Radiative corrections to $\tau^- \to P^- \nu_\tau$ and consequences for new physics

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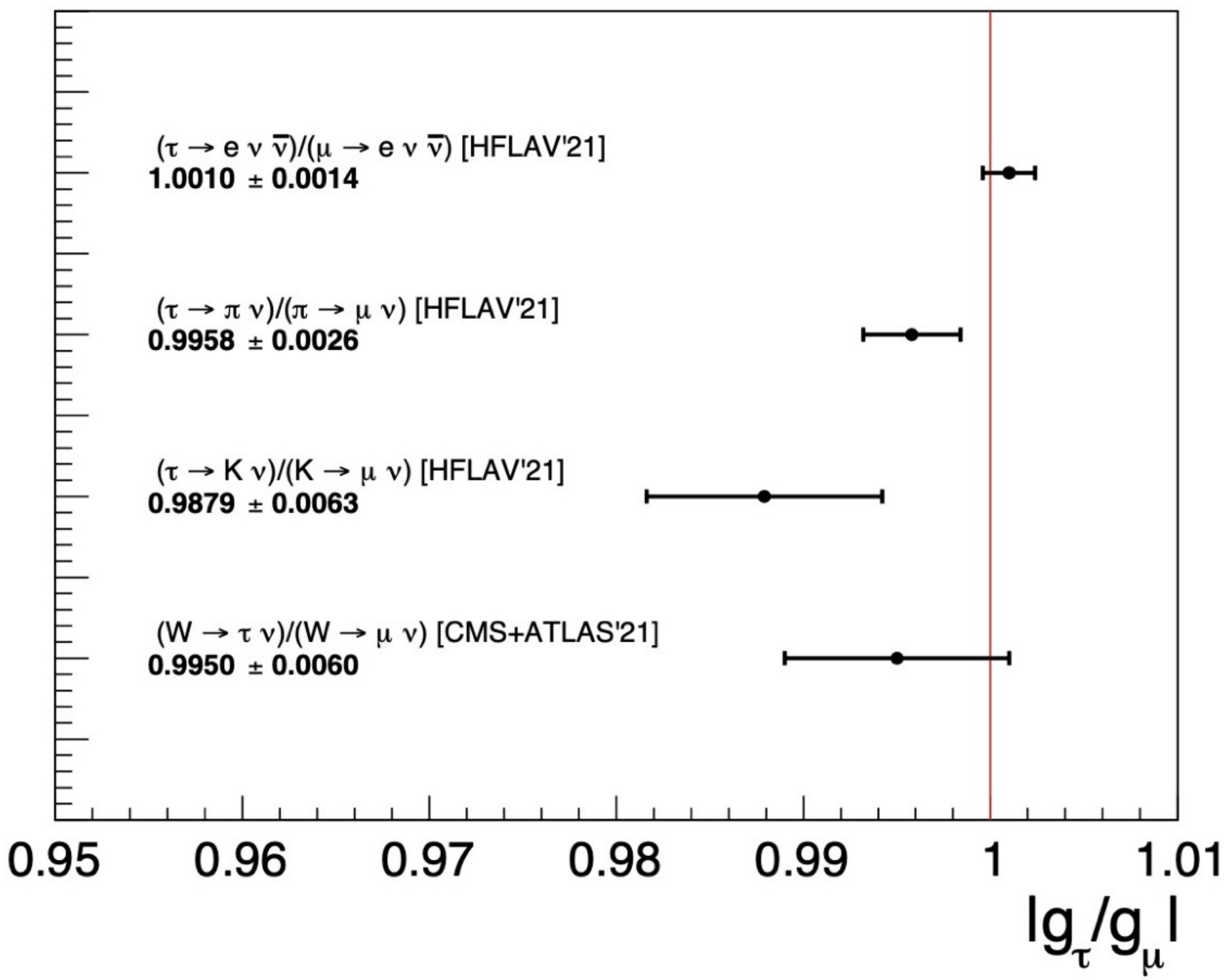
Motivation for $O(\alpha)$ radiative corrections in $\tau \to P\nu_\tau$

