



NAGOYA UNIVERSITY

Electric dipole moment of the tau lepton at Belle

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2021/9 at international workshop on tau physics

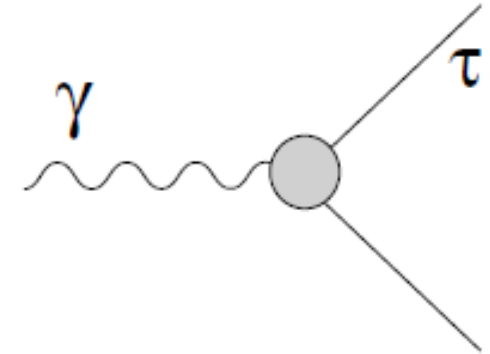
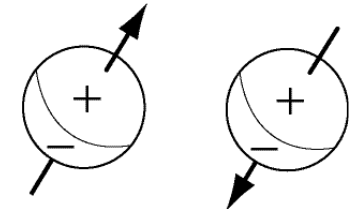
Ref: arXiv 2108.11543 [hep-ex]

- Charge asymmetry along spin direction
- CP/T violating effect in the interaction with electric field

$$\mathcal{H}_{\text{int}} = \rho_m \boldsymbol{\sigma} \cdot \mathbf{H} + \rho_e \boldsymbol{\sigma} \cdot \mathbf{E}$$

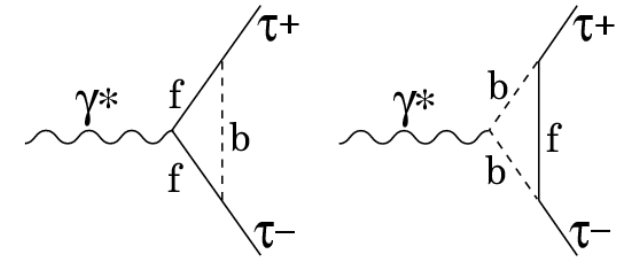
– Non zero EDM indicates P and T violation

- CP violation parameter in $\gamma\tau\tau$ vertex
- Standard Model prediction: $O(10^{-37})$ ecm
 - Far below the current sensitivity
- A non-zero EDM may arise from new physics
 - e.g. new particles in a loop diagram

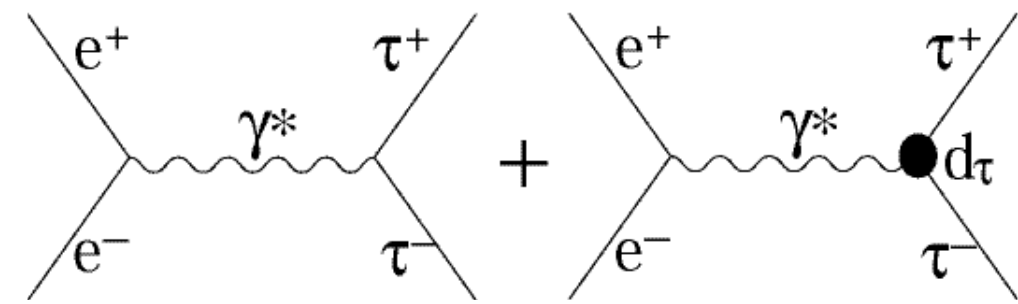


$$\mathcal{L}_{CP} = -\frac{i}{2} \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau d_\tau(s) F_{\mu\nu}$$

- Current limit
 - Belle; 29.5fb^{-1} data [PLB 551(2003)16]
 - $-2.2 < \text{Re}(d_\tau) < 4.5$ (10^{-17} e cm)
 - $-2.5 < \text{Im}(d_\tau) < 0.8$ (10^{-17} e cm)



- Effective Lagrangian with EDM term for $e^+e^- \rightarrow \tau^+\tau^-$

$$\mathcal{L}_{\text{eff}} = \bar{\psi}(i \not{\partial} - eQ \not{A})\psi - id_{\tau}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi\partial_{\mu}A_{\nu}$$


- Squared spin density matrix (proportional to cross section)

$$\chi_{\text{prod}} = \chi_{\text{SM}} + \text{Re}(d_{\tau})\chi_{\text{Re}} + \text{Im}(d_{\tau})\chi_{\text{Im}} + |d_{\tau}|^2\chi_{d^2}$$

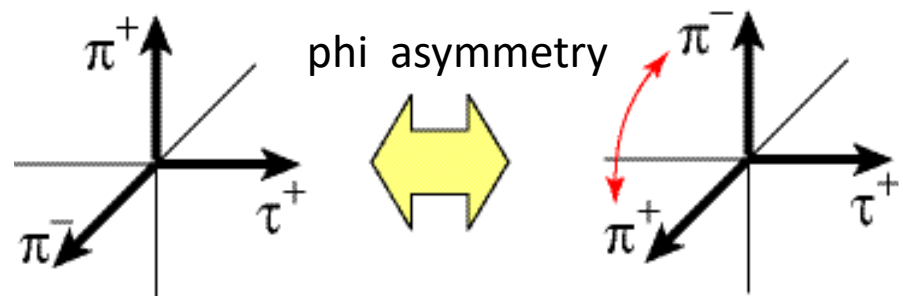
- Interference term between lowest order and EDM term

→ CP violating spin-momentum correlation

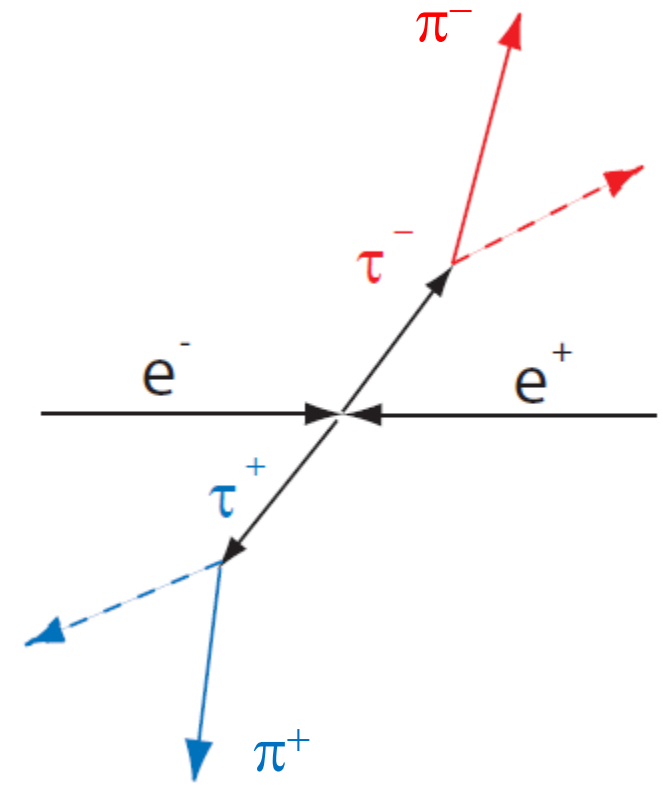
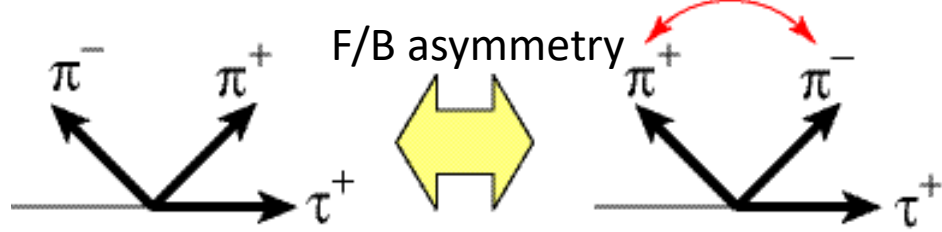
$$\begin{aligned} \chi_{\text{Re}} &\sim (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{p}} && : \text{CP-odd, T-odd} \\ \chi_{\text{Im}} &\sim (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{p}} && : \text{CP-odd, T-even} \end{aligned}$$

