Precision α_s measurements: perturbative and non-perturbative effects

"Interplay between perturbative and non-perturbative effects with the ARES method"
arXiv: 2303.01534v1

Andrea Banfi[a], Basem Kamal El-Menoufi[b], Ryan Wood[a]

[a] Department of Physics and Astronomy, University of Sussex

[b] Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester

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Perform precise determinations of the strong coupling constant, α_s

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Interaction Constants	Value	Relative Uncertainty
Strong - $\alpha_s(m_Z)$	0.1179 ± 0.0009	$\Delta\alpha_s / \alpha_s \approx 0.8\%$

Source: Particle Data Group Collaboration, R. L. Workman and Others, *Review of Particle Physics, PTEP* **2022** (2022) 083C01.

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Weak - $G_F / (\hbar c)^3$	$(1.1663788 \times 10^{-5}) \pm (6 \times 10^{-12})\ GeV^{-2}$	$\Delta[G_F / (\hbar c)^{3]} / [G_F / (\hbar c)^3] \approx (5 \times 10^{-5})\%$
QED - α	$(7.2973525693 \times 10^{-3}) \pm (1.1 \times 10^{-12})$	$\Delta\alpha / \alpha \approx (1.5 \times 10^{-8})\%$
Gravitational - G_N	$(6.67430 \times 10^{-11}) \pm (1.5 \times 10^{-15})\ m^3\ kg^{-1}\ s^{-2}$	$\Delta G_N / G_N \approx (2.2 \times 10^{-3})\%$

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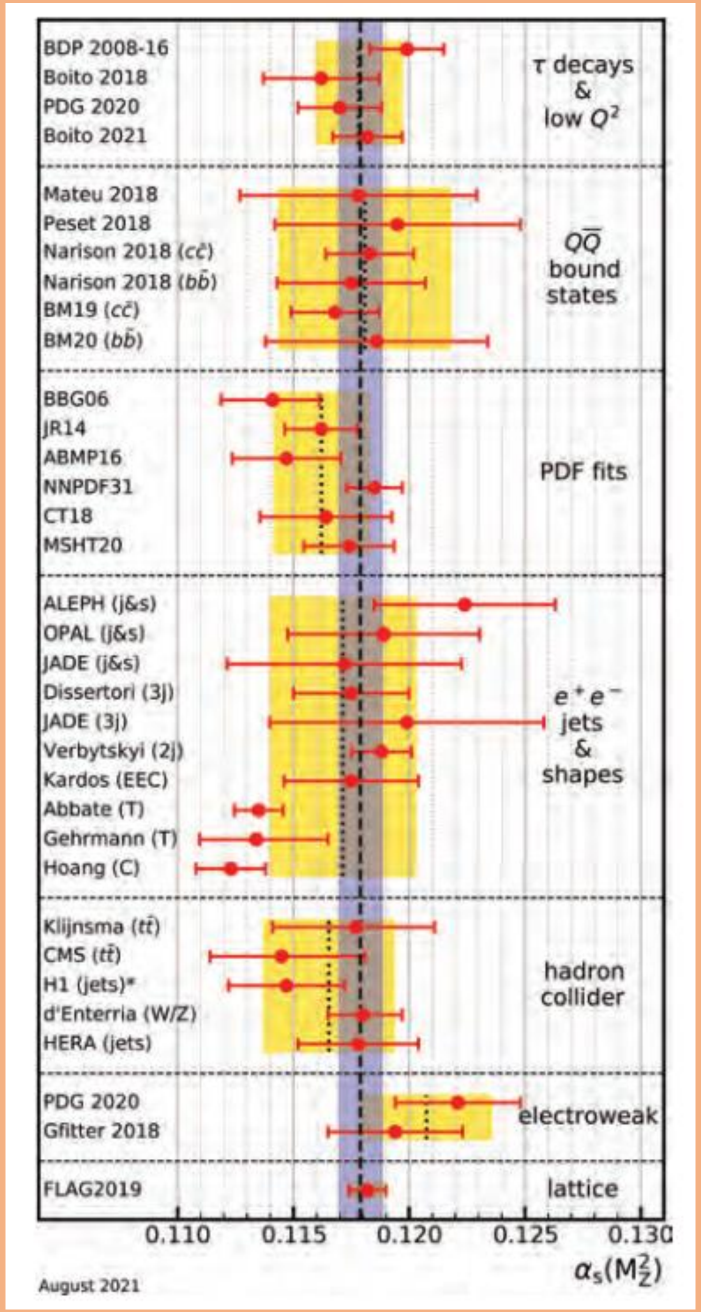
An example of the impact:

In the Higgs sector, our limited precision in the determination of α_s propagates into total final uncertainties of ~ 2 - 3% for key processes such as the Higgs gg - fusion and $t\bar{t}$ production cross-sections and of for the ~ 4% for the $H \to gg$ partial width.

Sources: [arXiv:1602.00695], [arXiv: 2010.04171], [arXiv: 1905.05078] and [arXiv: 2106.11802]

How?



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category	$\alpha_S(m_Z^2)$	relative $\alpha_S(m_Z^2)$ uncertainty
τ decays and low Q^2	0.1178 ± 0.0019	1.6%
$Q\overline{Q}$ bound states	0.1181 ± 0.0037	3.1%
PDF fits	0.1162 ± 0.0020	1.7%
e^+e^- jets & shapes	0.1171 ± 0.0031	2.6%
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lattice	0.1182 ± 0.0008	0.7%
world average (without lattice)	0.1176 ± 0.0010	0.9%
world average (with lattice)	0.1179 ± 0.0009	0.8%

Source: D. d'Enterria et al., The strong coupling constant: State of the art and the decade ahead (Snowmass 2021 White Paper), [arXiv:2203.08271 [hep-ph]].

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What are Event Shapes?

Event shapes are quantities which characterise a particular aspect of the 'shape' of the hadronic final states.

An example might be whether the distribution of final-state hadrons is pencil-like, spherical etc.

Event shapes were originally defined for e^+e^- annihilation to provide experimental test of QCD and have since been generalised to other colliders.

What are Event Shapes?