Talks by Marciano, Hertzog, Zupan, Martini, Celani,……

✱ Test of LFU in tau decays



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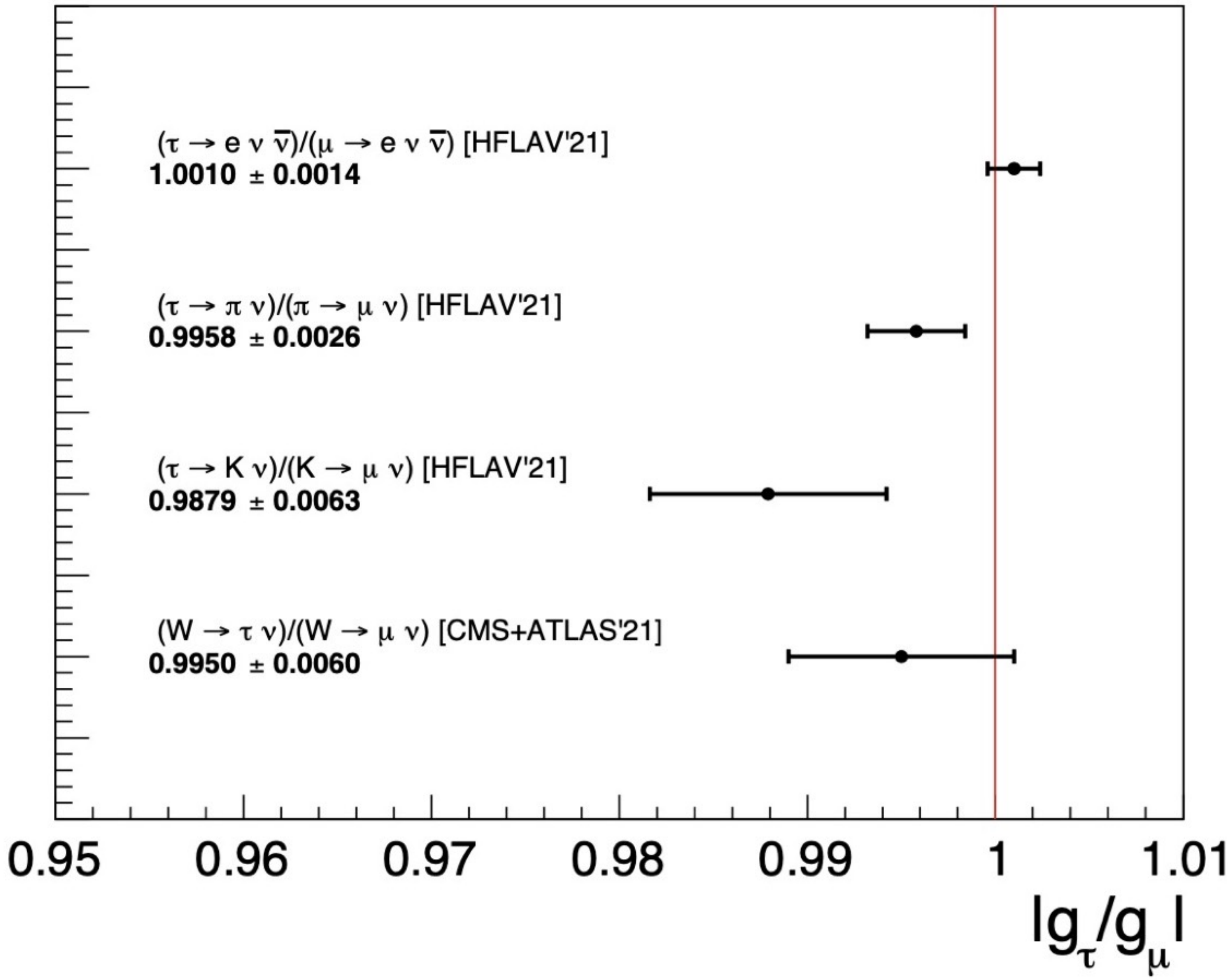
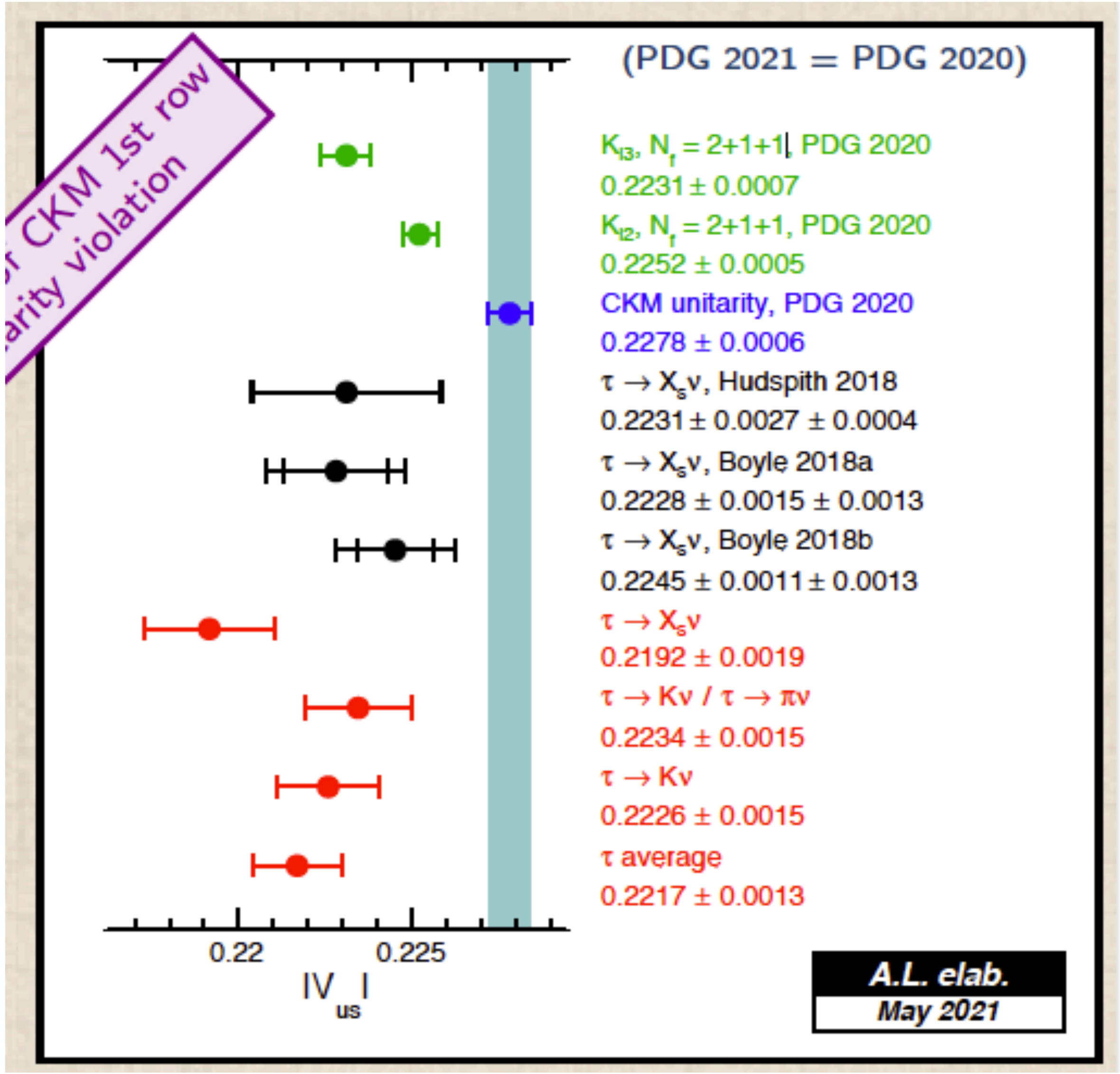


