Probing $0\nu\beta\beta$ Decay in Multiple Isotopes

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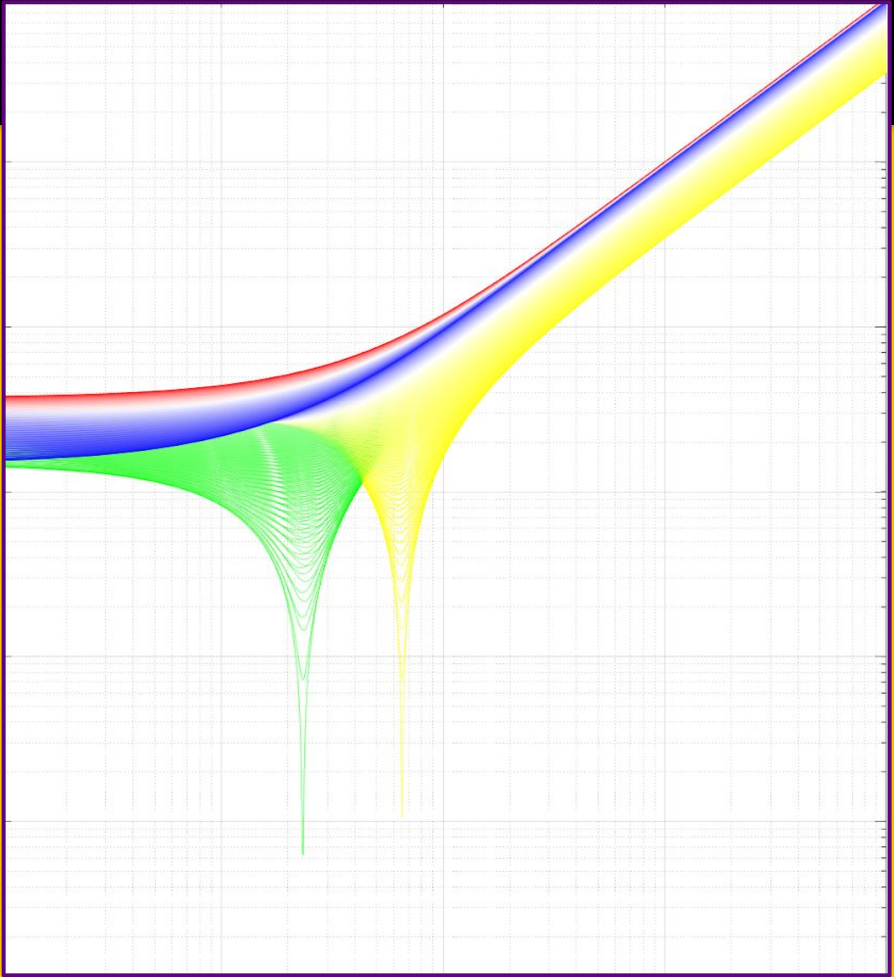

Probing $0\nu\beta\beta$ Decay in Multiple Isotopes: Quo Vadis?

