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Deriving experimental constraints on the scalar form factor in the second-class $\tau \to \eta\pi\nu$ mode

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based on work with:

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Introduction:

- Second-class currents [S.Weinberg, PR 112,1375(1958)] (V^{ud}_μ, A^{ud}_μ have definite G-parity): selection rules in isospin limit
  - → $\tau \to \eta\pi\nu_\tau$ mode is clean example

    $G^{-1} V^{ud}_\mu G = + V^{ud}_\mu$

    $G|\eta\pi\rangle = -|\eta\pi\rangle)$
  - → Isospin breaking in SM: $O((m_d - m_u)/m_s, e^2)$
  - → Exp. upper limit $BF_{\eta\pi} < 9.9 \times 10^{-5}$ [Babar, PR D83, 032002 (2010)]
  - → It would be very useful to have a good theoretical estimate of SM contribution

- A number of estimates already made:

$10^5 \times BF_v$	$10^5 \times BF_S$	$10^5 \times BF$	ref.
0.25	1.60	1.85	Tisserant, Truong (1982)
0.12	1.38	1.50	Bramon,Narison,Pich (1987)
0.15	1.06	1.21	Neufeld, Rupertsberger(1995)
0.36	1.00	1.36	Nussinov, Soffer (2008)
[0.2-0.6]	[0.2-2.3]	[0.4-2.9]	Paver, Riazuddin (2010)
0.435	0.04	0.47	Volkov,Kostunin (2012)
0.13	0.20	0.33	Descotes-Genon,B.M. (2014)
0.84	4.46	5.30	Escribano et al.(2016)

→ This talk: Constrain scalar form factor from $\gamma\gamma \to \eta\pi$, $\phi \to \gamma\eta\pi$ (using analyticity)

+Update on vector form factor

Why $\eta\pi$ form factors more difficult to predict than e.g. $K\pi$, $\pi\pi$ ones ?:

- Definition of vector $f_+^{P_1 P_2}$,scalar $f_0^{P_1 P_2}$ form factors:

$$\langle P_1(p_1)P_2(p_2)|\bar{u}\gamma^\mu d|0\rangle = C_{12} \times \Big\{ \left(p_1 - p_2 - \frac{m_1^2 - m_2^2}{s}(p_1 + p_2)\right)^\mu f_+^{P_1 P_2}(s) + \frac{m_1^2 - m_2^2}{s}(p_1 + p_2)^\mu f_0^{P_1 P_2}(s) \Big\}$$

→ In case of $K\pi$, unitarity is elastic

$$\mathrm{Im}\left[f_+^{K\pi}(s)\right] = e^{-i\delta_1^{K\pi}(s)} \sin \delta_1^{K\pi}(s) f_+^{K\pi}(s)$$
$$\mathrm{Im}\left[f_0^{K\pi}(s)\right] = e^{-i\delta_0^{K\pi}(s)} \sin \delta_0^{K\pi}(s) f_0^{K\pi}(s)$$

in energy region of light resonances $K^*(892)$, $K_0^*(600)$

- Analyticity+elastic unitarity FSI problem exactly soluble [R.Omnès(1958),J.Plemelj(1908),N.Muskhelishvili(1941)]

$$f_+^{K\pi}(s) \simeq f_+^{K\pi}(0)\,\exp\left[\frac{s}{\pi}\int_{(m_{m_\pi}+m_K)^2}^{\infty} ds'\,\frac{\delta_1^{K\pi}(s')}{s'(s'-s)}\right]$$

$$f_0^{K\pi}(s) \simeq f_+^{K\pi}(0)\,\exp\left[\frac{s}{\pi}\int_{(m_{m_\pi}+m_K)^2}^{\infty} ds'\,\frac{\delta_0^{K\pi}(s')}{s'(s'-s)}\right]$$