✱ "Anomalous" values of $|V_{us}|$



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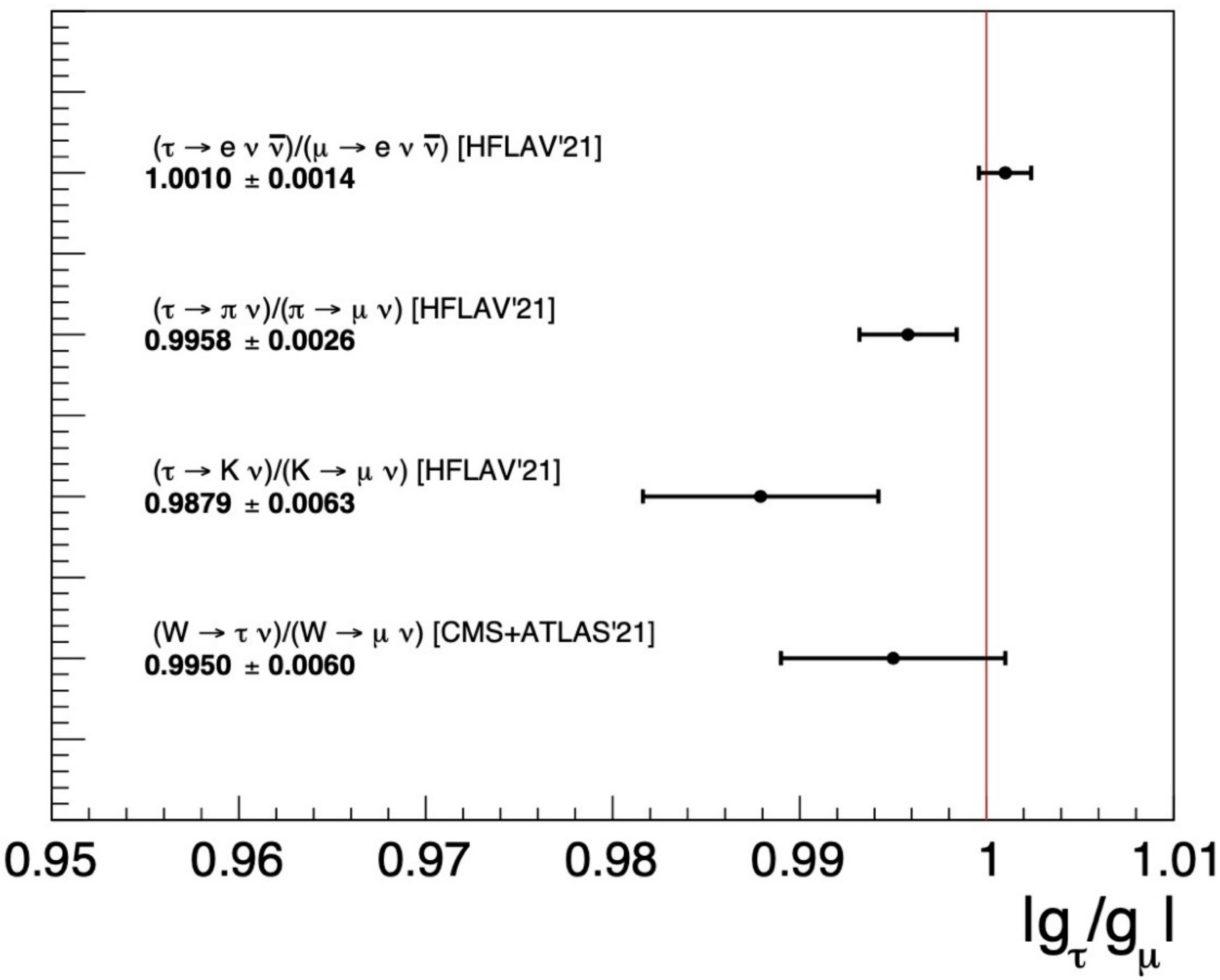

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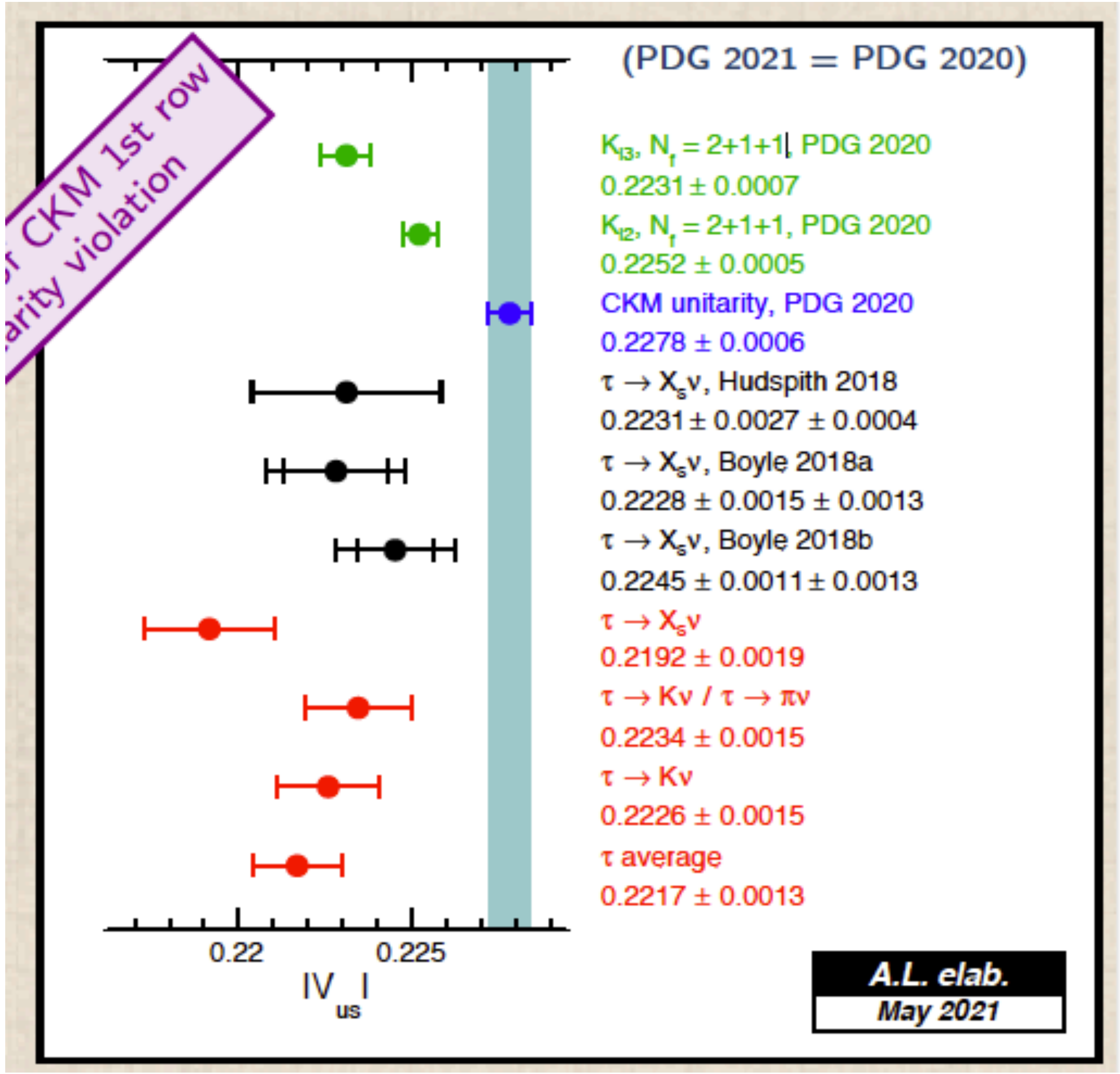