\mathbf{S}_{\pm} : Spin vectors of τ^{\pm}
 $\hat{\mathbf{k}}, \hat{\mathbf{p}}$: Momenta of τ^+ and e^+ beam

$$\chi_{\text{Re}} \sim (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{p}}$$

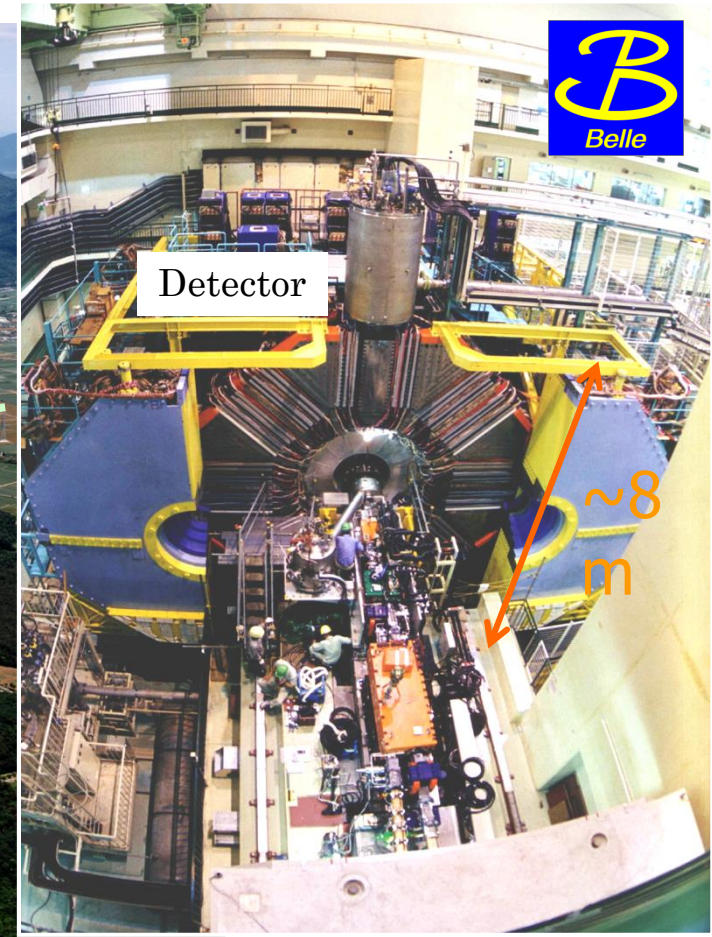
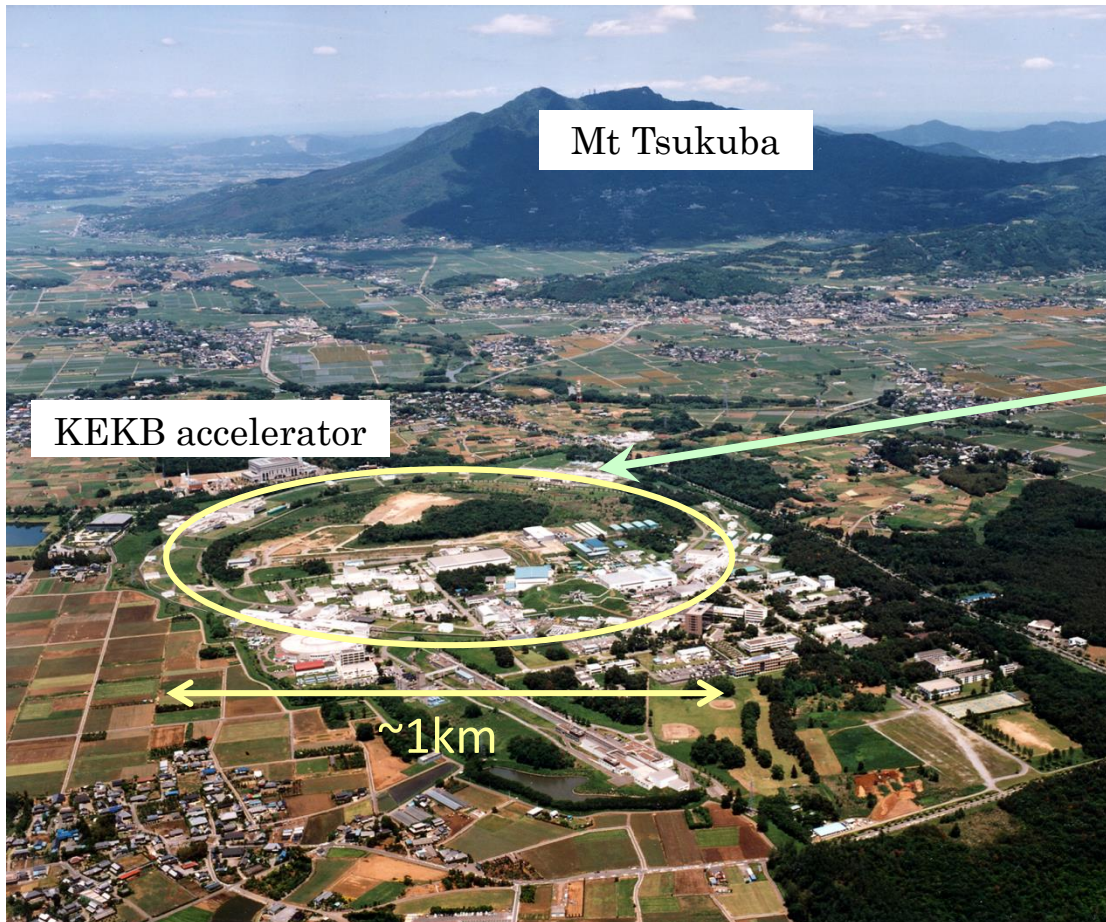


$$\chi_{\text{Im}} \sim (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{p}}$$

