Why FRG flow is hydrodynamic flow

Dirk H. Rischke

thanks to:
Adrian Koenigstein, Philipp Lakaschus, Martin J. Steil, Michael Buballa

Symposium on Contemporary QCD Physics and Relativistic Nuclear Collisions
in honor of the 70th birthday of
Jean-Paul Blaizot, Miklos Gyulassy, Larry D. McLerran
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Functional renormalization group (FRG)

FRG flow equation for the scale-dependent effective average action $\Gamma_k[\varphi]$:

Wetterich equation

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \mathrm{Tr} \left\{ \partial_k R_k \left[\Gamma_k^{(2)}[\varphi] + R_k \right]^{-1} \right\}$$

C. Wetterich, PLB 301 (1993) 90

- RG momentum scale k
- regulator function R_k, chosen such that
  - $\lim_{k \to \infty} \Gamma_k[\varphi] \equiv S[\varphi]$ classical action
    (in practice, $\Gamma_\Lambda[\varphi] \equiv S[\varphi]$, Λ sufficiently large UV cut-off)
  - $\lim_{k \to 0} \Gamma_k[\varphi] \equiv \Gamma[\varphi]$ full quantum 1PI effective action
- 2-point vertex function $\Gamma_k^{(2)}[\varphi]_{ij} \equiv \dfrac{\delta^2 \Gamma_k[\varphi]}{\delta\varphi_i \delta\varphi_j}$

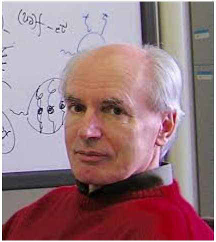
Wetterich equation is in principle exact, but cannot be solved without approximations

Reason: flow equation for $\Gamma_k^{(2)}[\varphi]$ depends on $\Gamma_k^{(2+n)}[\varphi]$, $n \geq 1$

$\Longrightarrow$ infinite tower of flow equations for the vertex functions $\Gamma_k^{(2+n)}[\varphi]$, $n \geq 0$

$\Longrightarrow$ need to truncate at some $n \geq 0$ by suitably approximating $\Gamma_k^{(2+n)}[\varphi]$

JPB's interest in FRG



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PHYSICS LETTERS B

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A new method to solve the non-perturbative renormalization group equations

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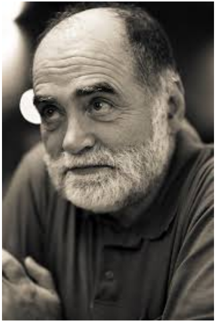
Abstract

We propose a method to solve the non-perturbative renormalization group equations for the n-point functions. In leading order, it consists in solving the equations obtained by closing the infinite hierarchy of equations for the n-point functions. This is achieved: (i) by exploiting the decoupling of modes and the analyticity of the n-point functions at small momenta: this allows us to neglect some momentum dependence of the vertices entering the flow equations; (ii) by relating vertices at zero momenta to derivatives of lower order vertices with respect to a constant background field. Although the approximation is not controlled by a small parameter, its accuracy can be systematically improved. When it is applied to the $O(N)$ model, its leading order is exact in the large-N limit; in this case, one recovers known results in a simple and direct way, i.e., without introducing an auxiliary field.



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PHYSICS LETTERS B

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The renormalization group equation for the color glass condensate

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Editor: P.V. Landshoff

Abstract

We present an explicit and simple form of the renormalization group equation which governs the quantum evolution of the effective theory for the Color Glass Condensate (CGC). This is a functional Fokker–Planck equation for the probability density of the color field which describes the CGC in the covariant gauge. It is equivalent to the Euclidean time evolution equation for a second quantized current–current Hamiltonian in two spatial dimensions. The quantum corrections are included in the leading log approximation, but the equation is fully non-linear with respect to the generally strong background field. In the weak field limit, it reduces to the BFKL equation, while in the general non-linear case it generates the evolution equations for Wilson-line operators previously derived by Balitsky and Kovchegov within perturbative QCD.

Gradient expansion

- consider $\varphi \equiv (\varphi_1, \ldots, \varphi_N)$ and $O(N)$ symmetry
- expand $\Gamma_k[\varphi]$ in gradients of the fields $\varphi_a\,,\ a = 1, \ldots N$

$$\Gamma_k[\varphi] = \int_x \left[V_k(\rho) + \frac{Z_k(\rho)}{2} \left(\partial_\mu \varphi_a\right)^2 + \frac{Y_k(\rho)}{4} \left(\partial_\mu \rho\right)^2 + \ldots \right]$$

J. Berges, N. Tetradis, C. Wetterich, Phys. Rept. 363 (2002) 223

- $\rho \equiv \frac{1}{2}\, \varphi_a^2$
- $V_k(\rho)$ effective average potential
- $Z_k(\rho)\,,\ Y_k(\rho)$ effective average (field-dependent) wave-function renormalization functions

In the following:

- local-potential approximation (LPA): $Z_k(\rho) \equiv 1\,,\ Y_k(\rho) = 0$

  $\Longrightarrow$ FRG flow equation for $\Gamma_k[\varphi]$ becomes partial differential equation for $V_k(\rho)$!
- $O(N)$ model: $V_\Lambda(\rho) = \lambda_2\, \rho + \frac{\lambda_4}{2}\, \rho^2 + \frac{\lambda_6}{3}\, \rho^3$

Flow equation for effective average potential I

- define "time" variable $t \equiv -\ln \frac{k}{\Lambda}$

  $\Longrightarrow$ "RG time" flows from $t = 0$ (UV) to $t \to \infty$ (IR)
- promote t to second independent variable besides ρ