Example: Thrust

$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} , \qquad \tau \equiv 1 - T ,$$

- $\{p_i\}$ are the momenta of each final-state particle in the event
- Q is the hard-scale of the process
- The vector $\vec{n}$ that maximises the sum defines the direction of the thrust axis, $\vec{n}_T$, which divides the event into two hemispheres $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$

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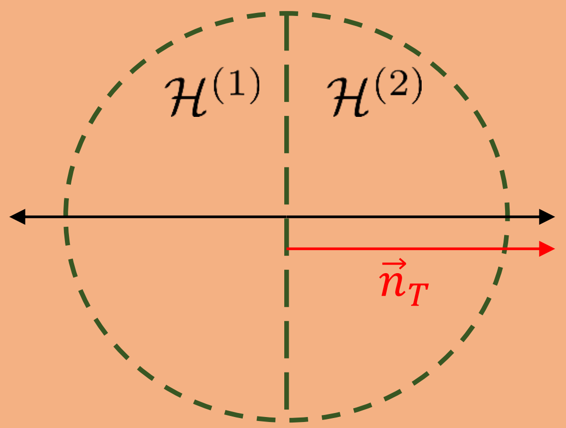

- In the two-jet limit with back-to-back hard $q\bar{q}$ pair (pencil-like geometry):

$$\tau = 0$$

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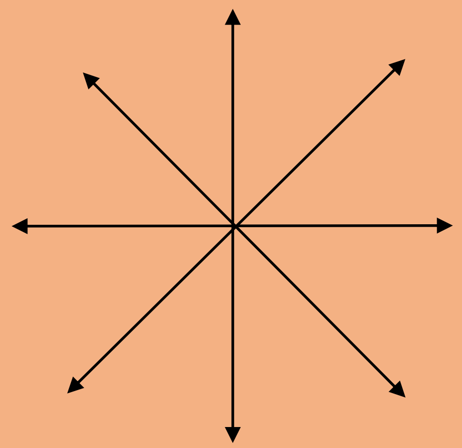


- With a spherical geometry of final state particles, $\vec{n}_T$ will be determined by the most energetic particle and

$\tau = \frac{1}{2}$

What are Event Shapes?

Heavy-jet Mass: $$\rho_H \equiv \max_{i=1,2} \frac{M_i^2}{Q^2} \,, \quad M_i^2 \equiv \left(\sum_{j\in\mathcal{H}^{(i)}} p_j\right)^2$$

C-parameter: $$C \equiv 3\left(1 - \frac{1}{2}\sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)}\right)$$

Jet Broadenings: $$B_L \equiv \sum_{i\in\mathcal{H}^{(1)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q} \,, \quad B_R \equiv \sum_{i\in\mathcal{H}^{(2)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q}$$

$$B_W \equiv \max\{B_L, B_R\}$$

$$B_T \equiv B_L + B_R$$

Thrust Major: $$T_M \equiv \max_{\vec{n}\cdot\vec{n}_T=0} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$, where the vector $\vec{n}$ that maximises the sum defining the thrust-major axis

Why are Event Shapes in $\mathrm{e^+e^-}$ annihilation useful?

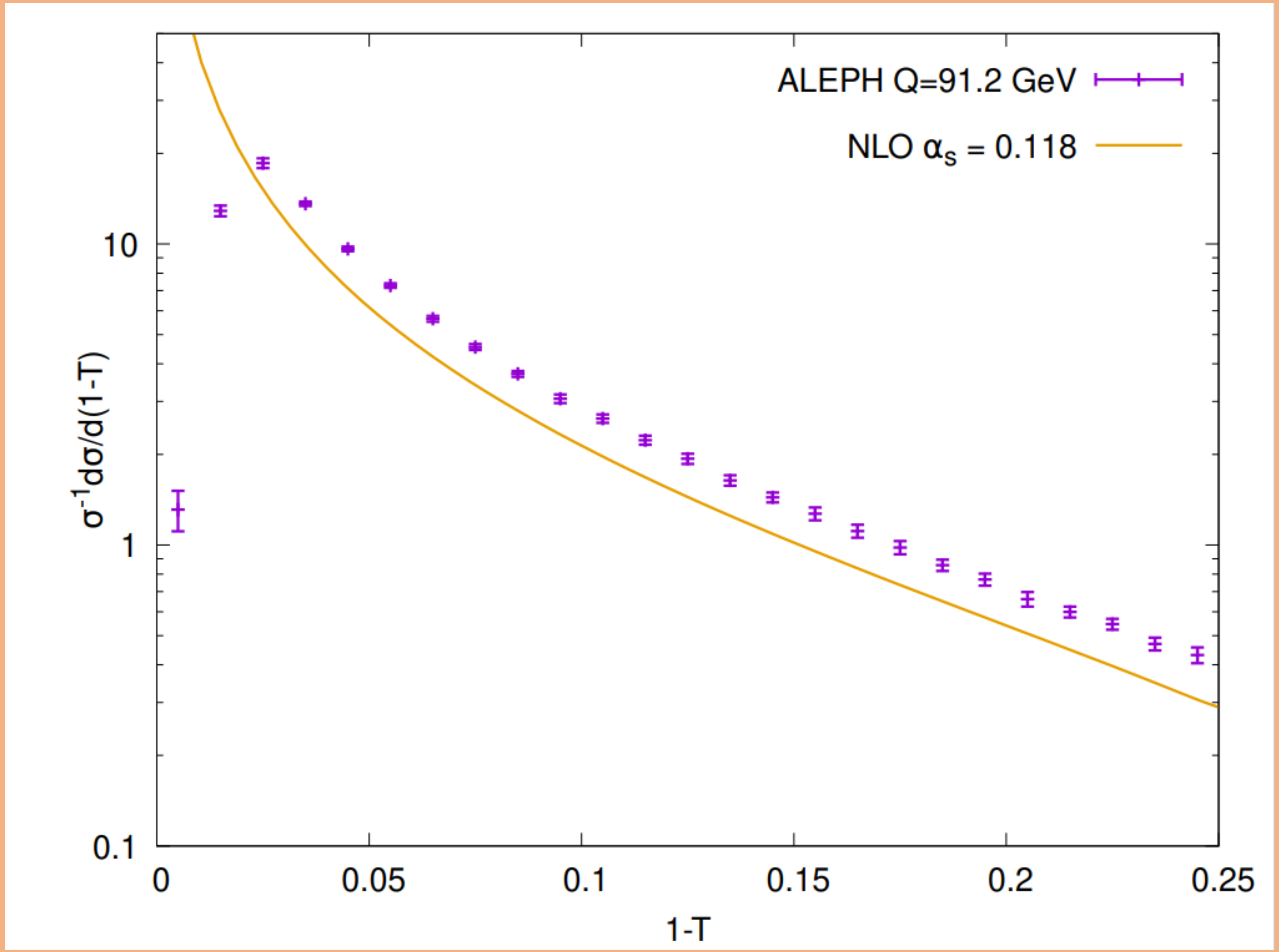
In principle, event shapes are the ideal testing ground for perturbative QCD:

- Span a wide range of energy scales in a continuous fashion
- Defined so as to be IRC safe and thus can be computed at fixed order in perturbative QCD
- At energy scales $Q \sim M_Z$, α_s is small and we therefore expect a well-behaved perturbative series
- With e^+e^- annihilation there are no additional uncertainties due to parton distribution functions

Recipe for determining $\boldsymbol{\alpha_s}$:

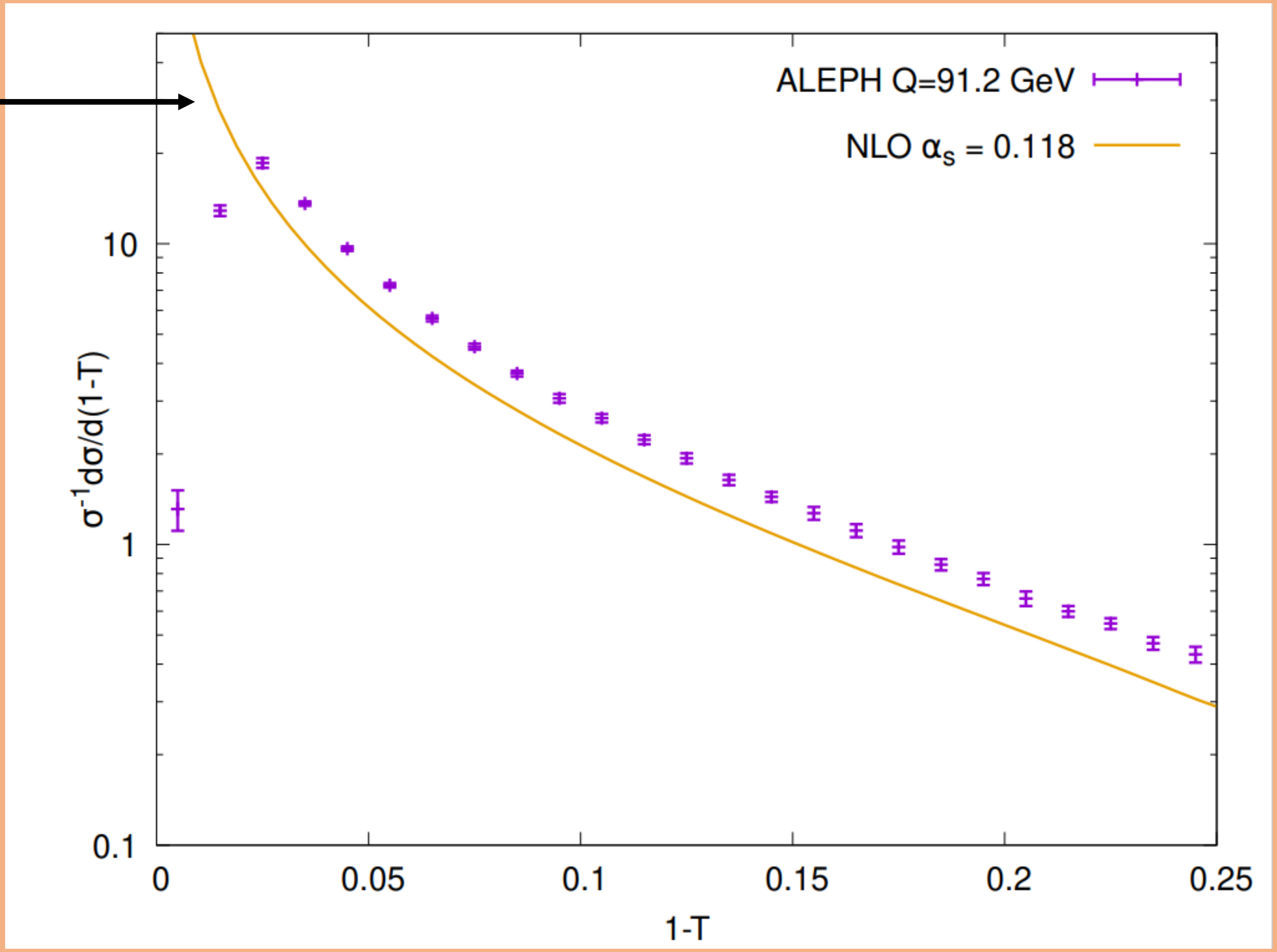
Produce a theoretical prediction that will depend upon α_s , vary α_s and fit to data

Example: Thrust



Example: Thrust

Problem:
Poor predictive power at small observable values
→
Require resummation in this region



