# Resonances in three-body decays of $\tau$

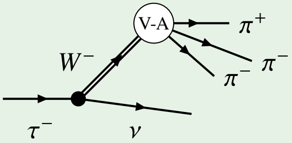
Mikhail Mikhasenko

ORIGINS Excellent Cluster

September 29$^{\text{th}}$, 2021

# $a_1(1260)$ state – ground axial vector

Clean $a_1$:
$J^{PC} = 1^{++}$

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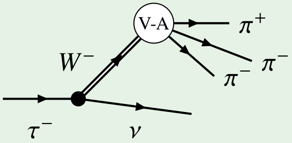


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## $a_1(1260)$ WIDTH

INSPIRE search

| VALUE (MeV) | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|
| **250 to 600** | **OUR ESTIMATE** | | | | |
| **389 ± 29** | **OUR AVERAGE** Error includes scale factor of 1.3. | | | | |
| $430 \pm 24 \pm 31$ | | DARGENT | 2017 | RVUE | $D^0 \to \pi^- \pi^+ \pi^- \pi^+$ |
| $367 \pm 9 ^{+28}_{-25}$ | 420k | ALEKSEEV | 2010 | COMP | $190\ \pi^- \to \pi^- \pi^- \pi^+ Pb'$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • | | | | | |
| $410 \pm 31 \pm 30$ | | [1] AUBERT | 2007AU | BABR | $10.6\ e^+ e^- \to \rho^0 \rho^\pm \pi^\mp \gamma$ |
| 520 - 680 | 6360 | [2] LINK | 2007A | FOCS | $D^0 \to \pi^- \pi^+ \pi^- \pi^+$ |
| $480 \pm 20$ | | [3] GOMEZ-DUMM | 2004 | RVUE | $\tau^+ \to \pi^+ \pi^+ \pi^- \nu_\tau$ |
| $580 \pm 41$ | 90k | SALVINI | 2004 | OBLX | $\bar{p} p \to 2\pi^+ 2\pi^-$ |
| $460 \pm 85$ | 205 | [4] DRUTSKOY | 2002 | BELL | $B^{(*)} K^- K^{*0}$ |
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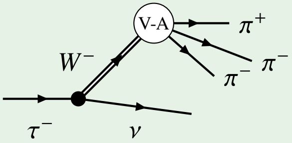


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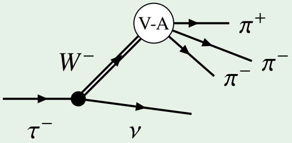


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# Three-body decay channels

The channels that $a_1$ can decay to

- $\pi^-\pi^-\pi^+$
- $\pi^-\pi^0\pi^0$
- $\pi^-K^-K^+$
- $\pi^-K^0\overline{K}^0$
- $K^-\pi^0K^0$

Subchannel resonances:

- $[\pi\pi]_S$: $f_0(500)$, $f_0(980)$
- $[\pi\pi]_P$: $\rho(770)$
- $[\pi\pi]_D$: $f_2(1260)$
- $[K\pi]_S$: $K_0^*(700)$
- $[\pi\pi]_D$: $K^*(892)$

TABLE III. Results of the nominal fit for the moduli $|\beta_i|$ and phases $\phi_{\beta_i}$ of the coefficients for the amplitudes listed in Eq. (6). The two errors shown are statistical and systematic, respectively. The branching fractions $\mathcal{B}$ are derived from the squared amplitudes (using the values of $|\beta_i|$), and are normalized to the total $\tau^-\to\nu_\tau\pi^-\pi^0\pi^0$ rate. These do not sum to 100%, due to interference between the amplitudes.

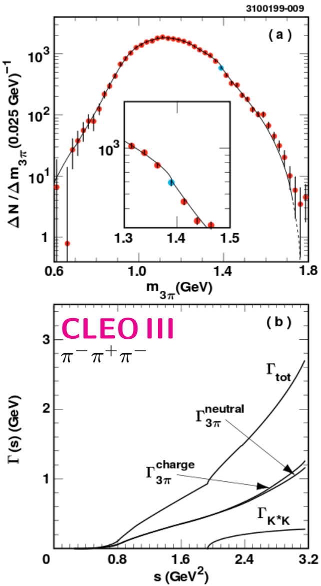
| | | Signif. | $\lvert\beta_i\rvert$ | $\phi_{\beta_i}/\pi$ | $\mathcal{B}$ fraction (%) |
|---|---|---|---|---|---|
| $\rho$ | $s$-wave | | 1 | 0 | 68.11 |
| $\rho(1450)$ | $s$-wave | $1.4\sigma$ | $0.12\pm0.09\pm0.03$ | $0.99\pm0.25\pm0.04$ | $0.30\pm0.64\pm0.17$ |
| $\rho$ | $d$-wave | $5.0\sigma$ | $0.37\pm0.09\pm0.03$ | $-0.15\pm0.10\pm0.03$ | $0.36\pm0.17\pm0.06$ |
| $\rho(1450)$ | $d$-wave | $3.1\sigma$ | $0.87\pm0.29\pm0.06$ | $0.53\pm0.16\pm0.06$ | $0.43\pm0.28\pm0.06$ |
| $f_2(1270)$ | $p$-wave | $4.2\sigma$ | $0.71\pm0.16\pm0.05$ | $0.56\pm0.10\pm0.03$ | $0.14\pm0.06\pm0.02$ |
| $\sigma$ | $p$-wave | $8.2\sigma$ | $2.10\pm0.27\pm0.09$ | $0.23\pm0.03\pm0.02$ | $16.18\pm3.85\pm1.28$ |
| $f_0(1370)$ | $p$-wave | $5.4\sigma$ | $0.77\pm0.14\pm0.05$ | $-0.54\pm0.06\pm0.02$ | $4.29\pm2.29\pm0.73$ |