  $\Longrightarrow$ $V_k(\rho) \equiv V(t, \rho)$
- define $V'(t,\rho) \equiv \partial_\rho V(t,\rho)\,,\;\; V''(t,\rho) \equiv \partial_\rho^2 V(t,\rho)$
- $\left[\Gamma_t^{(2)}(q)\right]_{ab} = [q^2 + V'(t,\rho)]\delta_{ab} + 2\rho V''(t,\rho)\,\delta_{aN}\delta_{bN}$

  $\Longrightarrow$ $\varphi_a\,,\; a = 1, \ldots, N-1$ are "pion" modes, φ_N is "sigma" mode

$$\Longrightarrow \quad \partial_t V(t,\rho) = \frac{1}{2}\int_q \left[\left(\frac{N-1}{q^2 + V'(t,\rho) + R_k(q)} + \frac{1}{q^2 + V'(t,\rho) + 2\rho V''(t,\rho) + R_k(q)}\right)\partial_t R_k(q)\right]$$

Flow equation for effective average potential II

- use Litim regulator $R_k(q) = (k^2 - q^2)\Theta(k^2 - q^2)$ $\Longrightarrow$ integral over q collapses!

- rescale $r \equiv \dfrac{\rho}{(N-1)\Lambda^{d-2}}$, $v(t,r) \equiv \dfrac{V(t,\rho)}{(N-1)\Lambda^d}$

  $\Longrightarrow$ $v' \equiv \dfrac{\partial v}{\partial r} = \dfrac{V'}{\Lambda^2}$, $\quad r\, v'' = r \dfrac{\partial^2 v}{\partial r^2} = \dfrac{\rho V''}{\Lambda^2}$

- define $A_d \equiv \dfrac{\Omega_d}{d(2\pi)^d}$, Ω_d volume of $d-1$–dimensional sphere

$\Longrightarrow$

$$\partial_t v(t,r) = -A_d e^{-(d+2)t} \left[\frac{1}{e^{-2t} + v'(t,r)} + \frac{1}{N-1} \frac{1}{e^{-2t} + v'(t,r) + 2r\, v''(t,r)} \right]$$

$\Longrightarrow$ standard method: solve this equation with finite-difference methods

New approach: take derivative with respect to r, define $u \equiv v'$

$\Longrightarrow$ $$\partial_t u(t,r) + \partial_r F(t,u) + \partial_r G(t,r,u,u') = 0$$

where $F(t,u) \equiv \dfrac{A_d e^{-(d+2)t}}{e^{-2t} + u(t,r)}$, $G(t,r,u,u') \equiv \dfrac{1}{N-1} \dfrac{A_d e^{-(d+2)t}}{e^{-2t} + u(t,r) + 2r\,u'(t,r)}$

For now, take large–N approximation, $G(t,r,u,u') \to 0$,

$\Longrightarrow$ $$\partial_t u(t,r) + \partial_r F(t,u) = 0$$

$\Longrightarrow$ advection equation known from hydrodynamics!

$\Longrightarrow$ solvable with algorithms used in hydrodynamics!

$\Longrightarrow$ E. Grossi, N. Wink, arXiv:1903.09503 [hep-th]
(talk presented at EMMI workshop "Functional Methods in Strongly Correlated Systems", Hirschegg, March 31 – April 7, 2019)

Digression I: method of characteristics

Idea:

solve partial differential eqs. by converting them into system of ordinary differential eqs.

$\Longrightarrow$ in our case: find characteristic curves $s(t,r)$ where $u(s) = const.$, i.e., $\dfrac{\mathrm{d}u(s)}{\mathrm{d}s} = 0$

Obviously, $$0 = \frac{\mathrm{d}u(s)}{\mathrm{d}s} = \frac{\mathrm{d}t(s)}{\mathrm{d}s}\,\partial_t u(t,r) + \frac{\mathrm{d}r(s)}{\mathrm{d}s}\,\partial_r u(t,r)$$

$\Longrightarrow$ comparison with advection equation

$$0 = \partial_t u(t,r) + \partial_r F(t,u) = \partial_t u(t,r) + \frac{\partial F(t,u)}{\partial u}\,\partial_r u(t,r)$$

yields

characteristic equations

$$\frac{\mathrm{d}t(s)}{\mathrm{d}s} = 1\ , \quad \frac{\mathrm{d}r(s)}{\mathrm{d}s} = \frac{\partial F(t,u)}{\partial u} = -\frac{A_d e^{-(d+2)t}}{[e^{-2t} + u(t,r)]^2}\ , \quad \frac{\mathrm{d}u(s)}{\mathrm{d}s} = 0\ .$$

Interpretation: 1st two eqs. determine characteristic curves $s(t,r)$ in (t,r)–plane, where $u(s) = const.$, i.e., where it keeps its initial value $u(0,r) \equiv u_0(r)$

$\Longrightarrow$ since $\dfrac{\mathrm{d}r(s)}{\mathrm{d}s} < 0$, with increasing s (or t) characteristic curves bend left (towards smaller values of r)

Digression II: characteristic curves in ideal hydrodynamics I

Energy-momentum tensor

$$T^{\mu\nu} = [\epsilon + p(\epsilon)] u^\mu u^\nu - p(\epsilon) g^{\mu\nu}$$

- ϵ energy density
- $p(\epsilon)$ pressure
- $u^\mu = \gamma(1, \mathbf{v})^T$ fluid 4-velocity,
  $\gamma = (1 - \mathbf{v}^2)^{-1/2}$ Lorentz-gamma factor