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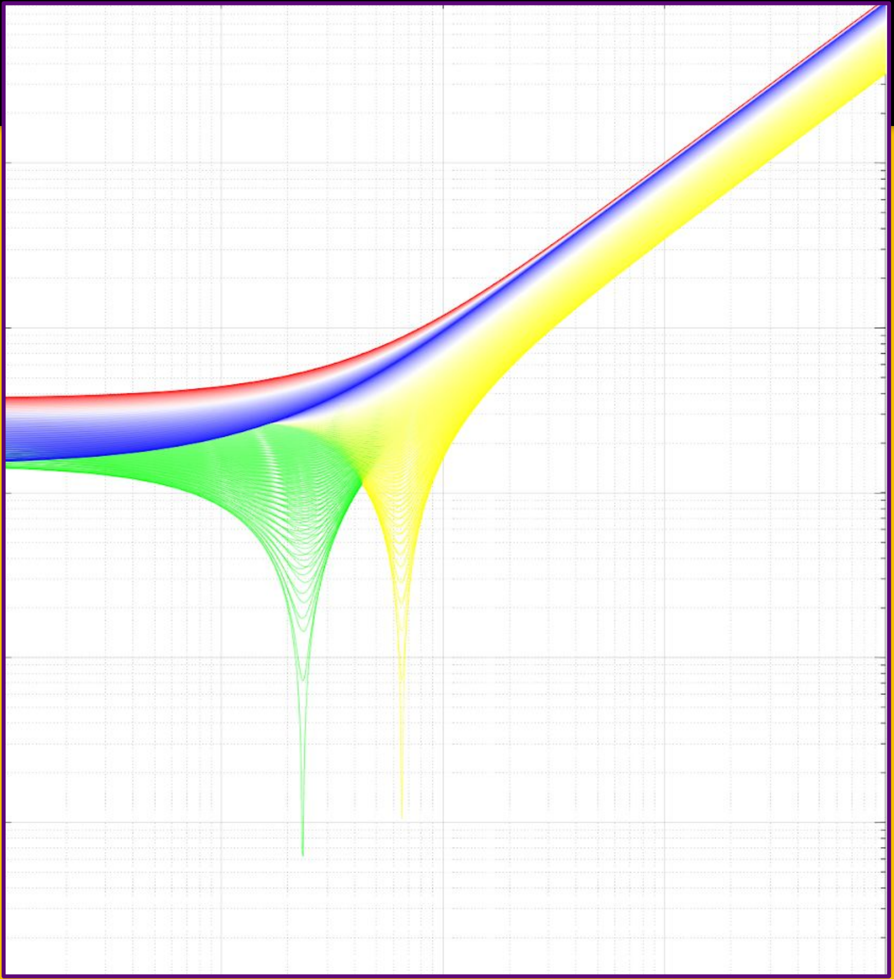


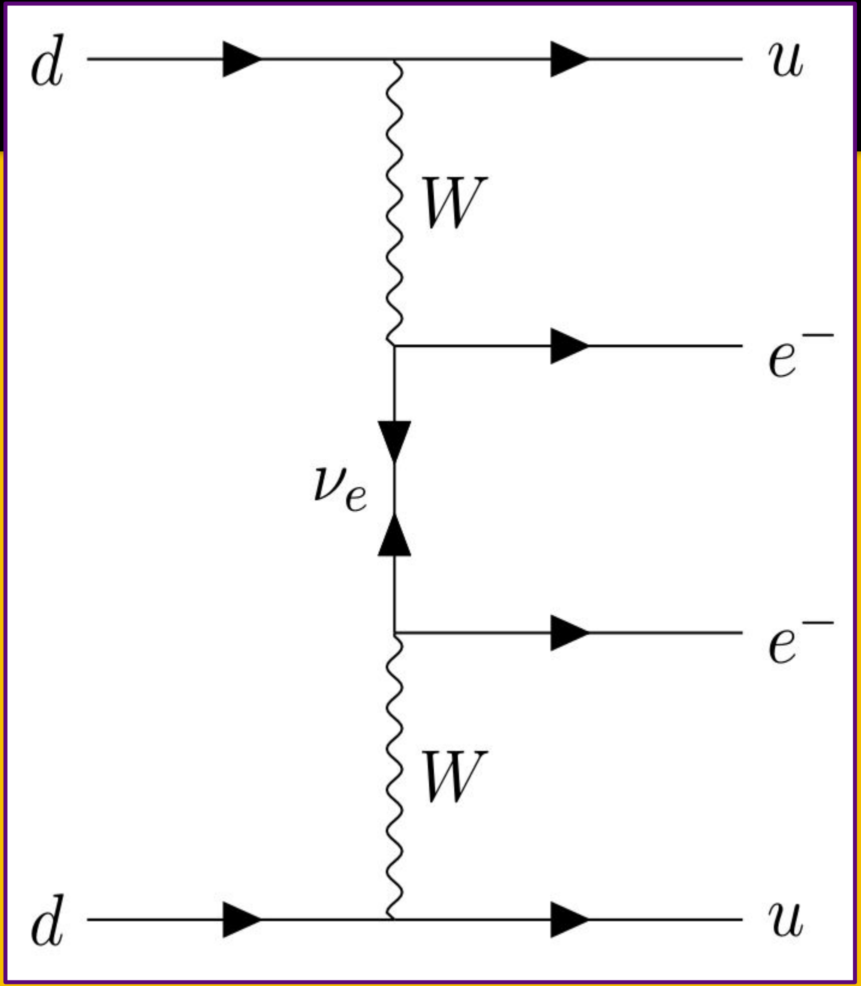
Neutrinos and $0\nu\beta\beta$

- **Flavour oscillation ⇒ massive neutrinos**

- Cosmology, oscillations, β-decay provide partial answers and constraints.

