Flavor violating ℓ_i decay into a ℓ_j and a light gauge boson, 'χ'

Alejandro Ibarra[1], Marcela Marín[2], Pablo Roig[2]

[1]Technische Universität München

[2]Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional

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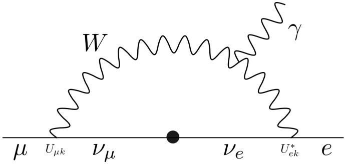

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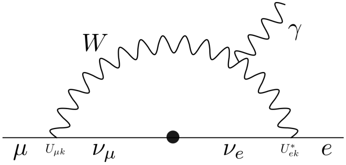

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⇒SM Predictions:

$\mathcal{B}r(Z \to \ell\ell') \sim 10^{-54}$ J. I. Illana & T. Riemann, '01
$\mathcal{B}r(H \to \ell\ell') \sim 10^{-55}$ E. Arganda, A. M. Curiel, M. J. Herrero & D. Temes, '05
$\mathcal{B}r(\mu \to 3e) \sim 10^{-54}$, $\mathcal{B}r(\tau \to 3\ell) \sim 10^{-55}$ Hernández-Tomé, López-Castro & Roig, '19
Blackstone, Fael & Passemar, '20

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In this work we will focus on light gauge bosons (χ), associated to the spontaneous breaking of an Abelian gauge symmetry, $U(1)_\chi$. We will show that in a renormalizable and gauge invariant theory the rate does not diverge when $m_\chi \to 0$.

Outline

Effective Theory $\ell_i \to \ell_j \chi$

The transition amplitude is given by $M = \bar{u}(p_j)\Gamma^\alpha(p_i, p_j)u(p_i)\epsilon^*_\alpha(p_\chi)$

$$\Gamma^\alpha = \left(\gamma^\alpha - \frac{\not{p}_\chi p^\alpha_\chi}{p^2_\chi}\right) F_1(p^2_\chi) + i\frac{\sigma^{\alpha\beta}p_{\chi\beta}}{m_i+m_j}F_2(p^2_\chi) + \frac{2p^\alpha_\chi}{m_i+m_j}F_3(p^2_\chi) + \left(\gamma^\alpha - \frac{\not{p}_\chi p^\alpha_\chi}{p^2_\chi}\right)\gamma^5 G_1(p^2_\chi) + i\frac{\sigma^{\alpha\beta}\gamma^5 p_{\chi\beta}}{m_i+m_j}G_2(p^2_\chi) + \frac{2p^\alpha_\chi}{m_i+m_j}\gamma^5 G_3(p^2_\chi)$$

$F_i(p^2_\chi)$ and $G_i(p^2_\chi)$ dimensionless scalar form factors

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$$\left(\gamma^{\alpha} - \frac{\not{p}_\chi p_\chi^{\alpha}}{p_\chi^2}\right)\gamma^5 G_1(p_\chi^2) + i\frac{\sigma^{\alpha\beta}\gamma^5 p_{\chi\beta}}{m_i+m_j}G_2(p_\chi^2) + \frac{2p_\chi^{\alpha}}{m_i+m_j}\gamma^5 G_3(p_\chi^2)$$

The Ward identities imply that $p_\chi^{\alpha} \cdot \epsilon^*_{\alpha}(p_\chi) = 0$

Effective Theory $\ell_i \to \ell_j \chi$

The decay rate can be expressed in terms of four form factors

$$\Gamma(\ell_i \to \ell_j \chi) = \frac{\lambda^{1/2}[m_i^2, m_j^2, m_\chi^2]}{16\pi m_i} \Bigg[\left(1 - \frac{m_j}{m_i}\right)^2 \left(1 - \frac{m_\chi^2}{(m_i - m_j)^2}\right) \Bigg(2\left|F_1(m_\chi^2) - F_2(m_\chi^2)\right|^2$$
$$+ \left|F_1(m_\chi^2)\frac{(m_i + m_j)}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2 \Bigg) + \left(1 + \frac{m_j}{m_i}\right)^2 \left(1 - \frac{m_\chi^2}{(m_i + m_j)^2}\right)$$
$$\Bigg(2\left|G_1(m_\chi^2) - G_2(m_\chi^2)\frac{(m_i - m_j)}{(m_i + m_j)}\right|^2 + \left|G_1(m_\chi^2)\frac{(m_i - m_j)}{m_\chi} + G_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2 \Bigg) \Bigg]$$

