Electron – Muon – Tau Universality
Theory Perspective and Overview

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Lepton Universality

Electroweak $SU(2)_L xU(1)_Y$+ doublet Higgs Sector

Flavor Universal vs Non-Universal

- e, $\mu, \tau \quad \nu_e \, \nu_\mu \, \nu\tau$

Universal bare gauge boson couplings g_1^0 & g_2^0 for all 3 generations vs ***very different*** Yukawa couplings in Higgs sector -> lepton masses hierarchy and *large* neutrino mixing

EXPECT: neutrino osc. & τ phenomena

Major playing fields for particle physics

Some "New Physics" Examples (beyond SM Higgs doublet) that break universality (often solve g_μ-2)

1) Heavy Lepton mixing eg. Vector like heavy leptons
2) Additional $U(1\,)_{L\tau\text{-}L\mu.}$ Light or Heavy Z' Boson, Dark Physics etc.
3) Additional Higgs doublets, singlets ?
4) Other?

Look for "new physics" in precision measurements (lepton g-2), universality violation, and neutrino osc., rare processes $\mu \to e\gamma, \mu N \to eN, \tau \to 3\mu \; etc$

Popular Experimental Hints of New Physics

Anomalous Magnetic Moment $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 251(41)(43)\text{x}10^{-11}$ **4.2σ!**

Δa_e (-2.4σ or +1.6σ) depends on α *from Cs vs Rb*

Δa_τ (is O(10^{-7}) precision possible?

- CKM Unitarity Violation. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$? 3 sigma
- $|V_{us}|/|V_{ud}|$ value? 0.2292(9) from K_{l3} or 0.2313(4) from $K_{\mu 2}$ 2 sigma difference
- Semi-Leptonic B decays & Possible e, μ, τ universality breakdown?

(Anticipating LHC, Belle II & proposed super tau-charm factory at high luminosity)

Determination of Fermi constant G_F

MuLan Lifetime: $\tau_\mu = 2.1969811(22)\text{x}10^{-6}$ sec most precise lifetime measurement (ever)

determines $\Gamma(\mu \rightarrow e\nu\nu(\gamma))^{-1}$ -> G_F called **G_μ=1.1663787(6)x10^{-5}GeV^{-2}** **spectacular**

Second best determination of G_F

- First row unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$?

  $G_{CKM} = 1.1655(3)\text{x}10^{-5}\text{GeV}^{-2}$ smaller than G_μ by 3 sigma

Other definitions use $\alpha, mZ, mW, sin^2\theta_W + natural\ relationship$, leptonic Z, W, τ partial decay widths etc. generally involve larger errors.

Comparison among various G_F determinations constrains "new physics". S, T and U, W*, Z', heavy leptons etc.

- ***W. Marciano PRD (1999)***

Recent Update and Global fit to electroweak data (see figure)

A. Crivellin, M. Hoferichter and C. Manzari PRL (2021)

Taken from: A. Crivellin, M. Hoferichter, and C. Manzari PRL 2021

M_W [GeV]	[8]	80.379(12)	A_ℓ	[10]	0.1513(21)
Γ_W [GeV]	[8]	2.085(42)	R_ℓ^0	[10]	20.767(25)
BR($W \to$ had)	[8]	0.6741(27)	$A_{\mathrm{FB}}^{0,\ell}$	[10]	0.0171(10)
BR($W \to$ lep)	[8]	0.1086(9)	R_b^0	[10]	0.21629(66)
$\sin^2\theta_{\mathrm{eff}(Q_{\mathrm{FB}})}$	[8]	0.2324(12)	R_c^0	[10]	0.1721(30)
$\sin^2\theta_{\mathrm{eff(Tevatron)}}$	[28]	0.23148(33)	$A_{\mathrm{FB}}^{0,b}$	[10]	0.0992(16)
$\sin^2\theta_{\mathrm{eff(LHC)}}$	[29–32]	0.23129(33)	$A_{\mathrm{FB}}^{0,c}$	[10]	0.0707(35)
Γ_Z [GeV]	[10]	2.4952(23)	A_b	[10]	0.923(20)
σ_h^0 [nb]	[10]	41.541(37)	A_c	[10]	0.670(27)
P_τ^{pol}	[10]	0.1465(33)			

TABLE I: EW observables included in our global fit together with their current experimental values.

Parameter	Prior
$\alpha \times 10^3$ [8]	7.2973525664(17)
$\Delta\alpha_{\mathrm{had}} \times 10^4$ [16, 17]	276.1(1.1)
$\alpha_s(M_Z)$ [8, 33]	0.1179(10)
M_Z [GeV] [8, 34–37]	91.1876(21)
M_H [GeV] [8, 38–40]	125.10(14)
m_t [GeV] [8, 41–44]	172.76(30)

TABLE II: Parameters of the EW fit together with their (Gaussian) priors.