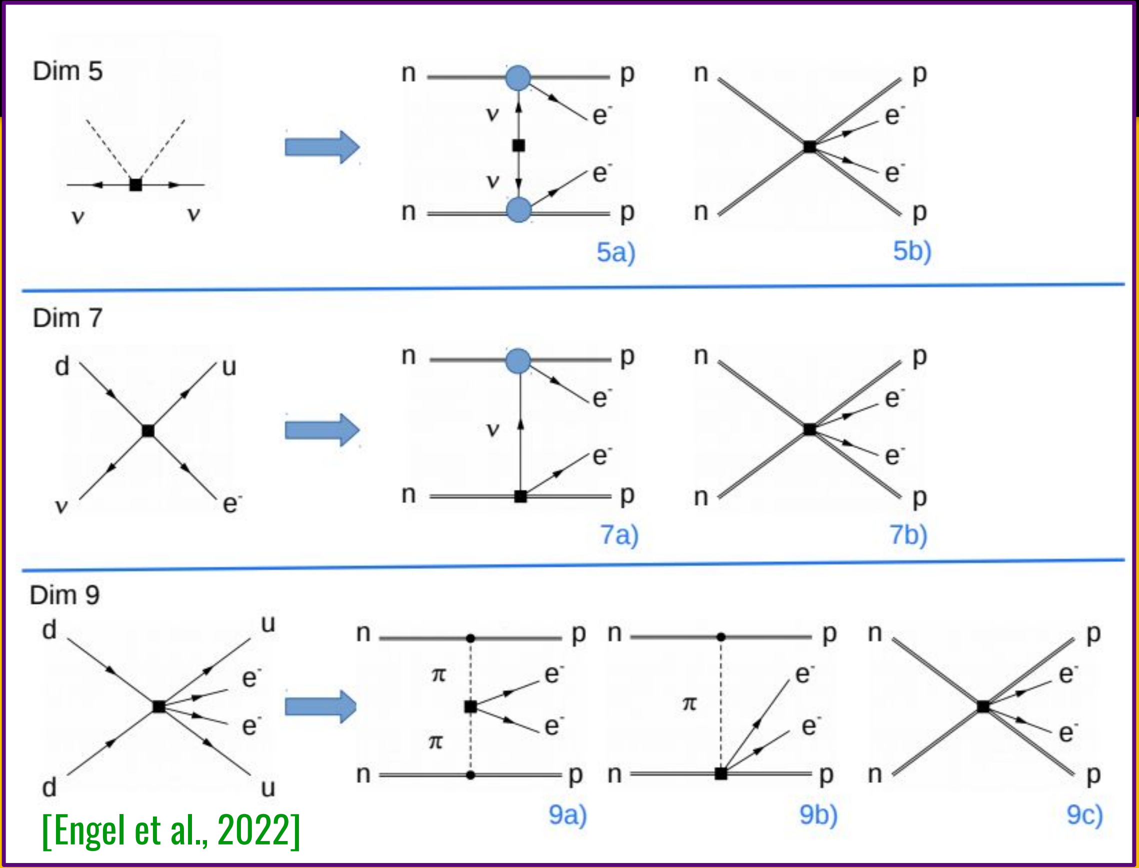
- $0\nu\beta\beta$ -- just another probe? No!
  - Sensitive to LNV
  - May differentiate mass mechanisms

- Current bounds (@90%)
  - GERDA: $T_{1/2} > 1.8 \cdot 10^{26}$ y [1]
  - KamLAND-Zen: $T_{1/2} > 2.3 \cdot 10^{26}$ y [2]

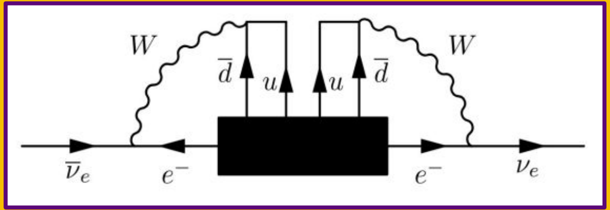


$0\nu\beta\beta$ Mechanisms

- $\Delta L=2$ from odd-dim SM-EFT operators [Kobach, 2016]
    - 5,7 classified, 9 partially
- $\Lambda_{LNV} >> \Lambda_{EW} \Rightarrow$ 5-dim only
- Schechter-Valle Theorem:
    - $\Delta L=2 \Rightarrow$ Majorana ν mass



[Engel et al., 2022]



[Schechter + Valle, 1982]

Searching for Short-Range Mechanisms

- **Question:** To what extent can multi-isotope observations constrain exotic mechanisms?
    - What is the impact of (correlated) NME uncertainties, and how to fold into analysis?

