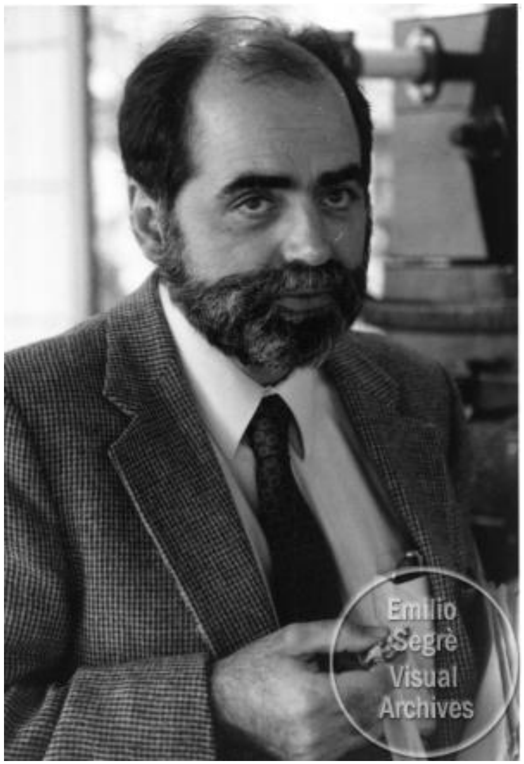
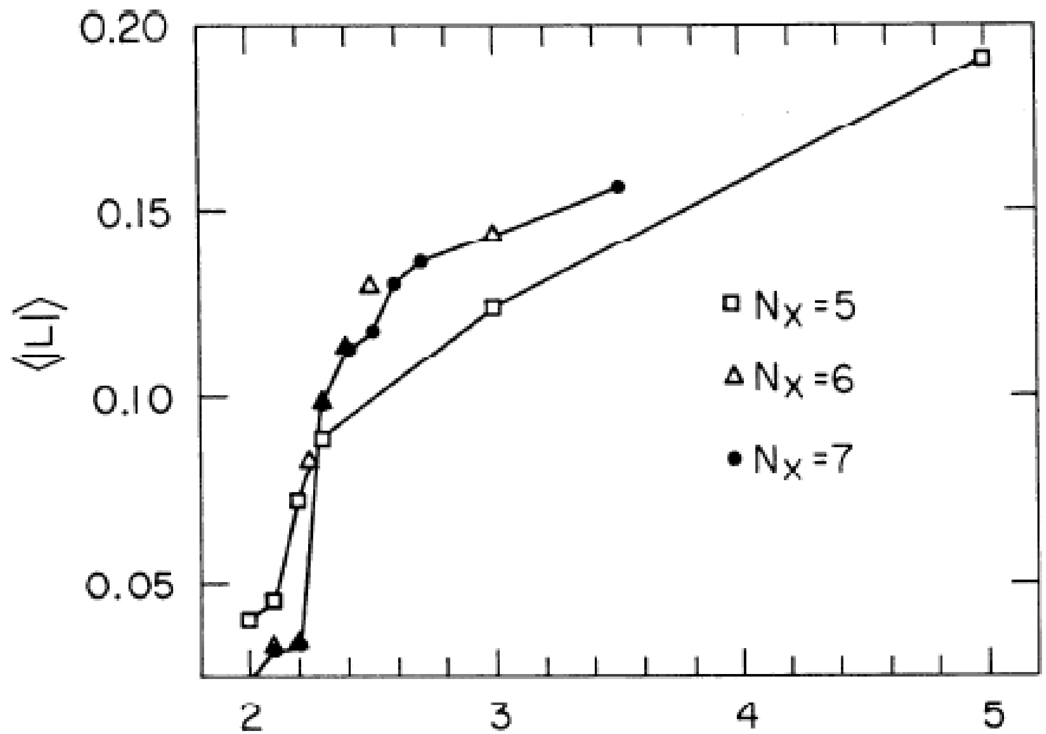
Professor Larry McLerran (1981)



"Larry, in collaboration with Ben Svetitsky, performed the first Monte-Carlo computations of an SU(2) gauge theory at finite temperature."

Larry D. McLerran & Benjamin Sveritsky:
Phys.Lett 98B (1981) 195, Phys.Rev. D24 (1981) 450



Polyakov loop and QCD critical dynamics

Professor Larry McLerran (2017)

Larry, in collaboration with Ben Svetitsky, introduced Polyakov loop as an order parameter for deconfinemnet in SU(N) gauge theory

- Deconfinement in SU(N) gauge theory and Polyakov loop Fluctuations
- Modelling deconfinement in the limit of heavy quarks at finite density and magnetic field
- Polyakov loop fluctuations in the presence of dynamical quarks

In coll. with: Bengt Friman, Olaf Kaczmarek, Pok. Man Lo, Larry McLerran, Chihiro Sasaki, Michal Szymanski

Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:

$$L \Rightarrow c_N L \qquad c_N = e^{2\pi i k/N} \in Z(N)$$

$$L^{\rm bare}_{\vec{x}} = \frac{1}{N_c} Tr \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}$$

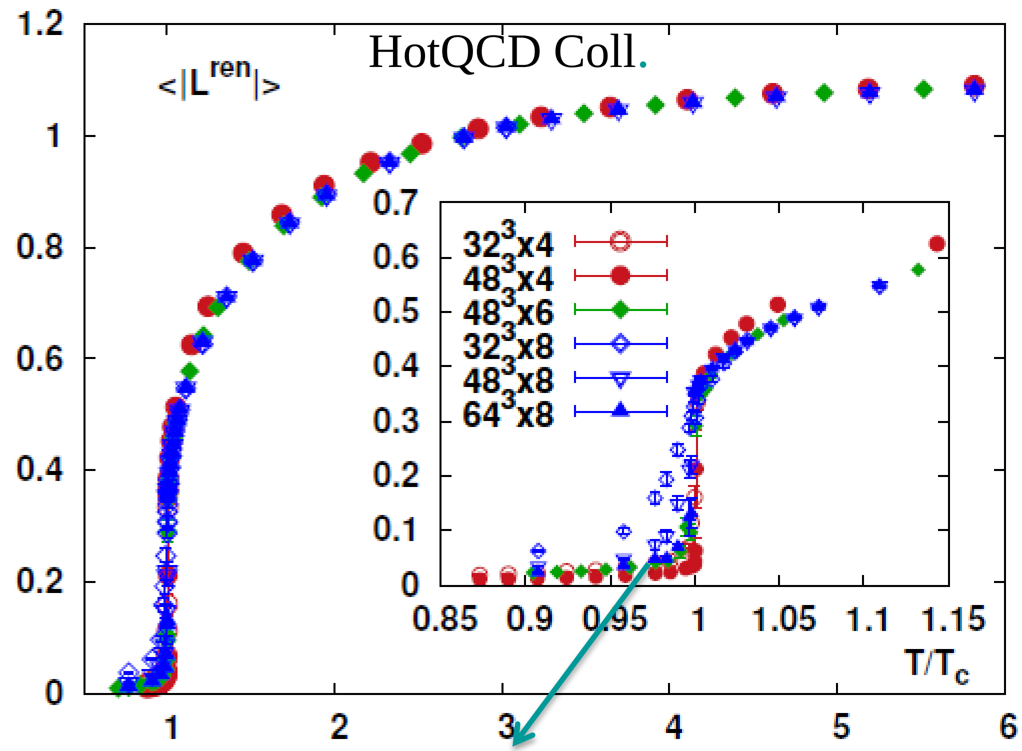
$$L^{\rm bare} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L^{\rm bare}_{\vec{x}}$$

$$<| L^{\rm ren} |> = e^{-\beta F_q^{\rm ren}} \rightarrow \begin{cases} \neq 0 \Leftrightarrow \text{deconfined } \ T > T_c \\ 0 \Leftrightarrow \text{confined } \ T<T_c \end{cases}$$

- Renormalized ultraviolet divergence

$$L^{\rm ren} = \left(Z(g^2)\right)^{N_\tau} L^{\rm bare}$$

- Usually one takes $\langle |L^{\rm ren}| \rangle$ as an order parameter



$1/\sqrt{V}$

To probe deconfinement : consider fluctuations

- Fluctuations of modulus of the Polyakov loop

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$

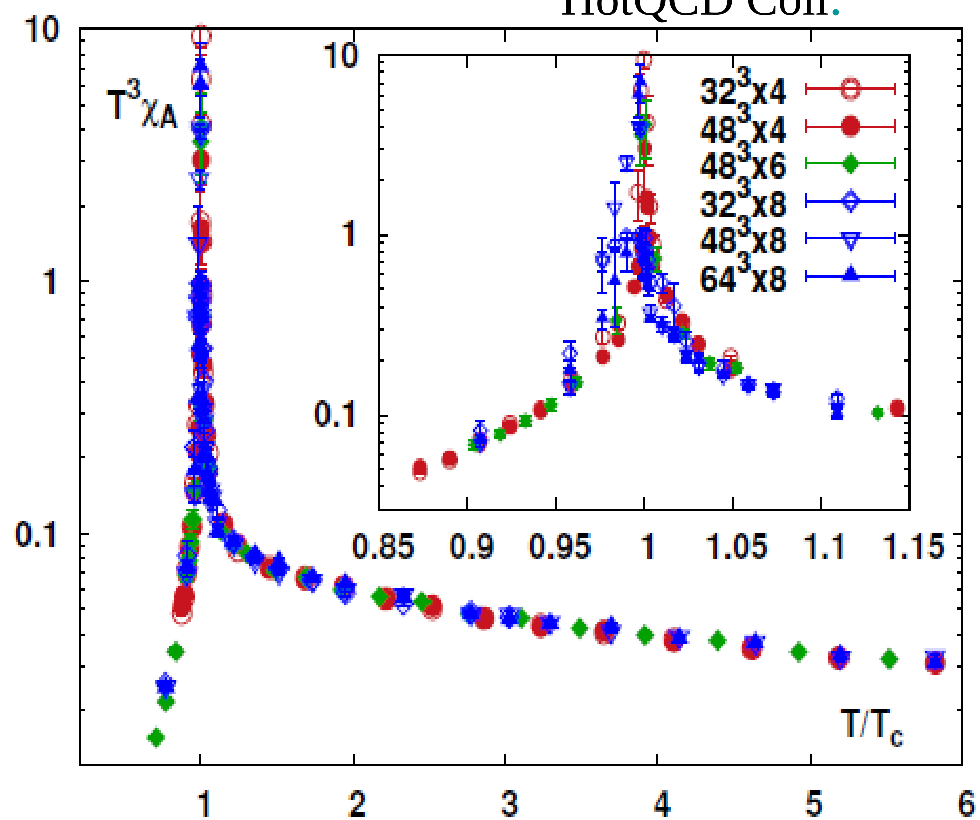
However, the Polyakov loop

$$L = L_R + iL_I$$

Thus, one can consider fluctuations of the real χ_R and the imaginary part χ_I of the Polyakov loop.

SU(3) pure gauge: LGT data

HotQCD Coll.