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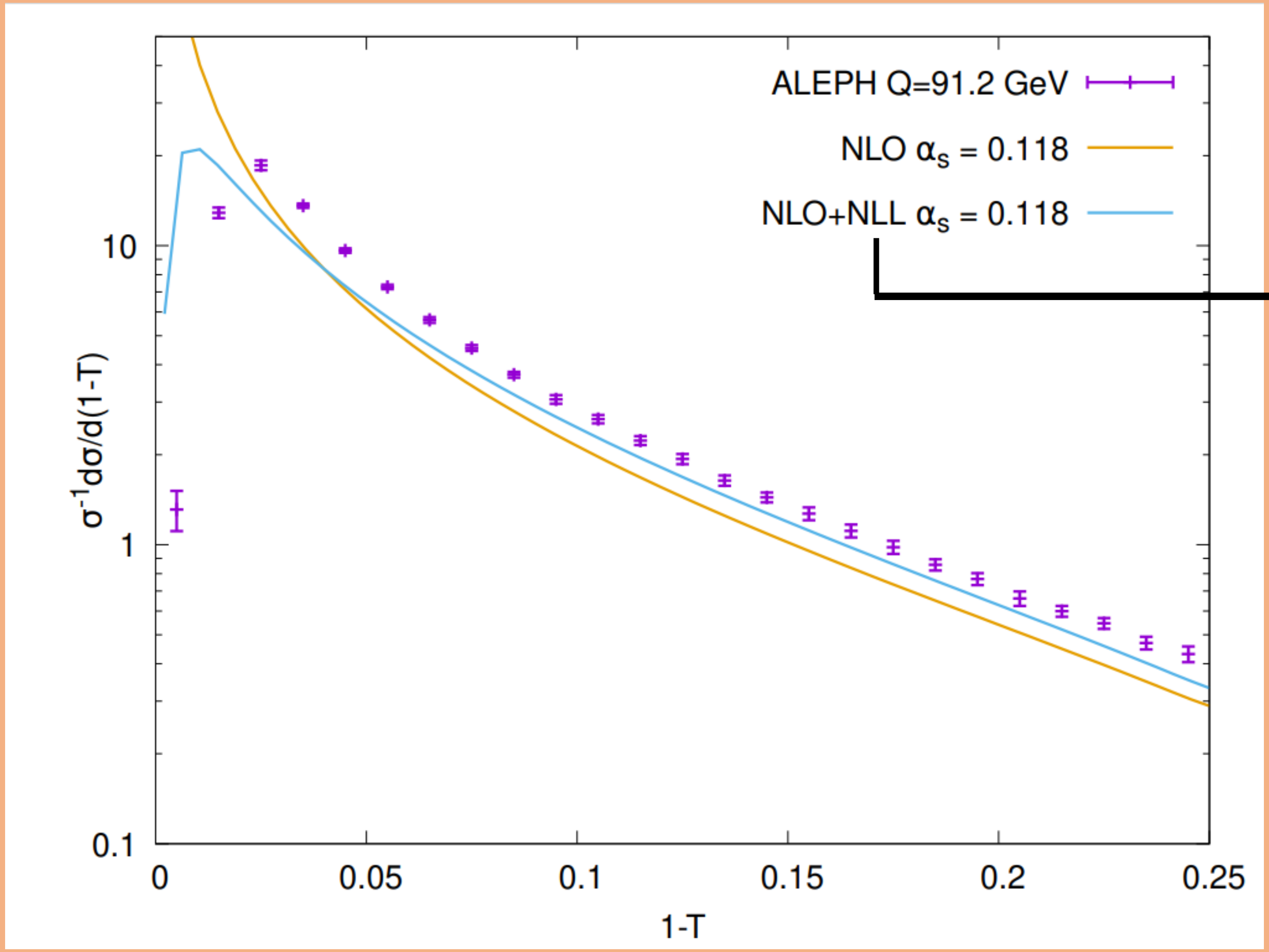

NLL means resuming all of the leading logarithmic (LL) and next-to-leading logarithmic (NLL) terms, i.e. those of the form: $\alpha_s^m L^{m+1}$ and $\alpha_s^m L^m$

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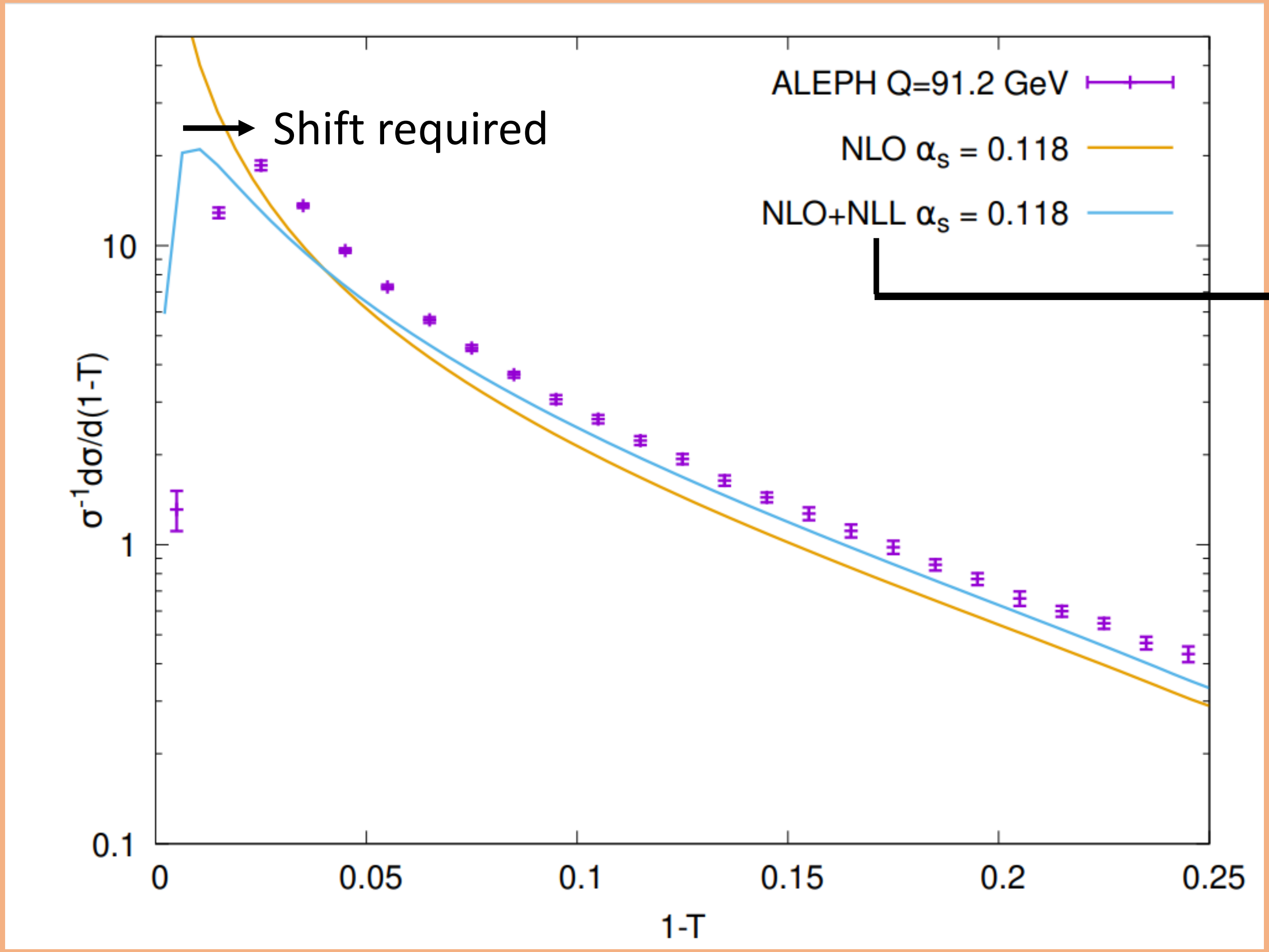