- Bounds on NSI in SMEFT, from exclusive semileptonic channels

Garcés et al 17; Cirigliano et al ’19; Gonzalez-Solís et al ‘20

Interest of precision in exclusive $\tau^- \to H^-\nu_\tau$ channels

$\tau \to H^-\nu$ H^-	Precision(BR) PDG 2020	Rad. Corr Calculation	Required for
π^-	0.5%	Yes †	LFU, NP
K^-	1.4%	Yes †	V_{us}, LFU, NP
$\pi^-\pi^0$	0.3%	Yes ⋆	ρ, ρ', ..., $(g-2)_\mu$, NP
$\overline{K^0}\pi^-$	1.7%	Yes •	K^*, $\not{C}P$, V_{us}, NP
$K^-\pi^0$	3.5%	Yes •	K^*, V_{us}, NP
$\pi^-\pi^+\pi^-$	0.5%	No	a_1
$\pi^-\pi^0\pi^0$	1.1%	No	a_1

† Decker+Finkemeier '95

⋆ Cirigliano et al'01, Flores-Tlalpa et al'06, Miranda+Roig '20

• Antonelli et al'13

Short-Distance corrections: Sirlin'78; Marciano-Sirlin'93

Main targets of radcorr in $\tau^- \to P^- \nu_\tau$

- LFU test:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right)$$

$$R^{(0)}_{\tau/P} = \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}$$

- $|V_{us}|$

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2 F_P^2}{8\pi} M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{ew}\left(1+\delta_{\tau P}\right)$$

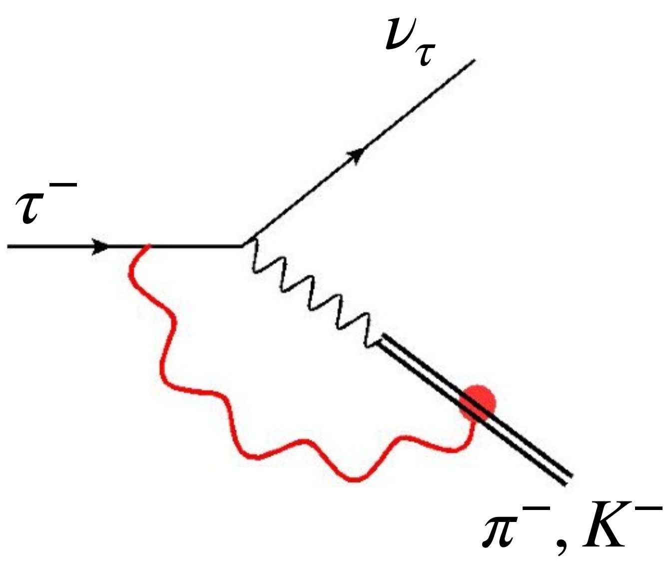
- $|V_{us}/V_{ud}|$

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

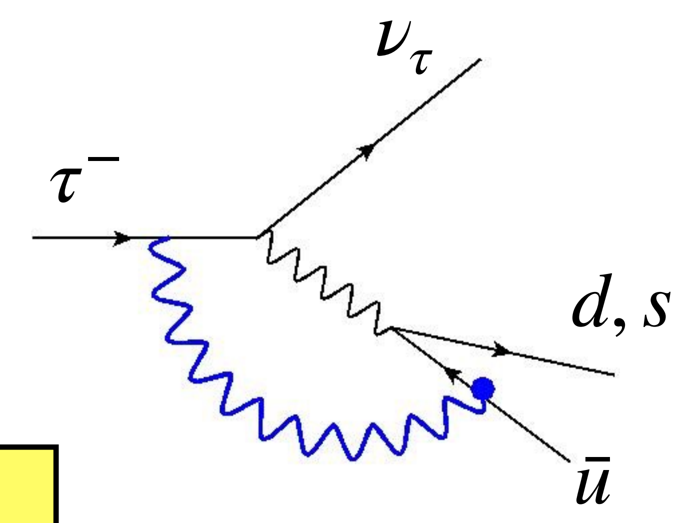
- Bounds on SMEFT parameters

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2|\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{ew}\left(1+\delta_{\tau P}+2\Delta^{\tau P}\right)$$

Difficulties with Radcorr in semileptonics ….



Λ_h



$k_\gamma \sim \Lambda_h$ non-perturbative region, resonances

- $k_\gamma \ll \Lambda_h$, long-distance (L-D)

- hadrons as DOF$\rightarrow$ Scalar QED

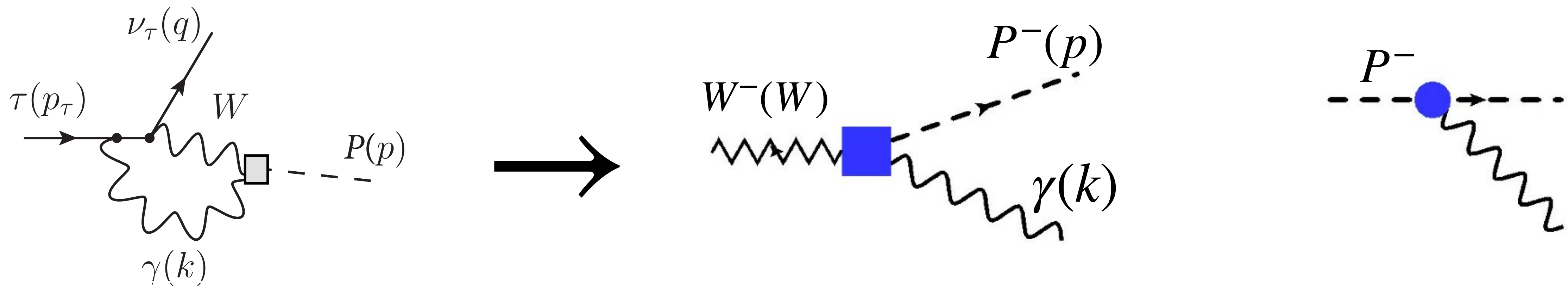
-$k_\gamma \gg \Lambda_h$,short-distance (S-D)

- quarks as DOF $\rightarrow$ SM

Necessary:

- A good model of γ-hadron interactions to interpolate L-D & S-D (VMD, RχTh, …)
- <u>Tree-level is purely axial hadronic current</u>. Axial and vector interactions get involved simultaneously in radcorr

γ-hadron interactions: structure-dependent FF + resonances



$$\sim -eg V_{uD}\Gamma^\nu \Big\{ iF_V^P(W^2,k^2)\epsilon_{\mu\nu\rho\sigma}k^\rho p^\sigma + F_A^P(W^2,k^2)\left[(W^2+k^2-m_P^2)g_{\mu\nu} - 2k_\mu p_\nu\right]$$
$$- A_2^P(k^2)k^2 g_{\mu\nu} + A_4^P(k^2)k^2(p+k)_\mu p_\nu \Big\}$$

Guevara et al '13
2107.04603

- Chiral constraints at low k^2;
- Good behavior at large k^2 (BL)
- M_V and M_A , large-N_C vector and axial resonance masses

$$F_V^P(W^2,k^2) = \frac{-N_C M_V^4}{24\pi^2 F_P (k^2 - M_V^2)(W^2 - M_V^2)}$$

$$F_A^P(W^2,k^2) = \frac{F_P}{2}\frac{M_A^2 - 2M_V^2 - k^2}{(M_V^2 - k^2)(M_A^2 - W^2)}$$

$$B(k^2) = \frac{F_P}{M_V^2 - k^2}$$

$$A_2^P(k^2) = A_4^P(k^2) * (W^2 - m_P^2) = -\frac{2F_P}{M_V^2 - k^2}$$

VMD approach: Decker and Finkemeier '93-'95

- Vector and axial FF in Vector Meson Dominance

$$F_V^\pi(t) = \frac{F_V^\pi(0)}{1+\sigma+\rho}\left[BW_\rho(t) + \sigma BW_{\rho'}(t) + \rho BW_{\rho''}(t)\right],$$

$$F_A^\pi(t) = F_A^\pi(0) BW_{a_1}(t)$$

- Corrections dependent on matching scale $\mu_{\rm cut}$ to separate S-D and L-D corrections

$$\frac{1}{k^2-\lambda^2} \to \underbrace{\frac{1}{k^2-\lambda^2}\cdot\frac{\mu_{\rm cut}^2}{\mu_{\rm cut}^2-k^2}}_{\text{“long−distance”}} + \underbrace{\frac{1}{k^2-\mu_{\rm cut}^2}}_{\text{“short−distance”}}$$

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- QCD constrains implemented for $F_V(t)$.
  Different FF for real and virtual corrections

- Uncertainties: hadronic parameters (FF), $\mu_{\rm cut} = 1 \sim 2$ GeV,

- **Results**: $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\,\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\,\%$

General form of corrected rate at $O(\alpha)$

$$\Gamma = \Gamma^0 + \Gamma^{\rm virtual}_{\rm EW} + \Gamma^{\rm virtual}_{\rm SI+SD} + \Gamma^{\rm real}_{\rm SI+SD}$$
$$= \Gamma^0 \left(1 + \delta_{\rm EW} + \delta_{\rm SI} + \delta_{\rm SD}\right)$$
$$= \Gamma^0 (1 + \delta_{\rm EW})(1 + \delta_{\rm SI})(1 + \delta_{\rm SD})$$

- $\delta_{\rm SI}$: scalar QED in virtual and real photon IB corrections
- $\delta_{\rm SD}$: structure-dependent (FF) virtual and real photon SD corrections
- $\delta_{\rm EW}$: short-distante electroweak corrections $\left[= \frac{2\alpha}{\pi} \ln\left(m_Z/m_\rho\right)\right]$ Sirlin'78 Marciano+ Sirlin '93
- Γ^0: leading order decay

  rate ($D = d, s;\ P = \pi, K$)

$$\Gamma^0(P \to \mu\nu) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{4\pi} m_P m_\mu^2 \left(1 - \frac{m_\mu^2}{m_P^2}\right)^2$$

$$\Gamma^0(\tau \to P\nu) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{8\pi} m_\tau^2 \left(1 - \frac{m_P^2}{m_\tau^2}\right)^2$$

1. Structure-Independent (SI) virtual+real photon corrections (IR-finite)

$$\delta_{SI}\big|_{\tau P} = \frac{\alpha}{\pi}\left[h_-\left(\frac{m_P^2}{m_\tau^2}\right) + \frac{3}{4} - \frac{m_P^2(2m_\tau^2 - 5m_P^2)}{4(m_\tau^2 - m_P^2)^2}\log\left(\frac{m_P^2}{m_\tau^2}\right)\right]$$

$$\delta_{SI}\big|_{P\mu} = \frac{\alpha}{\pi}\left[h_+\left(\frac{m_\mu^2}{m_P^2}\right) + \frac{3}{2}\log\left(\frac{m_\mu^2}{m_P^2}\right) - \frac{m_\mu^2(8m_P^2 - 5m_\mu^2)}{4(m_P^2 - m_\mu^2)^2}\log\left(\frac{m_\mu^2}{m_P^2}\right)\right]$$

$$h_\pm(x) = \left(\frac{1+x}{1-x}\log(1-x) - 2\right) + 2\left(\frac{1+x}{1-x}\right)\mathrm{Li}_2(\mathrm{x}) \mp \frac{x}{2(1-x)}\left(\frac{3}{2} \pm \frac{4\pi^2}{3}\right) + \frac{13}{8} - \frac{\pi^2}{3}$$

$$\delta R_{\tau/P}\big|_{SI} = \delta_{SI}\big|_{\tau P} - \delta_{SI}\big|_{P\mu} = \begin{cases} 1.05\ \%\ \text{for}\ P = \pi \\ 1.67\ \%\ \text{for}\ P = K \end{cases}$$

Agreement with analytical results of Decker & Finkemeier '95

2. Structure-Dependent (SD) radiative corrections

a) Real photon SD corrections: $|\mathrm{IB}+\mathrm{SD}|^2 = |\mathrm{IB}|^2 + |\mathrm{SD}|^2 + \text{interference}$

$$\delta_{\rm rSD}\big|_{P\mu} = \begin{cases} -1.3\times 10^{-8} \text{ for } \pi_{\mu 2}, \ \dagger \\ -1.7\times 10^{-5} \text{ for } K_{\mu 2}, \ \dagger \end{cases} \qquad \delta_{\rm rSD}\big|_{\tau P} = \begin{cases} +0.15\%, \quad \text{for } \tau_{\pi 2}, \ \star \\ +0.18(5)\%, \ \text{for } \tau_{K 2}, \ \star \end{cases}$$

$$\delta R_{\tau/P}\big|_{\rm rSD} = \delta_{\rm rSD}\big|_{\tau P} - \delta_{\rm rSD}\big|_{P\mu} = \begin{cases} 0.15\ \% \text{ for } P=\pi \\ 0.18(5)\ \% \text{ for } P=K \end{cases}$$