Fluctuations of the real and imaginary part of the renormalized Polyakov loop

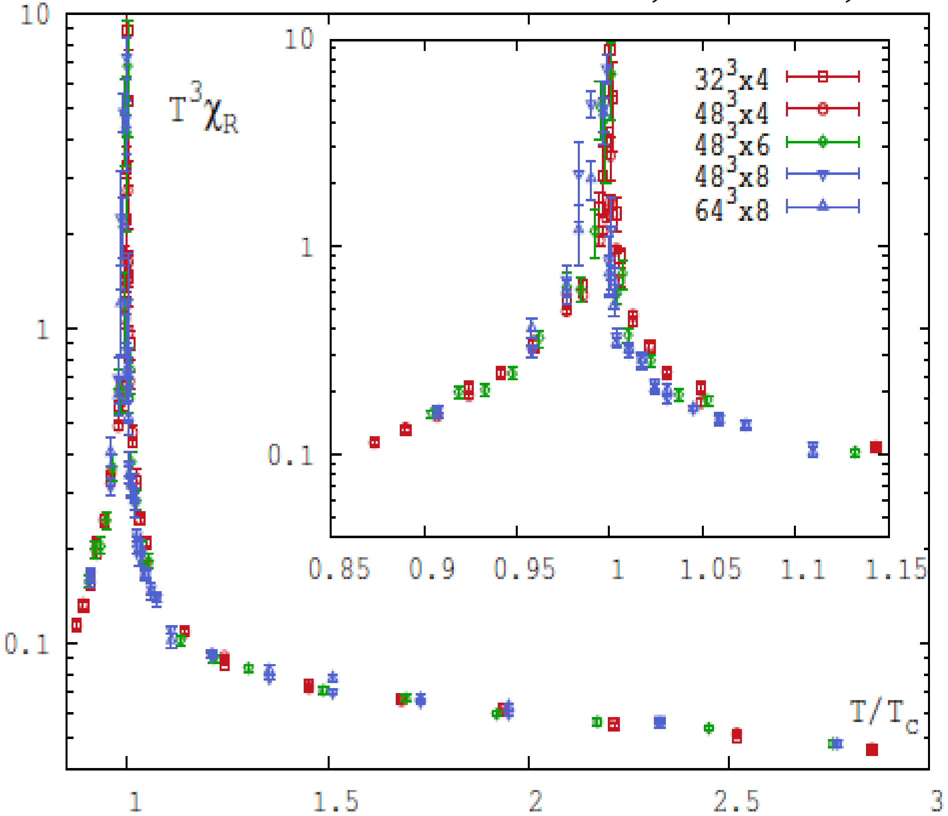
- Real part fluctuations

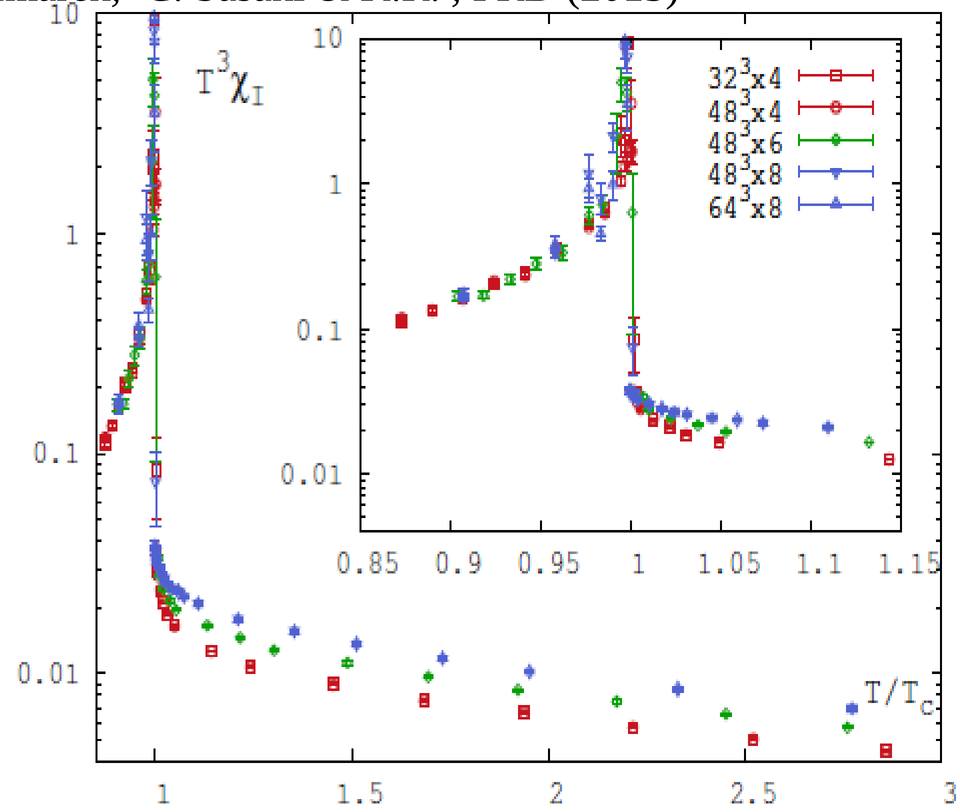
$$T^3\chi_R = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_R^{\rm ren})^2\rangle - \langle L_R^{\rm ren}\rangle^2\right]$$

- Imaginary part fluctuations

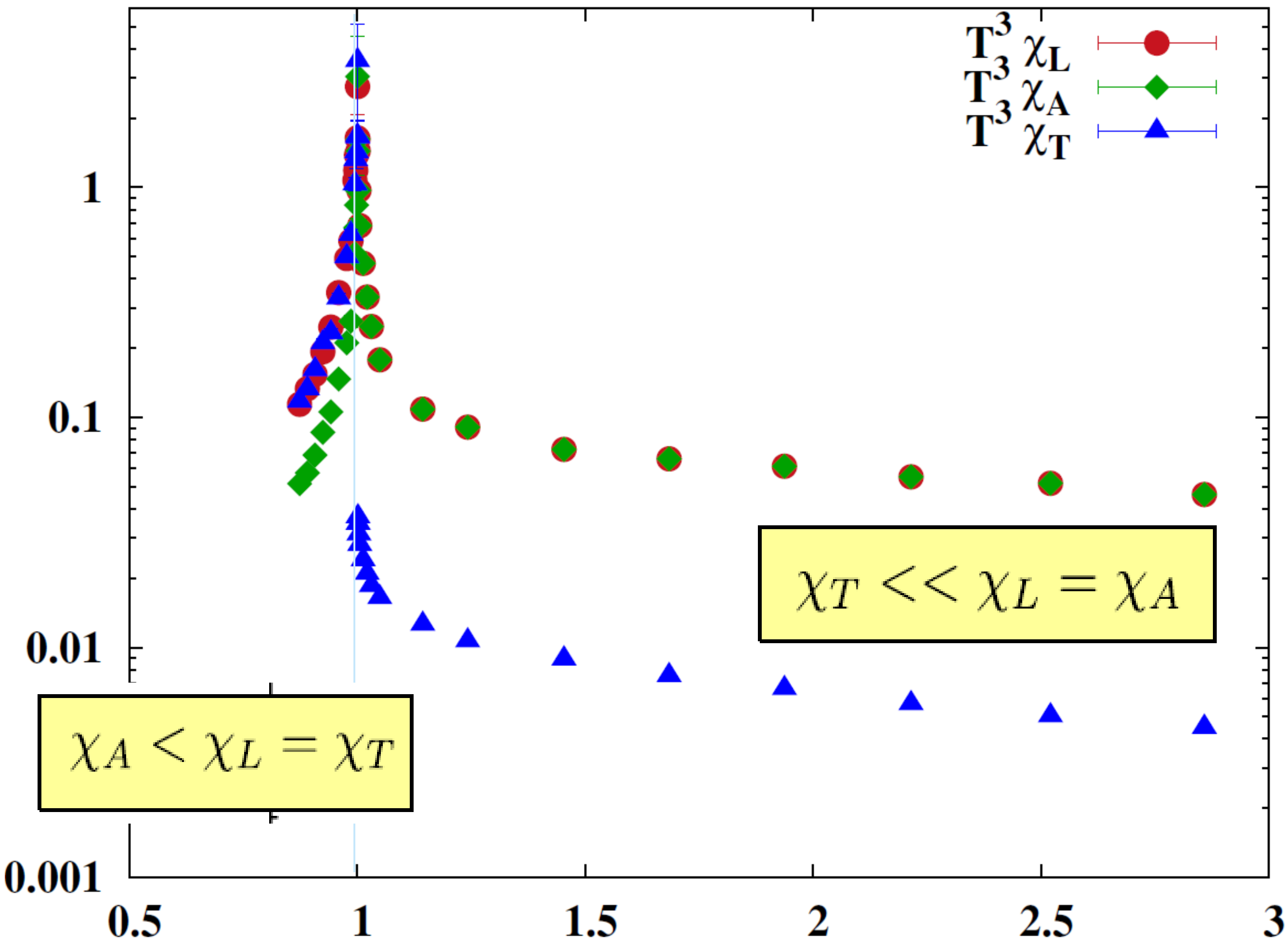
$$T^3\chi_I = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_I^{\rm ren})^2\rangle - \langle L_I^{\rm ren}\rangle^2\right]$$

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)





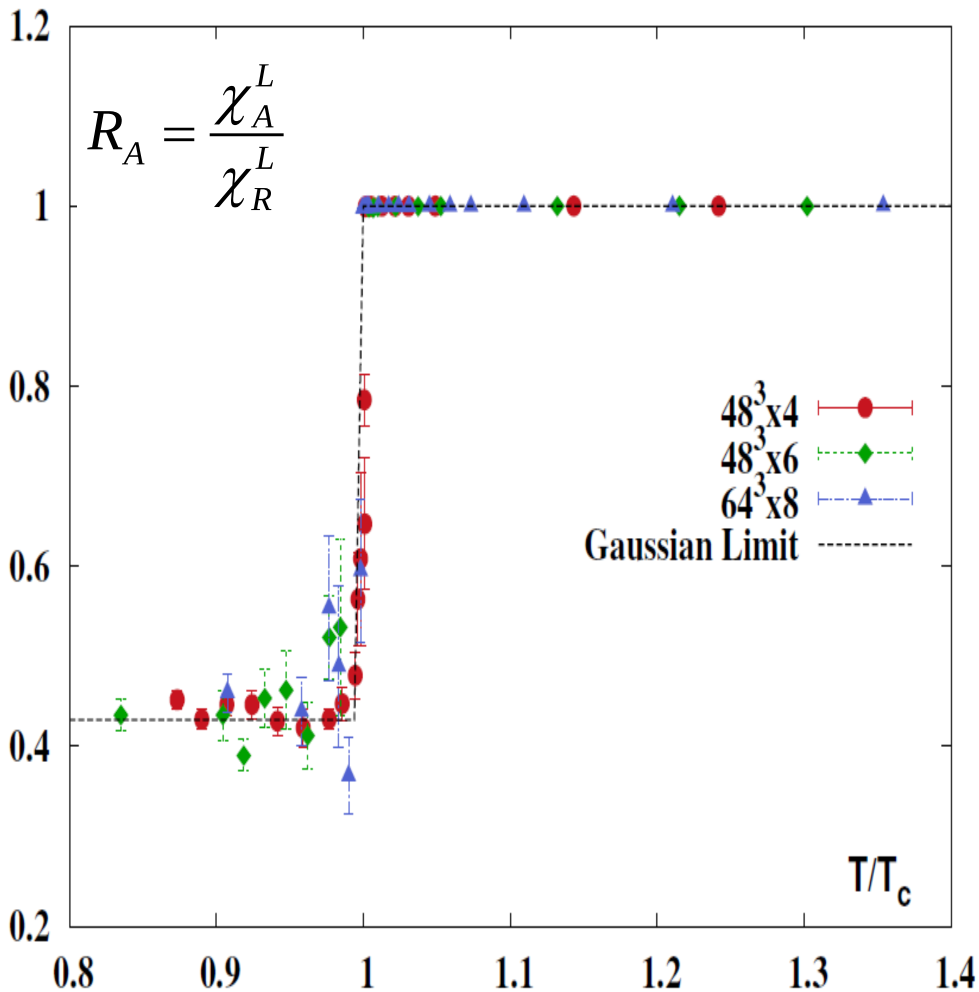
Compare different Susceptibilities:



- Systematic differences/similarities of the Polyakov lopp susceptibilities
- Consider their ratios!

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 = < L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \text{ thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$| L | = \sqrt{L_R{}^2 + L_I{}^2} \approx L_0 (1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2 L_0{}^2})$$

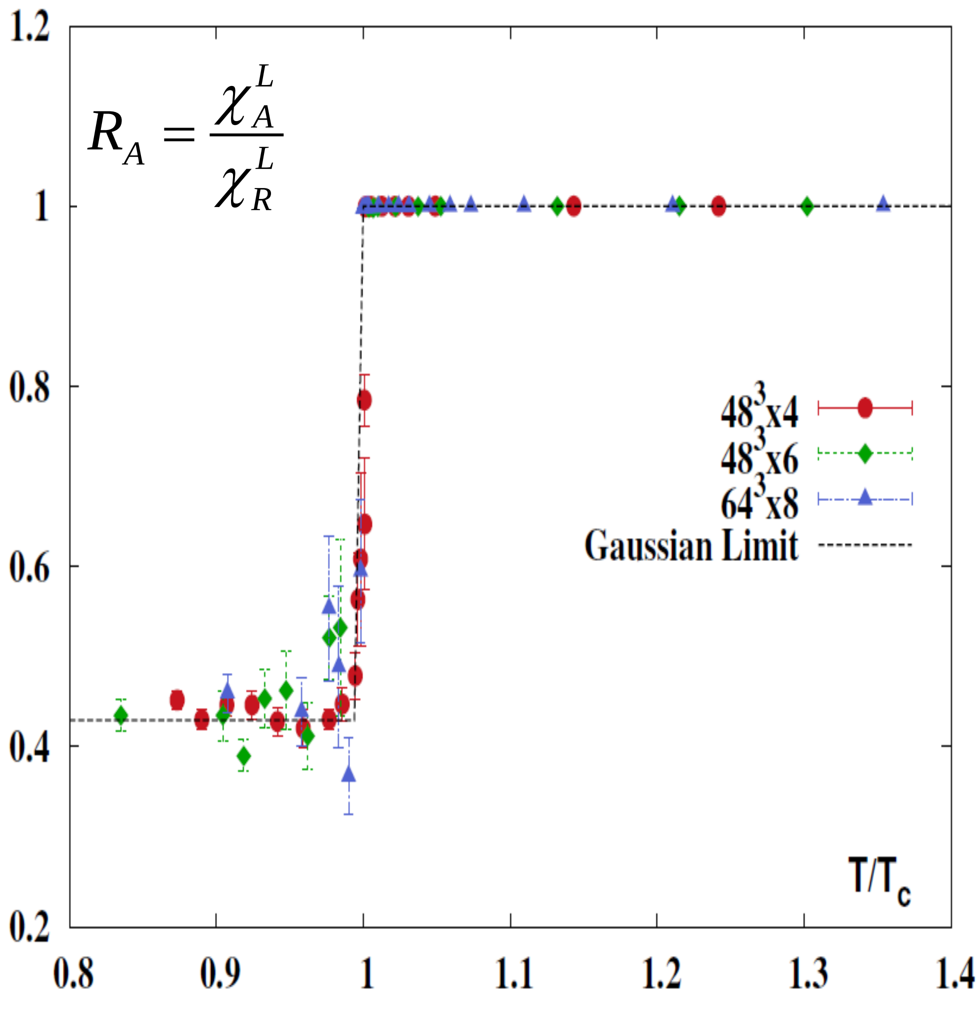
get in the leading order

$$<| L |^2> - <| L |>^2 \approx < (\delta L_R)^2 >$$

thus $\chi_A \approx \chi_R$

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase $R_A \approx 0.43$

Indeed, in the Z(3) symmetric phase, the probability distribution is Gaussian to the first approximation, with the partition function