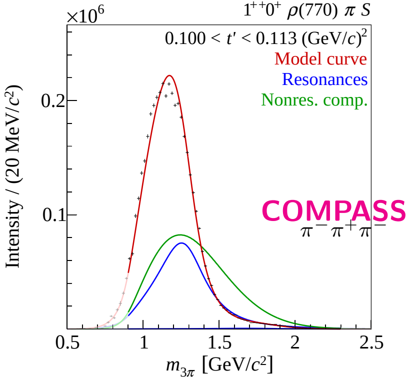
# $a_1$ studies

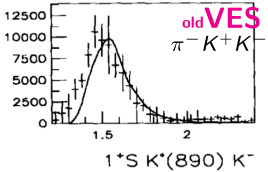
There is a list of measurements of $1^{+}+$ sector:

- diffractive production suffers from coherent physics background (Deck process)
- $\tau$-decay: limited statistics in old analysis

[Aster *et al.*, Phys.Rev. D, **60**, 0120002 (1999)]



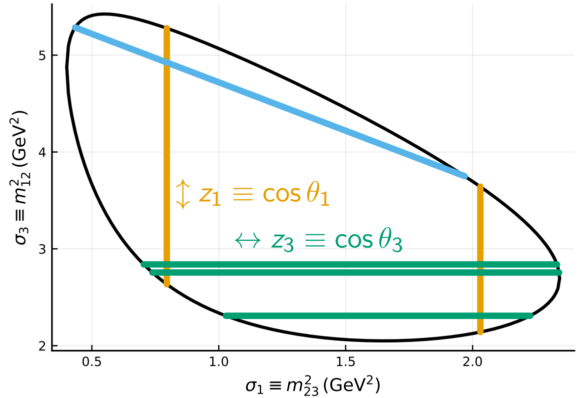




[Berdnikov *et al.*, Nuovo Cim. 107, 1941 (1994

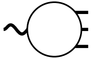
# Physics of the Dalitz plot

isobar model, rescattering, ladder of exchanges

# Three-body decay




Decay amplitude – $\langle p_1 p_2 p_3 | \hat{T} | p_0 \rangle$



$$= D^{J*}_{M\lambda}(\alpha, \beta, \gamma) F_\lambda(s, \sigma_1, \sigma_2)$$

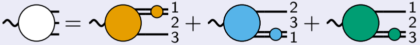
$$\xrightarrow[\text{scalars}]{} F(\sigma_1, \sigma_2)$$

## Dalitz plot variables

- Subchannel resonances are bands.
- Angular distribution along the bands determined by angular momenta.

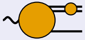
# Partial-waves vs Isobar representation

## Isobar representation



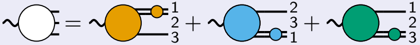

$$F(\sigma_1,\sigma_2) = F^{(1)}(\sigma_1,\sigma_2) + F^{(2)}(\sigma_1,\sigma_2) + F^{(3)}(\sigma_1,\sigma_2)$$

$$= \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_1) a_l^{(1)}(\sigma_1) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_{l}^{\text{few}} \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3).$$

Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)}\mathrm{BW}(\sigma_1) =$ .

# Partial-waves vs Isobar representation

## Isobar representation

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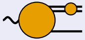
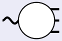
Simple **model**:  $= a_l^{(i)}(\sigma_1) \to c^{(i)}\text{BW}(\sigma_1) =$ .

## Partial-wave representation

 $= F(\sigma_1,\sigma_2) = \sum_l^{\infty} \sqrt{2l+1} P_l(z_1) f_l^{(1)}(\sigma_1)$

Why would someone do this? – theoretical constant to $f^{(1)}(\sigma_1)$ is straightforward.

# Two-body unitarity and Khuri-Trieman model

## Example of $f_0^{(1)}(\sigma_1)$ constraints:

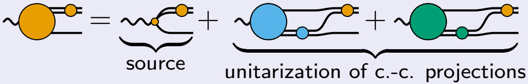
$$f_0^{(1)}(\sigma_1) = \underbrace{a_0^{(1)}(\sigma_1)}_{\text{same channel}} + \underbrace{\int_{-1}^{1} \frac{\mathrm{d}z_1}{2} \left( \sum_l \sqrt{2l+1} P_l(z_2) a_l^{(2)}(\sigma_2) + \sum_l \sqrt{2l+1} P_l(z_3) a_l^{(3)}(\sigma_3) \right)}_{\text{cross-channel(c.-c.) projections}}$$

Unitarity of $f_0^{(1)}(\sigma_1)$ – same RHC as $2 \to 2$ scattering amplitude, $\mathrm{BW}_0^{(1)}(\sigma_1)$

$\Rightarrow$ consistency relation the **direct term** and the **cross-channel projections**

$\Rightarrow$ $a_l^{(1)}(\sigma_1)$ obtains corrections from one seen in $2 \to 2$.

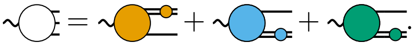
## KT model: analytic continuation of **two-body** unitarity



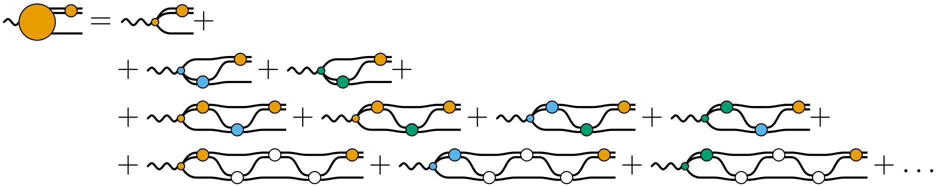

(the loop is a dispersive integral)