- Numerous parameterised mechanisms [Deppisch et al., Phys. Rev. D 102, 095016 (2020)]
- We consider a single exotic $0\nu\beta\beta$ mechanism with its own “heavy NME” (a la RPV SUSY):

$$T_{1/2}^{-1} = G_{iso}\left|\frac{m_{\beta\beta}}{m_e}M_{\nu,iso} + \epsilon M_{H,iso}\right|^2$$

- Any fixed $0\nu\beta\beta$ half-life corresponds to two parallel-line solutions in $\{m_{\beta\beta}, \varepsilon\}$

Ratio Degeneracy (or not)

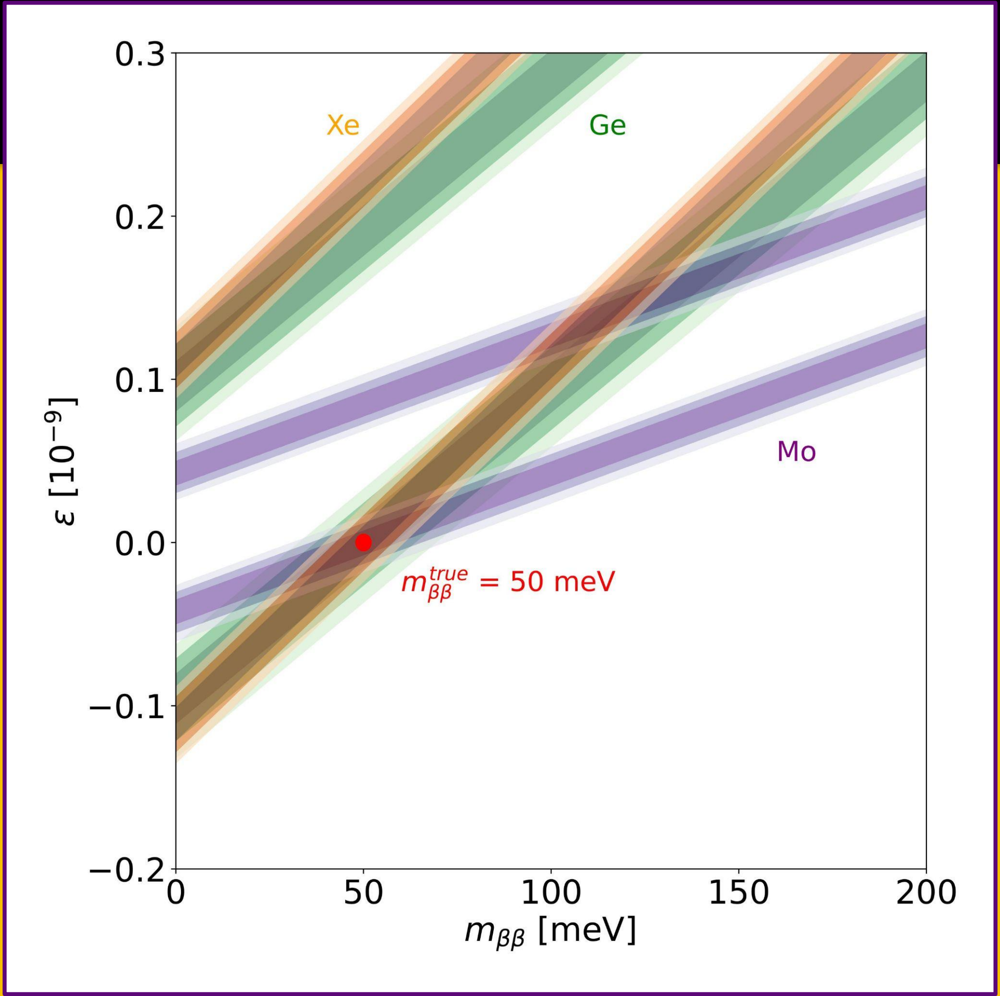
- Previous thinking: light/exotic NME ratio strongly degenerate across isotopes
    - QRPA in Ge/Se/Mo/Te [Faessler et al., 2011], Ge/Te/Xe [Lisi et al., 2015]
    - Big problem for multi-isotope analysis