$$Z = \int dL_R dL_I e^{VT^3[\alpha(T)(L_R{}^2 + L_I{}^2)]}$$

Thus $\chi_R = \frac{1}{2\alpha T^3}$, $\chi_I = \frac{1}{2\alpha T^3}$ and

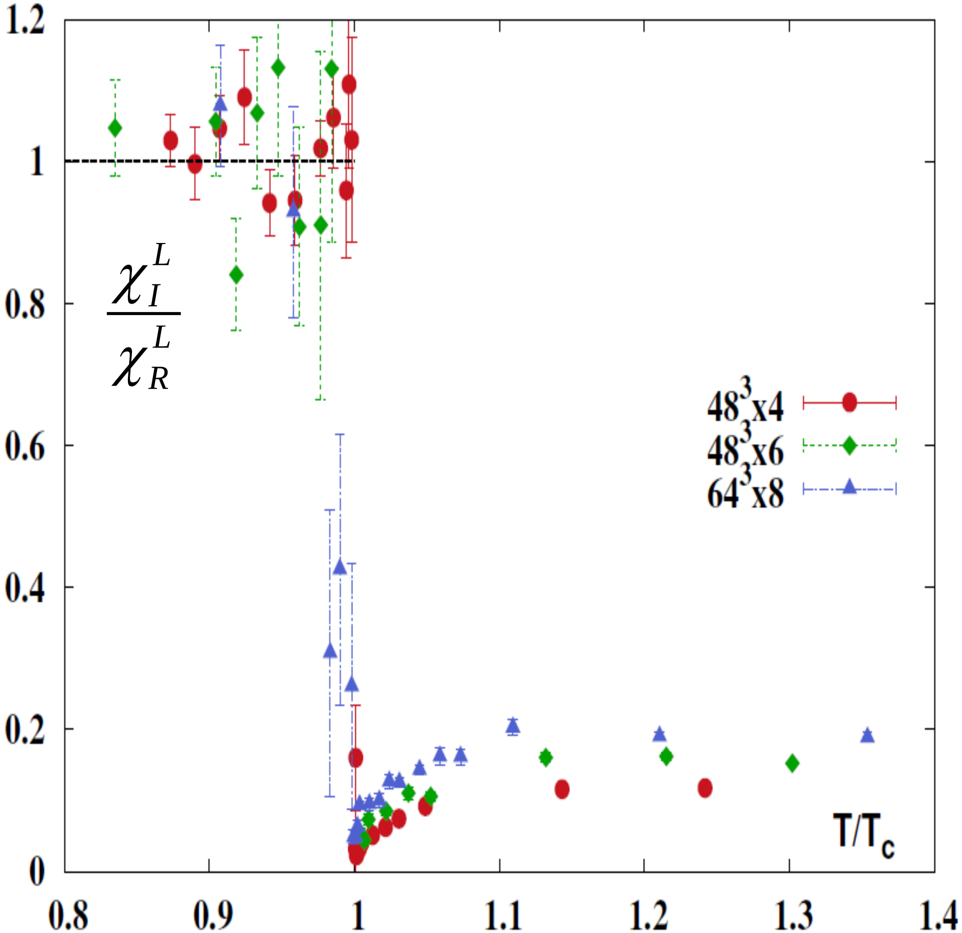
$\chi_A = \frac{1}{2\alpha T^3}(2 - \frac{\pi}{2})$, consequently

$$R_A^{SU(3)} = (2 - \frac{\pi}{2}) = 0.429$$

In the SU(2) case $R_A^{SU(2)} = (2 - \frac{2}{\pi}) = 0.363$ is in agreement with MC results

Ratio Imaginary/Real of Polyakov loop fluctuations

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase for any symmetry breaking operator its average vanishes, thus

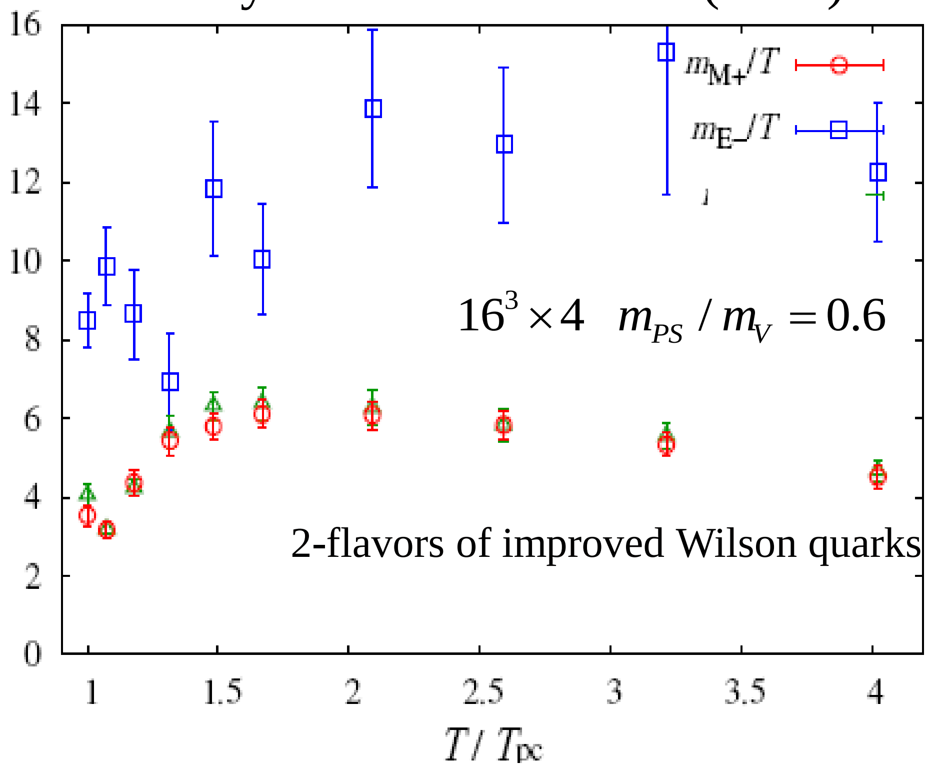
$$\chi_{LL} = < L^2 > - < L >^2 = 0$$ and

$$\chi_{LL} = \chi_R - \chi_I$$ thus $$\chi_R = \chi_I$$

- In deconfined phase the ratio of $\chi_I / \chi_R \neq 0$ and its value is model dependent

Ratio Imaginary/Real and gluon screening

WHOT QCD Coll:
Y. Maezawa[1], S. Aoki[2], S. Ejiri[3], T. Hatsuda[4], N. Ishii[4], K. Kanaya[2], N. Ukita[5] and T. Umeda[6]
Phys. Rev. D81 091501 (2010)



- In the confined phase

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

$$C_{R,(I)}(r) = < L_{R,(I)}(r) L_{R,(I)}(0) >_c$$

- WHOT QCD Coll. (Y. Maezawa et al.)

$$C_{R,(I)}(r)_{r\to\infty} \to \gamma_{R,(I)}(T) \frac{e^{-M_{R,(I)} r}}{rT}$$

and WHOT-coll. identified $M_{R(I)}$ as the magnetic and electric mass:

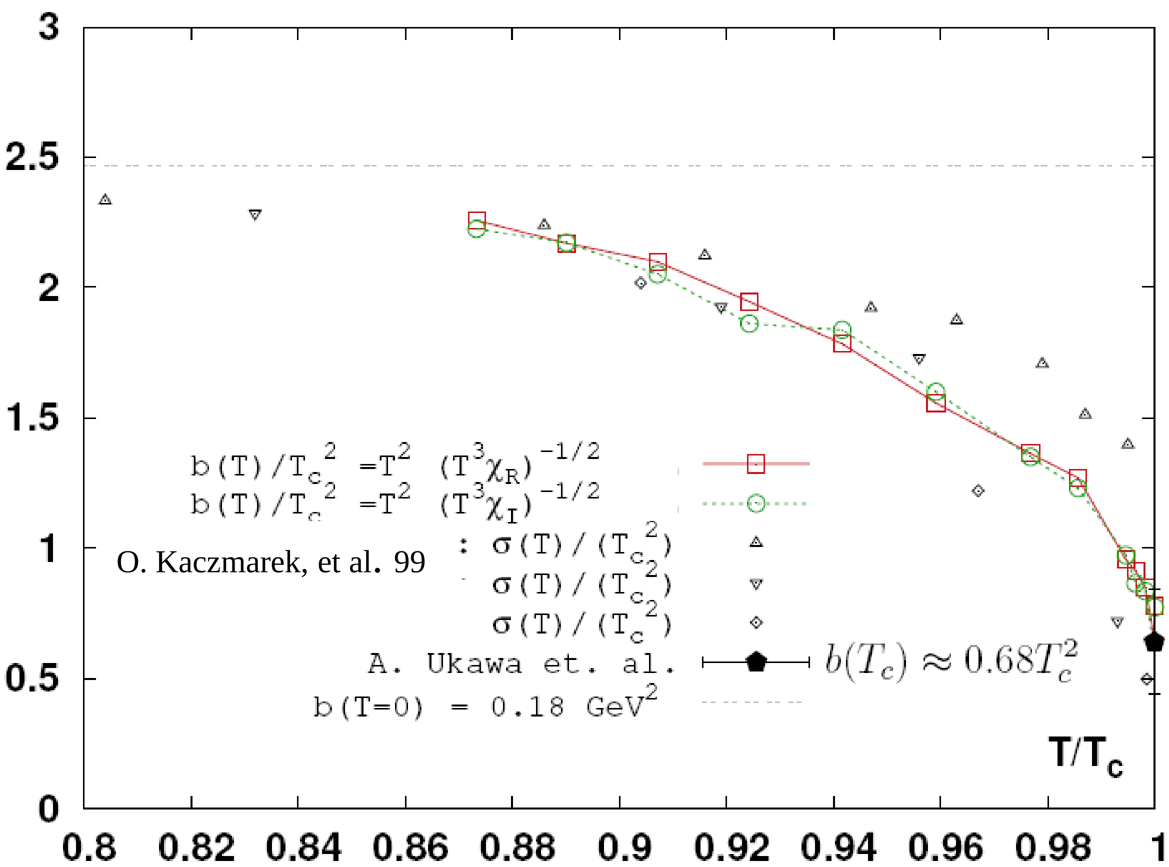
$$\chi_I \propto 1/m_E^2, \quad \chi_R \propto 1/m_M^2$$

Since

$$m_E^2 >> m_M^2 \Rightarrow \chi_I << \chi_R$$

String tension from the PL susceptibilities

Pok Man Lo, et al. (in preparation)