† Cirigliano+Rosell '07, Cirigliano-Neufeld'11; ★ Guo+Roig '10; • 2107.04603

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b) Virtual Photon SD corrections: form factors in loops **2107.04603**

$$\delta_{\mathrm{vSD}}\big|_{P\mu} = \begin{cases} +0.54(12)\ \% \text{ for } \pi_{\mu 2}, \ \dagger \\ +0.43(12)\ \% \text{ for } K_{\mu 2}, \ \dagger \end{cases} \qquad \delta_{\mathrm{vSD}}\big|_{\tau P} = \begin{cases} -0.48(56)\%, \quad \text{for } \tau_{\pi 2}, \ \bullet \\ -0.45(57)\%, \ \text{for } \tau_{K2}, \ \bullet \end{cases}$$

$$\delta R_{\tau/P}\big|_{\mathrm{vSD}} = \delta_{\mathrm{vSD}}\big|_{\tau P} - \delta_{\mathrm{vSD}}\big|_{P\mu} = \begin{cases} -1.02(57)\ \% \text{ for } P = \pi \\ -0.88(58)\ \% \text{ for } P = K \end{cases}$$

† **Cirigliano+Rosell '07, Cirigliano-Neufeld'11**; ★ **Guo+Roig '10**; ● **2107.04603**

Summary of results

Contribution	$\delta R_{\tau/\pi}(\%)$	$\delta R_{\tau/K}(\%)$	Ref
SI	$+1.05$	$+1.67$	D-F'95
rSD	$+0.15$	$+0.18(5)$	C-N'07, G-R'10
vSD	$-1.02(57)$	$-0.88(58)$	2107.04603
Total	$+0.18(57)$	$+0.97(58)$	2107.04603
	$[+0.16(14)]$	$[+0.90(22)]$	DF'95

- P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas τ decays within resonance effective approach [no matching to determine the counterterms].

- Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: ±0.22% and ±0.24% for the pion and the kaon case.

- Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: ±0.52%.

Consequences for tests of the SM and NP

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$

$$\Downarrow$$

1. LFU: input data PDG

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\text{th}} \pm 0.0025_{\text{exp}} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\text{th}} \pm 0.0072_{\text{exp}} = 0.9857 \pm 0.0078$$

Reduced 0.9σ (π) and 1.8σ (K)
from LU wrt HFLAV21
due to larger uncertainties

Consequences for tests of the SM and NP

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$$\Downarrow$$

1. LFU: input data PDG

Reduced 0.9σ (π) and 1.8σ (K) from LU wrt HFLAV21 due to larger uncertainties

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

Corrections to the decay rate

Short-distance EW corr.

Marciano- Sirlin '93

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{ew}\,(1+\delta_{\tau P})$$

$$\delta_{\tau P} = \frac{\alpha}{2\pi}\left(g\left(\frac{m_P^2}{M_\tau^2}\right)+\frac{19}{4}-\frac{2\pi^2}{3}-3\log\frac{m_\rho}{M_\tau}\right)+\delta_{\tau P}\big|_{\rm rSD}+\delta_{\tau P}\big|_{\rm vSD} = \begin{cases}\delta_{\tau\pi} = (-0.24\pm0.56)\% \\ \delta_{\tau K} = (-0.15\pm0.57)\%\end{cases}$$

2. Cabbibo angle anomaly

$$\frac{\Gamma(\tau\to K\nu_\tau[\gamma])}{\Gamma(\tau\to\pi\nu_\tau[\gamma])}=\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right|=0.2288\pm0.0010_{\rm th}\pm0.0017_{\rm exp}=0.2288\pm0.0020$$

2. Cabbibo angle anomaly

FLAG'20*:
$F_K/F_\pi = 1.1932 \pm 0.0019$

$$\delta = \frac{\alpha}{2\pi}\left\{g\left(\frac{m_K^2}{M_\tau^2}\right) - g\left(\frac{m_\pi^2}{M_\tau^2}\right)\right\} + \delta_{\tau K}\big|_{\rm rSD} - \delta_{\tau\pi}\big|_{\rm rSD} + \delta_{\tau K}\big|_{\rm vSD} - \delta_{\tau\pi}\big|_{\rm vSD} = +(0.10 \pm 0.80)\%$$

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

Compared to $|V_{us}/V_{ud}| = 0.2388 \pm 0.0014$ **from unitarity:** 2.1σ

3. Cabibbo angle from the decay rate

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi} |V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{ew} \; (1 + \delta_{\tau K})$$

3. Cabibbo angle from the decay rate

FLAG'20*:
$\sqrt{2}F_K$ = (155.7 ± 0.3) MeV

short-distance
EW correction
Sew=1.0232**

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{ew}\ (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\mathrm{th}} \pm 0.0016_{\mathrm{exp}} = 0.2220 \pm 0.0018$$

2.6σ away from CKM unitarity, considering $|V_{ud}|$=0.97373±0.00031

3. Bounds on SMEFT parameters

$$\mathcal{L}_{CC} = -\frac{G_F V_{uD}}{\sqrt{2}}(1+\epsilon_L+\epsilon_R)\Big[\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu\left[1-(1-2\hat{\epsilon}_R)\gamma_5\right]D$$

$$+\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}[\hat{\epsilon}_S-\hat{\epsilon}_P\gamma_5]D + 2\hat{\epsilon}_T\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}D\Big]$$

$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|\widetilde{V}_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{ew}\left(1+\delta_{\tau P}+2\Delta^{\tau P}\right)$$

$$\Delta^{\tau P} = \epsilon_L^\tau-\epsilon_L^e-\epsilon_R^\tau-\epsilon_R^e-\frac{m_P^2}{M_\tau(m_u+m_D)}\epsilon_P^\tau = \begin{cases}\Delta^{\tau\pi} = -(0.15\pm0.72)\cdot10^{-2}\\ \Delta^{\tau K} = -(0.36\pm1.18)\cdot10^{-2}\end{cases}$$