- However, nuclear structure correlations can break this degeneracy
    - Correlations suppress NME at <2 fm (in NSM/EDF) [Menéndez, 2018]
    - Therefore contribute less to exotic than to light nu exchange
    - Also verifies [F, 2011][L, 2015] by deselecting correlated nucleon states

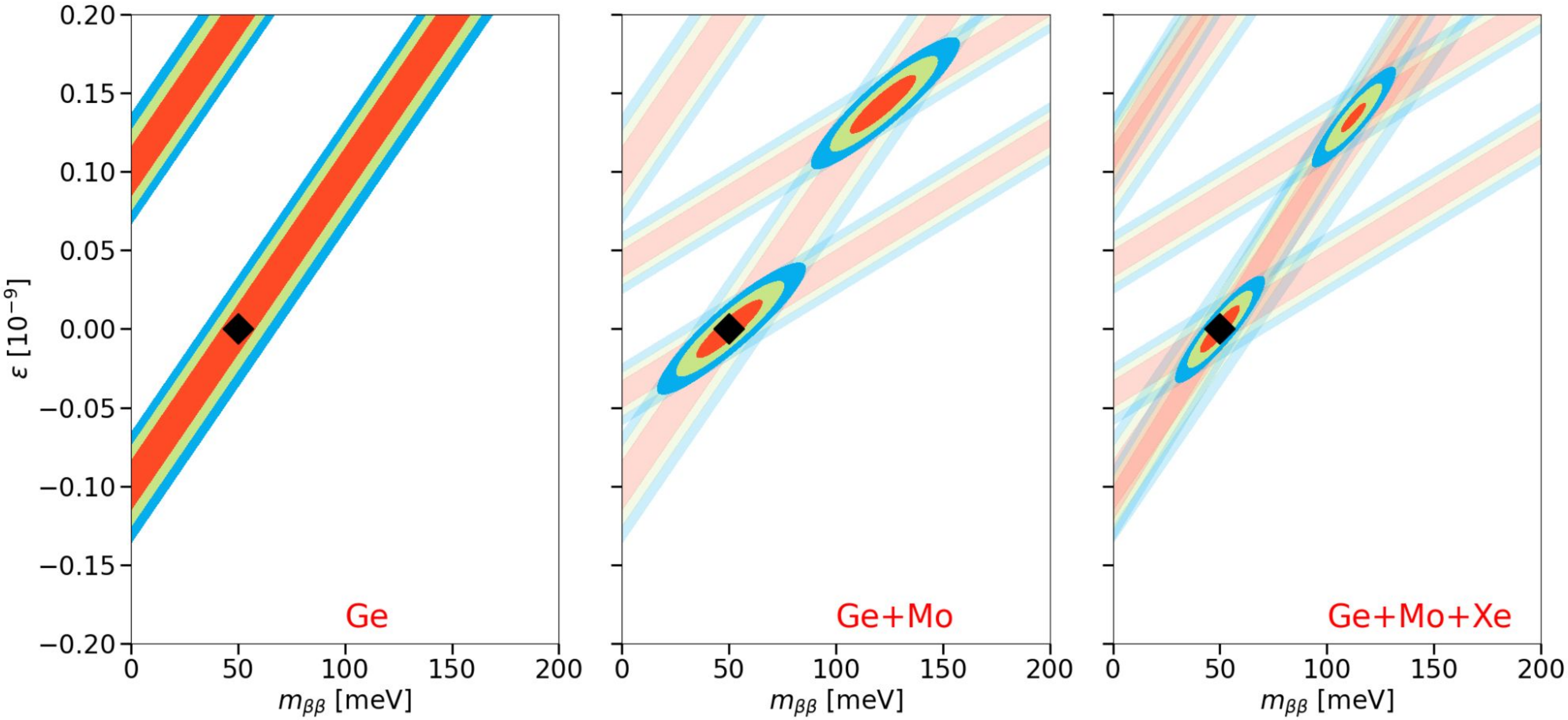
- Degeneracy broken in IBM-2 for five distinct exotic mechanisms [Deppisch et al., 2020]
    - Strongest deviation from degeneracy in ^{100}Mo