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Non-perturbative effects

- It is almost axiomatic that perturbation theory ***alone*** cannot give a full description of QCD (cf **Quarks/Gluons** in Theoretical Calculations vs **Hadrons** in Experimental Measurements)

- We must take into account the presence of non-perturbative corrections due to hadronisation
(N.B it transpires, in fact, that these corrections are suppressed by inverse powers of the centre-of-mass energy Q, and hence are often referred to as "**power corrections**")

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How do we do this when hadronisation is a phenomenon whose description lies beyond what can be achieved through perturbative QCD?

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- **Answer:**
  In the absence of a non-perturbative description of strong-interactions, some modelling is required (for which there are 2 possible avenues)

Non-perturbative effects

1) *Using Monte-Carlo (MC) parton shower event generators*

These are multi-purpose tools that simulate collider events down to the hadron level by using a hadronisation model, for example:
Lund String Model - PYTHIA
Cluster Model - HERWIG, SHERPA

Methodology:

1) Run the MC event generator down to both the parton and hadron level

2) Compute the ratio between an observable distribution or moment at both levels

3) Apply this correction to the corresponding perturbative prediction

4) One can estimate the uncertainty in the fit due to hadronisation by changing the event generator and/or the hadronisation model

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Limitations:

- For this approach to be reliable, it requires that parton-level distributions from the MC event generator are in reasonable agreement with the corresponding perturbative predictions

- Two problems:

1) The parton level of a MC simulation is not defined in a manner equivalent to that of a fixed-order calculation
2) MC event generators are tuned on less accurate perturbative calculations

Non-perturbative effects

2) *Employ analytic hadronisation models*

In particular, we consider the **dispersive model**.

This model is built around the observation that perturbative series in QFT show a factorial divergence and so cannot be pushed to arbitrarily high orders.

We are interested in probing low scales (or large distances) and, in this region, the divergence is known as an **infrared renormalon**.

From this viewpoint, infrared renormalons arise from the divergence of the perturbative expression for α_s at low scales, and the ambiguities associated with different ways of avoiding the renormalon poles in the Borel transform plane are resolved by specifying the infrared behaviour of α_s.

Non-perturbative effects

Specifying the infrared behaviour of α_s :

Analysis of infrared renormalon ambiguities suggests power corrections of the form λ_p/Q^p with $p \geq 1$. We shall consider leading hadronisation corrections only, $\sim 1/Q$, i.e $p = 1$.

We introduce an IR cutoff, μ_I, where $\Lambda_{\mathrm{QCD}} \leq \mu_I \ll Q$, and replace the strong coupling constant below the scale μ_I with an effective coupling, $\alpha_{\mathrm{eff}}(k)$, which we suppose to be finite in the infrared region down to $k \to 0$.

We define the average of this effective coupling up to the low scale μ_I as:

$$\frac{1}{\mu_I} \int_0^{\mu_I} dQ \, \alpha_{\mathrm{eff}}(Q) \equiv \alpha_0(\mu_I)$$

Non-perturbative effects

The leading $1/Q$ hadronisation corrections are "computed" using the matrix element for the emission of a single ultra-soft gluon where the QCD coupling, α_s , is replaced by the soft effective coupling, $\alpha_{\rm eff}(k)$.

The main result is that the $1/Q$ corrections are proportional to the average, $\alpha_0(\mu_I)$, of this effective coupling up to the low scale μ_I.

Therefore the dispersive model provides analytical predictions for the leading power corrections and only introduces a single new parameter, α_0 .

Our revised strategy is therefore to add these corrections to the corresponding perturbative event-shape distributions and means, and perform simultaneous fits of $\alpha_s(M_Z)$and $\alpha_0(\mu_I)$.

How do these corrections produce a shift?

We find that our observable cumulant can be written as:

$$\Sigma(v) \simeq \Sigma_{\rm PT}(v) - \langle \delta V_{\rm NP} \rangle \frac{d\Sigma_{\rm PT}}{dv} \simeq \Sigma_{\rm PT}(v - \langle \delta V_{\rm NP} \rangle)$$

where $\delta V_{\rm NP}$, is the change in the observable value due to the emission of an ultra-soft non-perturbative gluon.