- $T < T_c \Rightarrow \chi_I = \chi_R$

$$\chi_{R,(I)} = 4\pi \int dr\ r^2 C_{R,(I)}(r)$$

- Common mass scale for $C_{R,(I)}(r)$

$$C_{R,(I)}(r) \approx \frac{e^{-M\ r}}{4\pi r T}$$

- In confined phase a natural choose for M

$$M = b / T$$

string tension

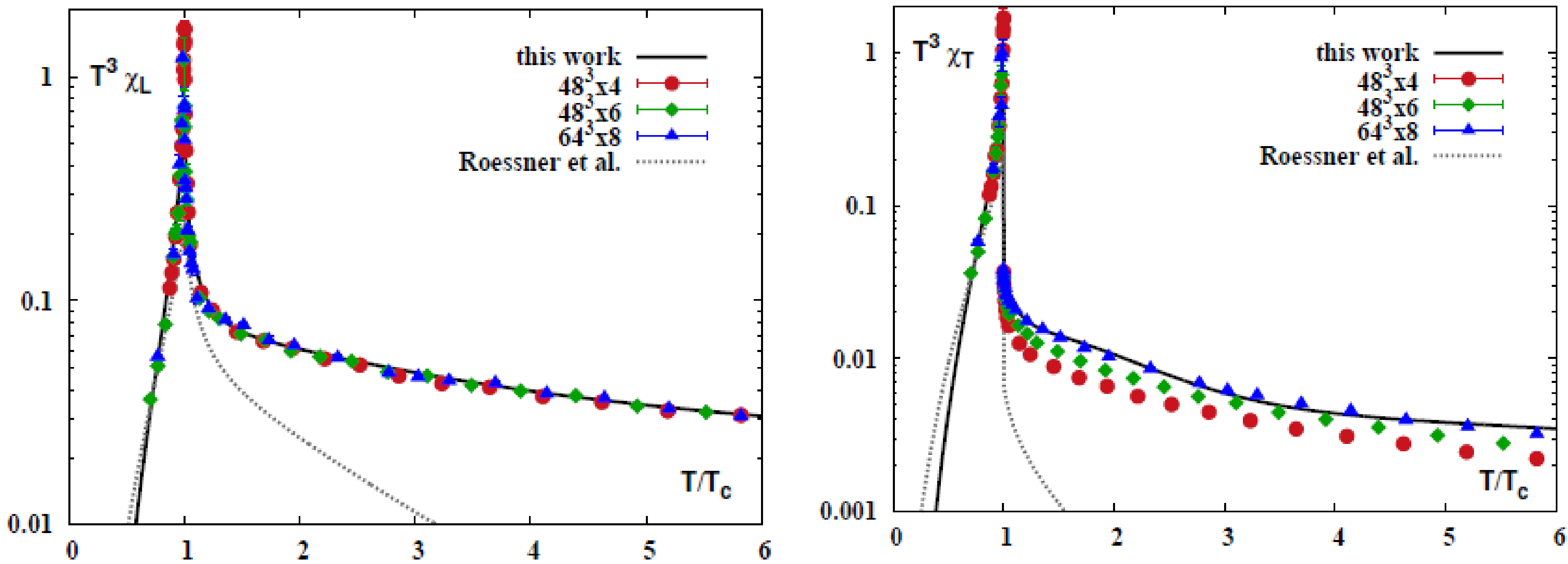
$$b(T)/T_c^2 \approx (T/T_c)^2 (T^3 \chi_{R,(I)})^{-1/2}$$

- The minimal potential needed to incorporate Polyakov loop fluctuations and SU(3) thermodynamics

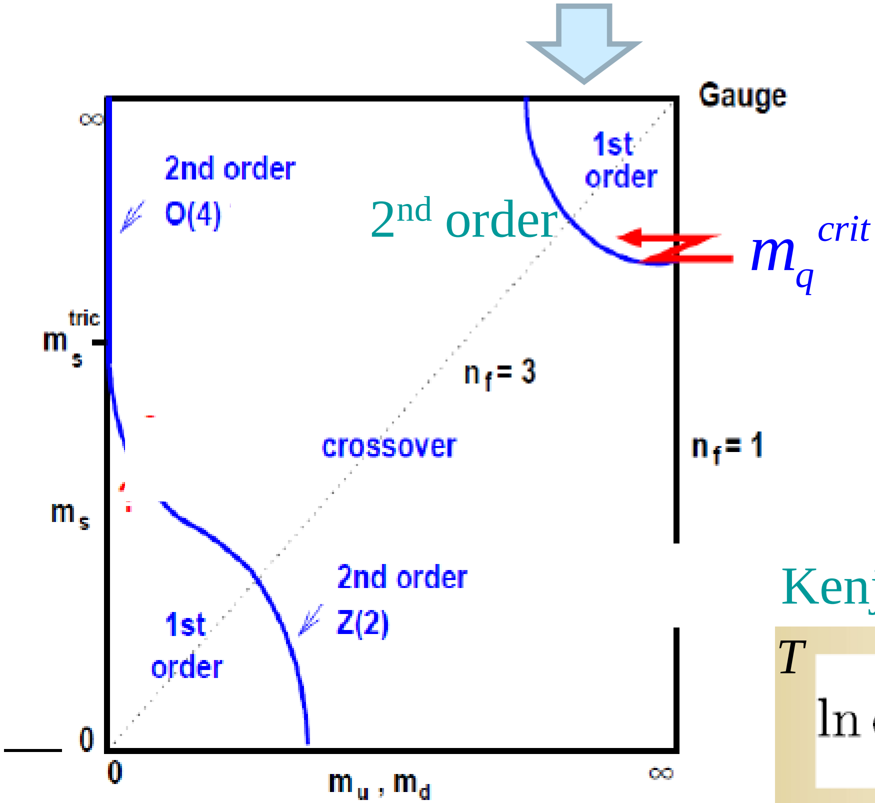
$$Z = \int dL dL^{+} e^{\beta V U(L, L^{+})}$$

$$\frac{U(L,\bar{L})}{T^4} = -\frac{1}{2}a(T)\bar{L}L + b(T)\ln M_H(L,\bar{L}) + \frac{1}{2}c(T)(L^3+\bar{L}^3) + d(T)(\bar{L}L)^2,$$

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



Deconfinement phase transition in the heavy quark region



- Modelling the partition function

$$Z = \int dL dL^+ e^{-\beta V U(L,L^+) + \ln \det[Q_f]}$$

- background field approach

$$\hat{Q}_F = (-\partial_\tau + \mu + igA_4)\gamma^0 + i\vec{\gamma}\cdot\nabla - M_Q$$

Kenji Fukushima

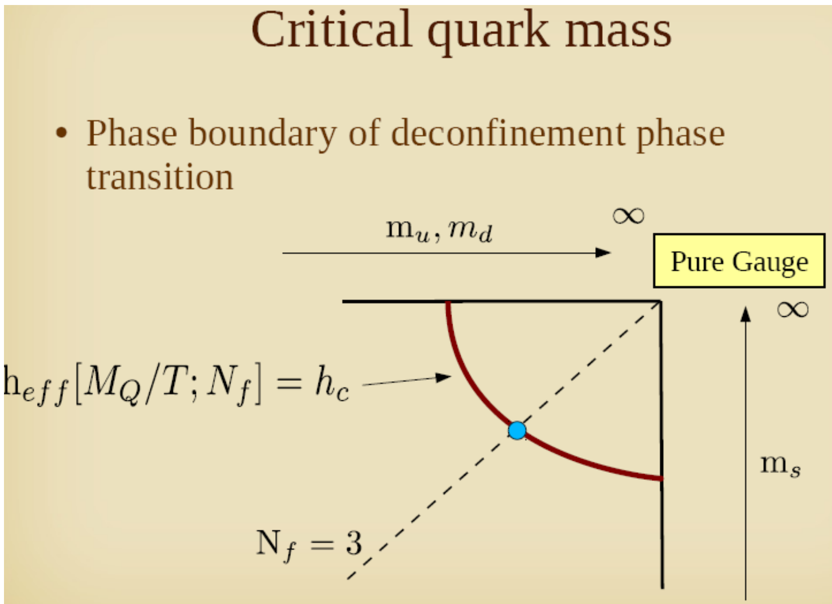
$$\ln \det[\hat{Q}_F] = V 2N_f \int \frac{d^3k}{(2\pi)^3} [3\beta E[k] + \ln g^+ \ln g^-]$$

$$g^\pm = 1 + 3\{L, \bar{L}\} e^{-\beta E^\pm} + 3\{\bar{L}, L\} e^{-2\beta E^\pm} + e^{-3\beta E^\pm}$$

$$E^\pm = E[k] \mp \mu$$

$$E[k] = (k^2 + M_Q^2)^{1/2}$$

PL and heavy quark coupling



- Effective potential

$$\ln \det[Q_f] = -VT^3 U_q[L, L^+; M_q]$$

- Tree level result $M_q >> T$

$$U_q = -h_{eff}(M_q, N_f) L_R$$

G. Green & F. Karsch (83)

$$U_G \to U_G - h_{eff} L_R$$

where

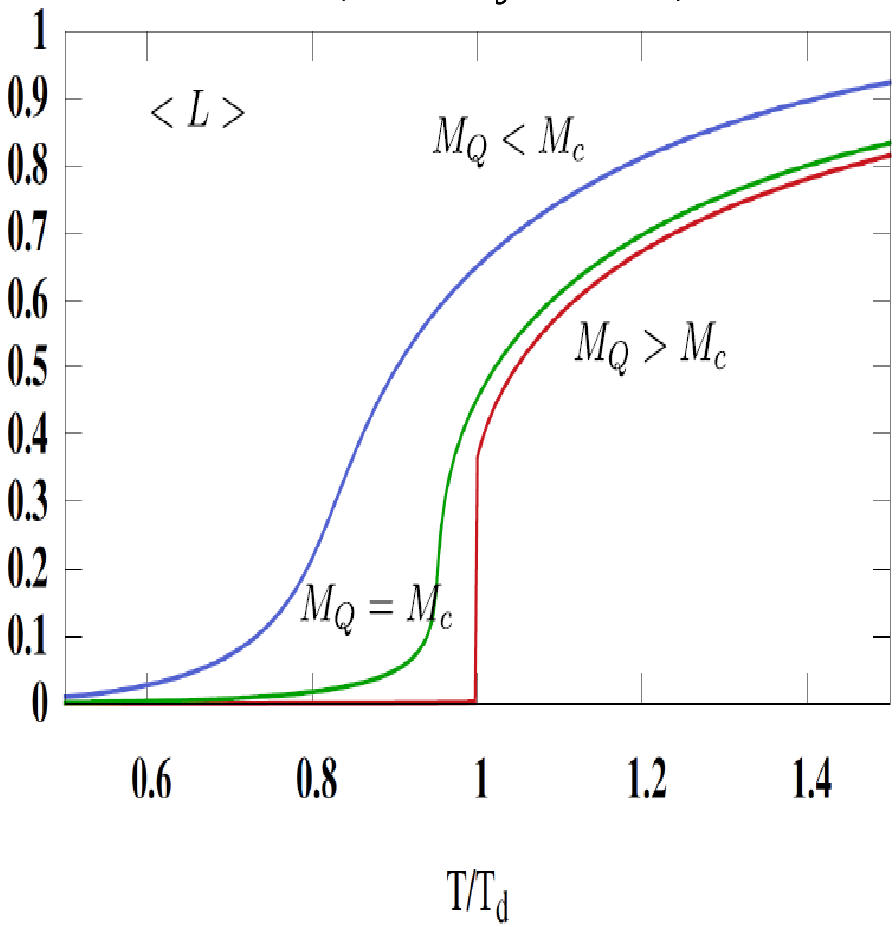
$$h_{eff} \approx N_f N_c (M_q/T)^2 K_2(M_q/T)$$

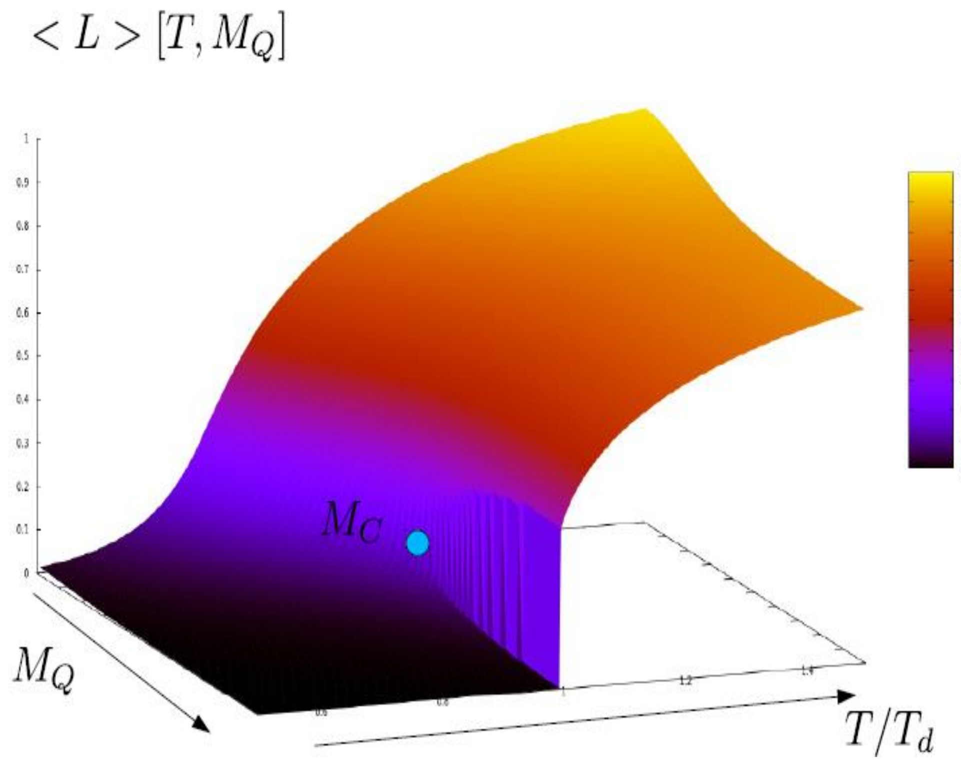
$$h_{eff}^{LGT} = (2N_f)(2N_c)(2\kappa(N_\tau))^{N_\tau} N_\tau^{\;3}$$