$\lambda[m_j^2, m_i^2, m_\chi^2]$ is the usual Källén function

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$$\left. + \left|F_1(m_\chi^2)\frac{(m_i + m_j)}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2\right) + \left(1 + \frac{m_j}{m_i}\right)^2 \left(1 - \frac{m_\chi^2}{(m_i + m_j)^2}\right)$$
$$\left.\left(2\left|G_1(m_\chi^2) - G_2(m_\chi^2)\frac{(m_i - m_j)}{(m_i + m_j)}\right|^2 + \left|G_1(m_\chi^2)\frac{(m_i - m_j)}{m_\chi} + G_2(m_\chi^2)\frac{m_\chi}{(m_i + m_j)}\right|^2\right)\right]$$

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$\Downarrow$

In an effective field theory approach, great care should be taken when considering decays into ultralight gauge bosons, since in a gauge invariant and renormalizable theory one generically expects the rate of $\ell_i \to \ell_j \chi$ to be finite

Outline

Tree Level Model

The particle content and the corresponding spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ are

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}
spin	$1/2$	$1/2$	$1/2$	$1/2$	0	0	0	0
$SU(2)_L$	2	2	1	1	2	2	2	2
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	Y_{11}	Y_{11}	Y_{21}	Y_{21}
$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

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$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

$L_i = (\nu_{L_i}, e_{L_i})$ and e_{R_i}, $i = 1, 2$, denote the Standard Model $SU(2)_L$ lepton doublets and singlets, respectively.

$\Downarrow$

We have restricted ourselves to the two generation case, although the extension to three generations is straightforward

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$U(1)_\chi$	q_{L_1}	q_{L_2}	q_{e_1}	q_{e_2}	$q_{\phi_{11}}$	$q_{\phi_{12}}$	$q_{\phi_{21}}$	$q_{\phi_{22}}$

ϕ_{jk} complex scalar fields and doublets under $SU(2)_L$. We assume that the hypercharge $Y_{jk} = 1/2$ and charge under $U(1)_\chi$ $q_{\phi_{jk}} = q_{L_j} - q_{e_k}$.

$$\Downarrow$$

We also assume that ϕ_{jk} acquire a vacuum expectation value $\Rightarrow \langle \phi_{jk} \rangle = v_{jk}$

Tree Level Model

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The kinetic term is:

$$\mathcal{L}_{\text{kin}} = \sum_{j=1}^{2} (\bar{i} L_j \not{D} L_j + i\bar{e}_{R_j} \not{D} e_{R_j}) + \sum_{j,k=1}^{2} (D_\mu \phi_{jk})^\dagger (D^\mu \phi_{jk}) ,$$

where D_μ denotes the covariant derivative

$$D_\mu = \partial_\mu + igW_\mu^a T_a + ig'YB_\mu + ig_\chi q\chi_\mu \text{ for the } SU(2)_L \text{ doublets} ,$$
$$D_\mu = \partial_\mu + ig'YB_\mu + ig_\chi q\chi_\mu \text{ for the } SU(2)_L \text{ singlets} ,$$

with g, g' and g_χ the coupling constants of $SU(2)_L$, $U(1)_Y$ and $U(1)_\chi$ respectively.

Tree Level Model

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The Yukawa interaction term is:

$$-\mathcal{L}_{\text{Yuk}} = \sum_{j,k=1}^{2} y_{jk} \overline{L}_j \phi_{jk} e_{R_k} + \text{h.c.}$$

Tree Level Model

The non-zero $\langle \phi_{jk} \rangle = v_{jk}$ generate a mass for the χ boson:

$$m_\chi^2 = g_\chi^2 (q_{\phi_{11}}^2 v_{11}^2 + q_{\phi_{12}}^2 v_{12}^2 + q_{\phi_{21}}^2 v_{21}^2 + q_{\phi_{22}}^2 v_{22}^2) \,.$$