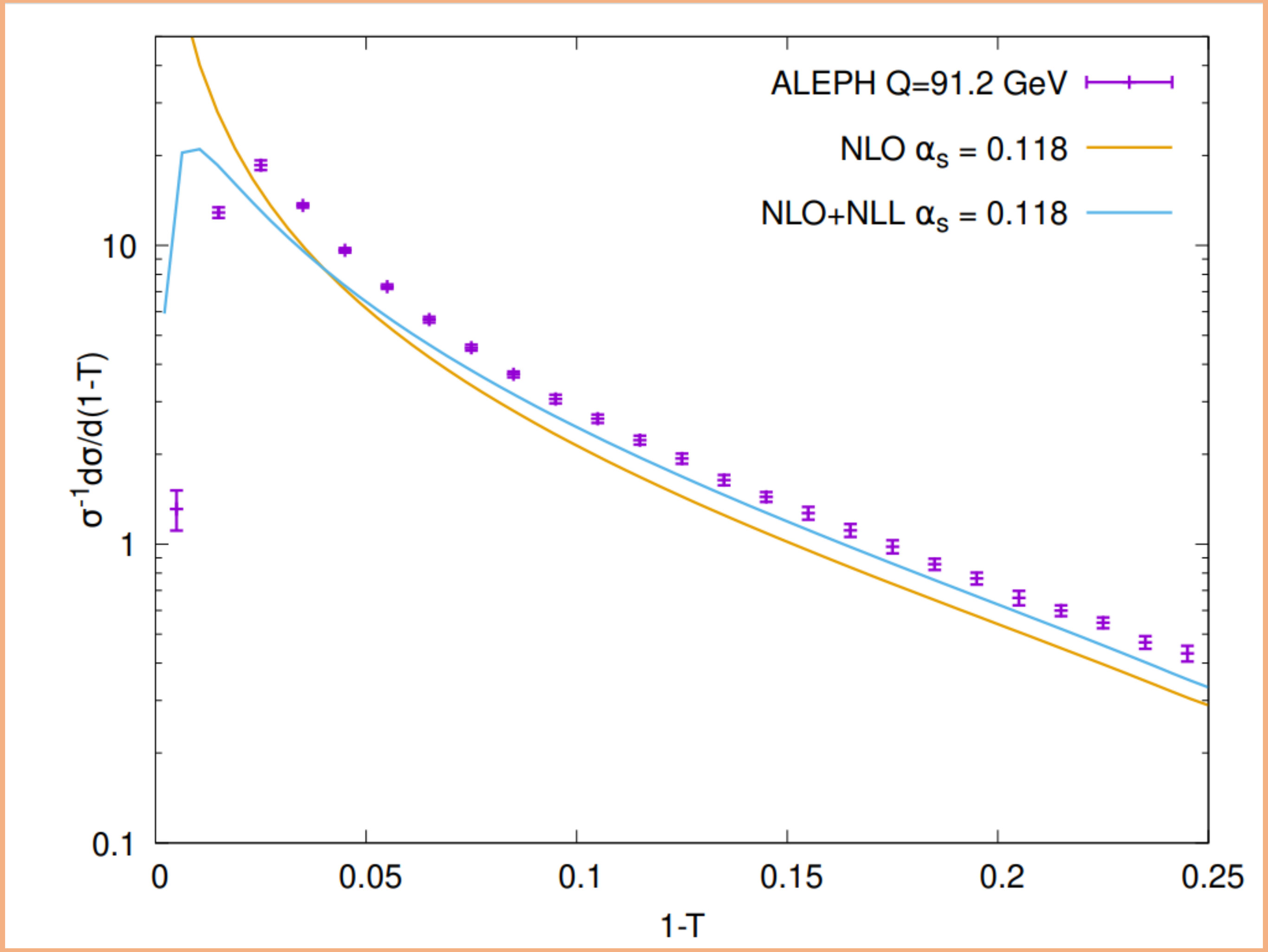
Therefore, non-perturbative corrections amount to a shift of the perturbative distribution by an amount given by the average $\langle \delta V_{\rm NP} \rangle$.

For the event shapes we have discussed:

$$\langle \delta V_{\rm NP} \rangle = \frac{\langle \kappa \rangle_{\rm NP}}{Q} \langle h_V \rangle \mathcal{M}$$

- $\mathcal{M} \simeq 1.49$ is the Milan Factor
- $\langle \kappa \rangle_{\rm NP} = \frac{4C_F}{\pi^2} \mu_I \left(\alpha_0(\mu_I) - \alpha_s - 2\beta_0 \alpha_s^2 \left(1 + \ln \frac{Q}{\mu_I} + \frac{K}{4\pi\beta_0} \right) \right)$
- $\langle h_V \rangle$ is observable dependent