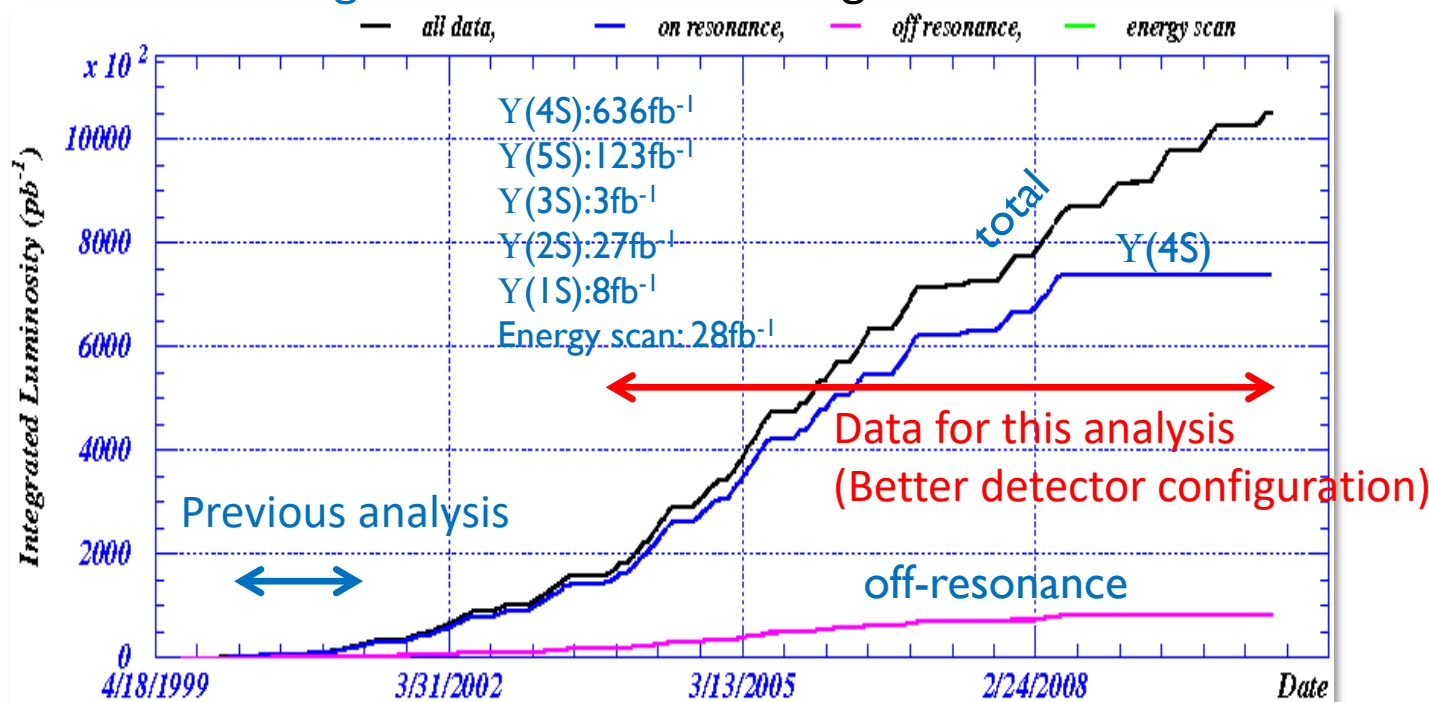


- Spin information is obtained by momentum of decay products.
- $\text{Re}(d_\tau)$: phi asymmetry, $\text{Im}(d_\tau)$: forward/backward asymmetry

- Electron(8GeV)-positron(3.5GeV) collider experiment at KEK Tsukuba Japan
- A B-factory is also a tau-factory. Collected $\sim 10^9$ τ pairs
- Belle detector; good tracking and particle identification, forward/backward asymmetric geometry.



- 833 fb⁻¹ of Belle data
 - 28 times larger than previous analysis
 - ~5 times less statistical error
 - **Improved detector understanding** compared to previous analysis
 - Better correction parameters for tracking, particle IDs
 - Improvement on the MC simulation
 - **More beam background** contribution to gammas



- Select 8 final modes exclusively
 - Tau-pair process

$$\tau\tau \rightarrow (e\nu\bar{\nu})(\mu\nu\bar{\nu}), (e\nu\bar{\nu})(\pi\nu), (\mu\nu\bar{\nu})(\pi\nu), (e\nu\bar{\nu})(\rho\nu),$$

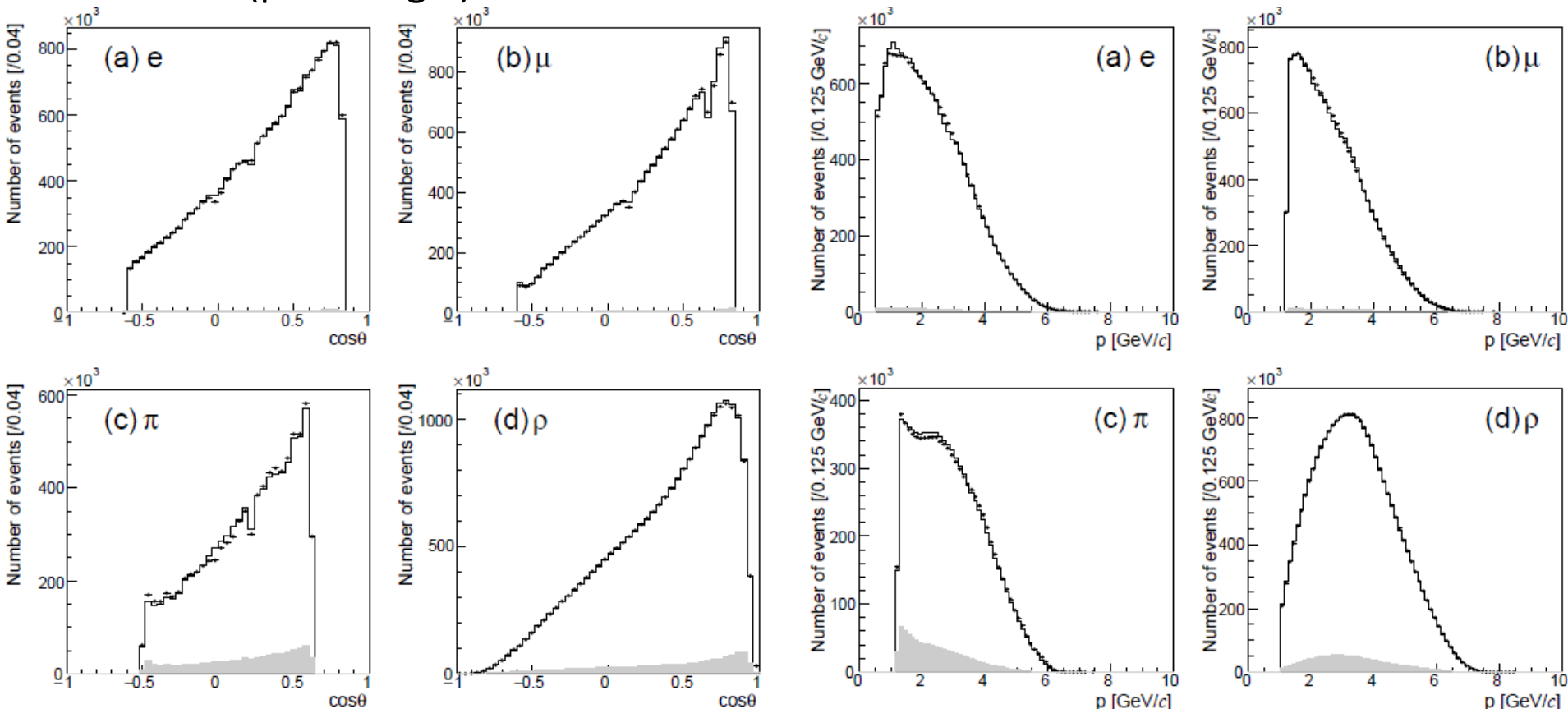
$$(\mu\nu\bar{\nu})(\rho\nu), (\pi\nu)(\rho\nu), (\rho\nu)(\rho\bar{\nu}), \text{ and } (\pi\nu)(\pi\bar{\nu})$$
 - PID for e, μ , π (+Kaon veto), ρ reconstructed from $\pi\pi^0(\rightarrow\gamma\gamma)$
 - Require high momentum and barrel region, to reduce systematic errors
- Total yield : 3.1×10^7 events, Averaged purity : 88.5%
- Background
 - Main : from tau decay : Multi- π^0 and mis-PID
 - Non- τ process: negligibly small

Note;
Final state of
 $\tau\tau \rightarrow (\pi\nu)(\rho\nu)$
express as
 $\pi\rho$, and so on.