$\langle \phi_{jk} \rangle = v_{jk}$ generates a mass term for the charged leptons. In the mass eigenstate basis

$$\begin{aligned}
m_\mu^2 &\simeq y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2 \,, \\
m_e^2 &\simeq \frac{(y_{11} v_{11} y_{22} v_{22} - y_{12} v_{12} y_{21} v_{21})^2}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \,, \\
\sin 2\theta_L &\simeq -2 \frac{(y_{11} v_{11} y_{21} v_{21} + y_{12} v_{12} y_{22} v_{22})}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \,, \\
\sin 2\theta_R &\simeq -2 \frac{(y_{11} v_{11} y_{12} v_{12} + y_{21} v_{21} y_{22} v_{22})}{y_{11}^2 v_{11}^2 + y_{12}^2 v_{12}^2 + y_{21}^2 v_{21}^2 + y_{22}^2 v_{22}^2} \,.
\end{aligned}$$

Tree Level Model

We recast the kinetic Lagrangian in terms of the mass eigenstates, and we find flavor violating terms of the form

$$-\mathcal{L} \supset \overline{e}_R i g_{e\mu}^{RR} \gamma^\rho \chi_\rho \mu_R + \overline{e}_L i g_{e\mu}^{LL} \gamma^\rho \chi_\rho \mu_L \,,$$

with

$$g_{e\mu}^{RR} = \frac{1}{2} g_\chi (q_{e_{R1}} - q_{e_{R2}}) \sin 2\theta_R \,,$$
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The rate for $\mu \to e\chi$ then reads:

$$\Gamma(\mu \to e\chi) = \frac{m_\mu}{16\pi} \left(\left| g_{e\mu}^{LL} \right|^2 + \left| g_{e\mu}^{RR} \right|^2 \right) \left(2 + \frac{m_\mu^2}{m_\chi^2} \right) \left(1 - \frac{m_\chi^2}{m_\mu^2} \right)^2 \,.$$

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Assuming $y_{22} \gg y_{11} \gg y_{12}, y_{21}$, $v_{ij} = v$, and $q_{ij} = Q$ the relevant parameters are:

$$m_\mu^2 \simeq y_{22}^2 v^2, \qquad m_e^2 \simeq y_{11}^2 v^2, \qquad m_\chi^2 \simeq 4 g_\chi^2 Q^2 v^2$$

$$\sin 2\theta_L \simeq -2\frac{y_{12}}{y_{22}}, \qquad \sin 2\theta_R \simeq -2\frac{y_{21}}{y_{22}}\ .$$

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$$\sin 2\theta_L \simeq -2\frac{y_{12}}{y_{22}}, \qquad \sin 2\theta_R \simeq -2\frac{y_{21}}{y_{22}} \; .$$

Therefore, the rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$ is given by

$$\Gamma(\mu \to e\chi)\Big|_{m_\chi \to 0} \simeq \frac{m_\mu}{16\pi}\frac{g_\chi^2}{y_{22}^2}\left(2 + \frac{y_{22}^2}{4g_\chi^2 Q^2}\right)\left(1 - \frac{4g_\chi^2 Q^2}{y_{22}^2}\right)^2 \left[y_{12}^2 (q_{e_{L1}} - q_{e_{L2}})^2 + y_{21}^2 (q_{e_{R1}} - q_{e_{R2}})^2\right] .$$

$\mu^- \to e^- e^+ e^-$ at tree level model

The decay $\mu^- \to e^- e^+ e^-$ is generated in this model at tree-level via the exchange of a virtual χ.

$$-\mathcal{L} \supset \bar{e}_R i g_{ee}^{RR} \gamma^\rho \chi_\rho e_R + \bar{e}_L i g_{ee}^{LL} \gamma^\rho \chi_\rho e_L \ ,$$

where

$$g_{ee}^{RR} = g_\chi \left(q_{e_{R2}} \sin^2 \theta_R + q_{e_{R1}} \cos^2 \theta_R \right) \ ,$$
$$g_{ee}^{LL} = g_\chi \left(q_{e_{L2}} \sin^2 \theta_L + q_{e_{L1}} \cos^2 \theta_L \right) \ .$$

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The decay $\mu^- \to e^- e^+ e^-$ is generated in this model at tree-level via the exchange of a virtual χ.

$$-\mathcal{L} \supset \bar{e}_R i g_{ee}^{RR} \gamma^\rho \chi_\rho e_R + \bar{e}_L i g_{ee}^{LL} \gamma^\rho \chi_\rho e_L \ ,$$

where

$$g_{ee}^{RR} = g_\chi \left(q_{e_{R2}} \sin^2\theta_R + q_{e_{R1}} \cos^2\theta_R \right) \ ,$$
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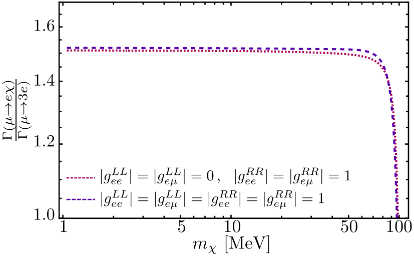
The ratio between $\Gamma(\mu \to e\chi)$ and $\Gamma(\mu \to 3e)$ is $\simeq 3/2$