Equations of motion: energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

In 1+1 dimensions, combine the two equations for $\nu = t, x$ to obtain

$$\left[\partial_t + \frac{v \pm c_s(\epsilon)}{1 \pm v\, c_s(\epsilon)}\, \partial_x\right] \mathcal{R}_\pm(\epsilon, v) = 0$$

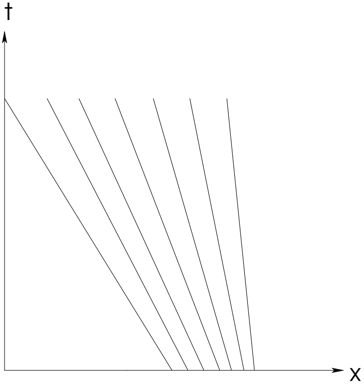
- $\mathcal{R}_\pm(\epsilon, v) \equiv y - y_0 \pm \int_{\epsilon_0}^{\epsilon} \mathrm{d}\epsilon' \, \frac{c_s(\epsilon')}{\epsilon' + p(\epsilon')}$
  Riemann invariants
- $c_s(\epsilon) \equiv \frac{\mathrm{d}p(\epsilon)}{\mathrm{d}\epsilon}$ velocity of sound
- $y \equiv \mathrm{Artanh}\, v$

$\Longrightarrow$ characteristic curves $s_\pm(t, x)$ defined by $\frac{\mathrm{d}t(s_\pm)}{\mathrm{d}s_\pm} = 1\,, \quad \frac{\mathrm{d}x(s_\pm)}{\mathrm{d}s_\pm} = \frac{v \pm c_s(\epsilon)}{1 \pm v\, c_s(\epsilon)}$

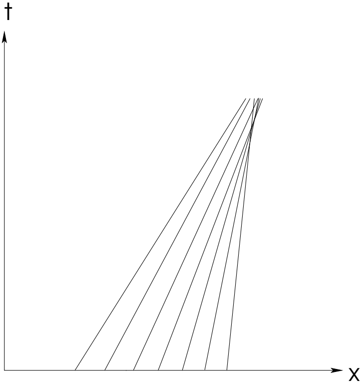
$\Longrightarrow$ sound waves travelling forward/backward in matter relative to v

$\Longrightarrow$ $\frac{\mathrm{d}\mathcal{R}_\pm}{\mathrm{d}s_\pm} = 0$, i.e., $\mathcal{R}_\pm = const.$ on $s_\pm(t, x)$

Digression II: characteristic curves in ideal hydrodynamics II



expansion: characteristics fan out



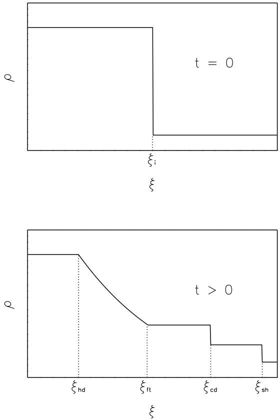
compression: characteristics cross

$\Longrightarrow$ formation of shock wave

$\Longrightarrow$ (analytical) method of characteristics not applicable when shock waves occur [(numerical) finite-difference methods do not converge to the correct solution]

But: shock waves can be treated using energy-momentum conservation across shock (Rankine–Hugoniot–Taub equations)

The Riemann problem



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$t = 0$: step in initial density distribution

$t > 0$:

- expansion wave travels with sound velocity into region of higher density
- plateau region between foot of expansion wave and
- contact discontinuity
- plateau region between contact discontinuity and
- shock wave, which travels into region of smaller density

Riemann problem completely analytically solvable!

- method of characteristics for expansion wave
- Rankine–Hugoniot–Taub equations for shock wave
- matching different regions

[$\Longrightarrow$ numerical: finite-volume methods converge to the correct solution]

Godunov and Godunov-type methods

Numerical solution of hydrodynamics on a grid:

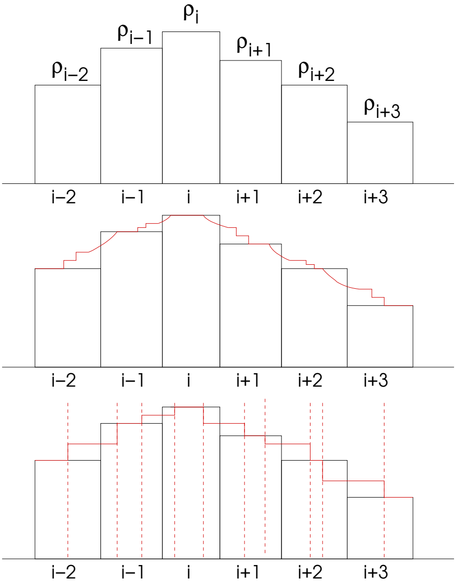
quantities are constant within a cell

Godunov methods:

solve Riemann problem
exactly between grid cells

Godunov-type methods:

solve Riemann problem
approximately between grid cells



The Harten–Lax–van Leer–Einfeldt (HLLE) algorithm I

HLLE solves advection equations of type $\partial_t u + \partial_x F(u) = 0$

- consider cell boundary at $x = 0$
- initial distribution of u at time $t = 0$: $$u(0,x) = \begin{cases} u_l \,, & x < 0 \\ u_r \,, & x \geq 0 \end{cases}$$
- for $t > 0$ discontinuity will decay and produce a distribution
$$u(t,x) = \begin{cases} u_l \,, & x < b_l t \\ u_{lr}(x) \,, & b_l t \leq x < b_r t \\ u_r \,, & x \geq b_r t \end{cases}$$
  where $b_l < 0$ and $b_r > 0$ are signal velocities with which the decay proceeds to the left and right of the cell boundary at $x = 0$
- In the exact solution of the Riemann problem, $u_{lr}(x)$ is a complicated function of x. In Godunov-type algorithms, approximate $u_{lr} = const.$
- u_{lr} is determined by integrating advection equation over finite volume $[x_{\min}, x_{\max}]$ with $x_{\min} < b_l t \,,\; x_{\max} > b_r t$:
$$u_{lr} = \frac{b_r\, u_r - b_l\, u_l - F(u_r) + F(u_l)}{b_r - b_l}$$
- $F(u_{lr})$ is determined by integrating advection equation over finite volume $[0, x_{\max}]$:

$$F(u_{lr}) = \frac{b_r b_l (u_r - u_l) + b_r F(u_l) - b_l F(u_r)}{b_r - b_l} \qquad (1)$$

The Harten–Lax–van Leer–Einfeldt (HLLE) algorithm II

CRC-TR 211

- advection equation in finite-difference form after time $t = (n+1)\Delta t$:

$$u_j^{n+1} = u_j^n - \lambda(G_{j+1/2} - G_{j-1/2}) \quad (2)$$

where $\lambda \equiv \dfrac{\Delta t}{\Delta x}$

- taking $G_{j\pm 1/2} \equiv F(u^n_{j\pm 1/2})$, with $F(u^n_{j\pm 1/2})$ given by Eq. (1), yields scheme which has first-order accuracy in time
- for second-order accuracy in time, compute half-step updated fluxes $G_{j\pm 1/2}$
- need to specify signal velocities $b^r_{j+1/2}\,,\ b^l_{j+1/2}$

$\Longrightarrow$ in relativistic hydrodynamics, use characteristics:

$$b^r_{j+1/2} \equiv \max\left(0, \frac{v^n_{j+1} + c^n_{s,j+1}}{1 + v^n_{j+1} c^n_{s,j+1}}\right)$$

$$b^l_{j+1/2} \equiv \min\left(0, \frac{v^n_j - c^n_{s,j}}{1 - v^n_j c^n_{s,j}}\right)$$

$\Longrightarrow$ causal signal propagation!

$\Longrightarrow$ if $\lambda < 1$, no quantity is propagated over distance larger than one cell size Δx in one timestep Δt! In practice, $\lambda = 0.99$ works very well

V. Schneider, U. Katscher, DHR, B. Waldhauser, J.A. Maruhn, C.D. Munz, J. Comput. Phys. 105 (1992) 93

DHR, S. Bernard, J.A. Maruhn, NPA 595 (1995) 346

MG and hydrodynamics

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Nuclear Physics A 608 (1996) 479–512

NUCLEAR PHYSICS A

The time-delay signature of quark–gluon plasma formation in relativistic nuclear collisions ★

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Received 26 June 1996

Abstract

The hydrodynamic expansion of quark–gluon plasmas with spherical and longitudinally boost-invariant geometries is studied as a function of the initial energy density. The sensitivity of the collective flow pattern to uncertainties in the nuclear matter equation of state is explored. We concentrate on the effect of a possible finite width, $\Delta T \sim 0.1 T_c$, of the transition region between quark–gluon plasma and hadronic phase. Although slow deflagration solutions that act to stall the expansion do not exist for $\Delta T > 0.08 T_c$, we find, nevertheless, that the equation of state remains sufficiently soft in the transition region to delay the propagation of ordinary rarefaction waves for a considerable time. We compute the dependence of the pion-interferometry correlation function on ΔT, since this is the most promising observable for time-delayed expansion. The signature of time delay, proposed by Pratt and Bertsch, is an enhancement of the ratio of the inverse width of the pion correlation function in out-direction to that in side-direction. One of our main results is that this generic signature of quark–gluon plasma formation is rather robust to the uncertainties in the width of the transition region. Furthermore, for longitudinal boost-invariant geometries, the signal is likely to be maximized around RHIC energies, $\sqrt{s} \sim 200 A \cdot \mathrm{GeV}$.

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- for FRG flow equation:

$$b^r_{j+1/2} \equiv 0$$
$$b^l_{j+1/2} \equiv \min\left[0, v_f\left(u^n_j\right)\right]$$

- $v_f(u) \equiv \frac{\partial F(u)}{\partial u}$

- FRG flow equation has only one set of characteristic curves
- $u(t,r)$ itself is associated Riemann invariant
- signals are always transported to the left only, towards smaller r
- Note: characteristic velocity $v_f(u(t,r)) \equiv v_f(t,r)$ can become much smaller than -1!

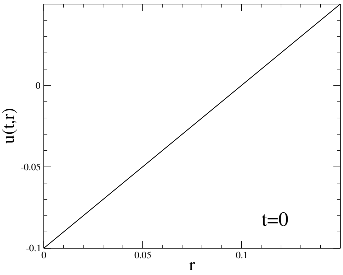
  $\Longrightarrow$ stiff differential equation!