Mode	Yield	Purity(%)	Background (%)
$e\mu$	6434268	95.8	$\gamma\gamma \rightarrow \mu\mu(2.5), \tau\tau \rightarrow e\pi(1.3)$
$e\pi$	2644971	85.7	$\tau\tau \rightarrow e\rho(6.5), e\mu(5.1), eK^*(1.3)$
$\mu\pi$	2503936	80.5	$\tau\tau \rightarrow \mu\rho(6.4), \mu\mu(4.9), \mu K^*(1.3), 2\gamma \rightarrow \mu\mu(3.1)$
$e\rho$	7218823	91.7	$\tau\tau \rightarrow e\pi\pi^0\pi^0(4.6), eK^*(1.7)$
$\mu\rho$	6203489	91.0	$\tau\tau \rightarrow \mu\pi\pi^0\pi^0(4.3), \mu K^*(1.6), \pi\rho(1.1)$
$\pi\rho$	2655696	77.0	$\tau\tau \rightarrow \rho\rho(6.7), \pi\pi\pi^0\pi^0(3.9), \mu\rho(5.1), \rho K^*(1.4), \pi K^*(1.4)$
$\rho\rho$	3277001	82.4	$\tau\tau \rightarrow \rho\pi\pi^0\pi^0(9.4), \rho K^*(3.1)$
$\pi\pi$	460288	71.9	$\tau\tau \rightarrow \pi\rho(11.3), \pi\mu(8.8), \pi K^*(2.5)$

Exp. data
 MC($d_\tau=0$)
 MC background

- $\cos\theta$ (polar angle) and momentum distribution



- Good visual agreement between data and MC
 - However, there are small mismatches in the distribution, which will be taken into account for the systematic error.

- Optimal observable [W. Bernreuther, O. Nachtmann, and P. Overmann; PRD 45(1992)2405]

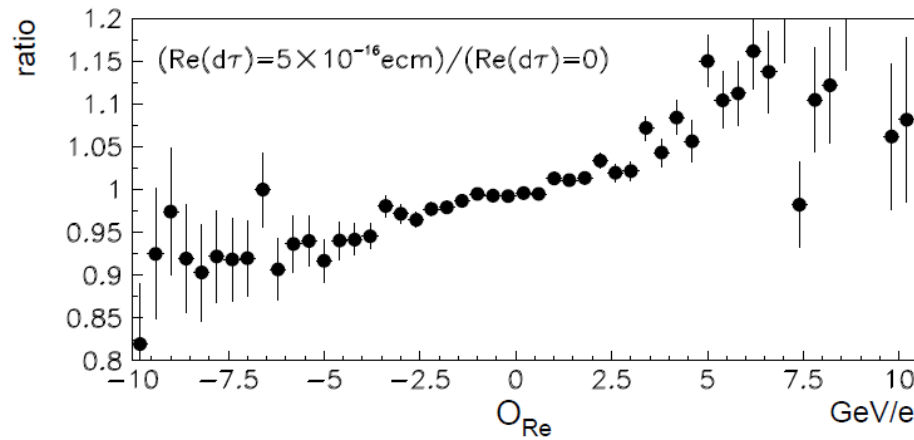
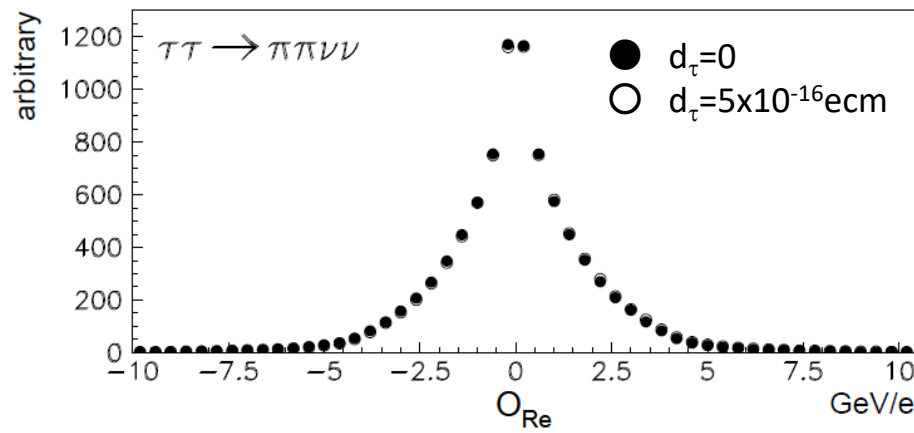
$$\mathcal{O}_{\text{Re}} = \frac{\chi_{\text{Re}}}{\chi_{\text{SM}}}, \quad \mathcal{O}_{\text{Im}} = \frac{\chi_{\text{Im}}}{\chi_{\text{SM}}}$$

$$\chi_{\text{prod}} = \chi_{\text{SM}} + \text{Re}(d_\tau) \chi_{\text{Re}} + \text{Im}(d_\tau) \chi_{\text{Im}} + |d_\tau|^2 \chi_{d^2}$$

- Maximize sensitivity (S/N)
- Calculate event-by-event
 - Using tau flight direction and spin direction (from decay products)
- Average value is proportional to EDM

$$\begin{aligned} \langle \mathcal{O}_{\text{Re}} \rangle &\propto \int \mathcal{O}_{\text{Re}} \chi_{\text{prod}} d\phi \\ &= \int \chi_{\text{Re}} d\phi + \text{Re}(d_\tau) \int \frac{(\chi_{\text{Re}})^2}{\chi_{\text{SM}}} d\phi \end{aligned}$$

MC simulation ($ee \rightarrow \tau\tau \rightarrow \pi\pi\nu\nu$)
with/without EDM ($5 \times 10^{-16} \text{ ecm}$)



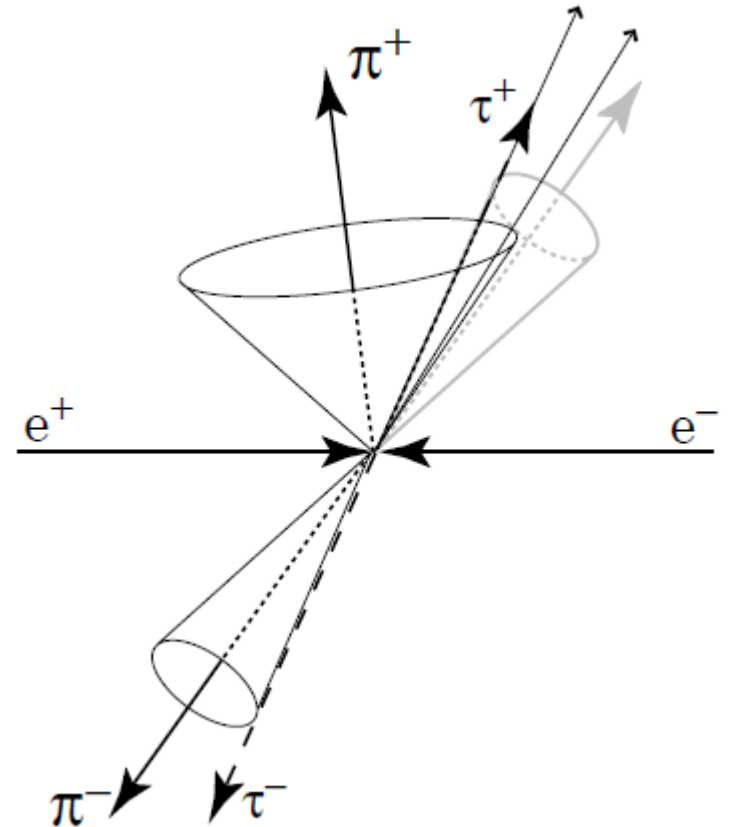
- Need tau flight direction
- Due to missing neutrinos from tau decays, there is uncertainty in the reconstructed tau direction
 - Two-fold ambiguity in case that both tau leptons decay hadronically

$$\cos \theta_i = \frac{2E_\tau E_i - m_i^2 - m_\tau^2}{2|\mathbf{k}||\mathbf{p}_i|}$$