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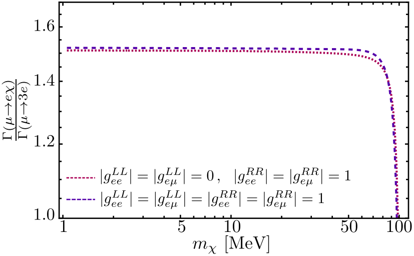

$\Downarrow$

This result can be understood analytically employing the narrow width approximation (NWA)

Ratio $\frac{\Gamma(\mu \to e\chi)}{\Gamma(\mu \to 3e)}$ at tree level model

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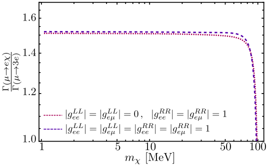

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$$\Gamma(\chi \to e^- e^+) \simeq \frac{m_\chi}{16\pi}\left(|g_{ee}^{LL}|^2 + |g_{ee}^{RR}|^2\right)$$

$$\Gamma(\mu \to 3e) = \frac{m_\mu}{24\pi}\left(|g_{e\mu}^{LL}|^2 + |g_{e\mu}^{RR}|^2\right)\left(2 + \frac{m_\mu^2}{m_\chi^2}\right)\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 + \frac{m_\chi}{32\pi}\left(|g_{ee}^{LL}|^2|g_{e\mu}^{LL}|^2 + |g_{ee}^{RR}|^2|g_{e\mu}^{RR}|^2\right)\frac{m_\chi}{m_\mu}\left(1 - 2\frac{m_\chi^2}{m_\mu^2}\right)$$

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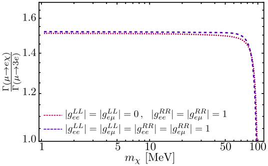

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$\Rightarrow$ subdominant contribution

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at tree level model

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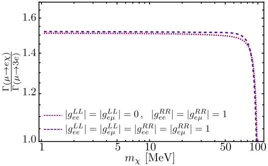

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$$\Gamma(\mu \to 3e) \backsimeq \frac{m_\mu}{24\pi}\left(|g_{e\mu}^{LL}|^2 + |g_{e\mu}^{RR}|^2\right)\left(2 + \frac{m_\mu^2}{m_\chi^2}\right)\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2$$

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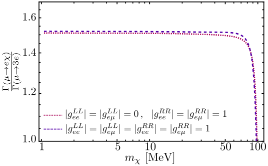

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Outline

One Loop Level Model

Spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ of the particles of the model

	L_1	L_2	e_{R_1}	e_{R_2}	ϕ	ψ	η
spin	1/2	1/2	1/2	1/2	0	1/2	0
$SU(2)_L$	2	2	1	1	2	1	1
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	$+1/2$	Y_ψ	Y_η
$U(1)_\chi$	q_L	q_L	q_e	q_e	q_ϕ	q_ψ	q_η

To violate the lepton flavor, we introduce a new Dirac fermion ψ and a new complex scalar η.
We assume that $q_e = q_\psi + q_\eta$ and $Y_e = Y_\psi + Y_\eta$.
We also assume that ϕ acquires a vacuum expectation value, but η does not

One Loop Level Model

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$SU(2)_L$	2	2	1	1	2	1	1
$U(1)_Y$	$-1/2$	$-1/2$	-1	-1	$+1/2$	Y_ψ	Y_η
$U(1)_\chi$	q_L	q_L	q_e	q_e	q_ϕ	q_ψ	q_η

