Hydrodynamization of QGP: from kinetic theory to moment equations

Li Yan

Institute of Modern Physics, Fudan University

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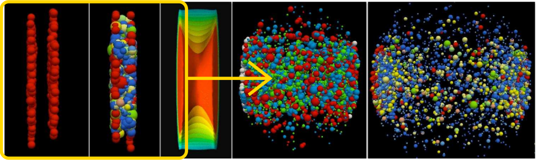
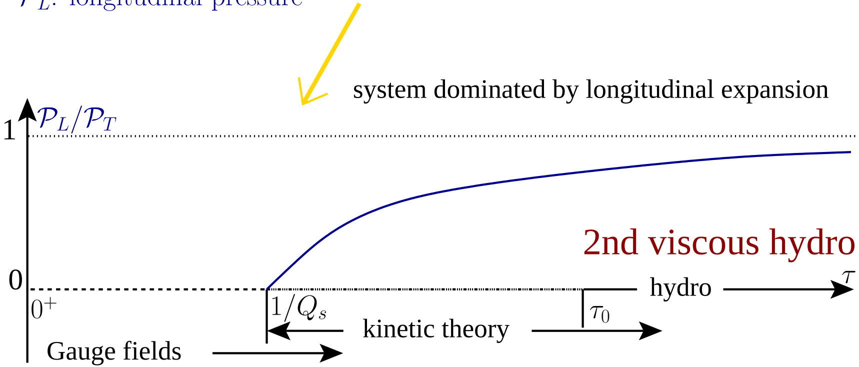
1712.03856, 1703.10694, 1904.08677, with Jean-Paul Blaizot

Thermalization of out-of-equilibrium QGP system

$\mathcal{P}_T$: transverse pressure

$\mathcal{P}_L$: longitudinal pressure





- A theoretically challenging question to have $\tau_0 \sim O(1)$ fm/c.
- Motivation from collective flow observation in small systems.

Kinetic theory description

Distribution function of quarks and gluons,

$$f(t,\mathbf{x},\mathbf{p}) \quad \Longleftrightarrow \quad \begin{cases} \int_{\mathbf{p}} p^\mu f(t,\mathbf{x},\mathbf{p}) = n^\mu & \sim \text{hydro} \\ \int_{\mathbf{p}} p^\mu p^\nu f(t,\mathbf{x},\mathbf{p}) = T^{\mu\nu} & \sim \text{hydro} \\ \int_{\mathbf{p}} p^\mu \ldots p^\nu f(t,\mathbf{x},\mathbf{p}) = M^{\mu\ldots\nu} & \end{cases}$$

- Distribution function satisfies kinetic equation,

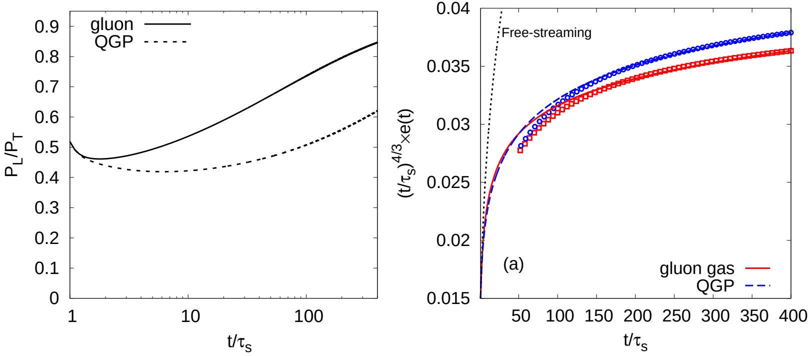
$$p^\mu \partial_\mu f(t,\mathbf{x},\mathbf{p}) = \mathcal{C}[f]$$

- Diffusion approximation for all QCD elastic scatterings: $gg \leftrightarrow gg$ ...
  [J-P. Blaizot, B. Wu, L. Yan]

$$\mathcal{C}[f] \to -\nabla_{\mathbf{p}} \cdot \mathcal{J}(\mathbf{p}) - \mathcal{S}(\mathbf{p})$$

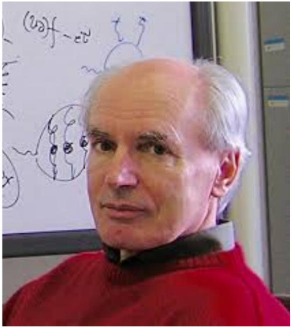
- Transient gluon BEC during thermalization, ... still an open question.
  [J-P. Blaizot, L. McLerran, J. Liao, R. Venugopalan, ...]