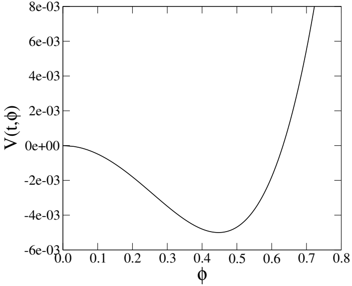
  $\Longrightarrow$ in principle, implicit method for time step required
- workaround: reduce λ, prevent transport over more than one cell in one timestep

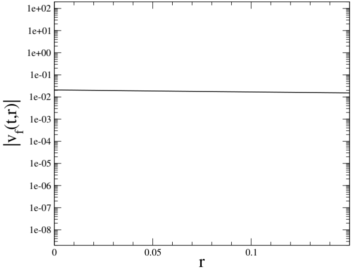
  $\Longrightarrow$ in practice, successive reduction of λ by constant factor < 1 works quite well

Results: second-order phase transition

$d = 3\,,\ \lambda_2 = -0.1\,,\ \lambda_4 = 1\,,\ \lambda_6 = 0$







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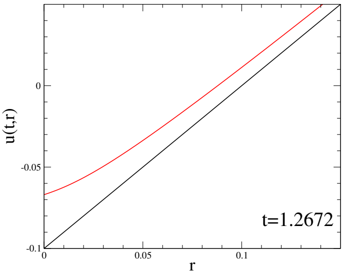

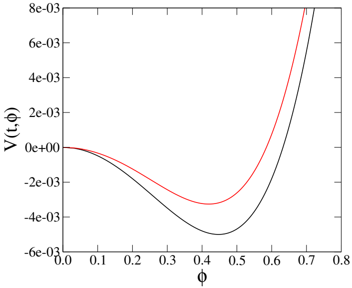

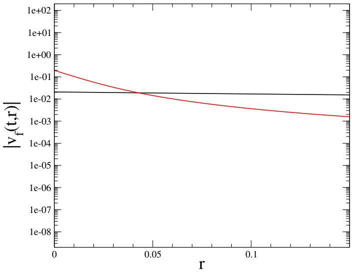

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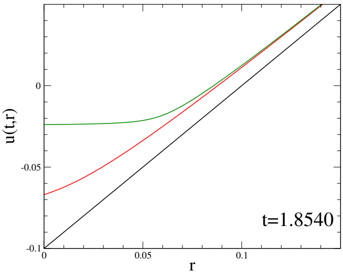

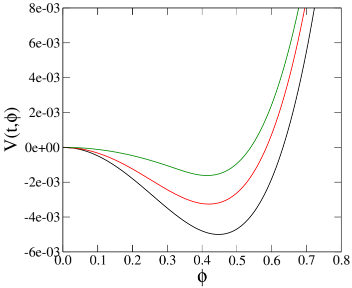

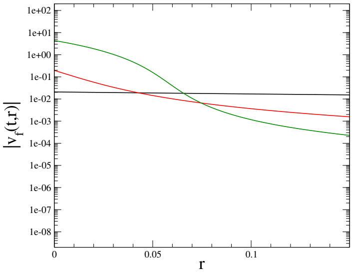

Results: second-order phase transition

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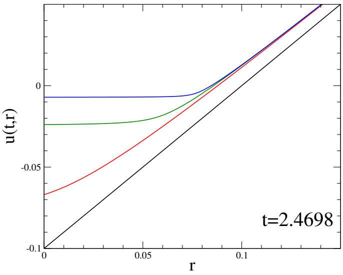

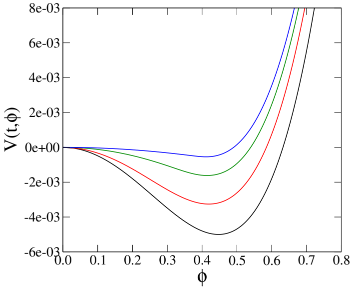

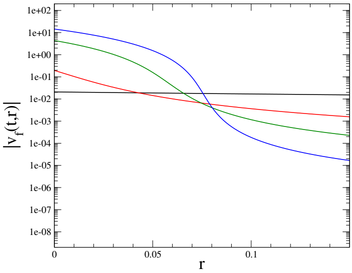

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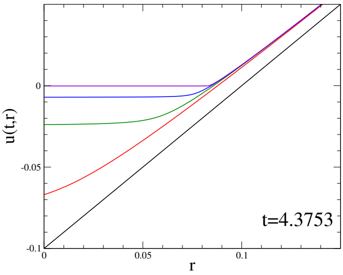
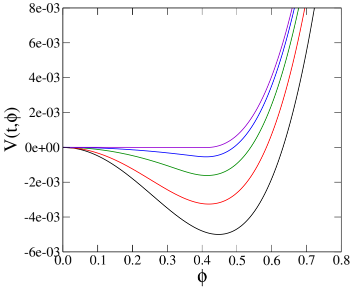
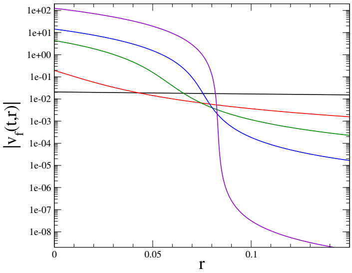


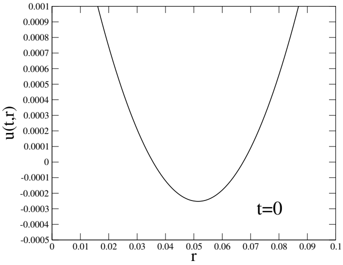


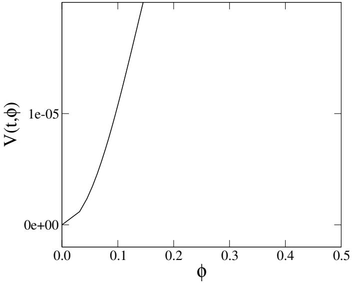


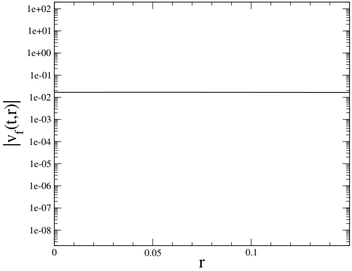
Remarks:

- locally concave potential
- signal velocity $|v_f(t, r)|$ can become $\gg 1$!