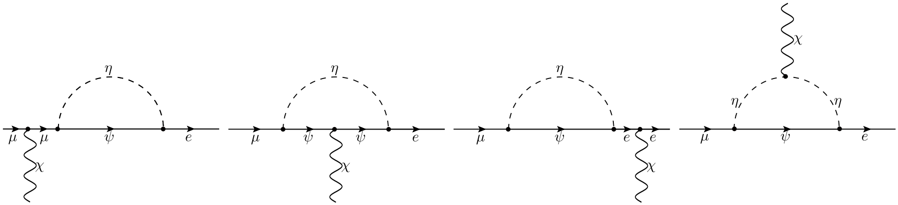
The interaction terms with the massive gauge boson χ in the mass eigenstates:

$$\mathcal{L} \supset -iq_e g_\chi \bar{e}\gamma^\nu e\chi_\nu - iq_\mu g_\chi \bar{\mu}\gamma^\nu \mu\chi_\nu - iq_\psi g_\chi \bar{\psi}\gamma^\nu \psi\chi_\nu - iq_\eta g_\chi \left[\eta^*(\partial_\nu\eta) - (\partial_\nu\eta^*)\eta\right]\chi^\nu + \text{h.c.}\,,$$

as well as a Yukawa coupling to the right-handed leptons:

$$\mathcal{L} \supset h_e \overline{e}_R \eta\psi + h_\mu \overline{\mu}_R \eta\psi + \text{h.c.}\,,$$

$\mu \to e\chi$ at the one loop level



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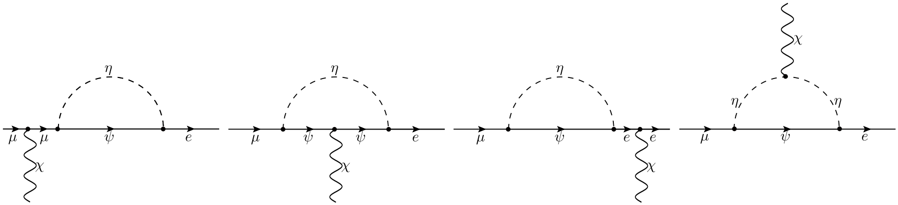

The form factors are finite and read:

$$F_1(m_\chi^2) = \quad G_1(m_\chi^2) = \frac{g_\chi y'_e y'_\mu}{384\pi^2} \boxed{\frac{m_\chi^2}{M_\eta^2}} \left[q_\eta \mathcal{F}_{1\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{1\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right)\right],$$

$$F_2(m_\chi^2) = -G_2(m_\chi^2) = \frac{g_\chi y'_e y'_\mu}{384\pi^2} \boxed{\frac{m_\mu^2}{M_\eta^2}} \left[q_\eta \mathcal{F}_{2\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{2\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right)\right],$$

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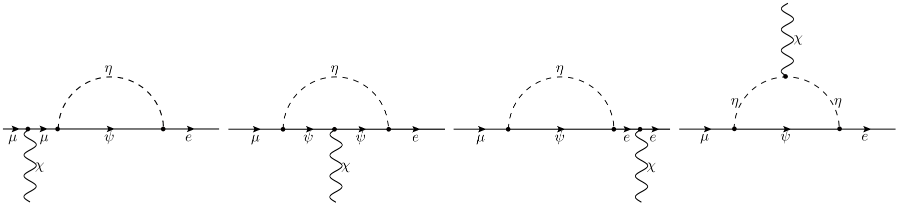

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where

$$\mathcal{F}_{1\eta}(x) = \frac{-2 + 9x - 18x^2 + x^3\,(11 - 6\ln x)}{3\,(1-x)^4}, \qquad \mathcal{F}_{2\eta}(x) = \frac{1 - 6x + 3x^2(1 - 2\ln x) + 2x^3}{(1-x)^4},$$

$$\mathcal{F}_{1\psi}(x) = \frac{16 - 45x + 36x^2 - 7x^3 + 6\,(2 - 3x)\ln x}{3\,(1-x)^4}, \quad \mathcal{F}_{2\psi}(x) = \frac{-2 - 3x(1 + 2\ln x) + 6x^2 - x^3}{(1-x)^4}. \tag{1}$$

$\mu \to e\chi$ at the one loop level

The decay rate reads for $\mu \to e\chi$:

$$\Gamma(\mu \to e\chi) \simeq \frac{m_\mu}{8\pi}\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 \left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2\left|F_1(m_\chi^2) - F_2(m_\chi^2)\right|^2\right]$$

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The form factor F_1 (and G_1) is proportional to m_χ^2/M_η^2