H. Saito, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya, Y. Maezawa, H. Ohno, and T. Umeda, Phys. Rev. D **84** (2011) 054502

The critical point of the 2nd order transition

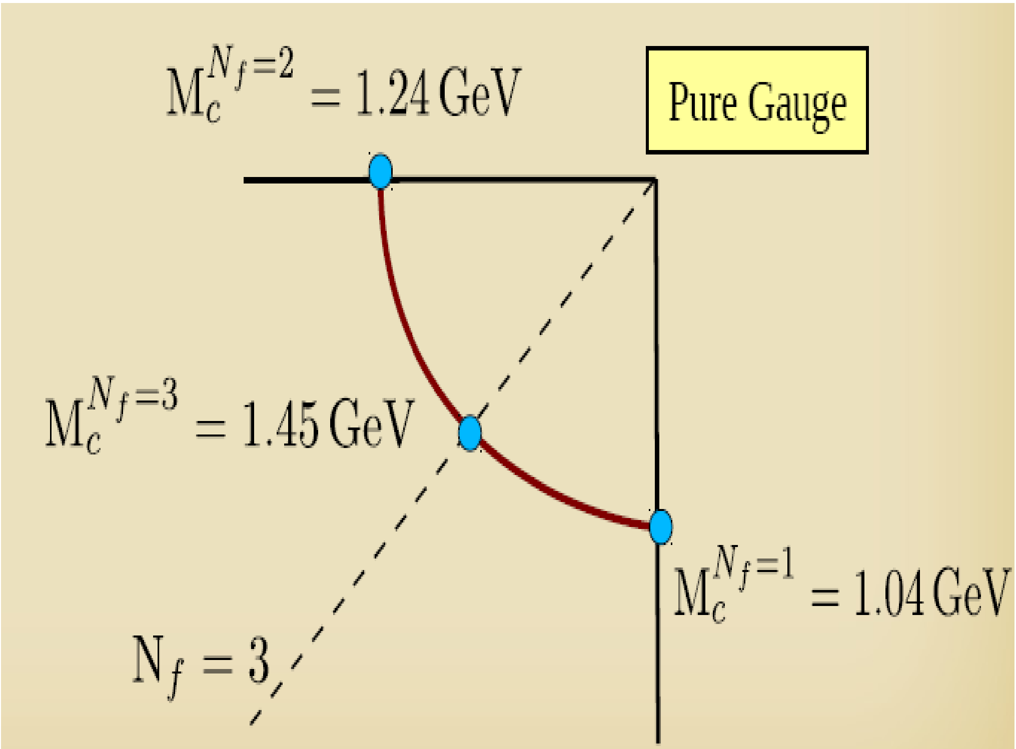
Pok Man Lo, M. Szymanski, C. Sasaki et al.

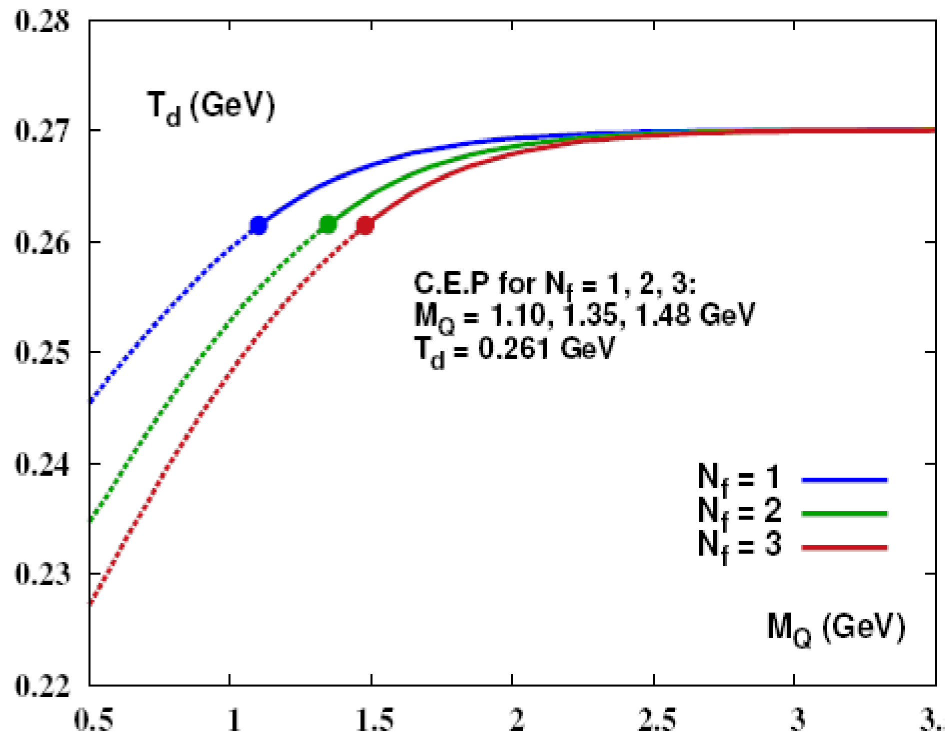




Critical masses and temperature values

Pok Man Lo, et al.





- Different values then in the matrix model by

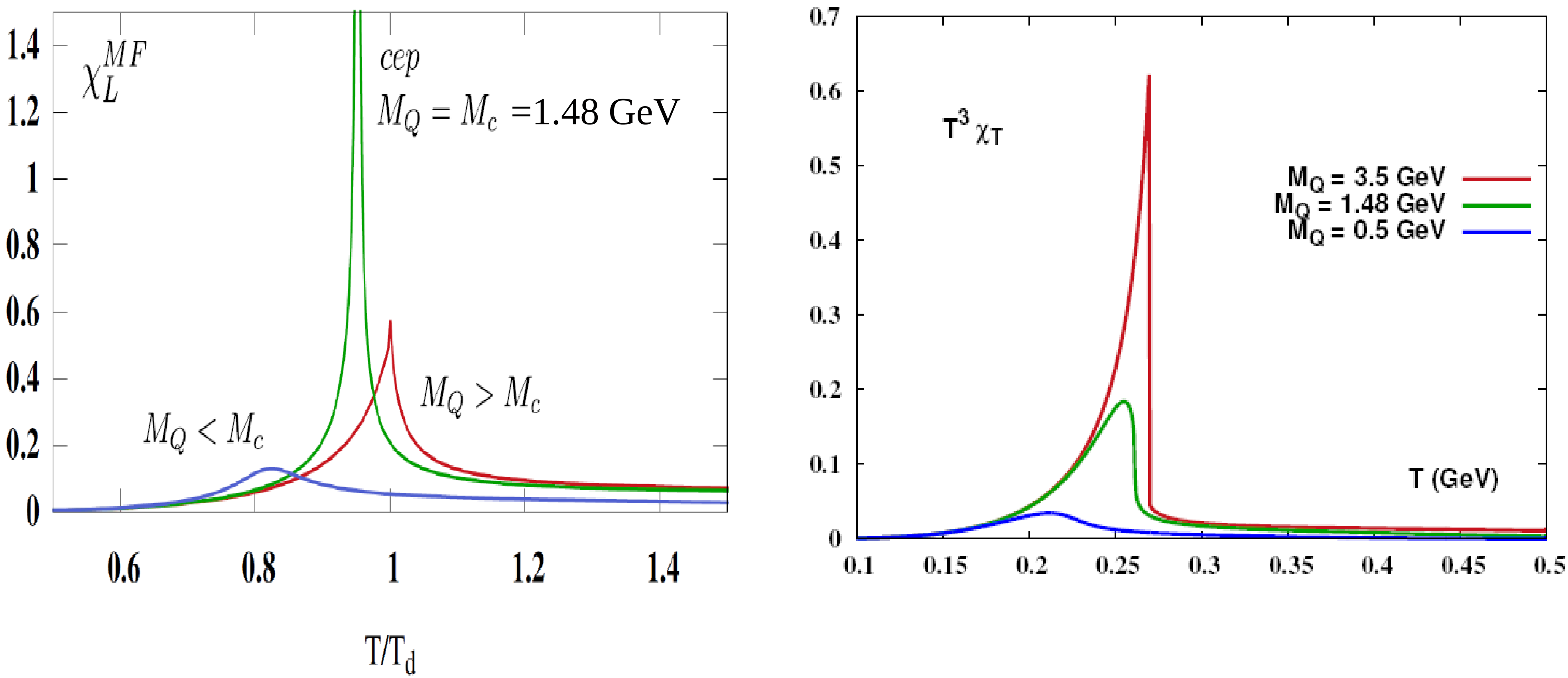
$$M_c^{N_f=3} \approx 2.5\,\text{GeV}$$

$$T_c^{de} \approx 0.27\,\text{GeV}$$

K. Kashiwa, R. Pisarski and V. Skokov,
Phys. Rev. D85 (2012)

LGT C. Alexandrou et al. (99) $M_c^{N_f=1} \approx 1.4 GeV$

Susceptibility at the deconfinement critical point



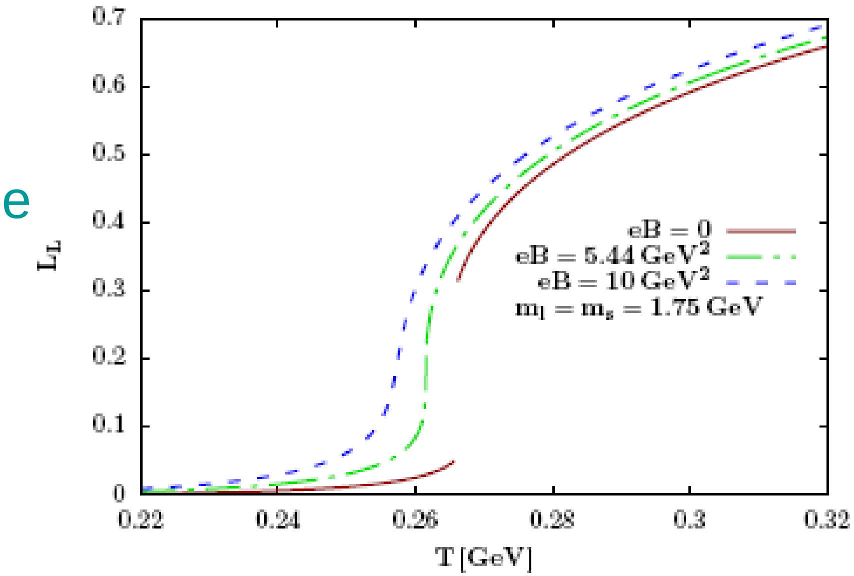
- Divergent longitudinal susceptibility at the critical point

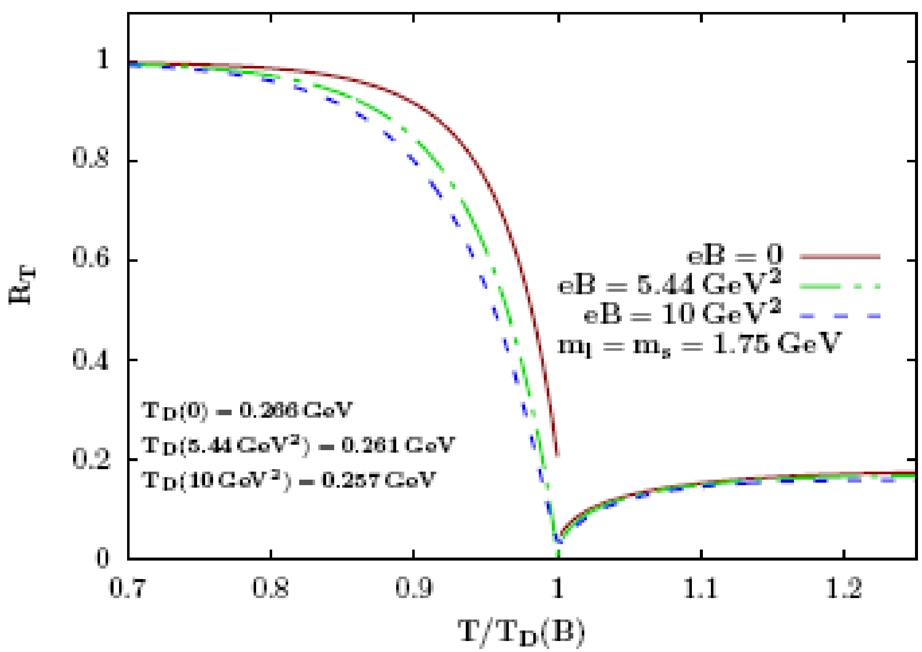
Deconfinement in the background magnetic field $|\vec{B}|$