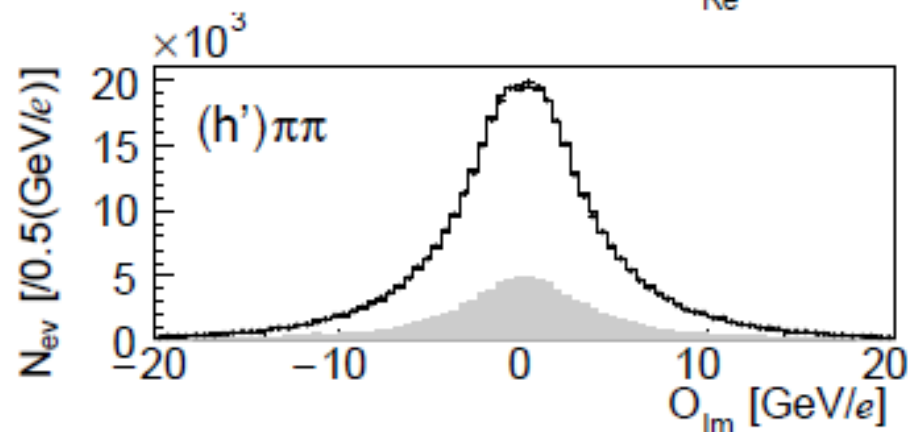
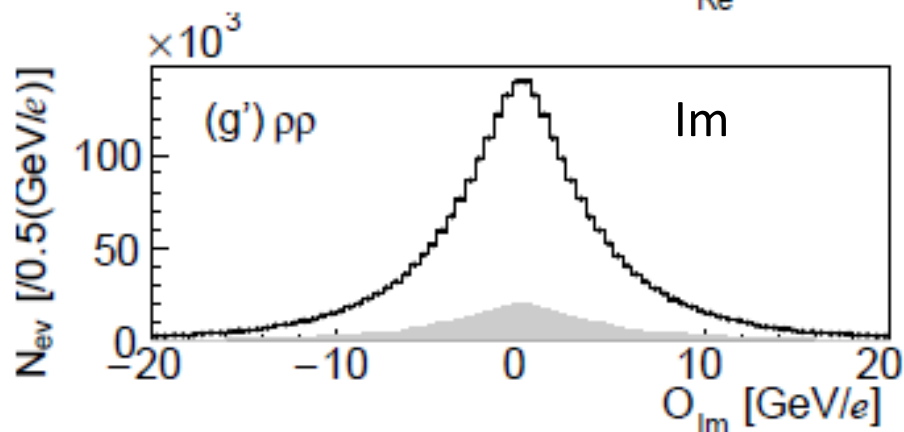
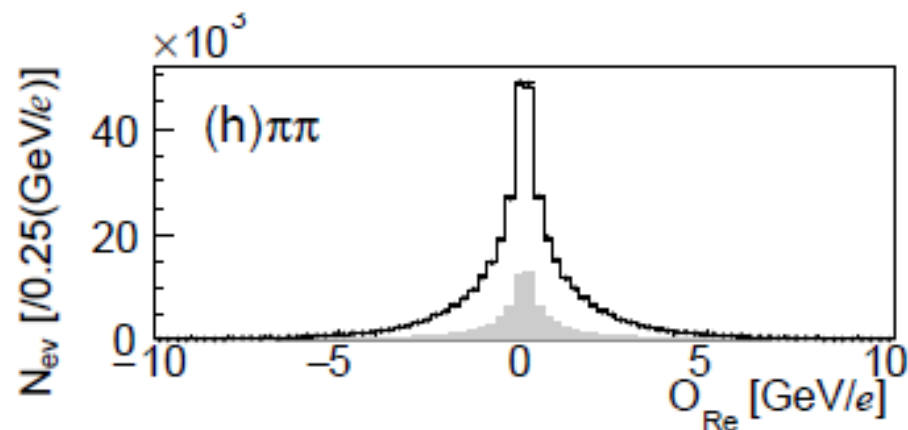
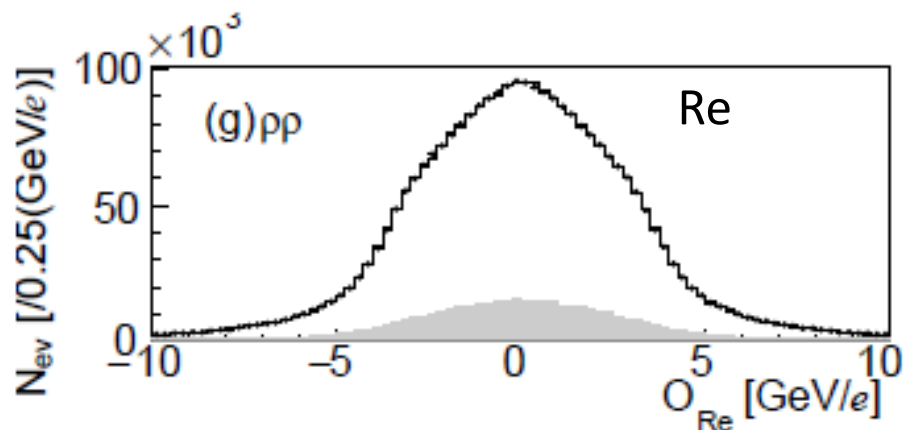
- Additional ambiguity ($m_{\nu\nu}^2$) if tau decays leptonically

$$\cos \theta_\ell = \frac{2E_\tau E_\ell - m_\ell^2 - m_\tau^2 + m_{\nu\nu}^2}{2|\mathbf{k}||\mathbf{p}_\ell|}$$