Returning to our example of the Thrust



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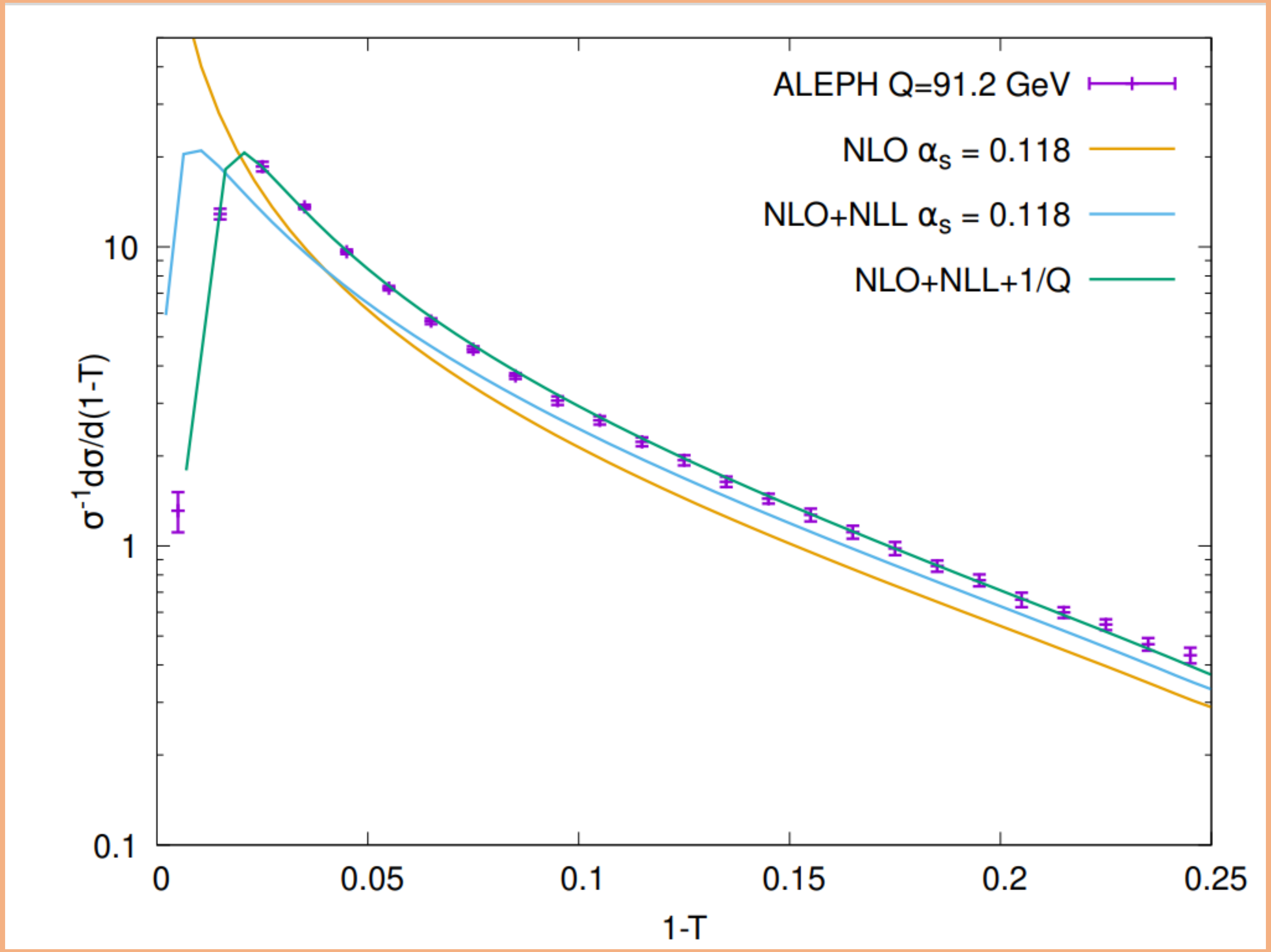
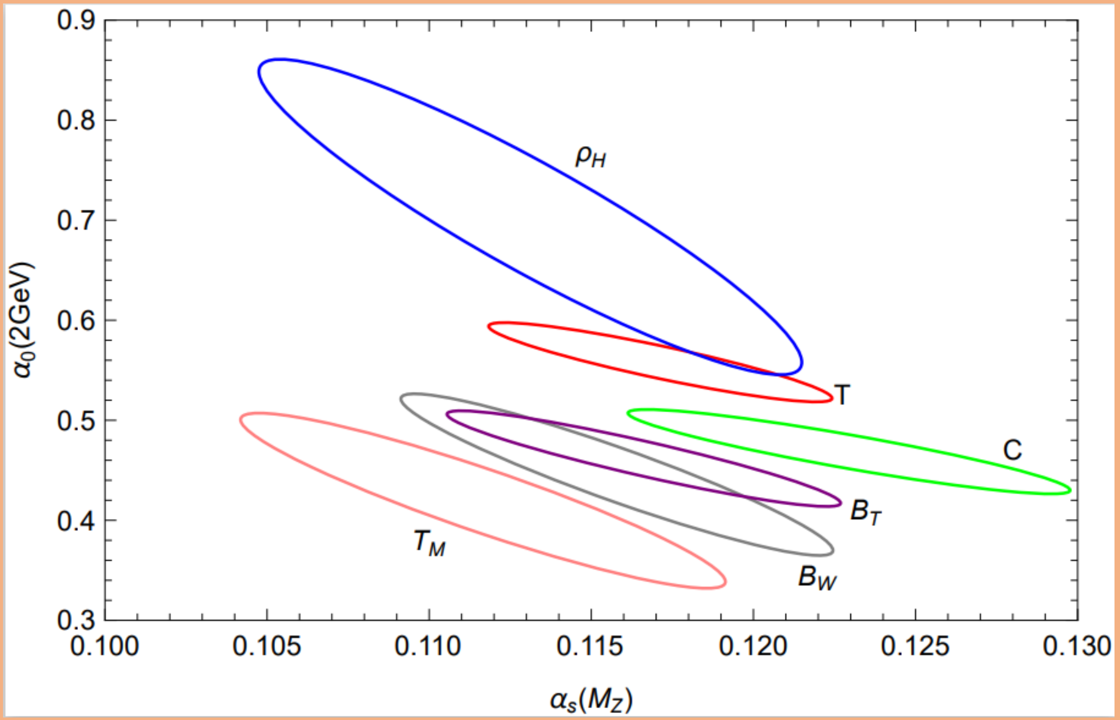
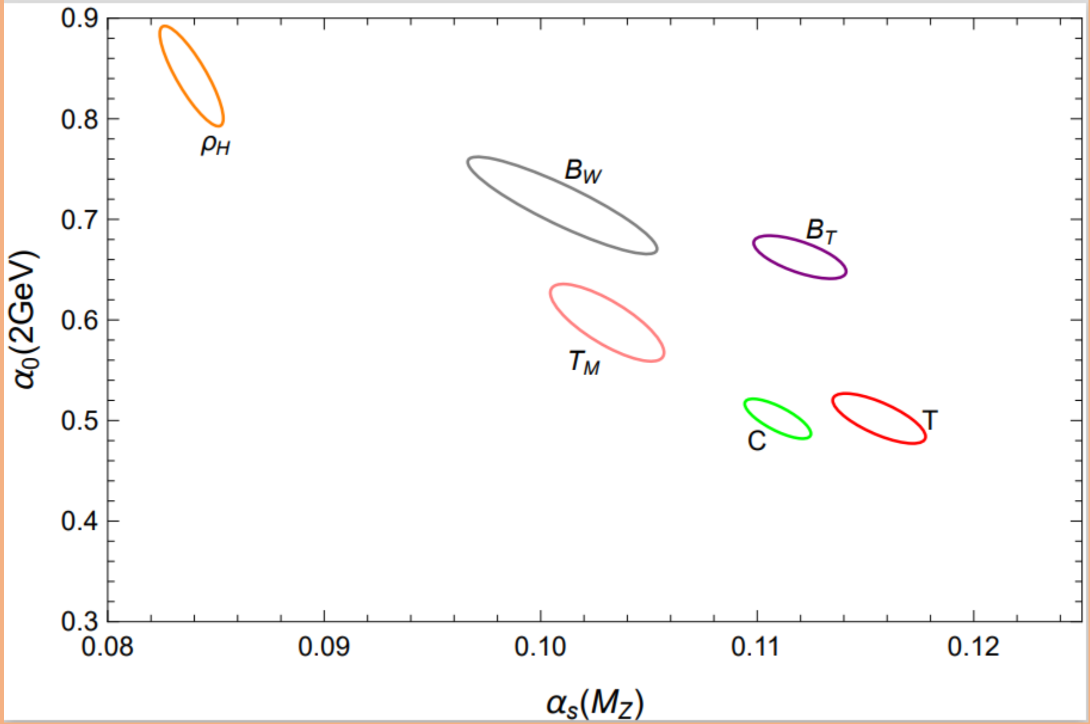