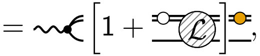
# Diagramatic representation

Isobar representation with $a_l^{(i)}(\sigma_i) = \hat{a}_l^{(i)}(\sigma_i)\,\mathrm{BW}_l^{(i)}(\sigma_i)$

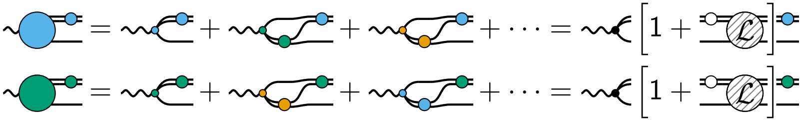


The amplitude prefactor is not constant: $a_l^{(i)}(\sigma_i) = c_l^i \mathrm{BW}_l^{(i)}(\sigma_i) + \dots$





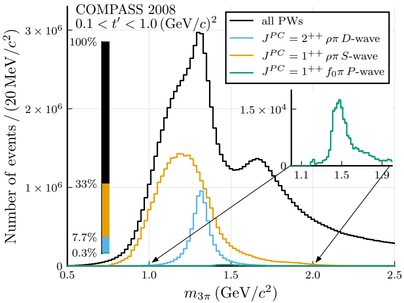
the ladder – a sum of all possible exchanges



Effect of the rescattering – modification of the resonance lineshape

# Observation of the $a_1(1420)$

[COMPASS, PRL 115 (2015) 082001]



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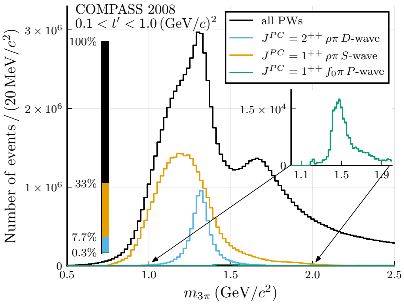


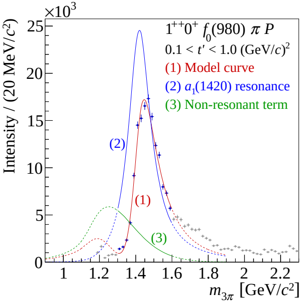

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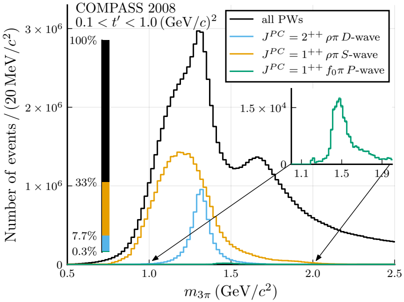
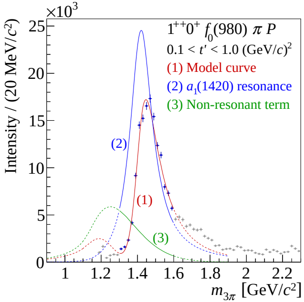
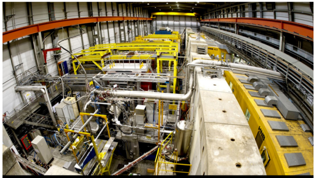



SCIENCE TICKER PARTICLE PHYSICS

## New particle may be made of four quarks

BY ANDREW GRANT 4:48PM, FEBRUARY 2, 2015



CERN's COMPASS installation has detected evidence of a particle that may be made up of four quarks.

### Not something ordinary

- Too close to the ground state $a_1(1260)$
- Its width is narrower than the ground state
- Close to threshold $K^*\bar{K}$, i.e. $(d\bar{s}) + (\bar{u}s)$, $E_{\text{th}} = 1.39\,\text{GeV}$.

# $a_1(1420)$ interpretations

## Possible scenaria

- **Pole** in the amplitude – Genuine resonance
- Singularity of the **non-pole** type

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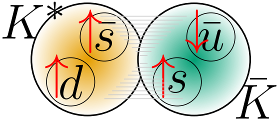


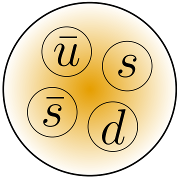

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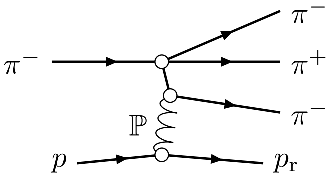


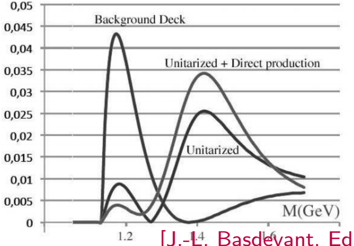


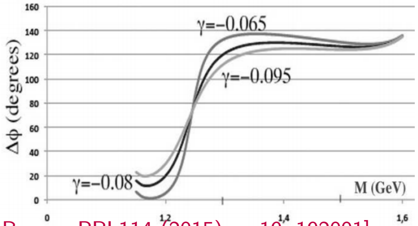


[J.-L. Basdevant, Ed. Berger, PRL114 (2015) no.19, 192001]

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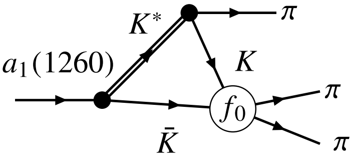
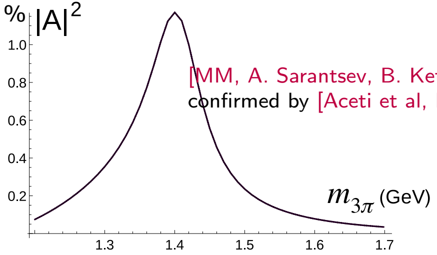
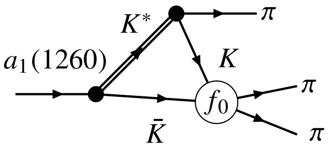
  - **Rescattering** from $K^*\bar{K}$ — Triangle singularity