- In the constant magnetic field background, the motion of charged particles undergoes the Landau quantization in the transverse plane

$$h(T,eB,m) = \frac{3|eB|}{2\pi^2 T}\sum_{s=\pm1}\sum_{k=0}^{\infty}\int dp_z e^{-E_B/T}$$

$$E_B = \sqrt{m^2 + p_z^2 + |eB|(2k+1-s)}$$





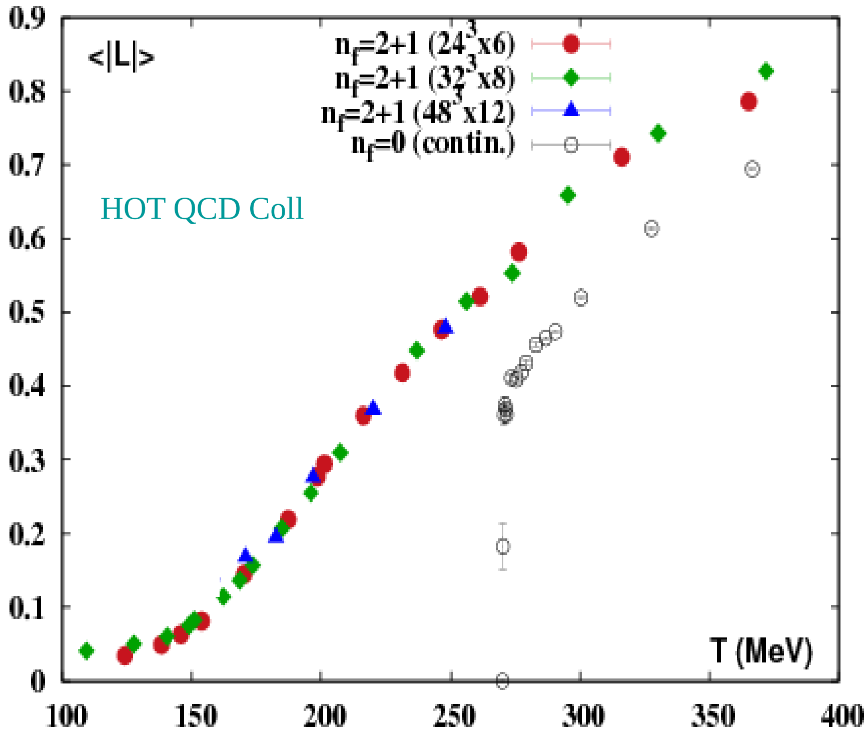
In the presence of Z(N) symmetry breaking term, the Polyakov loop fluctuation ratios exhibit similar "asymptotic" behavior as in the pure gauge theory.

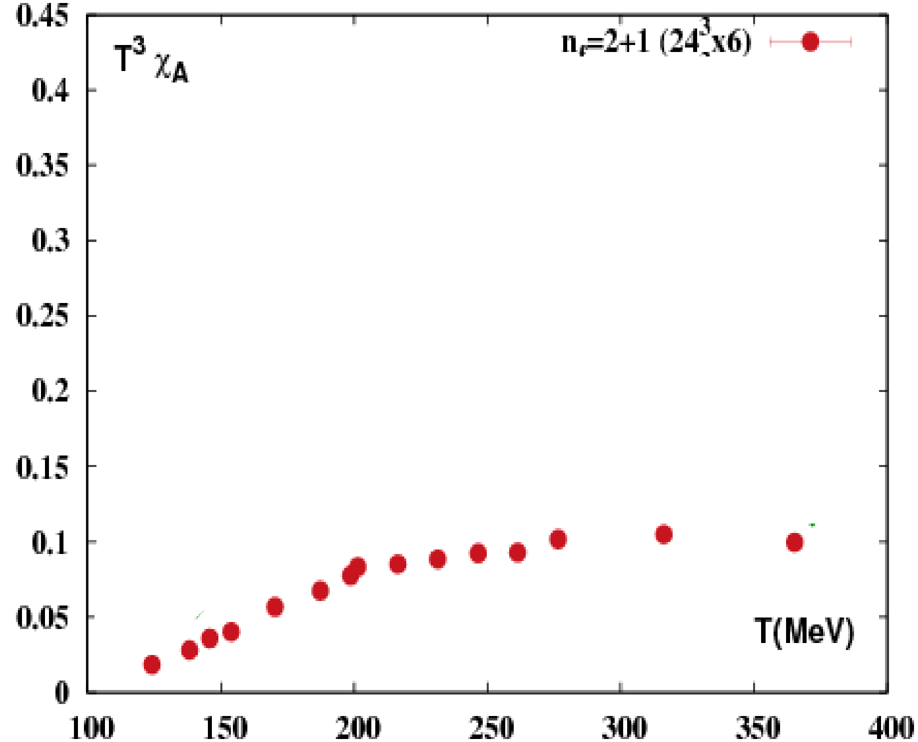
How these properties are modyfied by dynamical quarks?

Polyakov loop and fluctuations in QCD

- Smooth behavior for the Polyakov loop and fluctuations

  ⇨ difficult to determine deconfinement temp.



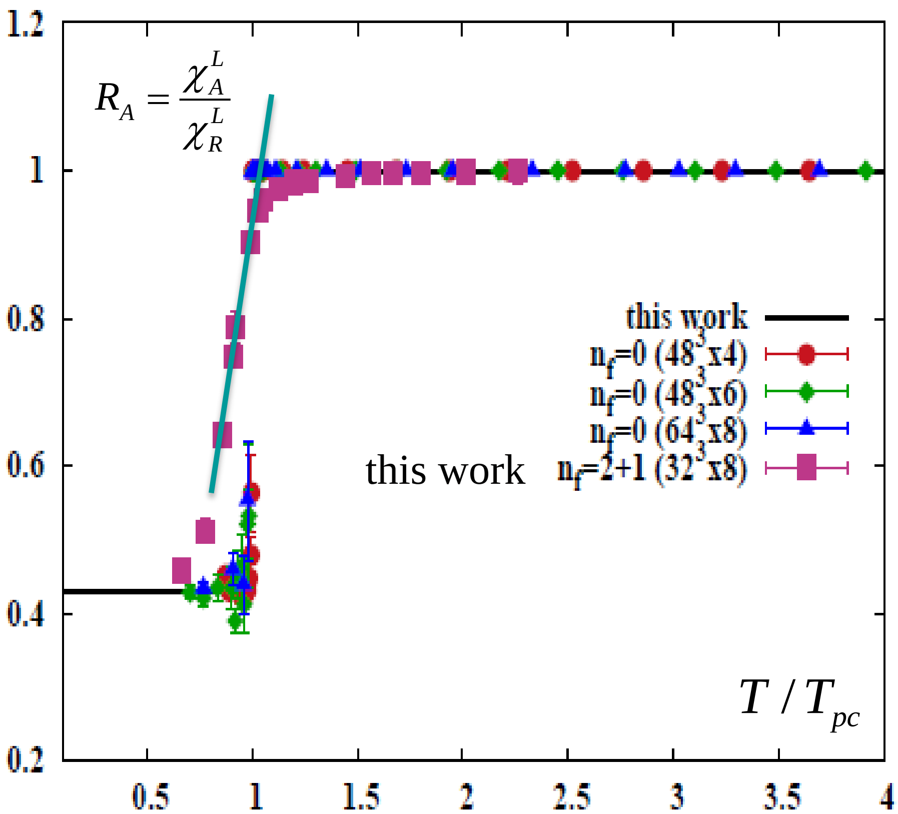


The inflection point at $T_{dec} \approx 0.22GeV$

The influence of fermions on the Polyakov loop susceptibility ratio

- Z(3) symmetry broken, however ratios still showing deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



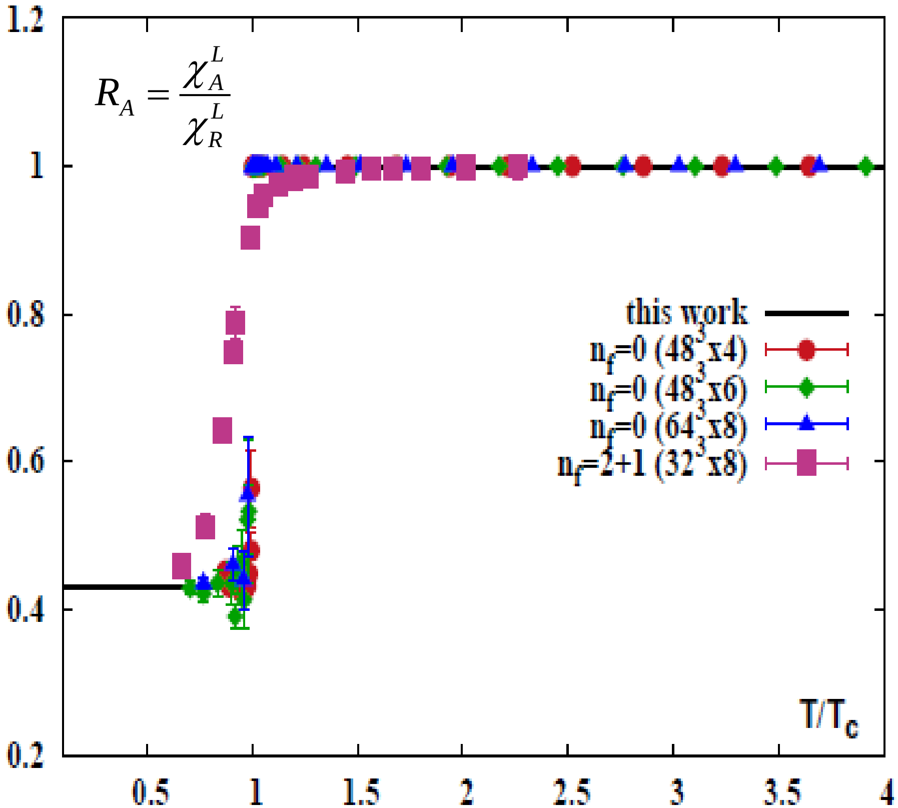
- Change of the slope in the narrow temperature range signals color deconfinement?

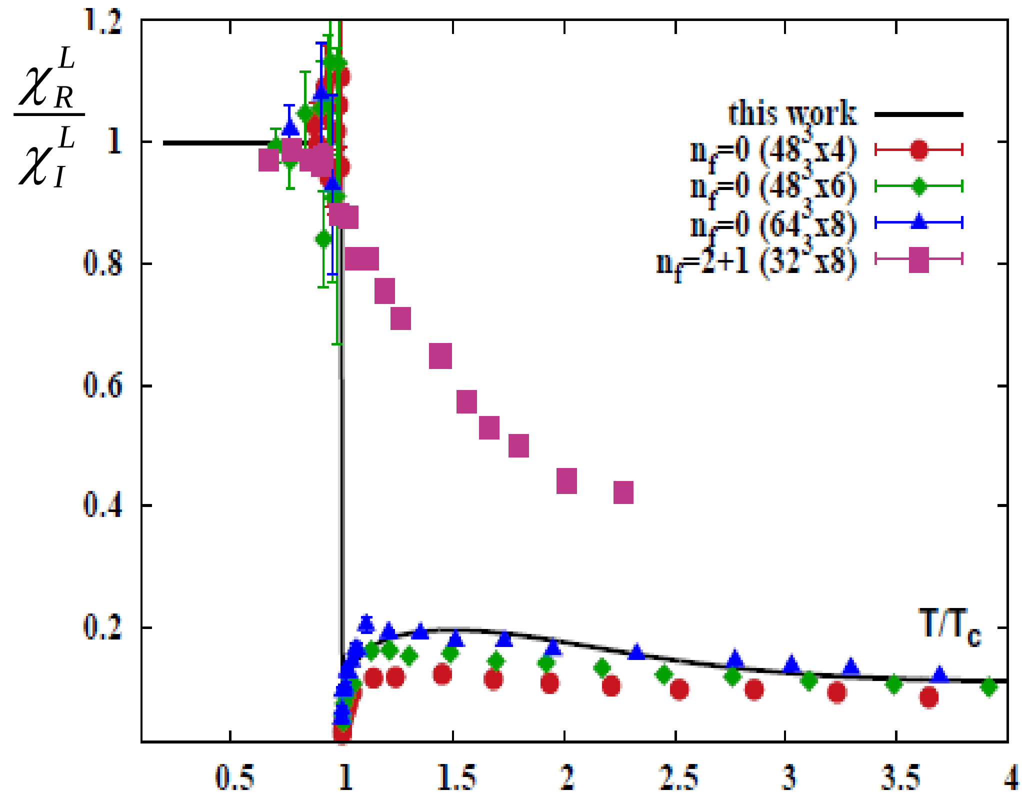
- Dynamical quarks imply smoothening of the susceptibilities ratio, between the limiting values as in the SU(3) pure gauge theory