Single Isotopes

- Proposed next-gen searches:
  - ^{76}Ge: LEGEND-1000
  - ^{100}Mo: CUPID
  - ^{136}Xe: nEXO
- Ratio degeneracy controls overlaps of per-isotope likelihoods
- Two-isotopes ⇒ secondary peak
  - Third suppresses this peak



χ^2 Analysis for Ge, Ge+Mo, Ge+Mo+Xe



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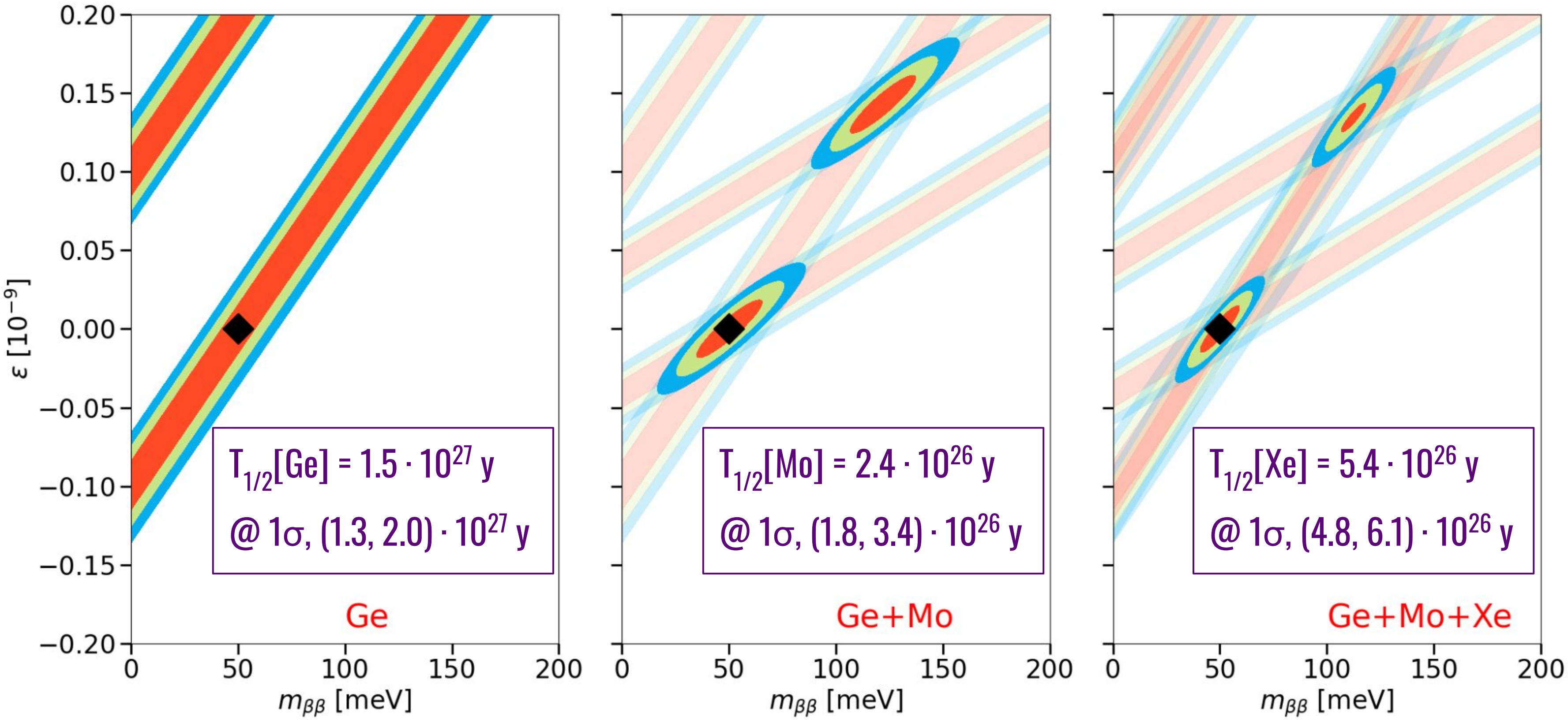