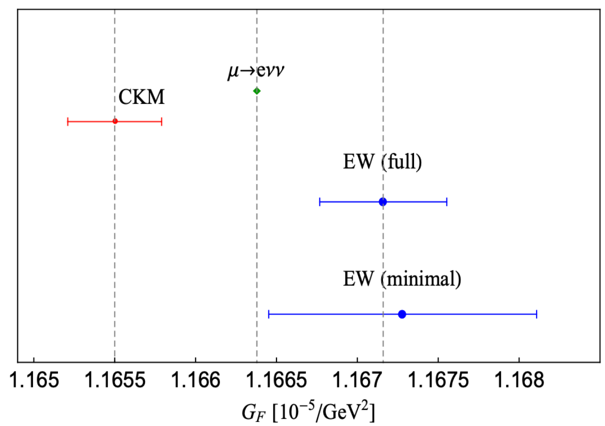

FIG. 1: Values of G_F from the different determinations.

As a third possibility, one can determine the Fermi constant from superallowed β decays, taking V_{us} from kaon or τ decays and assuming CKM unitarity ($|V_{ub}|$ is also needed, but the impact of its uncertainty is negligible). This is particularly interesting given recent hints for the apparent violation of first-row CKM unitarity, known as the Cabibbo angle anomaly (CAA). The significance of

QED Corrections to Radiative Inclusive Muon Decay

(First Full 3 loop Calculation: Fael, Schonwald, Steinhauser PRD 2021)

$$\tau_\mu^{-1} = \Gamma(\mu \to \text{all}) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)(1 + R.C.)\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ell n x$$

$$R.C. = \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{\alpha}{\pi}\left(\frac{2}{3}\ell n\frac{m_\mu}{m_e} - 3.7\right)\right)$$

Fael, Schonwald, Steinhauser 2021
three loop corrections

Kinoshita & Sirlin 1959
one loop

van Ritbergen & Stuart 2000
two loop

$\tau_\mu = 2.1969811(22)$x10^{-6}sec -> G_μ=1.1663787(6)x10^{-5}GeV^{-2}
most precise Fermi constant value by far

Application to $\Gamma\left(\tau \to \ell\nu\nu(\gamma)\right)$ $\ell = e, \mu$ *Radiative Inclusive*

- Use PDG averages as input
- $m_\tau = 1776.86(12)$MeV
- τ_τ = 290.3(5)x10^{-15}sec
- BR($\tau \to \mu\nu\nu$) = 0.1739(4) Radiative inclusive?
- BR($\tau \to e\nu\nu$) = 0.1782(4)

$\Gamma(\tau \to \mu\nu\nu(\gamma))$= 3.943(14)x$10^{-13}$GeV?

$\Gamma(\tau \to e\nu\nu(\gamma))$= 4.040(14)x$10^{-13}$GeV?

$G_{\tau\mu}$ =<u>1.1681</u>(20)x10^{-5}GeV^{-2} more interesting **$G_{\tau e}$ =1.1677(20)x10^{-5}GeV^{-2}**

Good agreement with G_μ=1.1663787(6)x10^{-5}GeV^{-2} at O(10^{-3})

Aim for factor of 5 improvement in future τ_τ & BR (Hard)!

Lepton Mass effects on QED Radiative Corrections & Precision

- m_e = 0.5109989461(31)MeV m_μ =105.6583745(24)MeV m_τ = 1776.86(12)MeV
- Smaller Mass-> larger bremsstrahlung, detector requirements differ for e, μ, τ

Experimental cuts can induce fake violations of lepton universality

Compare radiative inclusive safe quantities (eg lifetimes) or handle cut effects with care.

Bremsstrahlung mainly shifts charged particle spectrum to lower energy values

Integrating over entire spectrum free of mass singularities (KLN theorem)

Translate exp. measurements into radiative inclusive quantities

B decay anomalies need further study. Possible subtle acceptance effects.

Some other interesting universality tests in τ, K and π decays

see detailed talks by D. Hertzog, G. Lopez-Castro, A. Lusiani

$\frac{\Gamma(\tau\to\pi\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))}$ complements $\frac{\Gamma(\pi\to e\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))}$ = 1.2327(23)x10^{-4} (LOI for factor 10 improvement)

$$\frac{\Gamma(\tau\to K\nu_\tau[\gamma])}{\Gamma(\tau\to\pi\nu_\tau[\gamma])}=\frac{|V_{us}|^2F_K^2}{|V_{ud}|^2F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)\,,\quad\left|\frac{V_{us}}{V_{ud}}\right|=0.2288\pm0.0010_{\rm th}\pm0.0017_{\rm exp}$$

while $\frac{\Gamma(K\to\mu\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))}$ leads to $\frac{V_{us}}{V_{ud}}$ = 0.2313(4) differ by 1.25 sigma

Tau decays somewhat favor 1st row CKM uitarity violation $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2$ =0.9985(5)?
As well as smaller K_{l3} determination of $V_{us}\approx 0.2231(6)$

Questions: K/π *separation*? , *radiative inclusive*? *Improvement possible &warranted?*

Conclusions

- SM tested at precision level (better than O(10^{-3}). Improvement?
- Time to find origin of flavor universality violation. Why 3 generations?
- Tau physics at LHC, Belle II & tau-charm factory. **(High Luminosity)**
- Neutrino Oscillations potentially very important for further discovery
  υ_τ should play a major role in future tau meetings
- Look for Rare FCNC Events eg $\tau \rightarrow 3\mu$. Other rare reactions eg. $\mu N \rightarrow eN$

Expect: Tau Meetings to Remain Vibrant & Popular