Isotropization with diffusion approximation



$$\text{reduction of d.o.f. : } \underbrace{\text{kinetic theory}}_{\text{all moments}} \Longrightarrow \underbrace{\text{hydro}}_{\text{finite number of moments!}}$$

To capture the reduction of d.o.f.



Can we find some types of moments/modes, which are sensitive to the angular dependence of the distribution function, and are sufficient to describe system hydrodynamization ?

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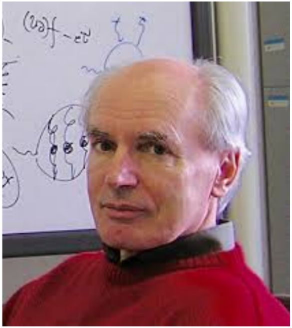

Can we find some types of moments/modes, which are sensitive to the angular dependence of the distribution function, and are sufficient to describe system hydrodynamization ?

- Dimension = energy-momentum tensor.
- Related to ϵ, $\mathcal{P}_L$ and $\mathcal{P}_T$.

$\mathcal{L}$-moment

p^2-moment weighted with Legendre Polynomial P_{2n}:

$$\mathcal{L}_n = \int \frac{d^3p}{(2\pi)^3 p^0} p^2 \underbrace{P_{2n}(v_z = p_z/p^0)}_{\text{Legendre Polynomial}} f(\tau, \vec{p}_\perp, p_z),$$

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- Mass dimension of $\mathcal{L}_n$ is same as $T^{\mu\nu}$.
- $n = 0 \quad \Leftrightarrow \quad$ energy density: $\mathcal{L}_0 = \epsilon$.
- $n = 1 \quad \Leftrightarrow \quad$ pressure anisotropy: $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$.
- $n \geq 2 \quad \Leftrightarrow \quad$ finer structure of f (or δf).

In the case of Bjorken expansion close to equilibrium: $\mathcal{L}_n = \sum_{m=n}^{\infty} \frac{c_m}{\tau^m}$

Equation of motion for $\mathcal{L}_n$

Transport equation with relaxation time approximation :

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p},\tau) = -\frac{f(\mathbf{p},\tau) - f_{\text{eq}}(p/T)}{\tau_R}, \qquad \tau_R = \tau_R(T) \sim \frac{\eta}{s}$$

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which leads to

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}), \quad n = 0, 1, ...$$

- a_n, b_n and c_n are constant coefficients.

$$a_0 = \frac{4}{3}, \quad a_1 = \frac{38}{21}, \quad \ldots$$

- τ_R/τ (Knudsen number) $\Leftrightarrow$ How far a system is away from equilibrium

Truncation of the coupled equations

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\,\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}) \quad n = 0, 1, ...$$

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- at $n = 0$

$$\frac{\partial \mathcal{E}}{\partial \tau} + \frac{4}{3}\frac{\mathcal{E}}{\tau} = 0 \quad \rightarrow \quad \mathcal{E} \sim \tau^{-4/3} \qquad \text{ideal hydro}$$

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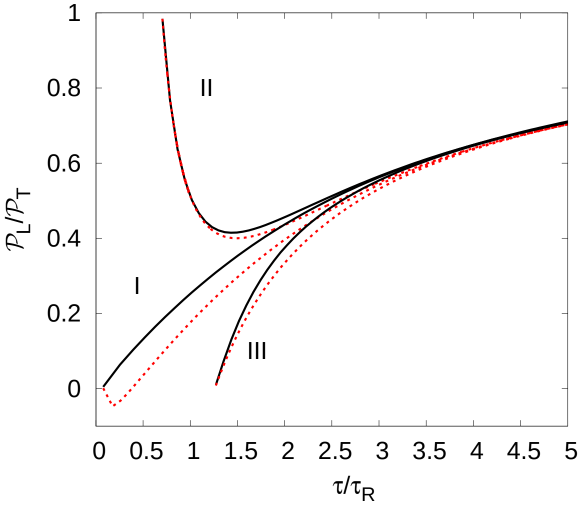
- at higher orders ...