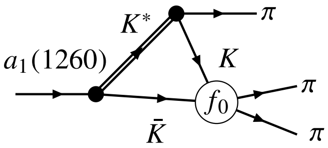

[MM, A. Sarantsev, B. Ketzer, PRD 91
confirmed by [Aceti et al, PRD 94, 096

# The key effect - the triangle rescattering graph

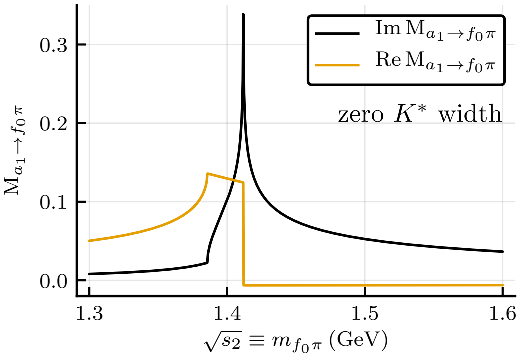



- $f_0$ is a resonance in $(K\bar{K})$ and also in $(\pi\pi)$ system.
- Ordinary $a_1$ decays to $K\bar{K}\pi$ via $K^*\bar{K}$
- $K\bar{K}$ form $f_0$ that decays to $\pi\pi$

# The key effect - the triangle rescattering graph




- $f_0$ is a resonance in $(K\bar{K})$ and also in $(\pi\pi)$ system.
- Ordinary $a_1$ decays to $K\bar{K}\pi$ via $K^*\bar{K}$
- $K\bar{K}$ form $f_0$ that decays to $\pi\pi$




- has a logarithmic singularity (divergence at a single point)
- $A \sim \log(s_0 - m_{3\pi}^2)$ with $s_0$ determined by masses of involved particles.

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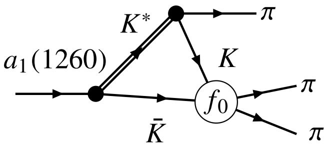


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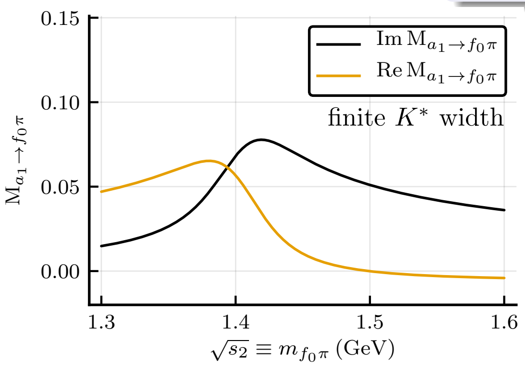
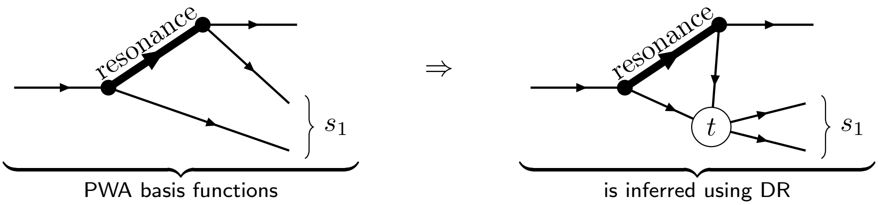



- has a logarithmic singularity (divergence at a single point)
- $A \sim \log(s_0 - m_{3\pi}^2)$ with $s_0$ determined by masses of involved particles.

# New calculation method: first iteration of the KT equations

## Dispersion approach based on unitarity

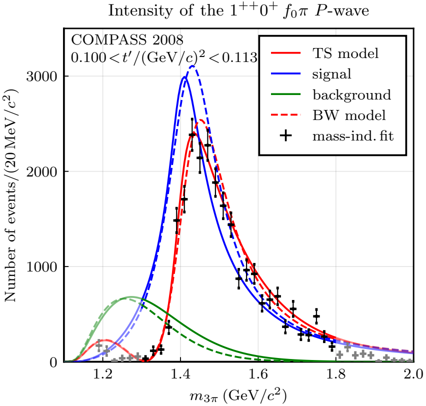
Main idea — relation between **cross–channel projection** and **rescattering** based on unitarity of $t(\sigma)$: $\pi\pi \leftrightarrow K\bar{K}$.

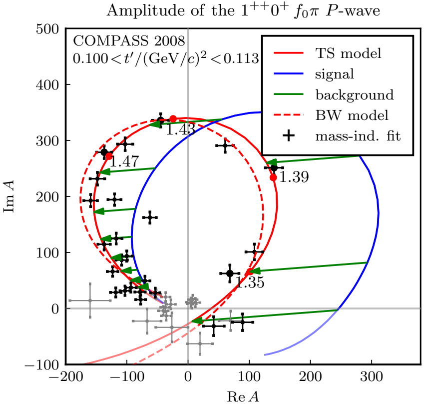



- Calculation of angular projections (the resonance is $K^*$)
- Unitarized using dispersion relations
- Attach $K\bar{K} \to \pi\pi$ interaction