To be compared

$$\Delta^{\tau P} = \begin{cases} -0.15(67)\times10^{-2}\ \dagger,\ -0.12(68)\times10^{-2}\ \bullet;\ \text{for } P=\pi\\ -0.41(93)\times10^{-2}\ \bullet;\ \ \text{for } P=K\end{cases}$$

† **Cirigliano et al'19**; • **González Solís et al '20**

Caveat: Double ratio test radcorr, free of CKM, FF's, LFU and other constants

$$\frac{\frac{\Gamma[\tau\to K\nu(\gamma)]}{\Gamma[\tau\to\pi\nu(\gamma)]}}{\frac{\Gamma[K\to\mu\nu(\gamma)]}{\Gamma(\pi\to\mu\nu(\gamma))}} = \left(\frac{m_K}{m_\pi}\right)^3 \cdot \left(\frac{m_\tau^2 - m_K^2}{m_\tau^2 - m_\pi^2}\right)^2 \cdot \left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 (1 + \delta_{\tau K\pi} - \delta_{\mu K\pi})$$

- $\delta_{\mu K\pi} = \delta_{\rm EM} = -\,(0.69 \pm 0.17)\,\%$ from **Cirigliano-Neufeld '11**
- Branching fractions from **PDG 2020** ⇒ $\delta_{\tau K\pi} = -\,(2.1 \pm 1.51)\,\%$

Compared to $\delta_{\tau K\pi} = +\,(0.10 \pm 0.80)\,\%$ from theory

Caveat: Double ratio test radcorr, free of CKM, FF's, LFU and other constants

$$\frac{\frac{\Gamma[\tau\to K\nu(\gamma)]}{\Gamma[\tau\to\pi\nu(\gamma)]}}{\frac{\Gamma[K\to\mu\nu(\gamma)]}{\Gamma(\pi\to\mu\nu(\gamma))}} = \left(\frac{m_K}{m_\pi}\right)^3 \cdot \left(\frac{m_\tau^2 - m_K^2}{m_\tau^2 - m_\pi^2}\right)^2 \cdot \left(\frac{m_\pi^2 - m_\mu^2}{m_K^2 - m_\mu^2}\right)^2 (1 + \delta_{\tau K\pi} - \delta_{\mu K\pi})$$

$$= \delta R_{\tau/K} - \delta R_{\tau/\pi}$$

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Compared to $\delta_{\tau K\pi} = +\,(0.10 \pm 0.80)\,\%$ from theory

- Also: $\delta R_{\tau/K} - \delta R_{\tau/\pi} = \begin{cases} (-1.37 \pm 1.35)\%, & \text{from data} \\ (+0.79 \pm 0.58)\%, & \text{from theory} \end{cases}$

Compatible w/errors but 2.2% difference (exp or th ?)

Final comments

- New calculation of L-D radcorrs to $\tau^- \to P^- \nu_\tau(\gamma)$. RChT framework

- Larger errors dominated by running scale, but Vector and Axial Vector Form Factors consistent with QCD.

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref
Total	$+0.18(57)\%$	$+0.97(58)\%$	2107.04603
	$[+0.16(14)\%]$	$[+0.90(22)\%]$	DF'95

Important contributions of SD corrrections (large cancellation with SI corrections)

- Minimal impact on the determination of $|V_{us}|$. Consistent with other determinations, but difficult to reconcile with CKM unitarity

Backup slides

2. $P \to \mu\, \nu_\mu\, [\gamma]$ ($P=\pi,K$)

✓ Calculated unambigously within the Standard Model (Chiral Perturbation Theory, ChPT*).

✓ Notation by Marciano & Sirlin** and numbers by Cirigliano & IR*** (D=d,s for π,K):

LO result — short-distance EW correction — structure independent (SI) contributions (point-like approximation)^

$$\Gamma(P\to\mu\nu_\mu[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{4\pi}\, m_P\, m_\mu^2 \left(1-\frac{m_\mu^2}{m_P^2}\right)^2 \left\{1+\frac{2\,\alpha}{\pi}\log\frac{m_Z}{m_\rho}\right\}\left\{1+\frac{\alpha}{\pi}F(m_\mu^2/m_P^2)\right\}\times$$

$$\left\{1-\frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P}+c_1^{(P)}+\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}+c_3^{(P)}+c_4^{(P)}(m_\mu/m_P)\right)-\frac{m_P^2}{m_\rho^2}\,\tilde{c}_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}\right]\right\}$$

structure-dependent (SD) contributions [coefficents reported in Cirigliano & IR'07]

✓ The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:

✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

* Weinberg'79
* Gasser & Leutwyler'84 '85
** Marciano & Sirlin'93
*** Cirigliano & IR'07
^ Kinoshita'59
† Ecker et al.'89
† Cirigliano et al.'06

3. $\tau \to P\,\nu_\tau\,[\gamma]$ (P=π,K)

- ✓ Calculated within an effective approach encoding the hadronization:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies*.
- ✓ We follow a similar notation to $P\to\mu\nu_\mu[\gamma]$ (D=d,s for π,K):

LO result — short-distance EW correction — structure independent (SI) contributions (point-like approximation)**

$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2\left\{1+\frac{2\,\alpha}{\pi}\log\frac{m_Z}{m_\rho}\right\}\left\{1+\frac{\alpha}{\pi}G(m_P^2/M_\tau^2)\right\}\times$$
$$\left\{1-\frac{3\alpha}{2\pi}\log\frac{m_\rho}{M_\tau}+\delta_{\tau P}\big|_{\rm rSD}+\delta_{\tau P}\big|_{\rm vSD}\right\}$$

real-photon structure-dependent (rSD) contributions — virtual-photon structure-dependent (vSD) contributions

- ✓ Real-photon structure-dependent (rSD) contributions from Guo & Roig'10***.
- ✓ Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

* Ecker et al.'89
* Cirigliano et al.'06
** Kinoshita'59
*** Guo & Roig'10

Amplitude for $\tau^- \to \pi^- \ell^+ \ell^- \nu_\tau$

Roig, Guevara, GLC '13

$$\mathcal{M}_{IB} = -iG_F V_{ud}\frac{e^2}{k^2}F_\pi M_\tau \bar{u}(p_-)\gamma_\mu v(p_+)\bar{u}(q)(1+\gamma_5)\left[\frac{2p^\mu}{2p\cdot k + k^2} + \frac{2p_\tau^\mu - \not{k}\gamma^\mu}{-2p_\tau \cdot k + k^2}\right]u(p_\tau),$$

$$\mathcal{M}_V = -G_F V_{ud}\frac{e^2}{k^2}\bar{u}(p_-)\gamma^\nu v(p_+)F_V(p\cdot k, k^2)\epsilon_{\mu\nu\rho\sigma}k^\rho p^\sigma \bar{u}(q)\gamma^\mu(1-\gamma_5)u(p_\tau),$$