$T_{1/2}[\mathrm{Ge}] = 1.5 \cdot 10^{27}$ y

@ 1σ, $(1.3, 2.0) \cdot 10^{27}$ y

$T_{1/2}[\mathrm{Mo}] = 2.4 \cdot 10^{26}$ y

@ 1σ, $(1.8, 3.4) \cdot 10^{26}$ y

$T_{1/2}[\mathrm{Xe}] = 5.4 \cdot 10^{26}$ y

@ 1σ, $(4.8, 6.1) \cdot 10^{26}$ y

Bayesian Methodology

- Update prior knowledge **$\pi(\theta)$** with likelihood **$L_X(\theta)$** to obtain posterior knowledge **$p(\theta)$**.

$$p(\boldsymbol{\theta}) = \frac{L_X(\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\int L_{\mathbf{X}}(\boldsymbol{\theta}')\pi(\boldsymbol{\theta}')d\boldsymbol{\theta}'_H} \equiv \frac{L_{\mathbf{X}}(\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{M^H_{\mathbf{X}}}$$

- Markov Chain Monte Carlo (MCMC) samples **$p(\theta)$** using only local distributions.
  - Locality helps to combat MC rejection problems at high-dimensionalities.

- Our parameter space: $\{m_{\beta\beta}, \varepsilon\}$ + $\{M_\nu, M_H\}$ for each isotope
  - Flat priors on $m_{\beta\beta}$, ε [Deppisch, **GVG** Phys. Rev. D 104, 055040 (2021)]
  - What priors to use on $\{M_\nu, M_H\}$, given sample estimates?

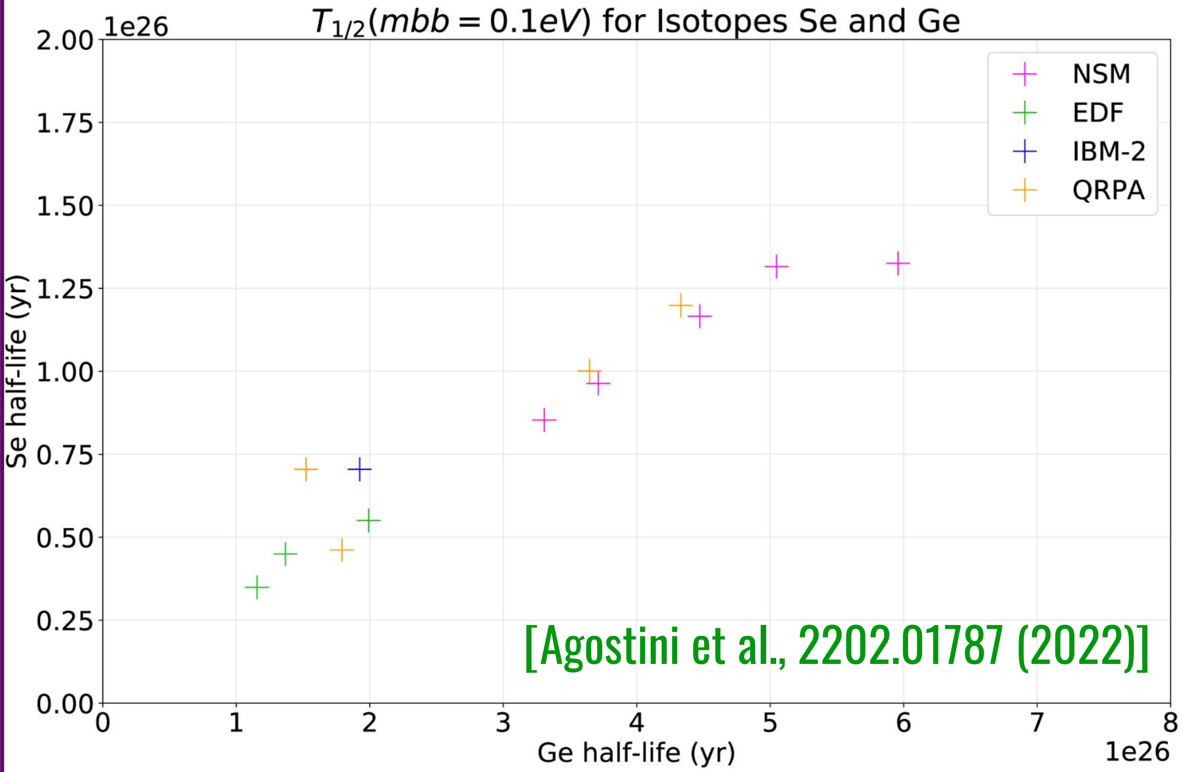
Nuclear Matrix Elements (NMEs)

- Recall $m_{\beta\beta}$ and half-life:

$$T_{1/2}^{-1} = G_{0\nu}|\mathbb{M}|^2 m_{\beta\beta}^2$$

- NMEs discrepancies are correlated for (some) pairs of isotopes.