- Take an average over the possible tau directions

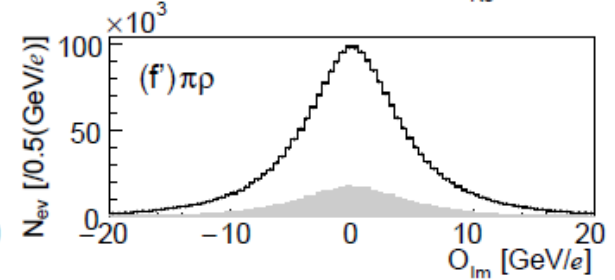
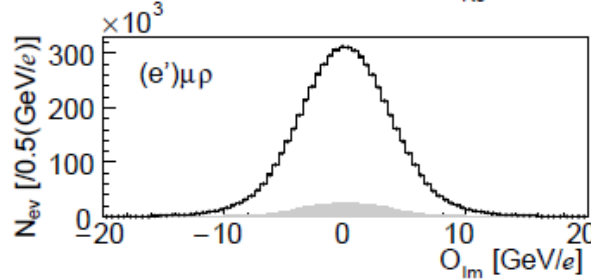
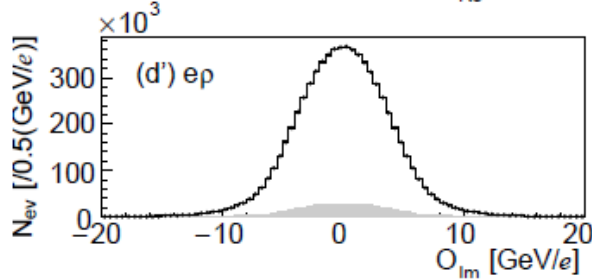
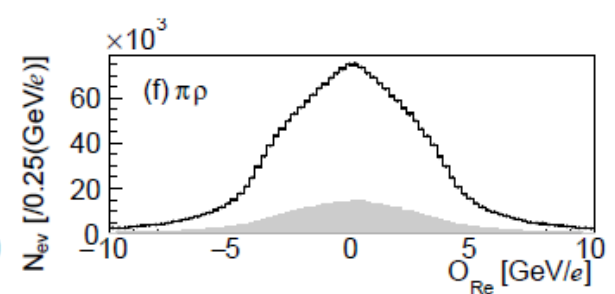
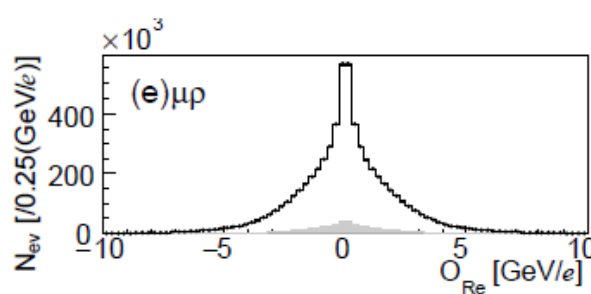
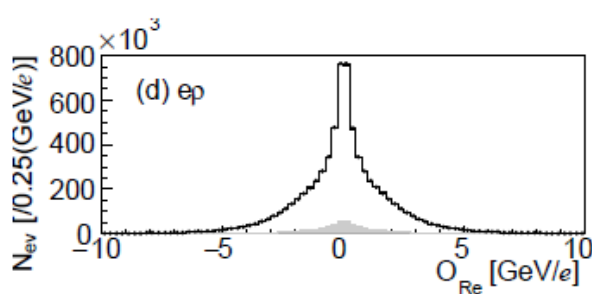
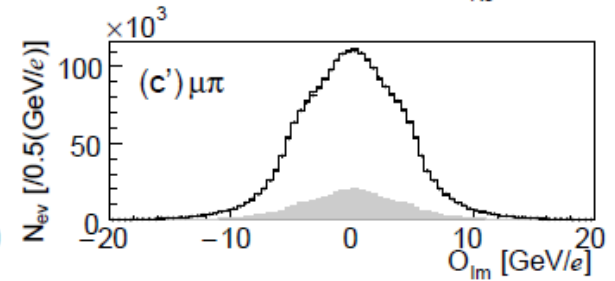
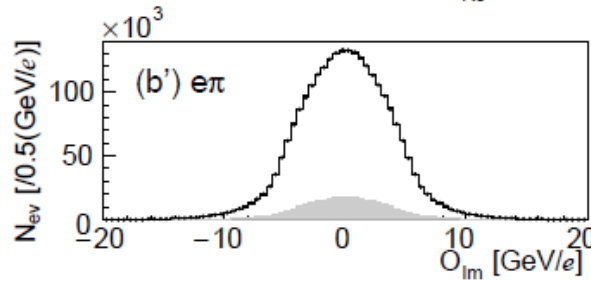
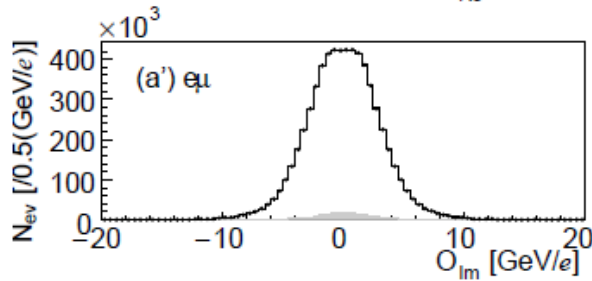
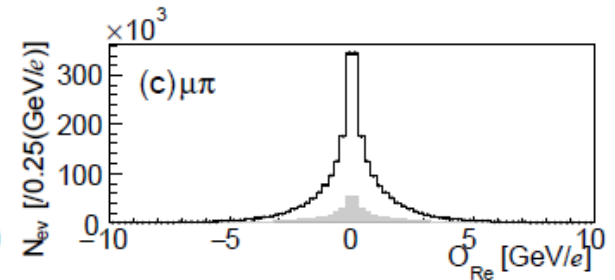
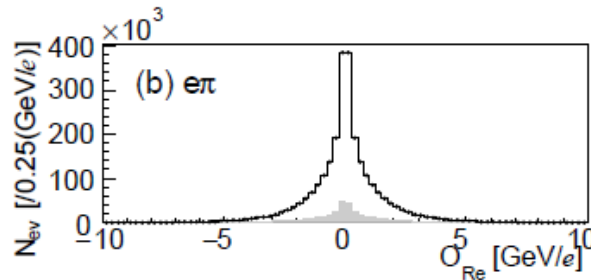
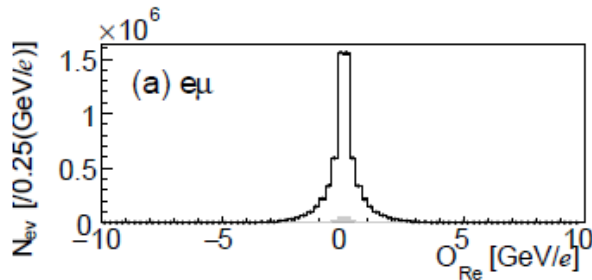