$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.103\,,\ \lambda_6 = 1$

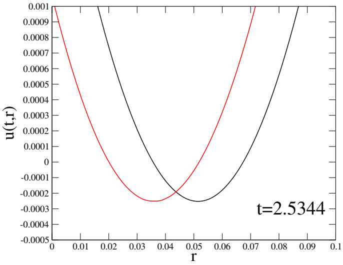


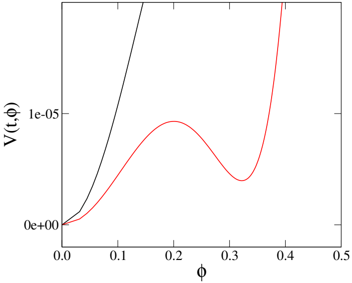


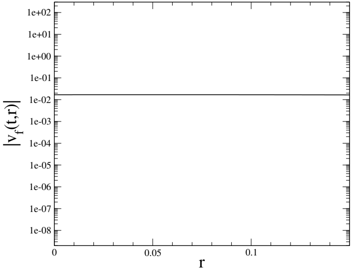


Results: first-order phase transition – symmetric phase

$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.103\,,\ \lambda_6 = 1$







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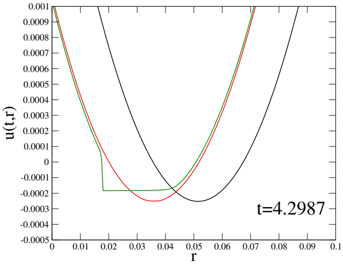
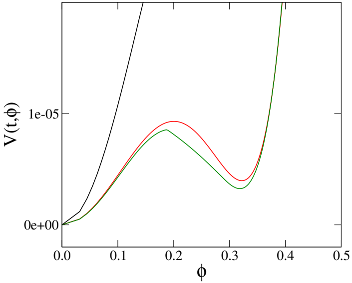
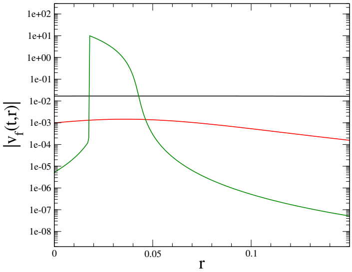




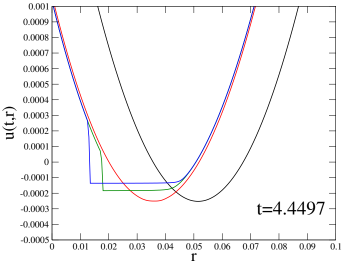


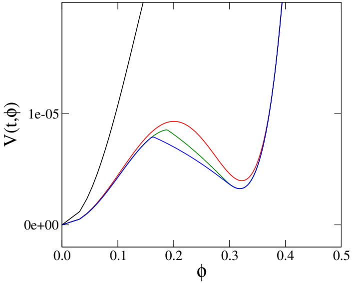
Shock discontinuity develops!

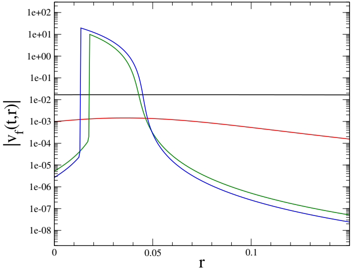
$\Longrightarrow$ cannot be resolved by finite-difference method!

$\Longrightarrow$ requires finite-volume method or other shock-capturing scheme!

$$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.103\,,\ \lambda_6 = 1$$







Results: first-order phase transition – symmetric phase

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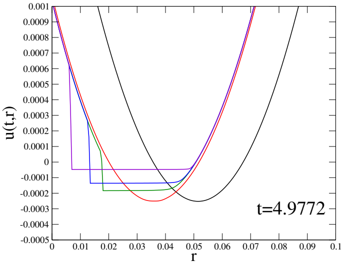

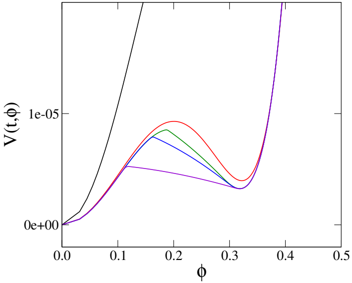

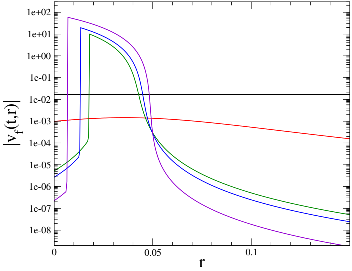

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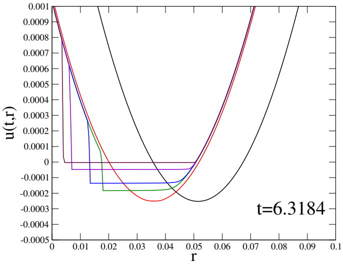
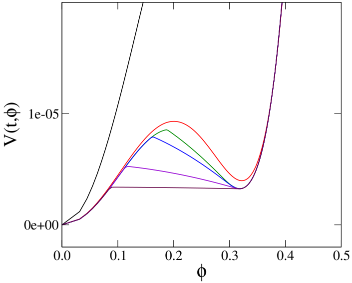
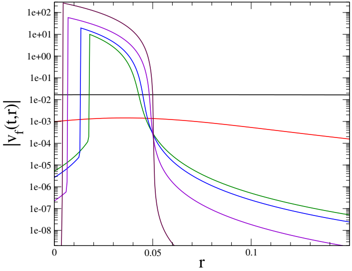
$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.103\,,\ \lambda_6 = 1$