The influence of fermions on ratios of the Polyakov loop susceptibilities

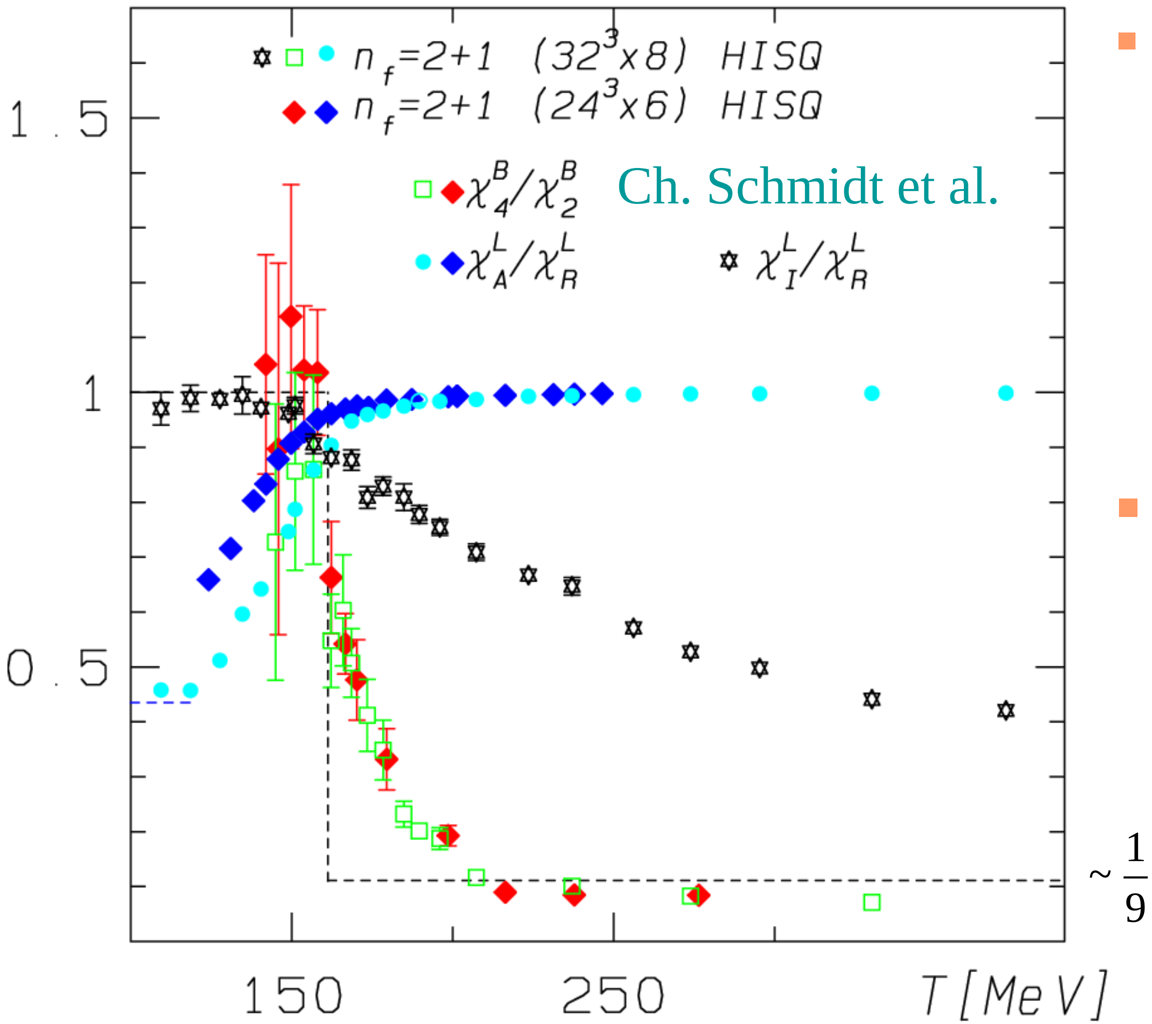
- Z(3) symmetry broken, however ratios still showing the transition
- Change of the slopes at fixed T

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.





Polyakov loop susceptibility ratios still away from the continuum limit:



- The renormalization of the Polyakov loop susceptibilities is still not well described: strong dependence on N_τ in the presence of quarks.

- Kurtosis of the net baryon number measures the squared of the baryon number carried by leading particles in a medium S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

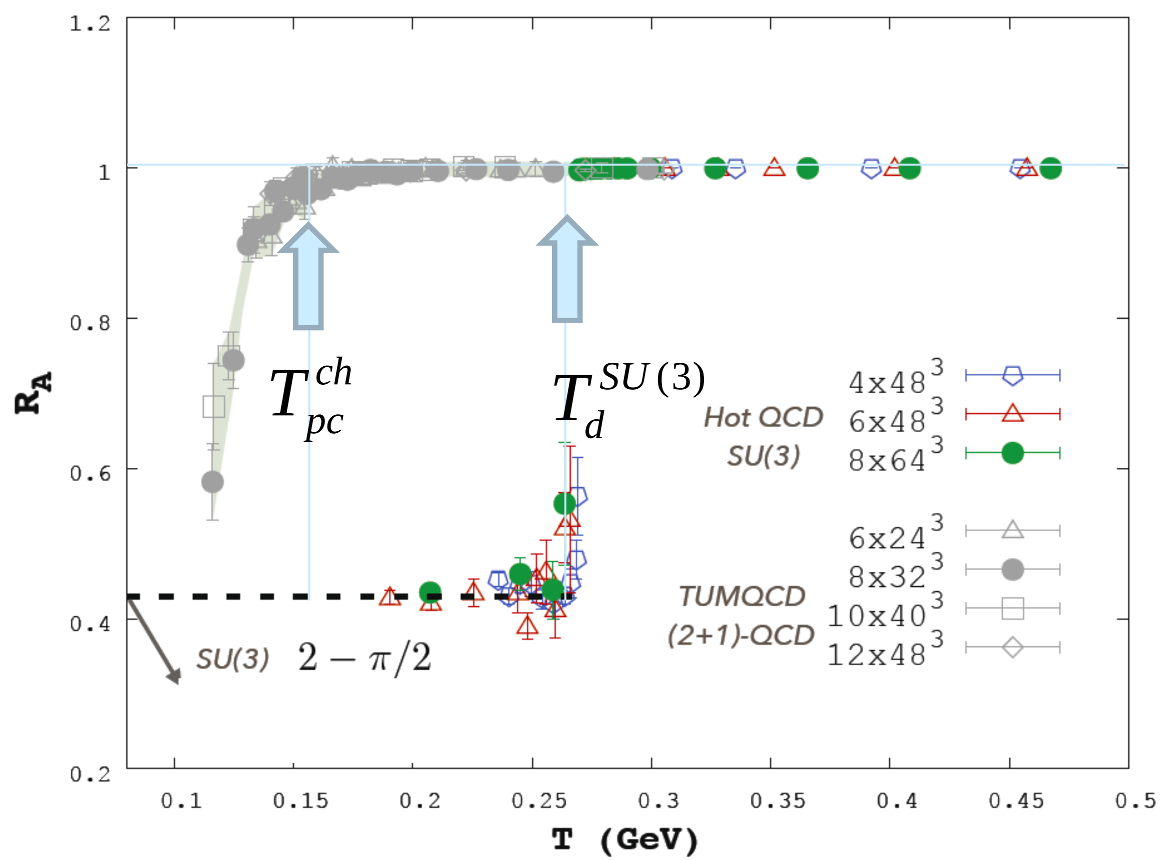
Susceptibility ratio renormalization with gradient flow

A. Bazavov, N. Brambilla, H. -T. Ding, P. Petreczky, H. -P. Schadler, A. Vairo, J.H. Weber, Phys.Rev. D93 (2016)

- For flow time $f = f_0$ there is no cutoff dependence in R_A , i.e. it looks to be renormalized quantity
- However, with increasing f the R_A increases towards unity at low T and loses information about inflection at $T \approx 0.155$ GeV
- Furthermore, R_A maybe intensive quantity

Pok Man Lo, et al. Phys. Rev.

Flow time $f_0 = 0.2129$ fm



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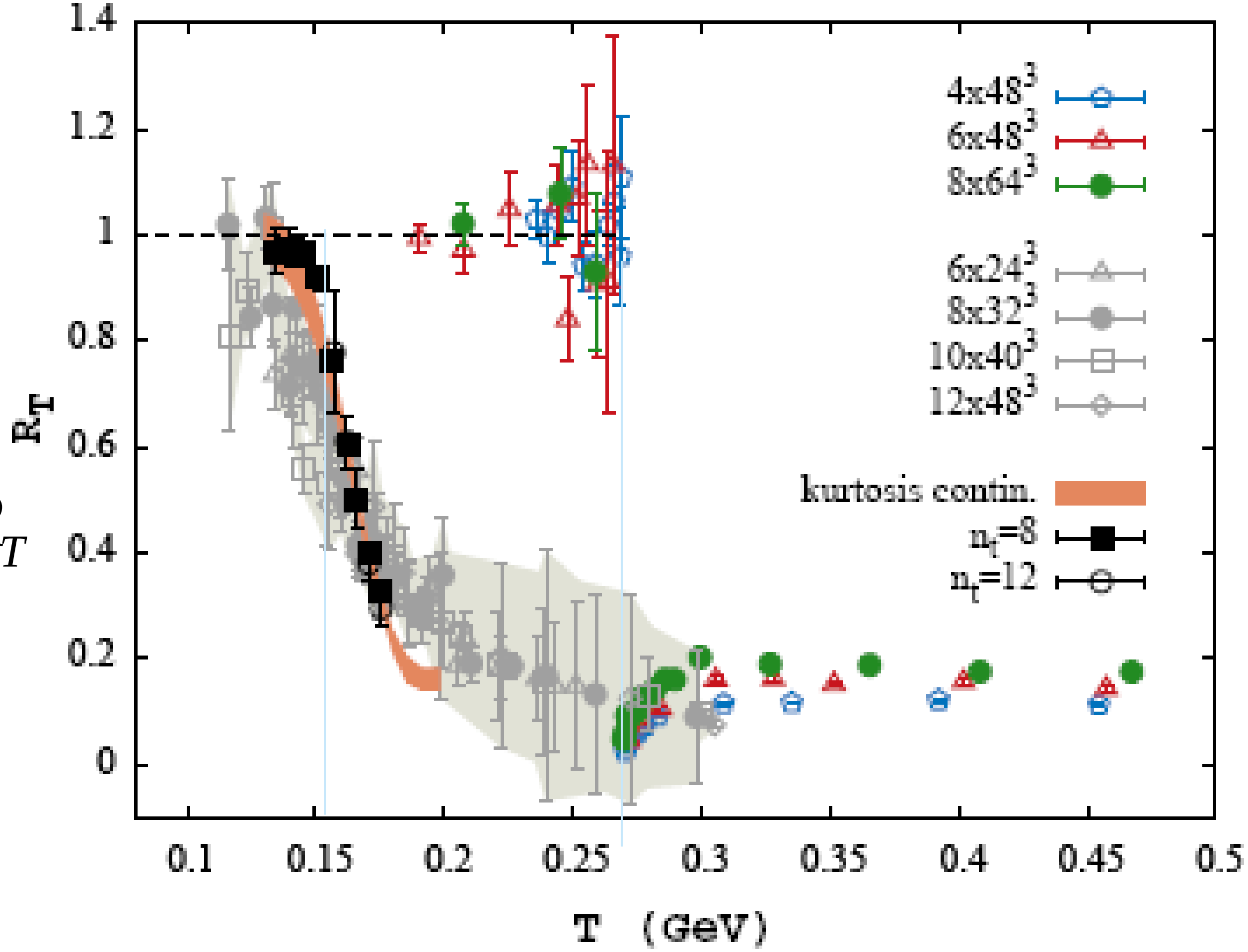
- R_T decouples from unity at $0.15 < T < 0.16$, thus is consistent with the chiral crossover T

- The rates of change of kurtosis of the net-baryon number and R_T ratio with temperature are similar