Exp. data
 MC($d_\tau=0$)
 MC background



- Good agreement in the distributions

Exp. data
 MC($d_\tau=0$)
 MC background



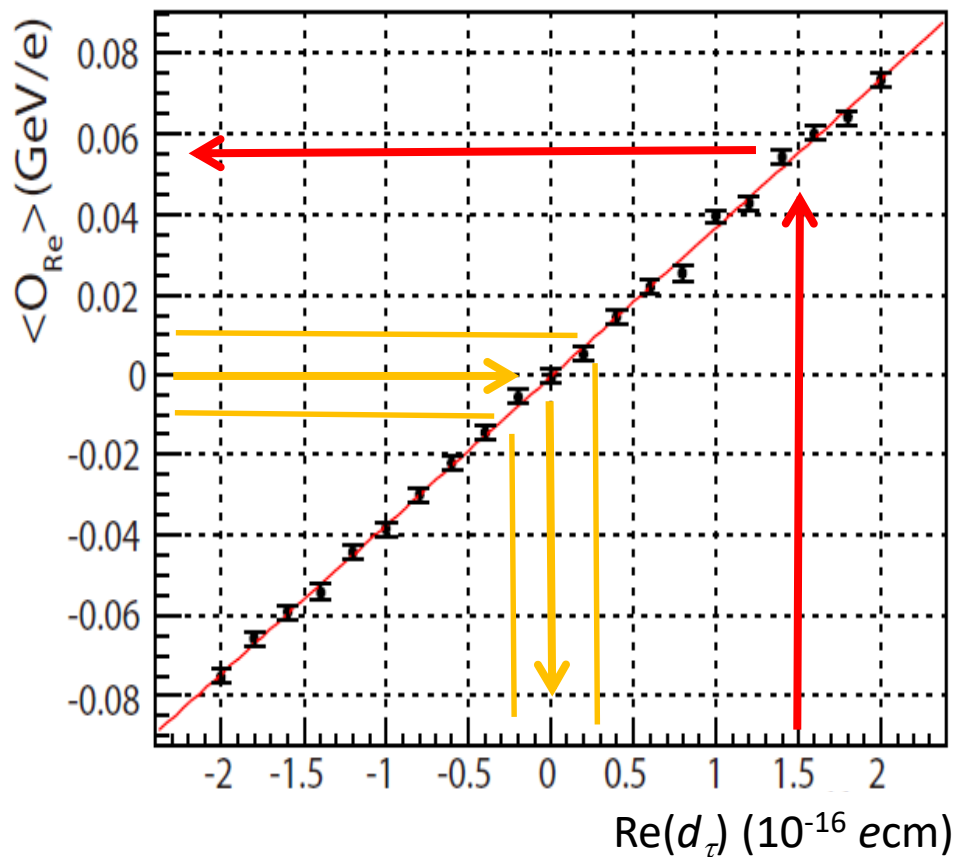
- EDM is extracted by

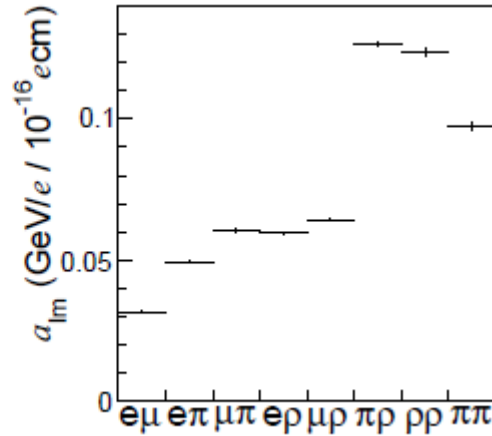
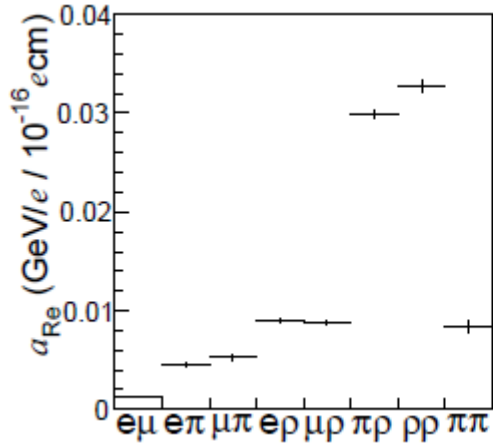
$$\langle \mathcal{O}_{Re} \rangle = a_{Re} \cdot \text{Re}(d_\tau) + b_{Re}$$

$$\langle \mathcal{O}_{Im} \rangle = a_{Im} \cdot \text{Im}(d_\tau) + b_{Im}$$

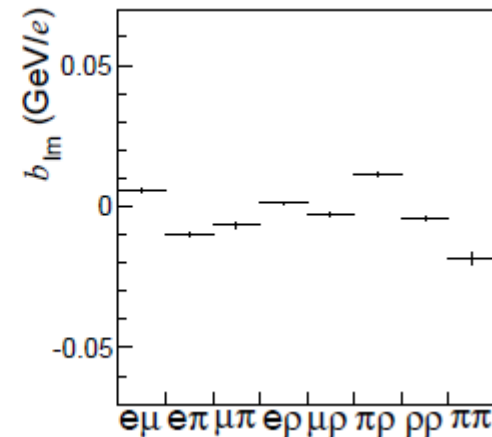
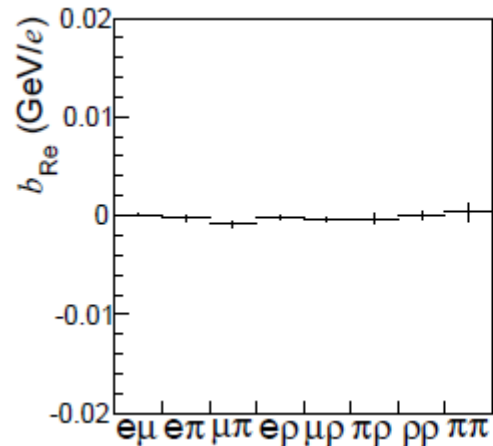
- Due to complicated detector acceptance distribution, parameters cannot be calculated analytically.
- Conversion parameters are obtained from MC.
 - Systematic error will come from the MC mismatch with data

$\langle \mathcal{O}_{Re} \rangle$ vs EDM d_τ from MC (example)





Coefficient a (\sim sensitivity)



Offset b

$$\langle \mathcal{O}_{Re} \rangle = a_{Re} \cdot Re(d_\tau) + b_{Re}$$

$$\langle \mathcal{O}_{Im} \rangle = a_{Im} \cdot Im(d_\tau) + b_{Im}$$

- The $\rho\rho$ and $\pi\rho$ modes have higher sensitivity, because of less neutrinos.
- Offset b_{Im} due to the forward/backward asymmetric acceptance

- Difference between data and MC make systematic uncertainty.
 - MC statistics for the systematic error estimation also contribute some amount.
 - Category of large uncertainty is similar with the previous analysis, although improved.

$\text{Re}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$	(10^{-17} ecm)
Detector alignment	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.3	
Momentum reconstruction	0.1	0.6	0.5	0.1	0.3	0.2	0.1	1.5	
Charge asymmetry	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	
Mismatch of distribution	3.2	4.8	3.8	0.9	2.2	0.9	0.9	3.6	←
Background variation	1.6	0.3	1.7	0.4	0.2	0.2	0.2	3.5	
Radiative effects	0.7	0.5	0.6	0.2	0.2	0.0	0.0	0.1	
Total	3.6	4.8	4.3	1.0	2.2	1.0	0.9	5.2	
$\text{Im}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$	
Detector alignment	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	
Momentum reconstruction	0.2	0.5	0.4	0.0	0.1	0.1	0.1	0.1	
Charge asymmetry	0.2	2.0	2.4	0.1	0.1	1.1	0.0	0.0	←
Mismatch of distribution	1.0	0.9	0.6	0.5	0.8	0.4	0.4	1.2	
Background variation	1.4	0.0	0.7	0.3	0.1	0.1	0.1	0.1	
Radiative effects	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0	
Total	1.8	2.2	2.6	0.6	0.8	1.2	0.4	1.2	