# Fit with the rescattering model [COMPASS, PRL(2021)]

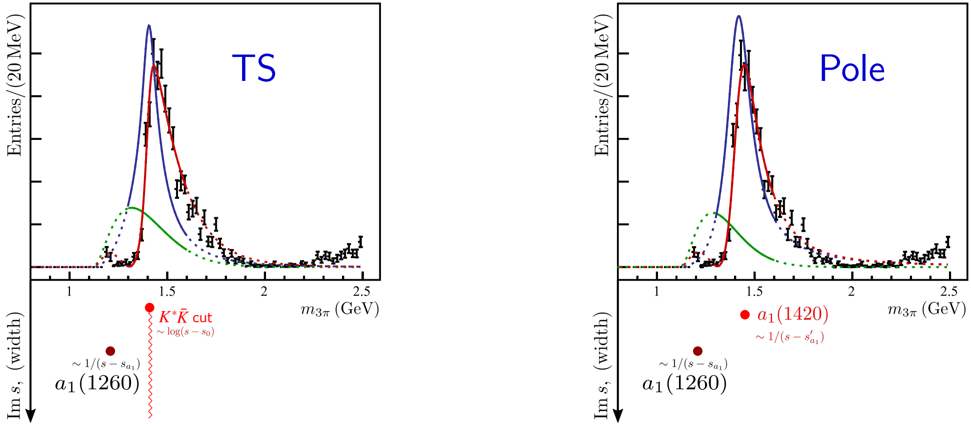
Fit perfectly describes the intensity and the phase motion




- No shape parameters for the signal component (TS)
- Background with constant phase is needed to shift the amplitude
- TS model shows a comparable quality to the resonance model (BW-model)

# Comment of the analytic structure [COMPASS, PRL 127, 082501 (2021) (2021)]



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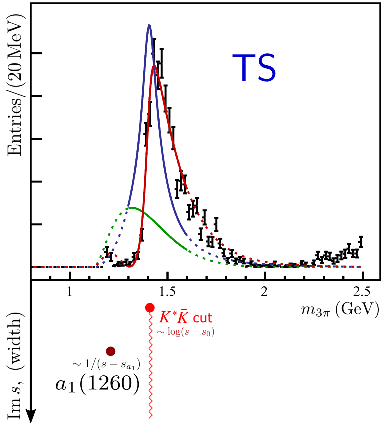
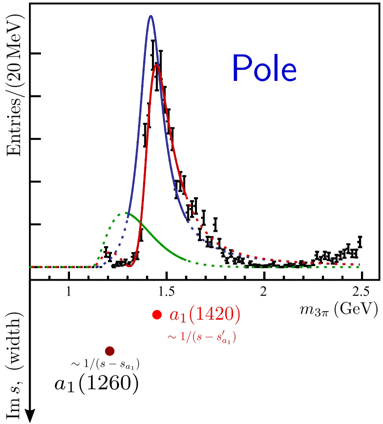
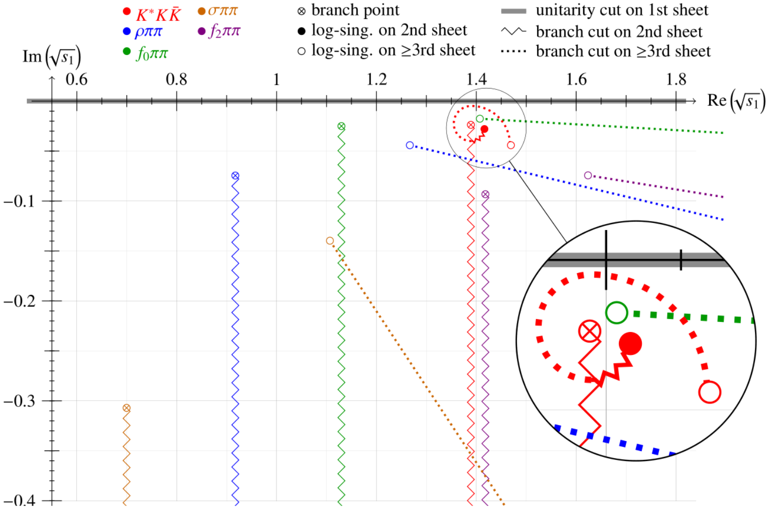



- $a_1(1420)$ signal can be described with $a_1(1260)$ as source for the **rescattering** via the triangle diagram $\Rightarrow$ the first clear observation of the TS
- An additional pole is not needed, although, not excluded

# Many more cuts in the complex plane

## But no TS next to the physics region



# Three-particle scattering: parameters of $a_1(1260)$

Three-body unitarity, Ladders and Resonances, short-range factorization

[JHEP 08 (2019) 080]

# Three-body-unitarity constraint

[MM (JPAC), JHEP 08 (2019) 080]

Three-body scattering amplitude must satisfy the integral equation

$$\mathcal{T}(\sigma', s, \sigma) - \mathcal{T}^\dagger(\sigma', s, \sigma) =$$

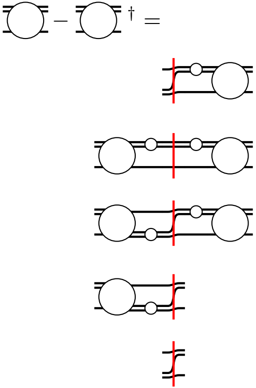
$$2i\,\frac{1}{\lambda_s^{1/2}(\sigma')}\,\frac{1}{8\pi}\int_{\sigma^-(\sigma',s)}^{\sigma^+(\sigma',s)} \mathrm{d}\sigma_3'\, t(\sigma_3')\mathcal{T}(\sigma_3', s, \sigma)$$

$$+\,\frac{i}{3}\int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} \frac{\mathrm{d}\sigma''}{2\pi}\mathcal{T}^\dagger(\sigma', s, \sigma'')\, t(\sigma'')t^\dagger(\sigma'')\,\rho(\sigma'')\rho_s(\sigma'')\,\mathcal{T}(\sigma'', s, \sigma)$$

$$+\,\frac{2i}{3}\,\frac{1}{(8\pi)^2}\iint_{\phi(\sigma_2'', s, \sigma_3'')>0} \frac{\mathrm{d}\sigma_2''\mathrm{d}\sigma_3''}{2\pi s}\,\mathcal{T}^\dagger\,(\sigma', s, \sigma_2'')t^\dagger\,(\sigma_2'')t(\sigma_3'')\mathcal{T}(\sigma_3'', s, \sigma)$$

$$+\,2i\,\frac{1}{\lambda_s^{1/2}(\sigma)}\,\frac{1}{8\pi}\int_{\sigma^-(\sigma,s)}^{\sigma^+(\sigma,s)} \mathrm{d}\sigma_2\,\mathcal{T}^\dagger(\sigma', s, \sigma_2)t^\dagger(\sigma_2)$$

$$+\,6i\,\frac{2\pi s}{\lambda_s^{1/2}(\sigma')\lambda_s^{1/2}(\sigma)}\,\theta^+(\phi(\sigma', s, \sigma)).$$