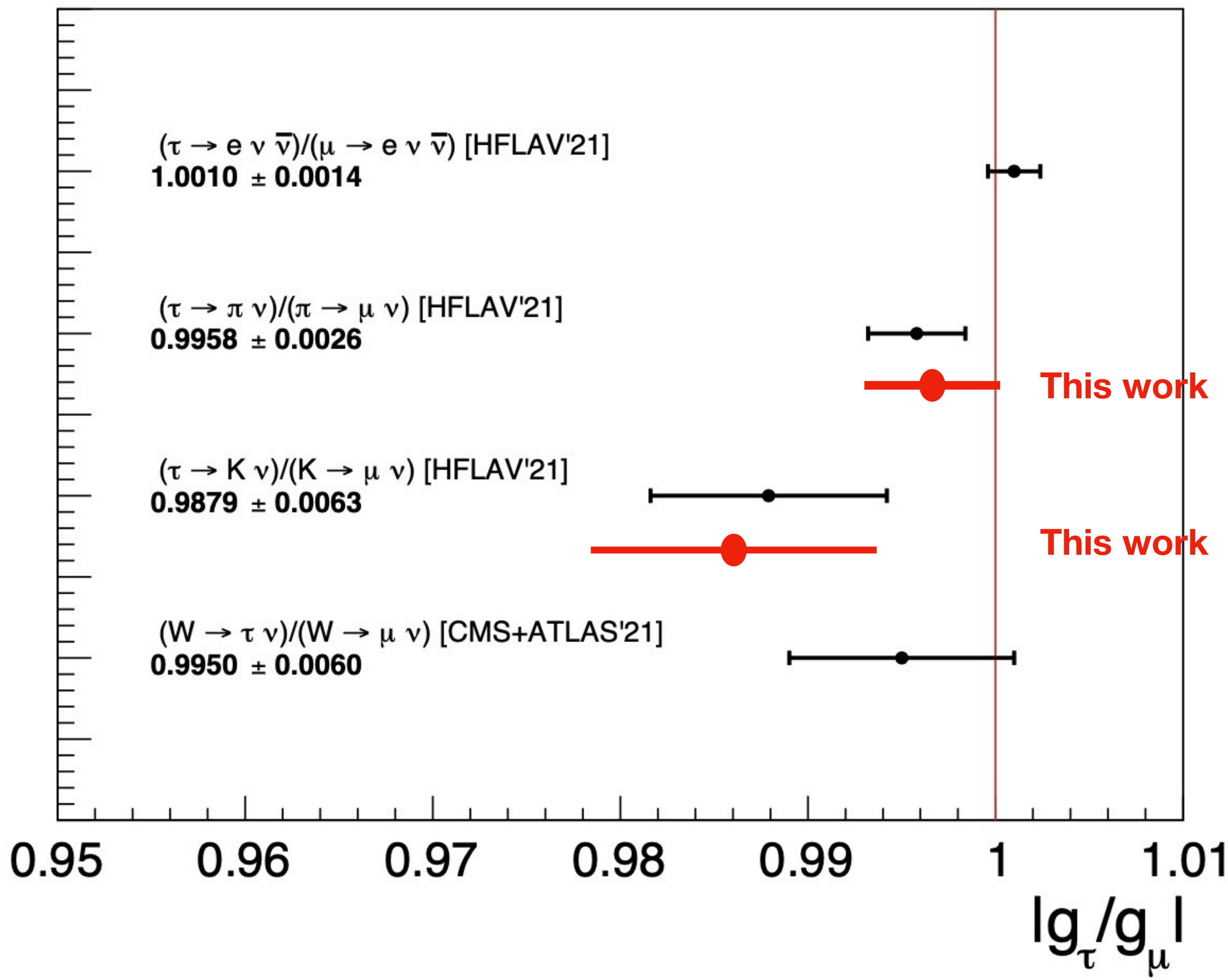
$$\mathcal{M}_A = iG_F V_{ud}\frac{2e^2}{k^2}\bar{u}(p_-)\gamma_\nu v(p_+)\Big\{F_A(p\cdot k, k^2)[(k^2 + p\cdot k)g^{\mu\nu} - k^\mu p^\nu] - \frac{1}{2}A_2(k^2)k^2 g^{\mu\nu} + \frac{1}{2}A_4(k^2)k^2(p+k)^\mu p^\nu\Big\}\bar{u}(q)\gamma_\mu(1-\gamma_5)u(p_\tau).$$

FIG. 2. Vector current contributions to the $W^{-*} \to \pi^- \gamma^*$ vertex.

FIG. 3. Axial-vector current contributions to the $W^{-*} \to \pi^- \gamma^*$ vertex.

Prediction in RChT in Good agreement with $\tau \to \pi e^+ e^- \nu$ observed by Belle in 2019

2107.04603


(τ → e ν ν̄)/(μ → e ν ν̄) [HFLAV'21]
1.0010 ± 0.0014
(τ → π ν)/(π → μ ν) [HFLAV'21]
0.9958 ± 0.0026
This work
(τ → K ν)/(K → μ ν) [HFLAV'21]
0.9879 ± 0.0063
This work
(W → τ ν)/(W → μ ν) [CMS+ATLAS'21]
0.9950 ± 0.0060
0.95
0.96
0.97
0.98
0.99
1
1.01
|g_τ/g_μ|

First row unitarity of CKM matrix:

$\lvert V_{ud}\rvert$ (SFT)	$\lvert V_{us}\rvert$ (K_{e3})	$\sum \lvert V_{uj}\rvert^2$	pull	PDG edition
0.9740 ± 0.0010	0.2196 ± 0.0023	0.9969 ± 0.0022	1.4σ	2000
0.9740 ± 0.0005	0.2200 ± 0.0026	0.9971 ± 0.0015	1.9σ	2004
0.97377 ± 0.00027	0.2257 ± 0.0021	0.9992 ± 0.0011	0.7σ	2006 •

In 2003-2006, measurements by BNL-865, KTeV, KLOE led to larger branching fractions of semileptonic Kaon decays