EoM of $\mathcal{L}$-moments and exact solutions



- Truncation at $n = 2$ already results in good description.
- Agreements of truncation at higher orders ($n \geq 5$) are remarkable.

The free-streaming fixed points: $\tau/\tau_R \to 0$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right]$$

For infinite n:

- $\mathcal{L}_0 = \mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \ldots$

$$\Rightarrow \mathcal{L}_n = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right)^2 \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = -2$$

- $\mathcal{L}_n(\tau) = P_{2n}(0)\mathcal{L}_0(\tau)$,

$$\Rightarrow \mathcal{L}_n(\tau) = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right) \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = -1$$

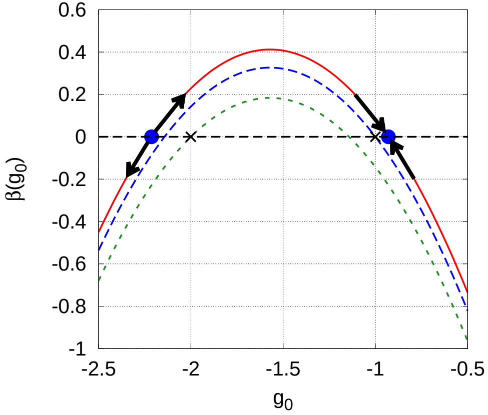
For finite n,

$$\begin{pmatrix} a_0 & c_0 & 0 & 0 & \ldots \\ b_1 & a_1 & c_1 & 0 & \ldots \\ 0 & b_2 & a_2 & c_2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} \Rightarrow \qquad \approx -2(\text{unstable}) \text{ and } \approx -1(\text{stable})$$

The free-streaming fixed points: $\tau/\tau_R \to 0$

- Define $g_n \equiv \partial \ln \mathcal{L}_n / \partial \ln \tau$, and beta function from moment EoM,

$$\beta(g_0, w) \equiv \tau \frac{\partial g_0}{\partial \tau} = -g_0^2 - (a_0 + a_1)\, g_0 - a_1 a_0 + c_0 b_1$$



The hydro fixed points: $\tau/\tau_R \to \infty$

Ansatz form of gradient expansion

$$\mathcal{L}_n = \sum_{m=0} \frac{\alpha_m^{(n)}}{\tau^n}$$

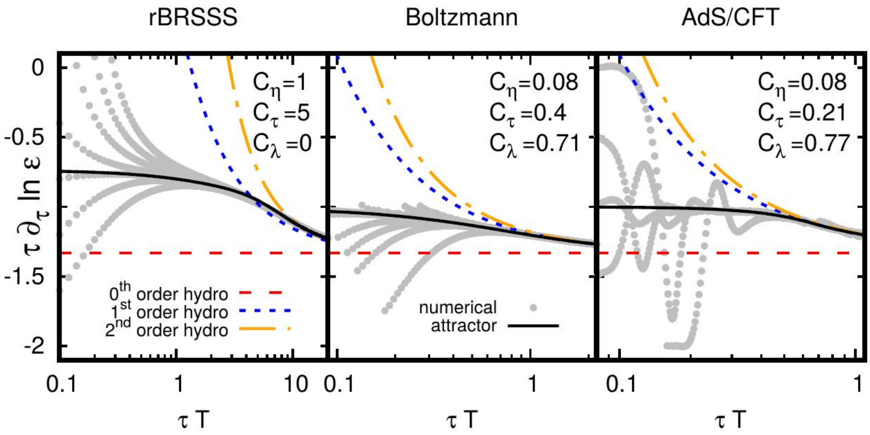
asymptotic decay rate determined by the leading term: $\mathcal{L}_n \sim \alpha_n^{(n)}/\tau$

$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+2n}{3} \quad (\tau_R \propto 1/T)$$

$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+3n}{3} \quad (\tau_R \text{ constant})$$

These are stable fixed points in the hydro regime.

