline{\mathrm{MS}}}$, full β-function



(b) $\delta^{(0)}_{W_\tau}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{(5/3)}$, full β-function



(c) $\delta^{(0)}_{W_\tau}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{(25/3)}$, full β-function



(d) $\delta^{(0)}_{W_\tau}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{(50/3)}$, full β-function

width of line
= FOPT Borel sum ambiguity
= renormalon ambiguity used
in previous literature

Agreement of CIPT series behavior with CIPT Borel sum can depend on the scheme.

Better agreement in schemes where $\alpha_s(m_\tau)$ is small.

Asymptotic separation provides quantitative description of CIPT-FOPT discrepancy for any given model !

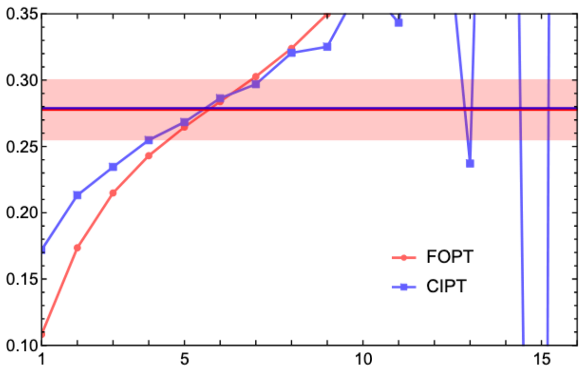
Moments w. suppressed Asymptotic Separation

Spectral function moments with small asymptotic separation

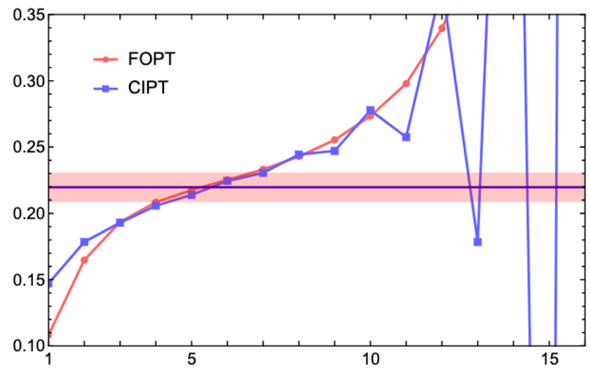
(Beneke/Jamin Borel Model `08)

vanishing asymptotic separation from gluon condensate renormalon in large-β_0

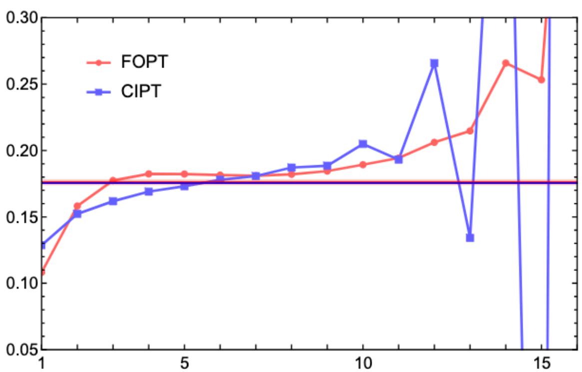
$$W_c(x) = (1-x)^2(1+cx+x^2)$$



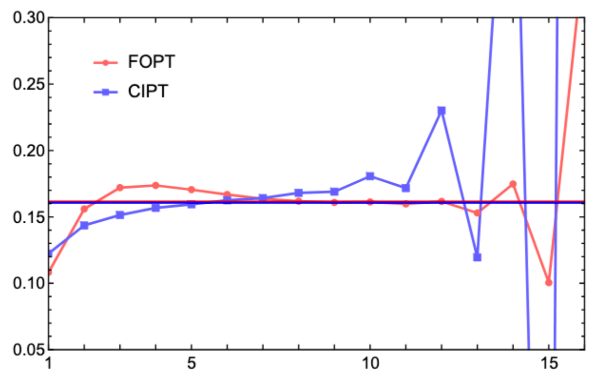
(a) $\delta^{(0)}_{W_{c=-1}}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{\overline{\mathrm{MS}}}$, full β-function