- EDM results

Mode	$\text{Re}(d_\tau)(10^{-17} \text{ ecm})$	$\text{Im}(d_\tau)(10^{-17} \text{ ecm})$
$e\mu$	$-3.2 \pm 2.5 \pm 3.6$	$0.6 \pm 0.4 \pm 1.8$
$e\pi$	$0.7 \pm 2.3 \pm 4.8$	$2.4 \pm 0.5 \pm 2.2$
$\mu\pi$	$1.0 \pm 2.2 \pm 4.3$	$2.4 \pm 0.5 \pm 2.6$
$e\rho$	$-1.2 \pm 0.8 \pm 1.0$	$-1.1 \pm 0.3 \pm 0.6$
$\mu\rho$	$0.7 \pm 1.0 \pm 2.2$	$-0.5 \pm 0.3 \pm 0.8$
$\pi\rho$	$-0.6 \pm 0.7 \pm 1.0$	$0.4 \pm 0.3 \pm 1.2$
$\rho\rho$	$-0.4 \pm 0.5 \pm 0.9$	$-0.3 \pm 0.3 \pm 0.4$
$\pi\pi$	$-2.2 \pm 4.3 \pm 5.2$	$-0.9 \pm 0.9 \pm 1.2$

- By adding the statistical and systematic errors quadratically, we obtain the weighted average of EDM and its error

$$\begin{aligned} \text{Re}(d_\tau) &= (-0.62 \pm 0.63) \times 10^{-17} \text{ ecm}, \\ \text{Im}(d_\tau) &= (-0.40 \pm 0.32) \times 10^{-17} \text{ ecm}. \end{aligned}$$

Previous results

$$\begin{aligned} \text{Re}(d_\tau) &= (1.15 \pm 1.70) \times 10^{-17} \text{ e cm}, \\ \text{Im}(d_\tau) &= (-0.83 \pm 0.86) \times 10^{-17} \text{ e cm}^1 \end{aligned}$$

- Consistent with zero EDM
- ~ 2.7 times smaller error than the previous results
- Systematic errors are comparable with the statistical errors.

- We have analyzed 833 fb^{-1} of Belle data to measure the electric dipole moment of tau lepton.
 - With optimal observable method
 - 28 times more data than in the previous analysis by Belle

- Obtained the result consistent with zero EDM

$$\text{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17} \text{ ecm},$$

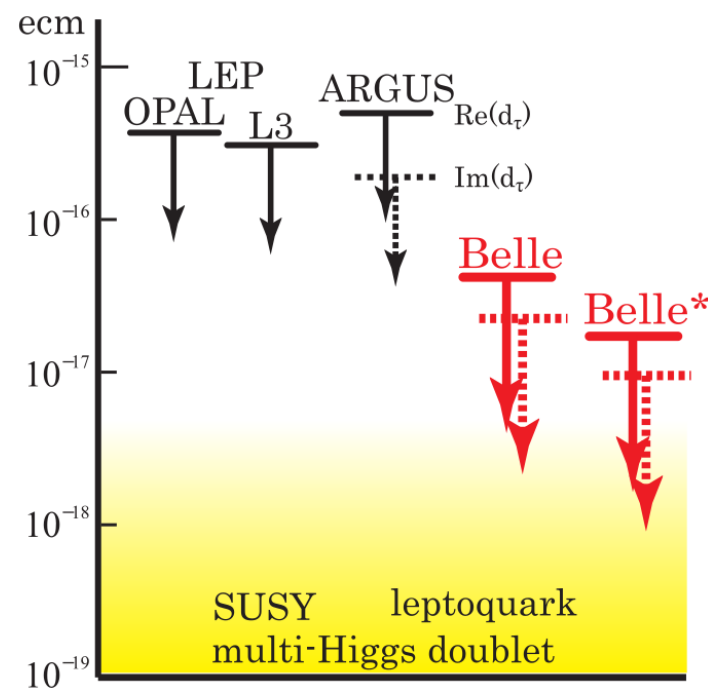
$$\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17} \text{ ecm}.$$

- 95% confidence intervals

$$-1.85 \times 10^{-17} < \text{Re}(d_\tau) < 0.61 \times 10^{-17} \text{ ecm},$$

$$-1.03 \times 10^{-17} < \text{Im}(d_\tau) < 0.23 \times 10^{-17} \text{ ecm}.$$

- Detector modeling limits our result
- Good event vertex resolution to obtain tau direction information will improve the sensitivity for future analysis.





$$e^+(\mathbf{p})e^-(-\mathbf{p}) \rightarrow \tau^+(\mathbf{k}, \mathbf{S}_+)\tau^-(-\tilde{\mathbf{k}}, \mathbf{S}_-)$$

$$\mathcal{M}_{\text{prod}}^2 = \mathcal{M}_{\text{SM}}^2 + \text{Re}(d_\tau)\mathcal{M}_{\text{Re}}^2 + \text{Im}(d_\tau)\mathcal{M}_{\text{Im}}^2 + |d_\tau|^2\mathcal{M}_{d^2}^2,$$

$$\begin{aligned} \mathcal{M}_{\text{SM}}^2 = & \frac{e^4}{k_0^2} [k_0^2 + m_\tau^2 + |\mathbf{k}^2|(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2 - \mathbf{S}_+ \cdot \mathbf{S}_- |\mathbf{k}|^2 (1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \\ & + 2(\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{k}} \cdot \mathbf{S}_-)(|\mathbf{k}|^2 + (k_0 - m_\tau)^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) + 2k_0^2(\hat{\mathbf{p}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) \\ & - 2k_0(k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})((\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) + (\hat{\mathbf{k}} \cdot \mathbf{S}_-)(\hat{\mathbf{p}} \cdot \mathbf{S}_+))], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{Re}}^2 = & 4\frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2)(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{k}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{p}}], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{Im}}^2 = & 4\frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2)(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{k}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{p}}], \end{aligned}$$

$$\mathcal{M}_{d^2}^2 = 4e^2 |\mathbf{k}|^2 \cdot (1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2)(1 - \mathbf{S}_+ \cdot \mathbf{S}_-),$$

$$\tau \rightarrow l\nu_l\nu_\tau$$

$$S_\pm = \frac{4c_\pm - m_\tau^2 - 3m_l^2}{3m_\tau^2 c_\pm - 4c_\pm^2 - 2m_l^2 m_\tau^2 + 3c_\pm m_l^2} \left(\pm m_\tau \mathbf{p}_{l\pm} - \frac{c_\pm + E_{l\pm} m_\tau}{k_0 + m_\tau} \mathbf{k} \right)$$

$$c_\pm = k_0 E_{l\pm} \mp \mathbf{k} \cdot \mathbf{p}_{l\pm}$$

$$\tau \rightarrow \pi\nu_\tau$$

$$S_\pm = \frac{2}{m_\tau^2 - m_\pi^2} \left(\mp m_\tau \mathbf{p}_{\pi\pm} + \frac{m_\tau^2 + m_\pi^2 + 2m_\tau E_{\pi\pm}}{2(E_\tau + m_\tau)} \mathbf{k} \right)$$

$$\tau \rightarrow \rho\nu_\tau \rightarrow \pi\pi^0\nu_\tau$$

$$S_\pm = \mp \frac{1}{(k_\pm H_\pm) - m_\tau^2 (p_{\pi^\pm} - p_{\pi^0})^2} \left(\mp H_0^\pm \mathbf{k} + m_\tau \mathbf{H}^\pm + \frac{\mathbf{k}(\mathbf{k} \cdot \mathbf{H}^\pm)}{(E_\tau + m_\tau)} \right)$$

$$(H^\pm)^\nu = 2(p_{\pi^\pm} - p_{\pi^0})^\nu (p_{\pi^\pm} - p_{\pi^0})^\mu (k_\pm)_\mu + (p_{\pi^\pm} + p_{\pi^0})^\nu (p_{\pi^\pm} - p_{\pi^0})^2$$