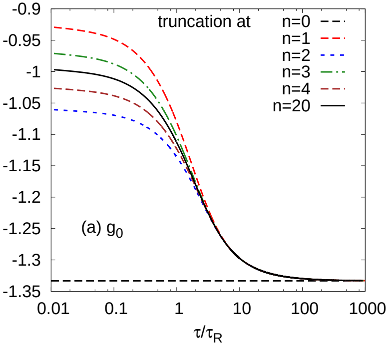
Attractor solutions



[M.Heller, M. Spalinski, R. Janik, G. Basar, G. Dunne, P. Romatschke, ...]

Fixed points and hydro attractor

Define: $g_n = \tau \partial_\tau \ln \mathcal{L}_n$



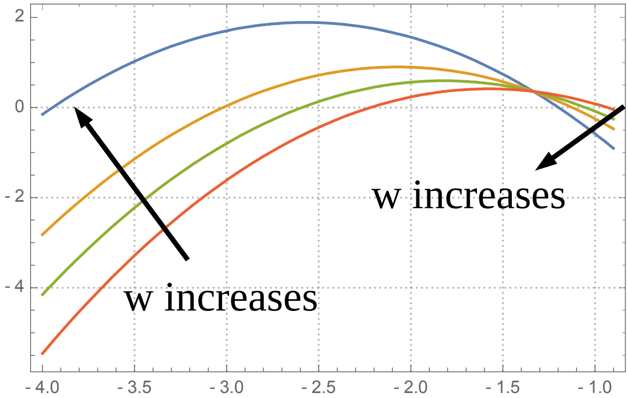
- Free-streaming limit: fixed points of all g_n degenerate ≈ -1.
- Hydro limit: fixed points of g_n split according to n.
- System evolves between these two types of fixed points $\Rightarrow$ *attractor*.

e.g., ideal hydro is a trivial attractor solution: $g_0 = \text{const.} = -4/3$

Attractors as smooth connections between fixed points

- Truncation at $n = 2$:

$$\beta(g_0, w) \equiv \tau \frac{\partial g_0}{\partial \tau} = -g_0^2 - (a_0 + a_1 + w)\, g_0 - a_1 a_0 + c_0 b_1 - a_0 w$$



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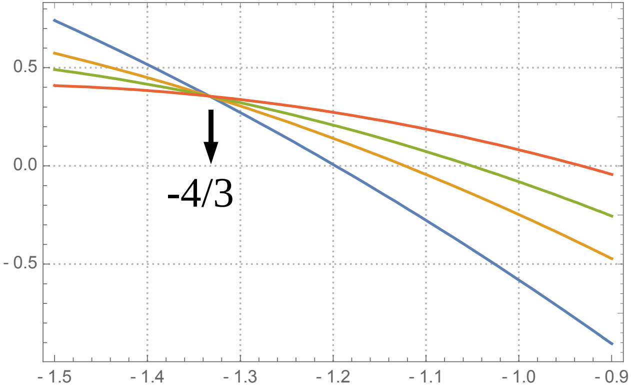


How attractor connects these fixed points?

Adiabatic evolution of pre-hydro mode:

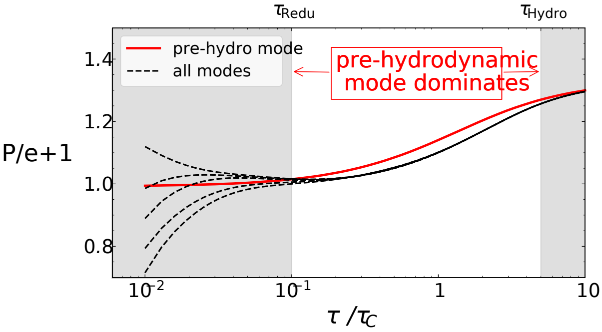
[J. Brewer, L. Yan, Y. Yin]

- Angular spectrum maps to moments $\mathcal{L}_n$, with gaps in eigenvalues.

$$f(t, \mathbf{x}, \mathbf{p}) \longrightarrow \Psi = (\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \ldots)$$

$$\text{so that } \Rightarrow \tau\partial_\tau \Psi = \mathcal{H}\Psi$$

- Ground state as the slowest mode during evolution.