(b) $\delta^{(0)}_{W_{c=0}}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{\overline{\mathrm{MS}}}$, full β-function



(c) $\delta^{(0)}_{W_{c=0.75}}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{\overline{\mathrm{MS}}}$, full β-function



(d) $\delta^{(0)}_{W_{c=1}}(m_\tau^2)$, $B_{\hat{D},\mathrm{mr}}$, $\alpha_s^{\overline{\mathrm{MS}}}$, full β-function

Spectral function moments with small CIPT-FOPT discrepancy can be designed.

Reconciling CIPT and FOPT

Benitez-Rathgeb, Boito, Jamin, AHH: w.i.p

Predictions of the asymptotic separation:

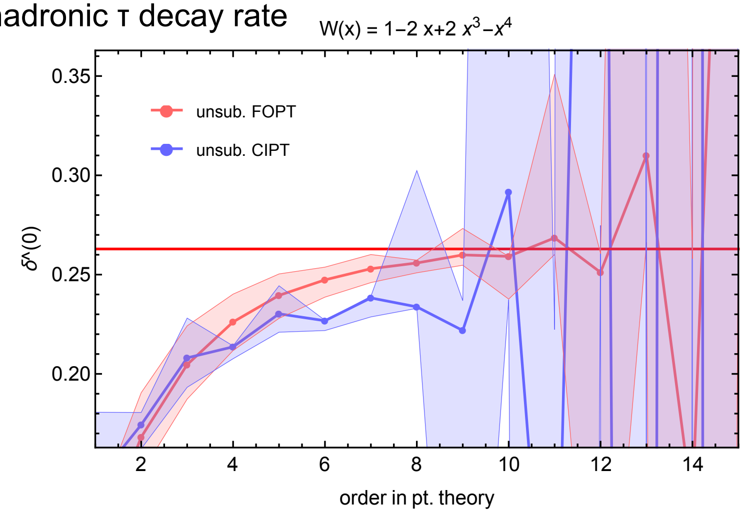
- Asymptotic separation vanishes if IR renormalons are absent
- CIPT and FOPT consistent for IR-subtracted perturbation theory

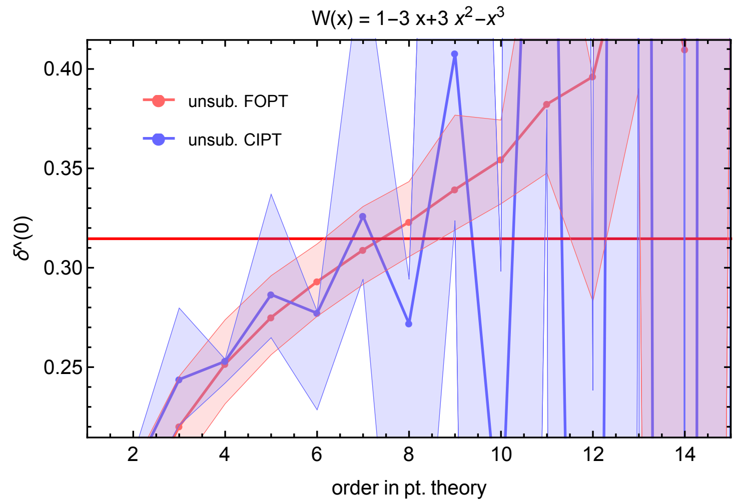
Borel function of Euclidean Adler function(large-β_0):

$$B[\hat{D}](u) \;=\; \frac{128}{3\beta_0}\, e^{5u/3}\Bigg\{\frac{3}{16(2-u)} + \sum_{p=3}^{\infty}\left[\frac{d_2(p)}{(p-u)^2} - \frac{d_1(p)}{p-u}\right] - \sum_{p=-1}^{-\infty}\left[\frac{d_2(p)}{(u-p)^2} + \frac{d_1(p)}{u-p}\right]\Bigg\}$$