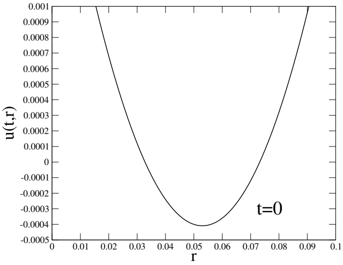
Remarks:

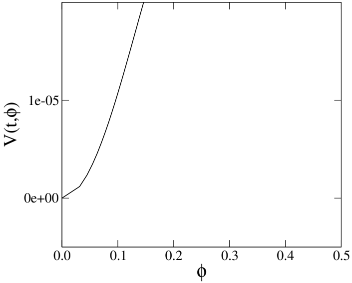
- signal velocity $|v_f(t,r)| \gg 1$ at shock discontinuity!
- shock discontinuity stalls!
- locally concave potential
- minimum at $\phi = 0$

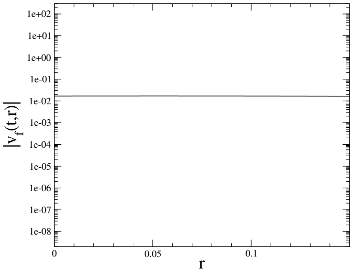
$\Longrightarrow$ symmetric phase!

Results: first-order phase transition – broken phase

$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.106\,,\ \lambda_6 = 1$







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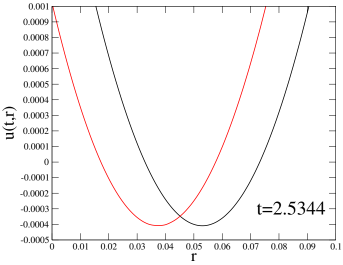

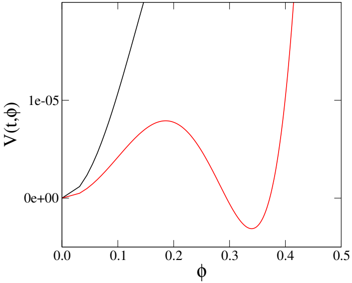

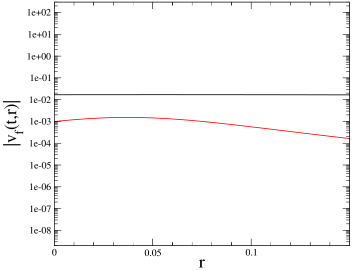

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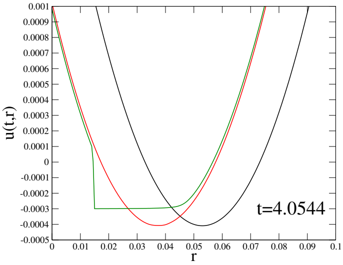
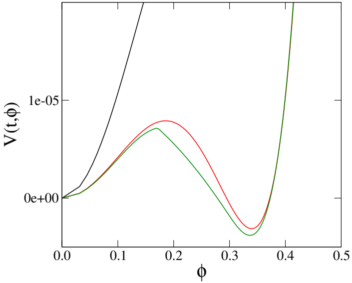
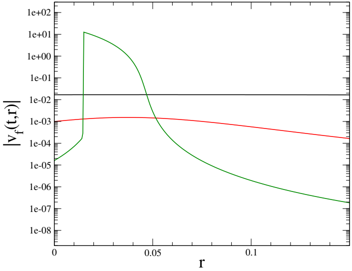
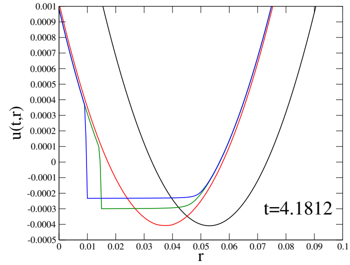
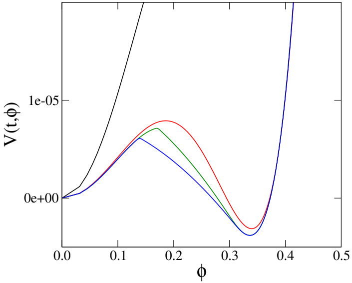