In a short form: [G.Fleming(1964), Aaron-Amada(1977), Mai et al. EPJ A53 (2017), Jackura et al. EPJ C79 (2019)]:

$$\mathcal{T} - \mathcal{T}^\dagger = \mathcal{D}\tau\mathcal{T} + \mathcal{T}^\dagger(\tau - \tau^\dagger)\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D}\tau\mathcal{T} + \mathcal{T}^\dagger\tau^\dagger\mathcal{D} + \mathcal{D},$$

# Factorization of the resonance kernel

- Strictly, one more (weak) assumption – **Factorization**

$$\widehat{\mathcal{R}}(\sigma', s, \sigma) = k_f(\sigma')\widehat{\mathcal{R}}(s)\ k_i(\sigma) \quad \text{OR}$$
$$= \widehat{\mathcal{R}}_{00}(s) + \sigma'\widehat{\mathcal{R}}_{10}(s) + \widehat{\mathcal{R}}_{01}(s)\sigma + \sigma'\widehat{\mathcal{R}}_{11}(s)\sigma + \dots,$$

$\Rightarrow$ Unitarity requirement is algebraic!

$$\widehat{\mathcal{R}}(s) - \widehat{\mathcal{R}}^\dagger(s) = i\,\widehat{\mathcal{R}}^\dagger(s)\Sigma(s)\,\widehat{\mathcal{R}}(s),$$

with $\Sigma \equiv \mathcal{K}^\dagger(\tau - \tau^\dagger)\mathcal{K} + \mathcal{K}^\dagger\tau^\dagger\mathcal{D}\tau\mathcal{K},$

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$\mathcal{K}$ is the modification of the isobar lineshape due to the rescattering

## An approximate-three-body unitarity

$$\widehat{\mathcal{R}}(s) = \frac{g^2}{m^2 - s - ig^2/2\left[\ \cdots\ \right]}$$

contains effect of the subchannel-resonances **interference**

## The quasi-two-body approximation

[J.Basdevant, Ed Berger, PRD19 (1979) 239]

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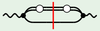
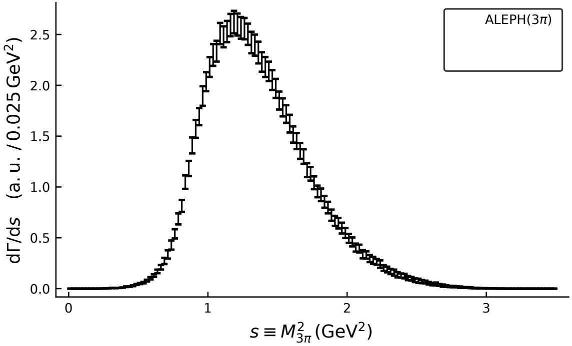
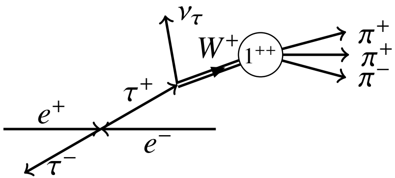


naively accounts for the subchannel-resonance decay

# Analysis of data (ALEPH measurements)

[data from Phys.Rept.421 (2005)]



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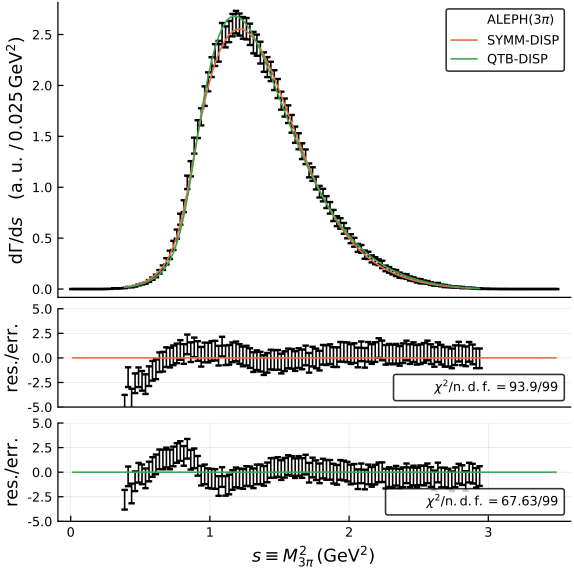
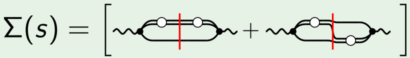
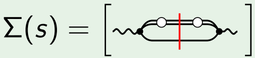



## Two models of $\rho\pi$ scattering:

- SYMM-DISP: Approximate three-body unitarity (includes interference)



- QTB-DISP: Quasi-two-body unitarity



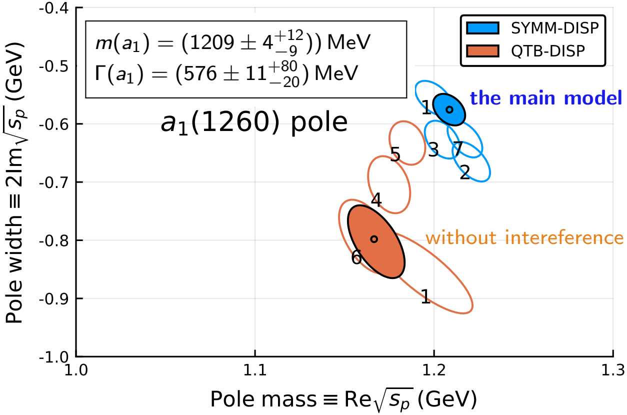
both neglect rescattering, $\mathcal{K} \to 1$.