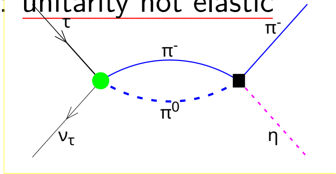
with Omnès function

→ $\delta_J^{K\pi}$: measured scattering phase-shifts

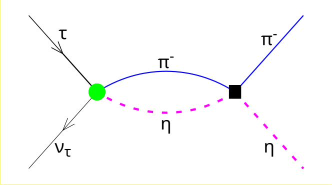
→ Improvement: use Watson theorem ($s' \lesssim 1$ GeV^2), QCD($s' \to \infty$)[P.Yndurain, PL B578,99 (2004)]

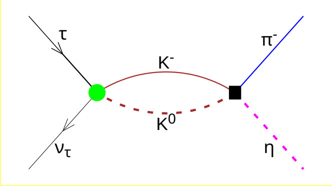
- Case of $\eta\pi$ form factors: unitarity not elastic

→ $\mathrm{Im}\,[f_+^{\eta\pi}(s)]$:
(in $\rho(770)$ region)



→ $\mathrm{Im}\,[f_0^{\eta\pi}(s)]$:
(in $a_0(980)$ region)





- Scalar form factor in terms of Omnès-Muskhelishvili matrix

$$\begin{pmatrix} f_0^{\eta\pi}(s) \\ \epsilon f_0^{K\overline{K}}(s) \end{pmatrix} \simeq \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} f_0^{\eta\pi}(0) \\ \epsilon f_0^{K\overline{K}}(0) \end{pmatrix}$$

with: $\epsilon = \dfrac{m_{K^0}^2 - m_{K^+}^2}{\sqrt{2}(m_\eta^2 - m_{\pi^+}^2)}$

Encodes two-channel unitarity + dipersion relation associated with right-hand cut.

→ Values at $s = 0$ (ChPT)

$$f_0^{\eta\pi}(0) = \frac{1}{\sqrt{3}} \left\{ \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^+}(0)} - 1 - \frac{3e^2}{4(4\pi)^2} \log \frac{m_K^2}{m_\pi^2} \right\}$$

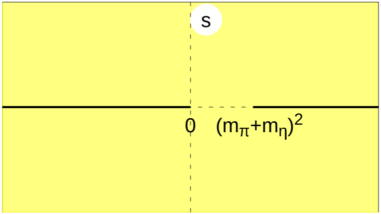
$$f_0^{K\overline{K}}(0) = 1$$

[H.Neufeld,H.Rupertsberger, ZP C68,91 (1995)]

Dispersive MO repres. of $\gamma\gamma$ $J = 0$ amplitudes

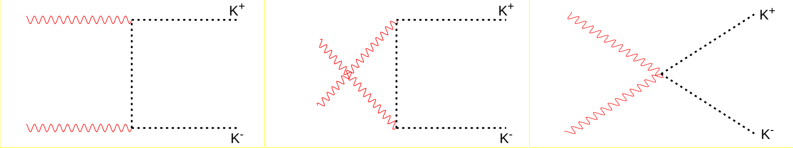
[Junxu Lu,B.M.,EPJ C80,436(2020)]

- $\gamma\gamma \to \pi^0\eta, K^0\overline{K}^0$: $L_{\lambda\lambda'}(s,t,u)$, $K_{\lambda\lambda'}(s,t,u)$,
  - → S-waves $l_{0++}(s), k_{0++}(s)$ dominate when $\sqrt{s} \lesssim 1.1$ GeV.
  - → Analytic structure for complex s:



  - → LH cut: from integrating over t

- Illustration: QED Born amplitudes



→ Easily computed:

$$K_{++}^{Born}(s,t,u) = \frac{2e^2(m_{K^+}^2 s)}{(t-m_{K^+}^2)(u-m_{K^+}^2)}$$

$J=0,\ I=1$ projection

$$k_{0++}(s) = -\frac{2\sqrt{2}e^2 m_{K^+}^2}{s\sigma_{K^+}(s)} \log\frac{1+\sigma_{K^+}(s)}{1-\sigma_{K^+}(s)},$$

with $\sigma_K(s)=\sqrt{1-4m_K^2/s}$. Has a cut on $[-\infty,0]$.