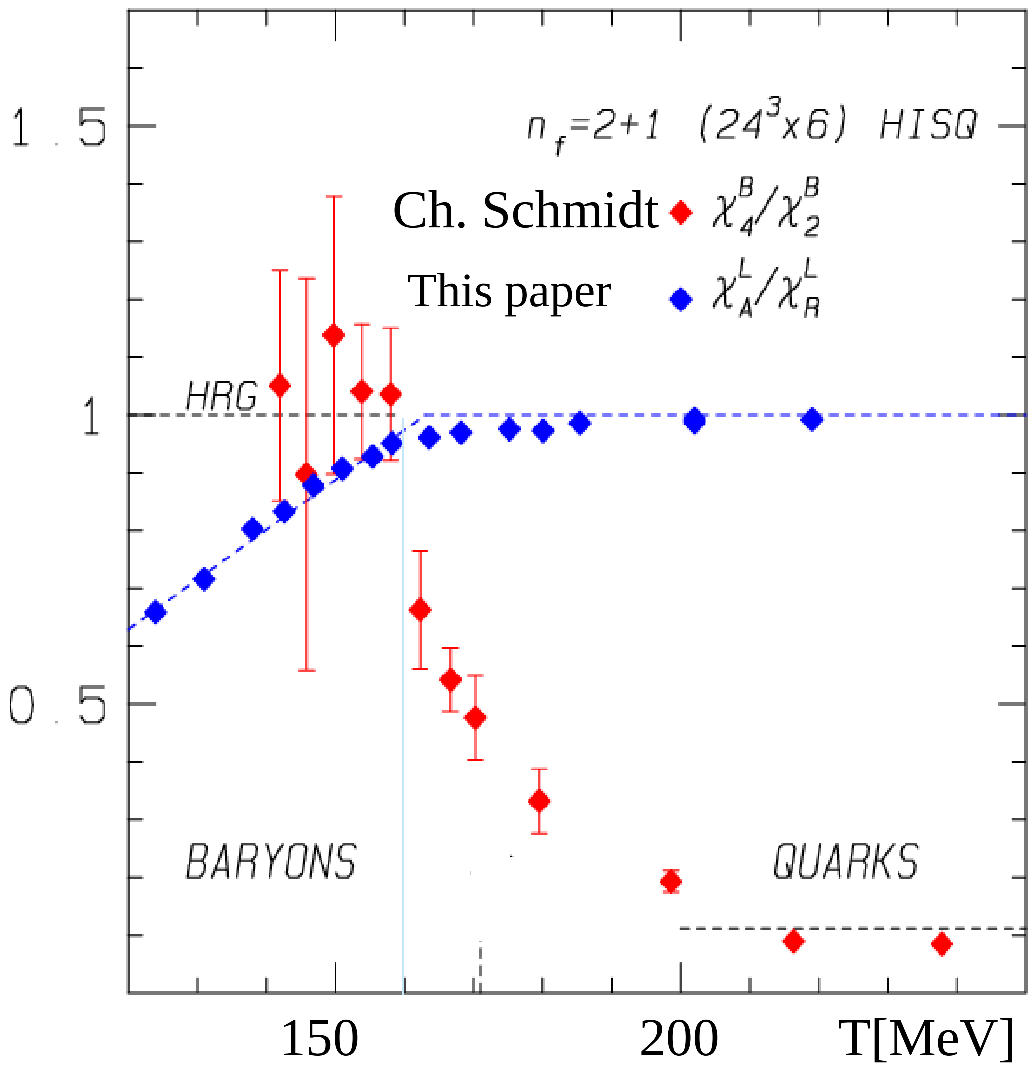


Conclusions:

- Ratios of 2nd order Polyakov loop susceptibilities absolute to real R_A and imaginary to real R_T are excellent probes of deconfinement in SU(N) gauge theory
- The R_T ratio can be used to signal deconfinement in QCD
- Further MC studies of these quantities is needed to understand renormalization and quark mass dependence of R_T
- Interplay of the Polyakov loop and quark number fluctuations is not to be excluded ?

Probing deconfinement in QCD

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



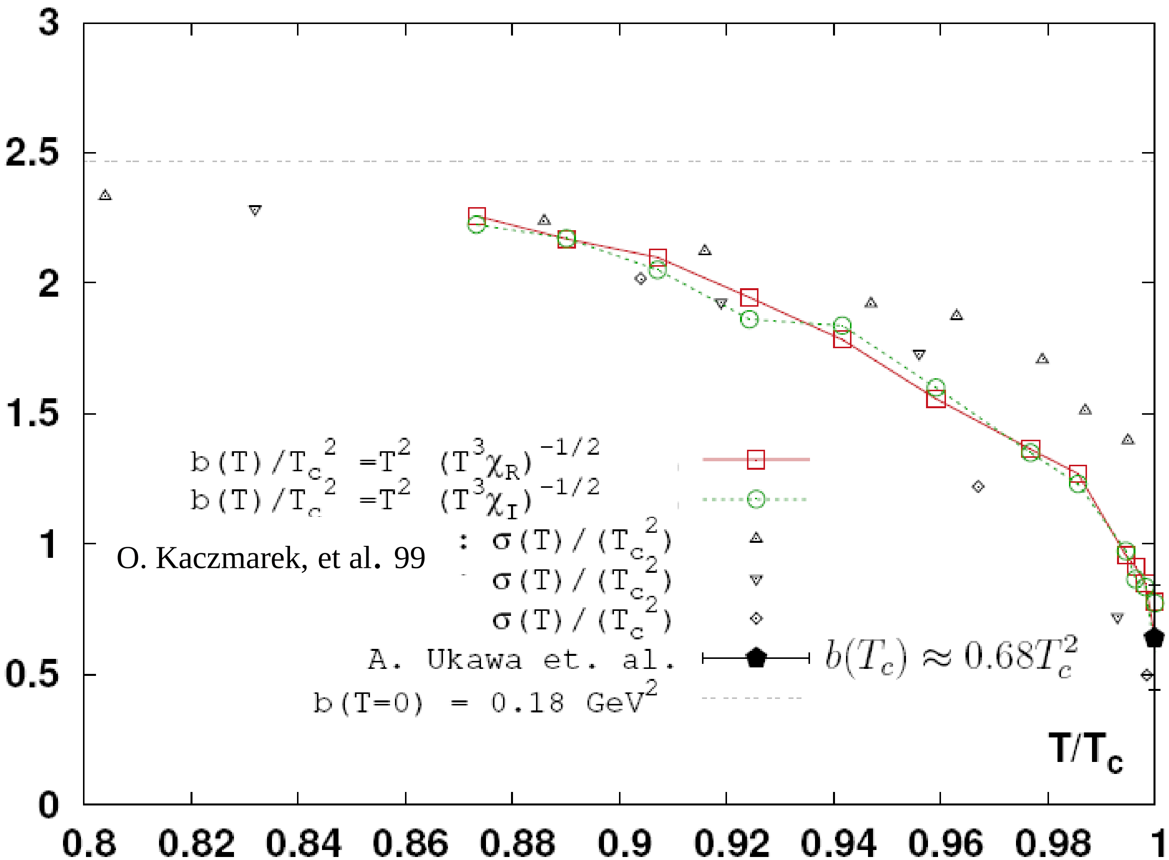
Change of the slope of the ratio of the Polyakov loop susceptibilities χ_A^L / χ_R^L appears at the same T where the kurtosis drops from its HRG asymptotic value

- In the presence of quarks there is "remnant" of Z(N) symmetry in the χ_A^L / χ_R^L ratio, indicating deconfi- nement

Still the lattice finite size effects need to be studied

String tension from the PL susceptibilities

Pok Man Lo, et al. (in preparation)



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- In confined phase a natural choose for M

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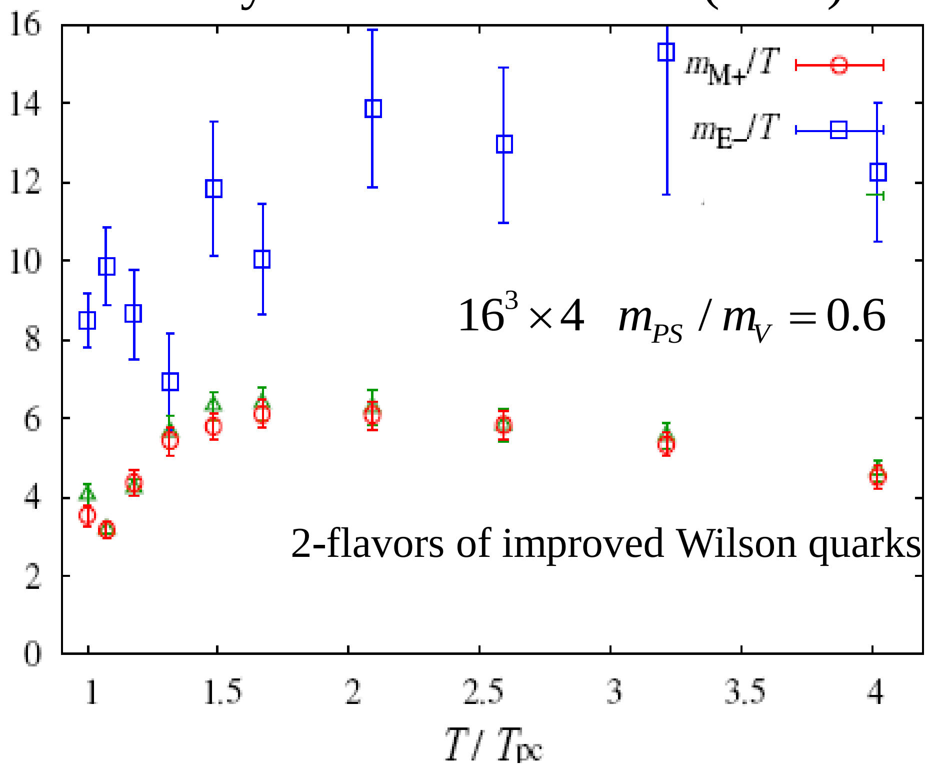
Y. Maezawa[1], S. Aoki[2], S. Ejiri[3], T. Hatsuda[4], N. Ishii[4], K. Kanaya[2], N. Ukita[5] and T. Umeda[6]
Phys. Rev. D81 091501 (2010)



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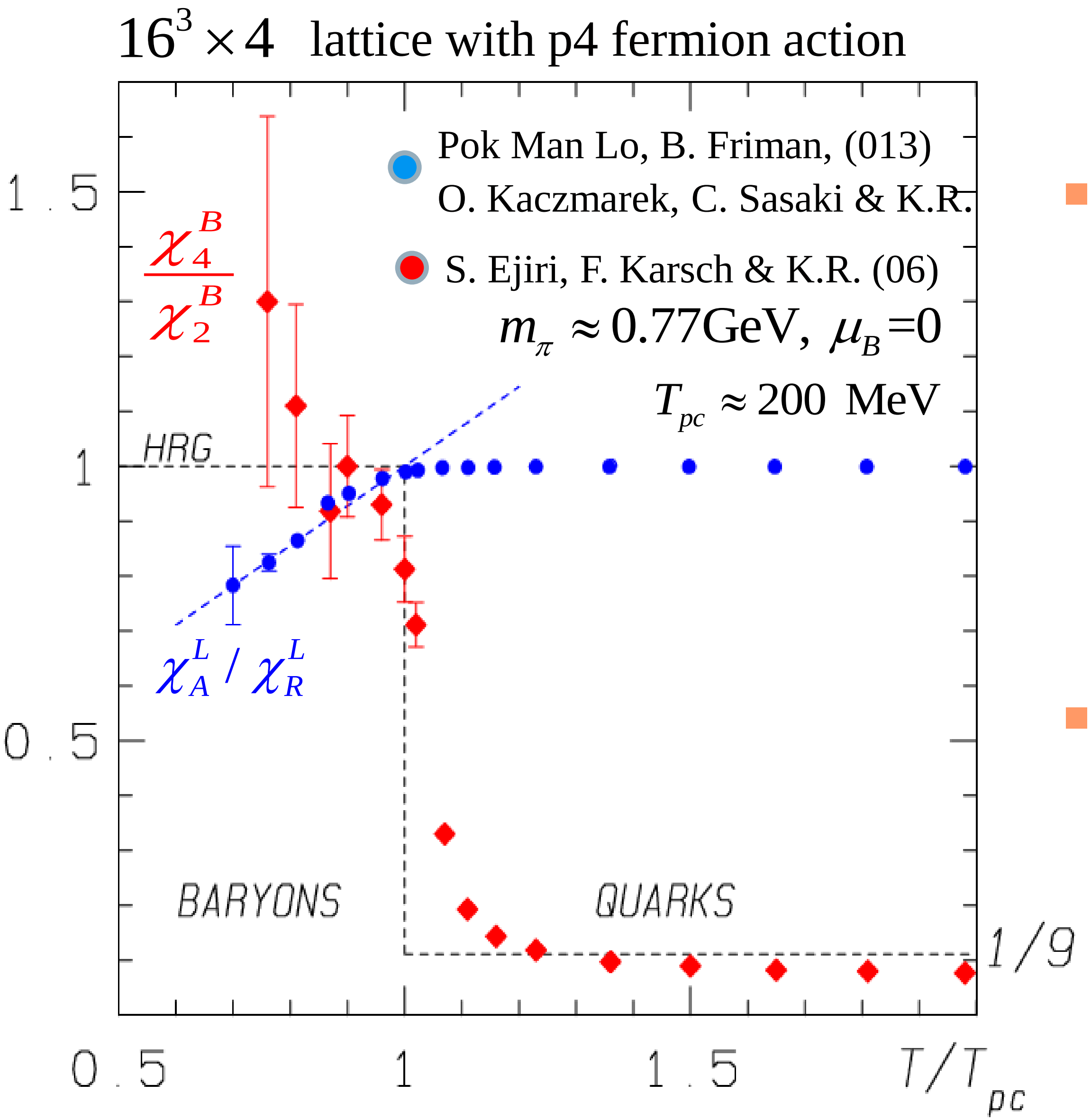
Probing deconfinement in QCD



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