## Fit to the public data

- Stat. cov. matrix is used in the fit
- Syst. cov. matrix – in the bootstrap

# Determination of the $a_1(1260)$ parameters

Non-trivial analytic continuation [MM et al. (JPAC), PRD98 (2018), 096021]




| # | Fit studies |
|---|---|
| 1 | $s < 2\ \mathrm{GeV}^2$ |
| 2 | $R' = 3\ \mathrm{GeV}^{-1}$ |
| 3 | $m'_\rho = m_\rho + 10$ MeV |
| 4 | $m'_\rho = m_\rho - 10$ MeV |
| 5 | $m'_\rho = m_\rho - 20$ MeV |
| 6 | $\Gamma'_\rho = \Gamma_\rho + 5$ MeV |
| 7 | $\Gamma'_\rho = \Gamma_\rho - 30$ MeV |

- Large systematic uncertainties due to disregard of rescattering effects
- Effect of the subchannel-resonances interference is very important

## Summary of the talk

- unitarity constraints to two-body resonances
- unitarity constraints to three-body resonances
- $a_1 \to f_0(980)\pi$ is consistent with TS
- $a_1(1260)$ parameters are extracted

$\tau$ decay is the cleanest environment to study hadron dynamics in $J^{PC} = 1^{++}$ sector

- Dalitz-plot distribution in $\tau \to hhh\,\nu$:
  - **Triangle Singularity in $\tau \to f_0\pi\,\nu$ is strongly anticipated (Belle II !!)**
  - **Changes of the lineshape** of $\rho$, $K^*$ in $\tau \to 3\pi$, $\tau \to KK\pi$

    will validate our understanding of the three-hadron dynamics.
- $a_1(1260)$ parameters need to be extracted using unitary model
  - First analysis shows promising results
  - Finer data are needed
  - Model requires extension with more subchannel resonances

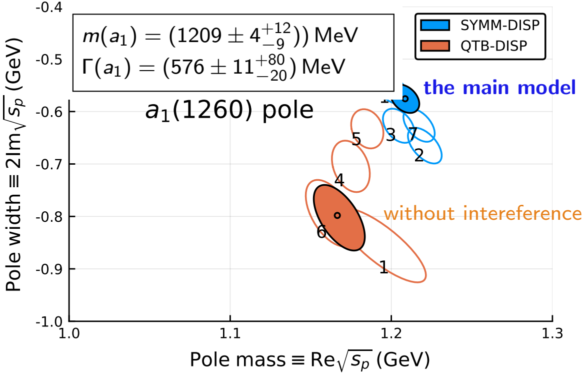
# Thank you for attention

Thanks to my collaborators:

- **JPAC group:** Yannick Wundelich, Andrew Jakura, Alessandro Pilloni, Vincent Mathieu, Miguel Albaladejo, Cesar Fernandez, Adam Szczepaniak, Sergi Gonzales, Daniel Winney, Lukasz Bibrzycki
- **COMPASS collaborators:** Mathias Wagner, Bernhard Ketzer, many others.

# Backup

# Systematic studies

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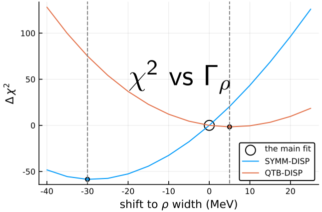


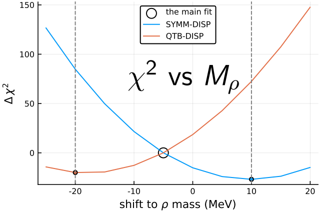


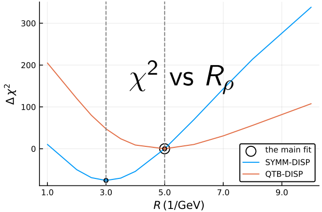

# Tour to the complex plane

[MM, JPAC, PRD98 (2018), 096021]

## Analytical continuation

$$|t_{II}^{-1}(s)| = \left| \frac{m^2 - s}{g^2} - i\left(\frac{\tilde{\rho}(s)}{2} + \rho(s)\right) \right|.$$

- Analytical continuation of $\rho(s)$: integral over the Dalitz plot for the complex energy

$$\rho(s) = \sum_\lambda \int \mathrm{d}\Phi_3 \left| f_\rho(\sigma_1) d_{\lambda 0}(\theta_{23}) - f_\rho(\sigma_3) d_{\lambda 0}(\hat{\theta}_3 + \theta_{12}) \right|^2$$

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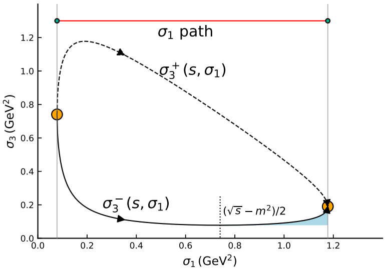

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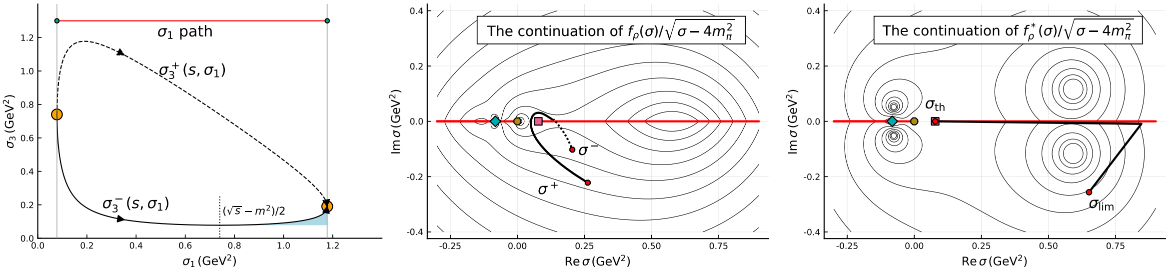