→ Further contributions: $\rho,\ \omega,\ \phi,\ K^*$ exchanges

- MO dispersive representation

$$\begin{pmatrix} l_{0++}(s) \\ k_{0++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_{0++}^{Born}(s) \end{pmatrix} + s \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} b_l + L_1^V(s) + R_1^{Born}(s) \\ b_k + L_2^V(s) + R_2^{Born}(s) \end{pmatrix}$$

→ Two subtraction constants b_l, b_k

→ $L_i^V(s)$, $R_i^{Born}(s)$: integral involving $\mathrm{Im}\,[l_{0++}^V]$, $\mathrm{Im}\,[k_{0++}^V]$, k_{0++}^{Born} and $(\Omega^{-1})_{ij}$.

→ Exact soft-photon theorem[Low(1958)] at $s = 0$ implemented + Adler zero $s_A = O(m_\pi^2)$ in l_{0++}.

Determination of Ω matrix

- Start with parametrization of T-matrix: [M.Albaladejo,B.M.,EPJ C75,488 (2015)]
  - → Unitary (2 channels: $\pi\eta$, $(K\overline{K})_{I=1}$),symmetric
  - → K-matrix approach

$$\boldsymbol{T}(s) = (1 - \boldsymbol{K}(s)\boldsymbol{\Phi}(s))^{-1}\boldsymbol{K}(s)$$

with chiral constraints $\boldsymbol{K} = \boldsymbol{K}_{(2)} + \boldsymbol{K}_{(4)} + \ldots$ [A.Dobado,J.R.Pelaez (1992),J.A.Oller,E.Oset (1997)]. Uses 6 parameters to be determined.

  - → $\boldsymbol{\Omega}(s)$ obtained by solving

$$\boldsymbol{\Omega}(s) = \frac{1}{\pi}\int_{(m_\pi+m_\eta)^2}^{\infty} ds' \frac{\boldsymbol{T}^*(s')\Sigma(s')\boldsymbol{\Omega}(s')}{s'(s'-s)}$$

- Experimental $\gamma\gamma$ data:

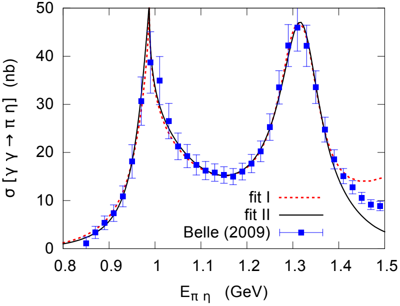
  - → $d\sigma_{\gamma\gamma\to\pi^0\eta}/dz$: 448 data points in range $[0.85 - 1.39]$ GeV
    [S. Uehara et al.(Belle), PR D80,032001 (2009)]

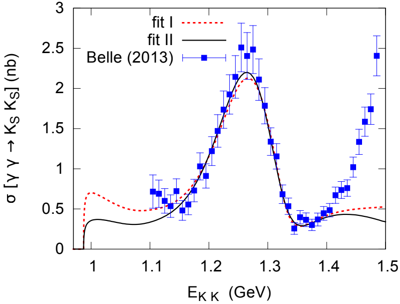
  - → $d\sigma_{\gamma\gamma\to K_S K_S}/dz$: 240 data points in range $[1.105 - 1.395]$ GeV
    [S. Uehara et al.(Belle), PTEP 12,123C01 (2013)]

  - → Also considered $\sigma_{\gamma\gamma\to K^+K^-}$: 7 data points
    [H. Albrecht et al. (ARGUS), Z.Phys. C48,183 (1990)]

    NOTE: must combine $\gamma\gamma \to (K\overline{K})_{I=1}$ with $\gamma\gamma \to (K\overline{K})_{I=0}$, taken from [R. García-Martín, B.M., EPJ C70,155 (2010)]