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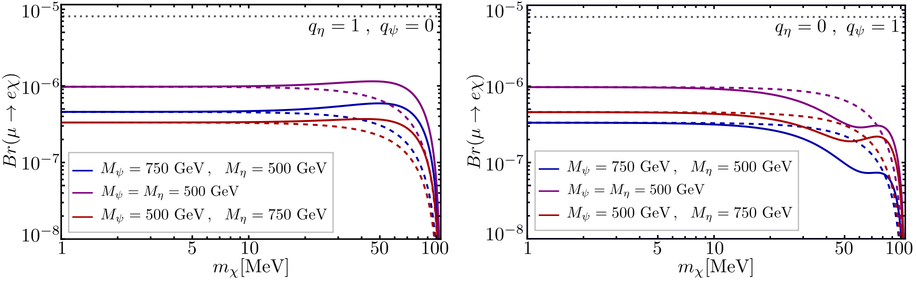
$\Downarrow$

The factors $1/m_\chi$ from the emission of the longitudinal polarization cancel with the factors m_χ^2 implicit in the form factor F_1, yielding a finite rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$.

$\Downarrow$

We are assuming $M_\eta, M_\psi \gg m_\mu$, it follows that the rate in the limit $m_\chi \to 0$ will depend mostly on the form factors F_2 and G_2.

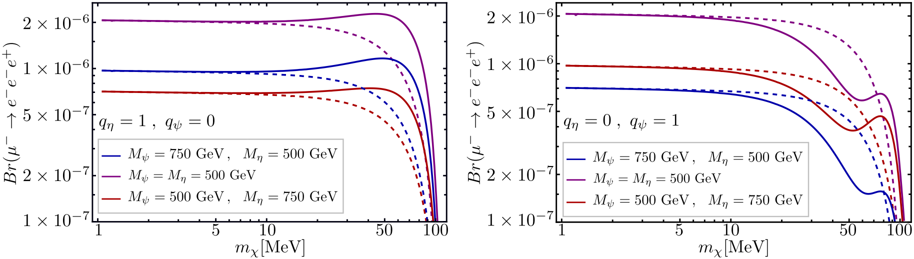
Branching ratio of the process $\mu \to e\chi$ as a function of m_χ for the one loop model. For Simplicity, we took the Yukawa-type couplings equal to one.



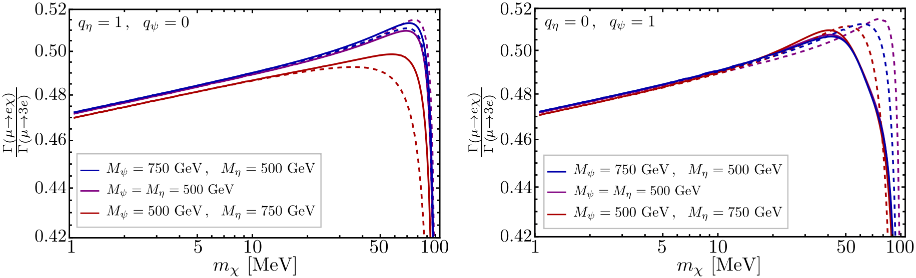
The solid lines show the full result, while the dashed lines assume $F_1 = G_1 = 0$. As apparent for the plot, while for $m_\chi \ll m_\mu$ the form factors F_1 and G_1 can be neglected, they modify the rate when $m_\chi/m_\mu \gtrsim 0.1$, especially close to the threshold.

$\mu^- \to e^- e^- e^+$ at the one loop level

The process $\mu^- \to e^- e^- e^+$ is generated through χ-penguin and through box diagrams. Assuming $h_e \ll g_\chi$, the decay will be dominated by the penguin diagrams. For Simplicity, we took the Yukawa-type couplings equal to one.

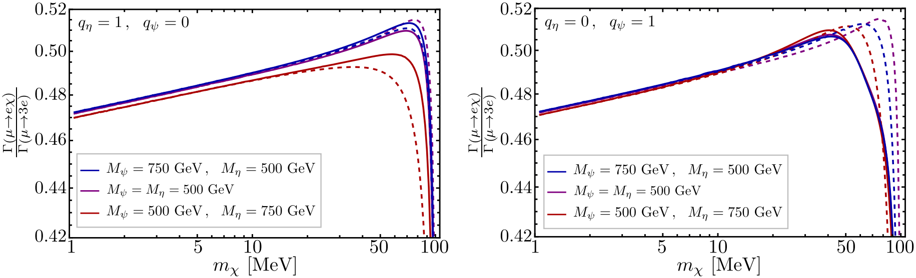


Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



The ratio is $\sim 1/2$.