Renormalization of η/s

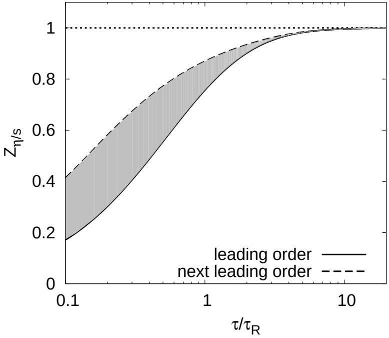
Effects from higher order moments/viscous hydro (leading order):

$$
\begin{aligned}
\partial_\tau \mathcal{L}_0 &= -\frac{1}{\tau}\left(a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1\right), \\
\partial_\tau \mathcal{L}_1 &= -\frac{1}{\tau}\left(a_1 \mathcal{L}_1 + b_0 \mathcal{L}_0\right) - \underbrace{\left[1 + \frac{c_1 \tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}\right]}_{Z^{-1}_{\eta/s}} \frac{\mathcal{L}_1}{\tau_R} \qquad \text{(2nd hydro)},
\end{aligned}
$$

$$
g_2(\tau/\tau_R) = -\, a_2 - b_2 \frac{\mathcal{L}_2}{\mathcal{L}_1} - \frac{\tau}{\tau_R}\,.
$$

- Taking attractor solution for g_2: Borel-resummed gradients.
- Off-equilibrium effects w.r.t. 2nd order hydro $\Leftrightarrow$ effective η/s !

Effective η/s

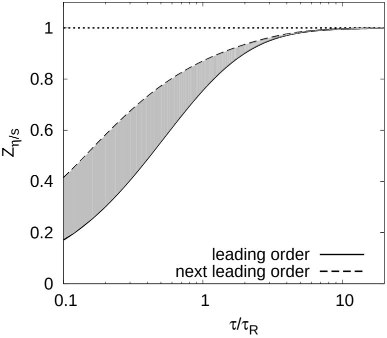


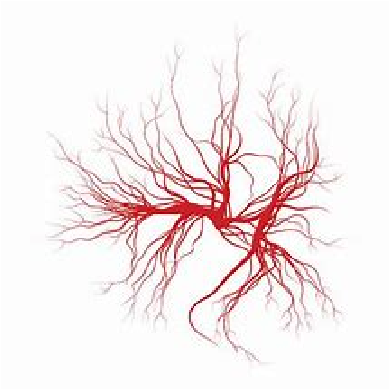
- Out-of-equilibrium physics effectively reduce $\eta/s \to \eta/s(\mathrm{Kn})$.

  [E. Shuryak, M. Lublinsky, P. Romatschke, M. Martinez et al., J. Noronha (QM19)]
- QGP as non-Newtonian liquid, shear-thinning (like blood flow in vein)

$$\begin{cases} \text{size change from AA to pA in HIC} : O(10), \\ \text{size change in vein} : O(10^3) \end{cases}$$

Effective η/s





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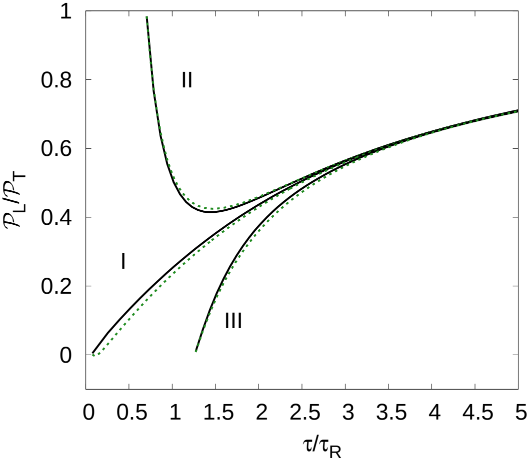
Numerical test of η/s renormalization



2nd order viscous hydro using effective η/s

Summary

- $\mathcal{L}$-moments are proposed to quantify system thermalization.

  $\Rightarrow$ fluid dynamics for out-of-equilibrium system

- Attractor solutions smoothly connect fixed points of $\mathcal{L}_n$ in two limits.
- Hydro can be (has been) used in out-of-equilibrium with $\eta/s(Kn)$:

  * Hydro starts much earlier in heavy-ion collisions.

  * Physical value of $\eta/s >$ phenomenological expectations $\sim O(1/4\pi)$.

- Why off-equilibrium effects effectively reduce η/s?
- Effects on the search of QCD critical point.