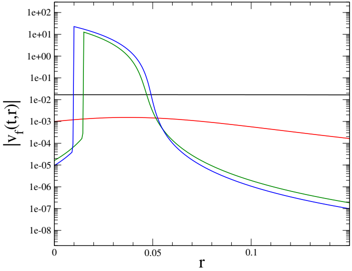




$$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.106\,,\ \lambda_6 = 1$$

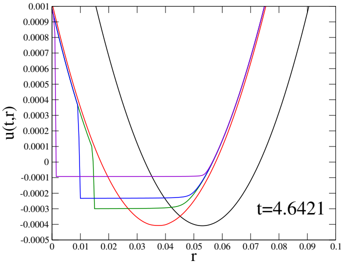


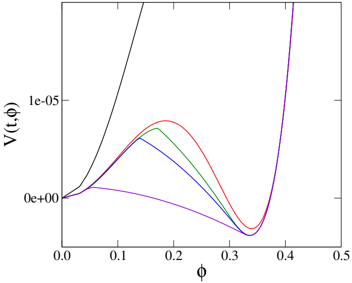


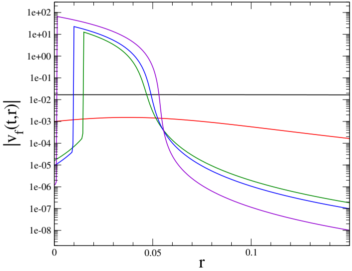


Results: first-order phase transition – broken phase

$d = 3\,,\ \lambda_2 = 0.0024\,,\ \lambda_4 = -0.106\,,\ \lambda_6 = 1$







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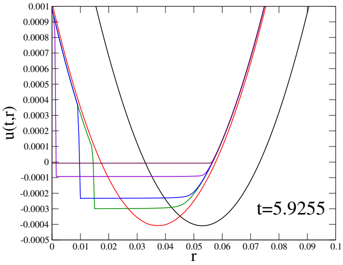
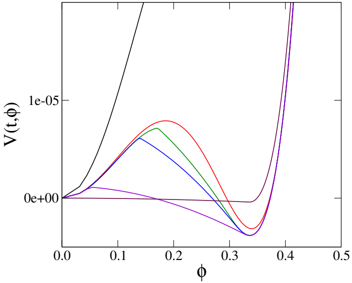
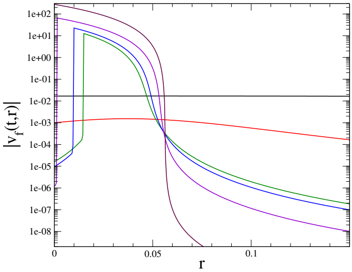






Remarks:

- signal velocity $|v_f(t,r)| \gg 1$ at shock discontinuity!
- shock discontinuity moves out of grid!
- locally concave potential
- minimum at $\phi \neq 0$

$\Longrightarrow$ broken phase!

Recent developments

- Problem of $|v_f| \gg 1$: use implicit method for time step
- Beyond large–N approximation:

$\Longrightarrow$ $$\partial_t u(t,r) + \partial_r F(t,u) + \partial_r G(t,r,u,u') = 0$$

where $G(t,r,u,u') \equiv \dfrac{1}{N-1} \dfrac{A_d e^{-(d+2)t}}{e^{-2t} + u(t,r) + 2\, r\, u'(t,r)}$

$\Longrightarrow$ diffusion term!

$\Longrightarrow$ HLLE algorithm in principle able to handle this, but more robust and flexible: Kurganov–Tadmor (KT) algorithm

A. Kurganov, E. Tadmor, J. Comput. Phys. 160 (2000) 241

also used in 3+1-d heavy-ion collision modelling! (MUSIC, BESHYDRO, ...)

- Include fermions:

$\Longrightarrow$ $$\partial_t u(t,r) + \partial_r F(t,u) + \partial_r G(t,r,u,u') = H(r)$$

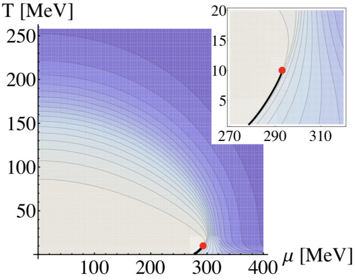
where $H(r)$ is simple r–dependent source term

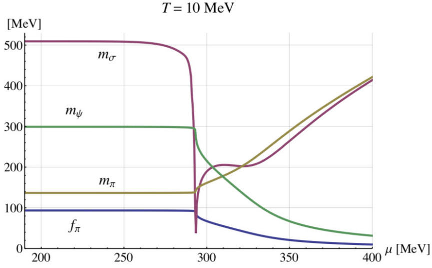
$\Longrightarrow$ treat with standard Sod's operator-splitting method

$\Longrightarrow$ Do results previously obtained with finite-difference methods change?

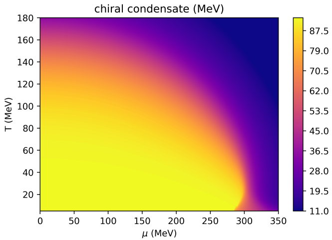
Phase diagram of quark-meson model

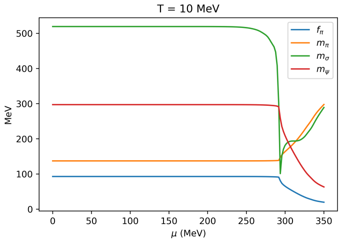
R.-A. Tripolt, N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D89 (2014) no.3, 034010





P. Lakaschus, DHR, in preparation





Conclusions and Outlook

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- FRG flow equation is equivalent to hydrodynamic flow equation
- standard finite-difference methods fail to produce correct solution in case of shock discontinuities
- standard finite-volume methods (HLLE, KT, . . .) used in hydrodynamics are able to obtain correct solution of FRG flow equation (i.e., resolving shock discontinuities)
- previous results on phase diagram and masses of quark-meson model seem unchanged (minimum of potential not affected by shock discontinuity in solution?)

- extend beyond LPA
- compute phase boundary to inhomogeneous phase by finding $\Gamma^{(2)}(k) = 0$ in $T - \mu$–plane (A. Koenigstein)
- chiral density wave ansatz to study inhomogeneous phase (M.J. Steil)
- extend to more than one order parameter, e.g. chiral plus color-superconducting (P. Lakaschus)