Simultaneous Fits

Mean Values:



Distributions:



Outlook

- A natural extension is to employ this method to provide predictions for the two-jet rates.
- With event shapes in the two-jet limit and the two-jet rate then under control, matching to the three-jet region could then be considered.

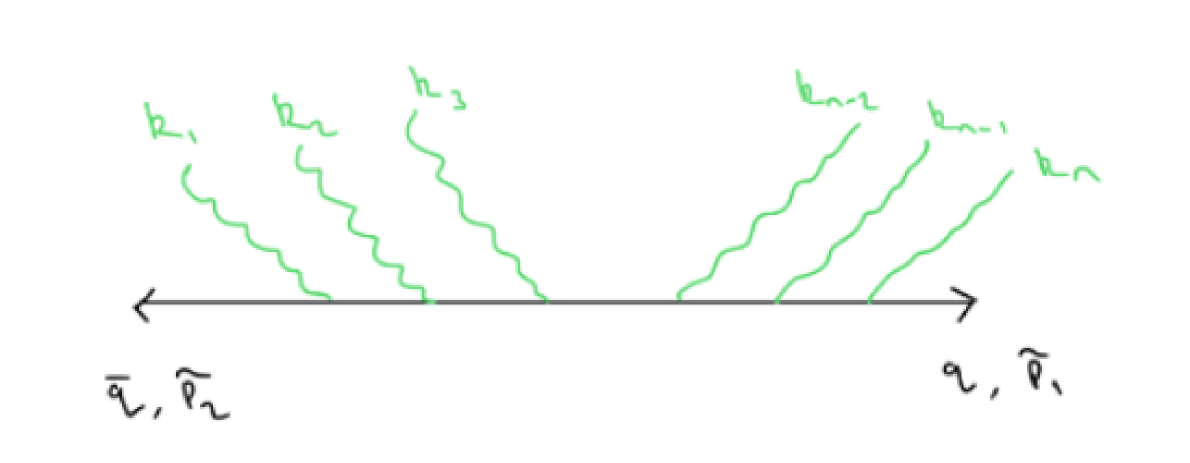
Sources:

- F. Caola, S. Ferrario Ravasio, G. Limatola, K. Melnikov, P. Nason and M. A. Ozcelik, Linear power corrections to e^+e^- shape variables in the three-jet region, [arXiv: 2204.02247].
- P. Nason and G. Zanderighi, Fits of α_s using power corrections in the three-jet region, [arXiv: 2301.03607].

Additional Material

The need for Resummation

Our event consists of a hard back-to-back $q\bar{q}$ pair (with momenta $\tilde{p}_1$ and $\tilde{p}_2$), accompanied by secondary emissions (with momenta k_i) emitted from either leg, as shown below:



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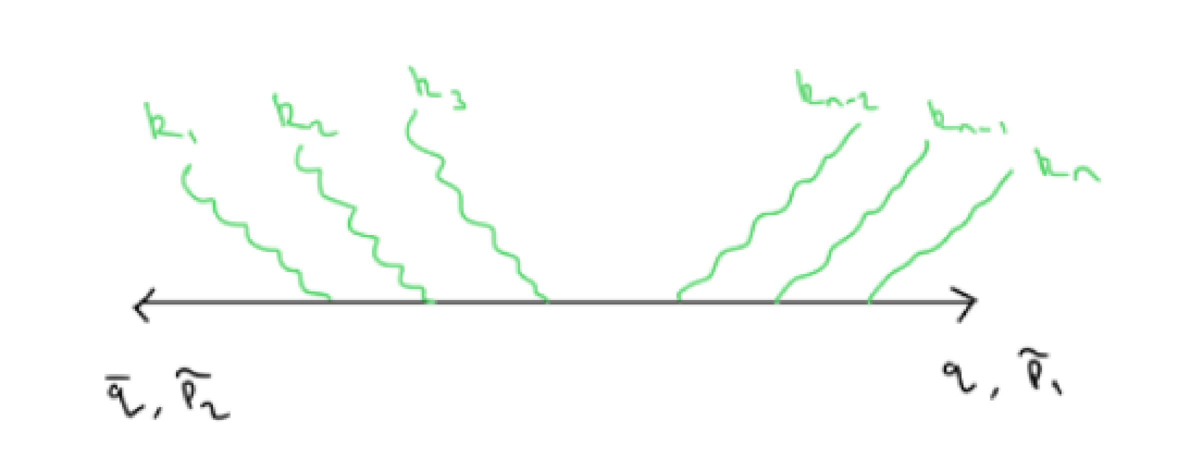

When our event-shape value is small there is an implicit constraint placed on these secondary emissions:

They must be soft and/or collinear to their emitter so as not to affect the value of the observable too far away from its Born value.

The need for Resummation

The soft-gluon emission probability is given by:

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

- Diverges for $E \to 0$: **soft divergence**
- Diverges for $\theta \to 0\,,\pi$: **collinear divergence**

Therefore each power of α_s may be accompanied by up to two logs (one corresponding to the soft divergence and one to the collinear divergence).

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Problem:

For small values of the event shape, $\alpha_s \ln\frac{1}{v} \sim 1$ and the perturbative series breaks down

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Solution: Resummation

For small values of the event shape, perform a schematic reshuffling of terms giving dominant logarithmic terms priority rather than those with the fewest powers of α_s