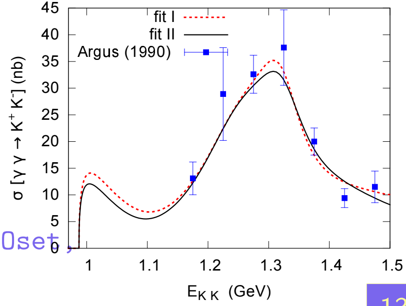
- Illustration of the fits:



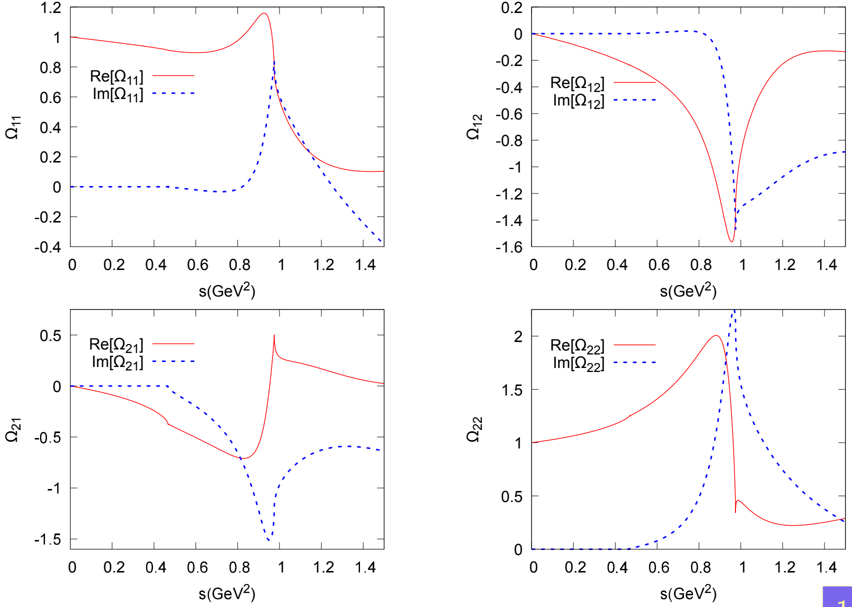


→ D-waves also included, $S-D$ interference probed (altogether $6+7$ params.)

→ Cross-section $\gamma\gamma \to K_S K_S$ very small, $I=0/I=1$ interference [J.A.Oller,E.Oset, NP A629,739 (1998)]



- Illustration of the resulting Ω_{ij} functions:



Further probe of Ω_{ij}: $\phi \to \gamma\pi^0\eta$

- Good data from ϕ factories[CMD-2 (1999),KLOE (2002), KLOE PL B681,5 (2009)]
- S-wave completely dominant
- Two-channel MO representation:

$$\begin{pmatrix} l_{0++}(s,q^2) \\ k_{0++}(s,q^2) \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_B \left(\frac{\beta(q^2)}{s-q^2} + \gamma(q^2) \right) \end{pmatrix} \leftarrow \text{sing. part of Born}$$

$$+(s-q^2) \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} a_1 + I_1^{LC}(s,q^2) + I_1^{RC}(s,q^2) \\ a_2 + I_2^{LC}(s,q^2) + I_2^{RC}(s,q^2) \end{pmatrix}$$

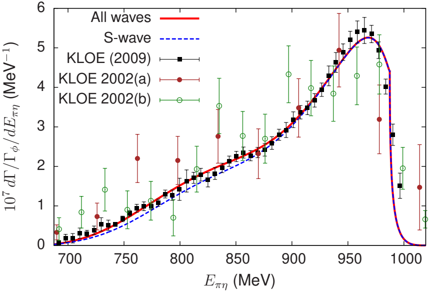
$q^2 = m_\phi^2$ (soft photon point $s = q^2$)

I_i^{LC} integrals over left-cut (now complex!)