$\mu^2 = \xi m_\tau^2 \quad \text{for} \quad \xi \in [0.5, 2]$

Total hadronic τ decay rate





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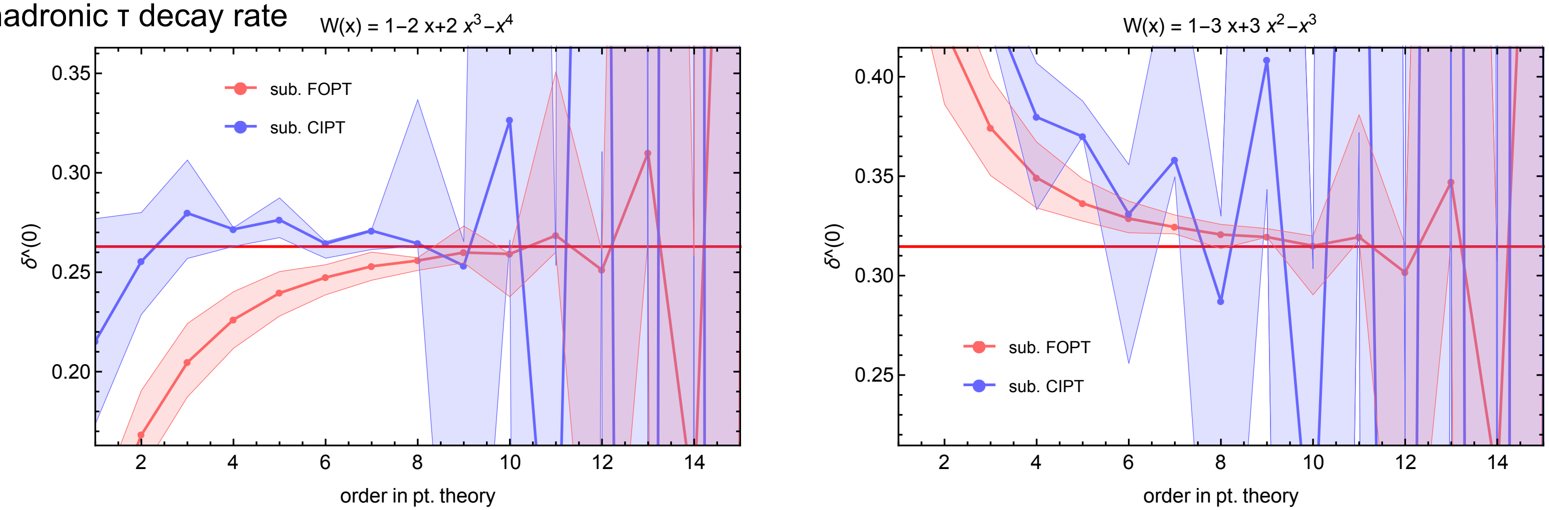
Subtract from perturbative expansion & Add back **FOPT Borel sum** to each coefficient

$$\mu^2 = \xi m_\tau^2 \quad \text{for} \quad \xi \in [0.5, 2]$$

Total hadronic τ decay rate



Summary and Conclusions

- The spectral function moment series in CIPT and FOPT are two different expansions that cannot be converted into each other by a change of scheme of the strong coupling!

- FOPT and CIPT Borel representations and their Borel sums differ
  $\longrightarrow$ **Asymptotic Separation**

- Discrepancy between FOPT and CIPT described well by asymptotic separation (if 5-loop series correction is dominated by the gluon condensate renormalon).

- Asymptotic separation implies that the OPE correction for CIPT and FOPT spectral function moments differ

- CIPT expansion not compatible with the standard OPE corrections.

- Discrepancy between CIPT and FOPT can be reconciled within IR subtracted perturbation theory.
  $\longrightarrow$ **Prospects for more precise determinations of** α_s **[w.i.p.]**