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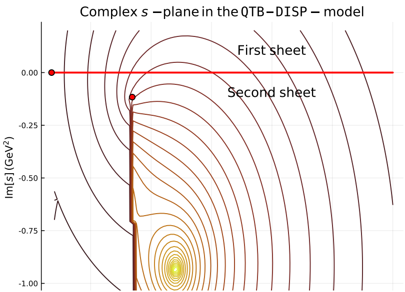


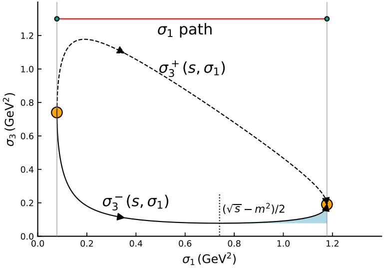


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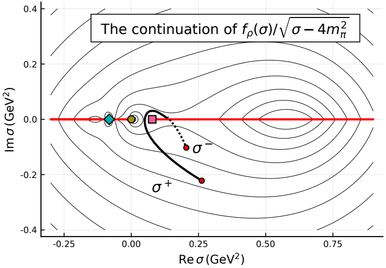


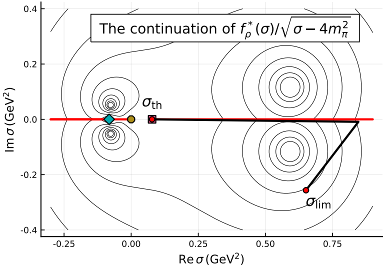

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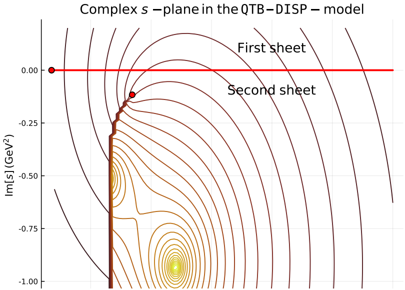


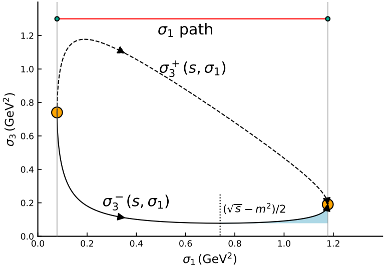


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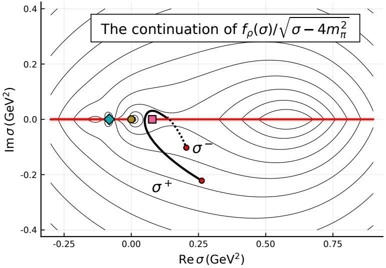


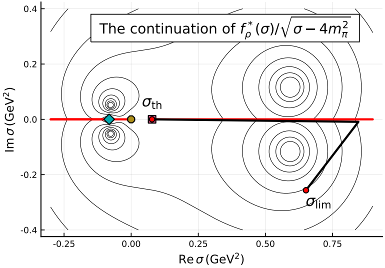

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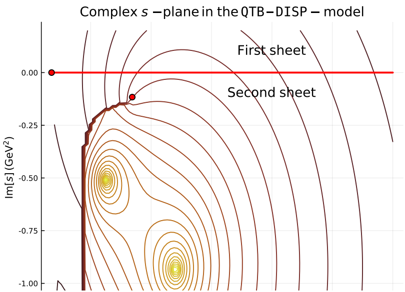


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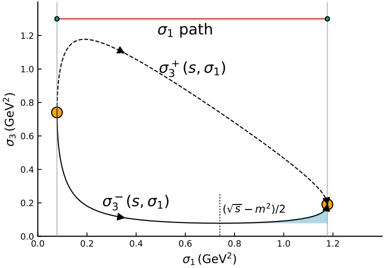


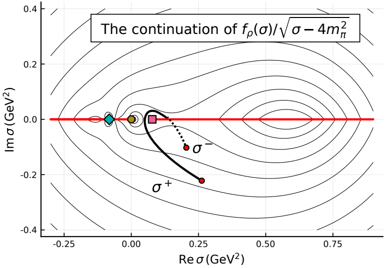


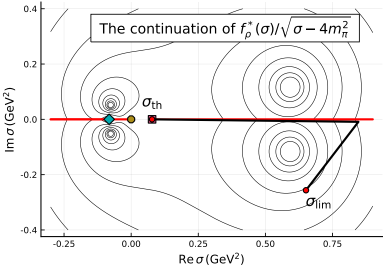

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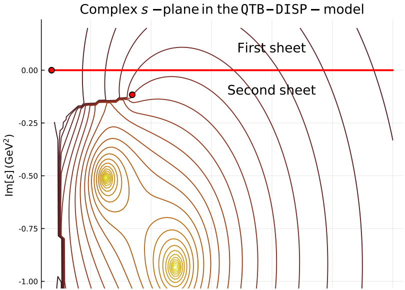


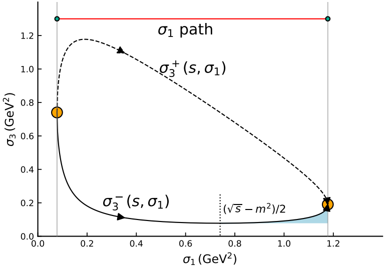


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