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level



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$\Downarrow$

This result can be understood analytically employing the narrow width approximation (NWA)

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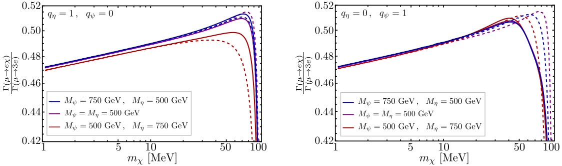

The ratio is $\sim 1/2$.

This result can be understood analytically employing the NWA.

$$\Gamma(\chi \to e^- e^+) \simeq \frac{|g_\chi|^2 (q_\eta + q_\psi)^2}{12\pi} m_\chi$$

$$\Gamma(\mu \to 3e) \simeq \frac{m_\mu}{4\pi}\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 \left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2|F_1(m_\chi^2) - F_2(m_\chi^2)|^2\right]$$
$$+ \frac{(q_\eta + q_\psi)^2}{16\pi}\frac{m_\chi^2}{m_\mu}\left(1 - 2\frac{m_\chi^2}{m_\mu^2}\right)\left(2|F_1(m_\chi^2)|^2 - |F_2(m_\chi^2)|^2\left(2 - \frac{m_\chi^2}{m_\mu^2}\right)\right).$$

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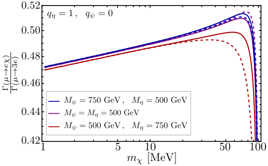

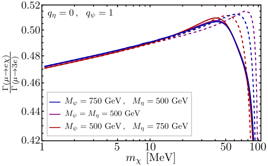

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$$\Gamma(\chi \to e^- e^+) \simeq \frac{|g_\chi|^2 (q_\eta + q_\psi)^2}{12\pi} m_\chi$$

$$\Rightarrow F_1 \sim m_\chi^2$$

$$\Gamma(\mu \to 3e) \simeq \frac{m_\mu}{4\pi}\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 \left[\boxed{\left| F_1(m_\chi^2)\frac{m_\mu}{m_\chi}\right.} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\Big|^2 + 2|F_1(m_\chi^2) - F_2(m_\chi^2)|^2\right]$$
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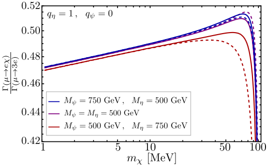

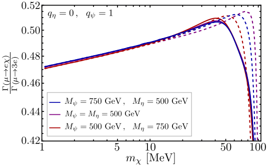

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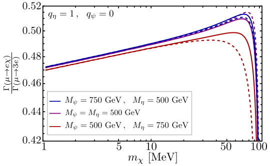
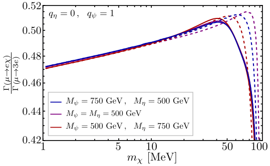
$\Rightarrow$ subdominant contribution .

Ratio $\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)}$ at the one loop level





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$$\Gamma(\mu \to e\chi) \simeq \frac{m_\mu}{8\pi}\left(1 - \frac{m_\chi^2}{m_\mu^2}\right)^2 \left[\left|F_1(m_\chi^2)\frac{m_\mu}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_\mu}\right|^2 + 2|F_1(m_\chi^2) - F_2(m_\chi^2)|^2\right]$$

$$\boxed{\frac{\Gamma(\mu\to e\chi)}{\Gamma(\mu\to 3e)} \sim \frac{1}{2}}$$

Outline

Conclusions

- To investigate the limit $m_\chi \ll m_\mu$ we have constructed two explicit renormalizable models where the decay $\mu \to e\chi$ is generated either at tree level or at the one-loop level. In both cases, we have found a finite rate for $\mu \to e\chi$ in the limit $m_\chi \to 0$.
- For the tree-level model we find that the decay is dominated by coupling terms proportional to γ^μ and $\gamma^5\gamma^\mu$.
- For the one-loop model the decay is mediated by interaction vertices proportional to γ^μ, $\gamma^5\gamma^\mu$, $\sigma^{\mu\nu}p_{\chi\nu}$ and$\gamma^5\sigma^{\mu\nu}p_{\chi\nu}$, although the latter two give the dominant contributions for $m_\chi \to 0$.

Thank you!