I_i^{RC} integrals over Born piece

a_1, a_2: subtraction constants

- Rather good fit of data with two parameters [B.M.,arxiv:2107.14147]



→ Size of a_1, a_2 seems "reasonable" (amplitude has Adler zero $s_A = O(m_\pi^2)$)

→ Cusp at $E = 2m_{K^+}$ too sharp ? ($m_{K^0} \neq m_{K^+}$ not accounted for)

→ Confirms Ω_{ij} determination. Apply now to $f_0^{\eta\pi}$

DR estimate of $\eta\pi$ vector form factor

- Dispersion relation:

$$f_+^{\eta\pi}(s) = f_+^{\eta\pi}(0) + \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,[f_+^{\eta\pi}(s')]}{s'(s'-s)}$$

- Unitarity:

$$\mathrm{Im}\,[f_+^{\eta\pi}(s)] = \theta(s-4m_\pi^2)\times \frac{4m_\pi^2 - s}{\sqrt{\lambda_{\eta\pi}(s)}} F_V^\pi(s) \times t_{J=1}^{\pi^0\pi^+\to\eta\pi^+}(s) + \cdots$$

→ $F_V^\pi(s)$: pion form factor (well known from $\tau \to \pi\pi\nu$)

→ Elastic contrib. prop. $t_{J=1}^{\eta\pi\to\eta\pi}$ should be small ($J^{PC} = 1^{-+}$, but $\pi_1(1400)$ resonance ?)

- $t_{J=1}^{\pi^0\pi^+\to\eta\pi^+}(s)$:use solutions of Khuri-Treiman type equations[S.Descotes-Genon,B.M. (2014)]
- Fix four parameters using experimental $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot[KLOE, JHEP 0805,006 (2008)]
- Now updated using[KLOE, JHEP 05,019 (2016)]

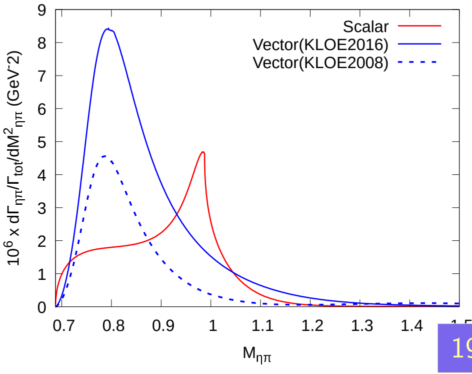
$\tau \to \eta\pi\nu$ decay width

- Energy distribution

$$\frac{d\Gamma_{\tau\to\eta\pi\nu}}{ds} = \frac{G_F^2 V_{ud}^2 S_{EW}}{384\pi^3} \frac{\sqrt{\lambda_{\eta\pi}(s)}\, m_\tau^3}{s^3} \left(1 - \frac{s}{m_\tau^2}\right) \left[|f_+^{\eta\pi}(s)|^2 \lambda_{\eta\pi}(s) \left(1 + \frac{2s}{m_\tau^2}\right) + 3 |f_0^{\eta\pi}(s)|^2 \Delta_{\eta\pi}^2 \right]$$

- Results:

$$BF_V \simeq 2.6 \times 10^{-6}$$
$$BF_S \simeq 1.5 \times 10^{-6}$$



Conclusions

- Using general FSI methods when unitarity involves finite number of two-body channels
- Derive relations between $\eta\pi$ scalar form factor and $\eta\pi$ production amplitudes via Ω matrix.

  In addition use inputs on $f_0^{\eta\pi}(0)$, $\frac{d}{ds}f_0^{\eta\pi}(0)$ from ChPT
- Result is somewhat smaller than early estimates. But approach limited to $E_{\eta\pi} \lesssim 1.2$ GeV
- Vector form factor: $\pi\pi \to \eta\pi$ P-wave evaluated using detailed input from $\eta \to 3\pi$ Dalitz plot.