- Source: systematic uncertainties across many-body methods



[Agostini et al., 2202.01787 (2022)]

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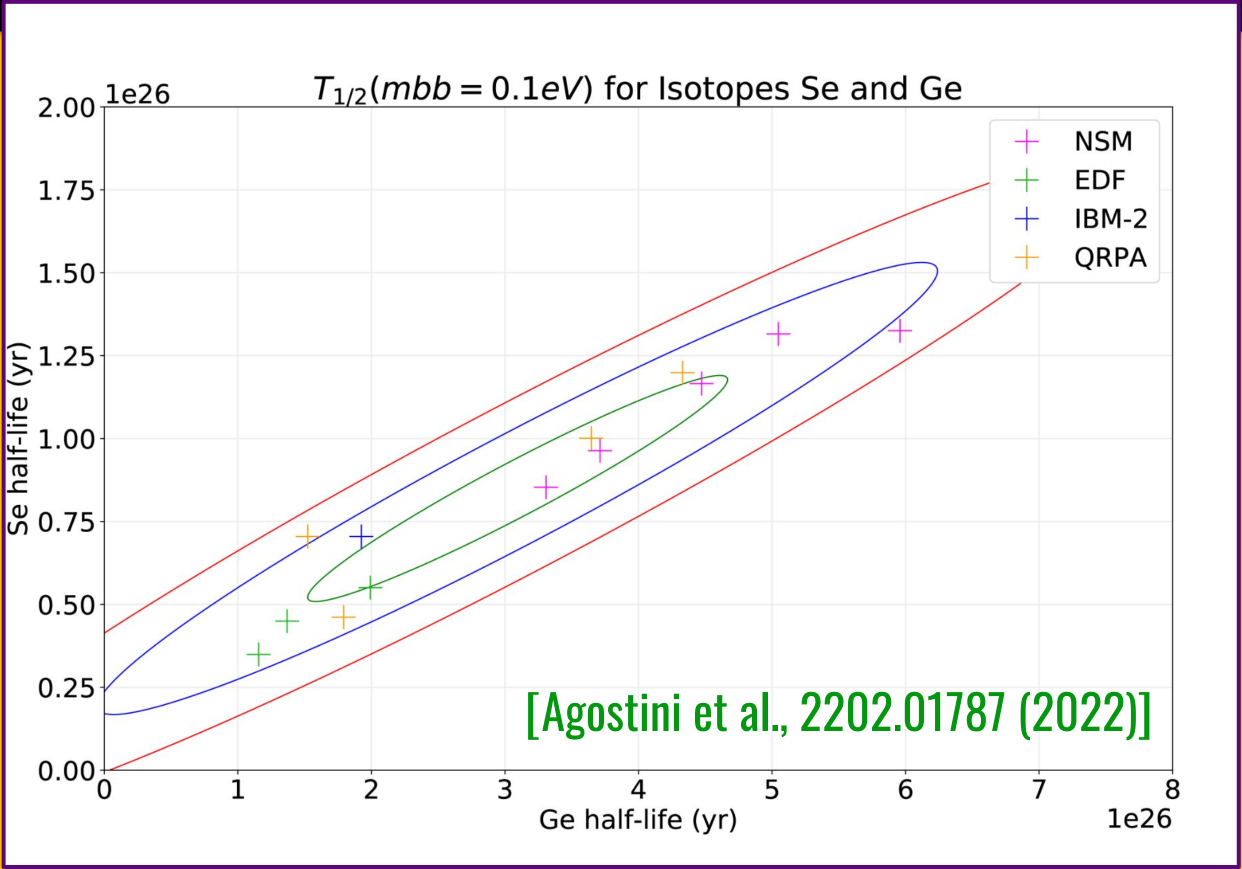


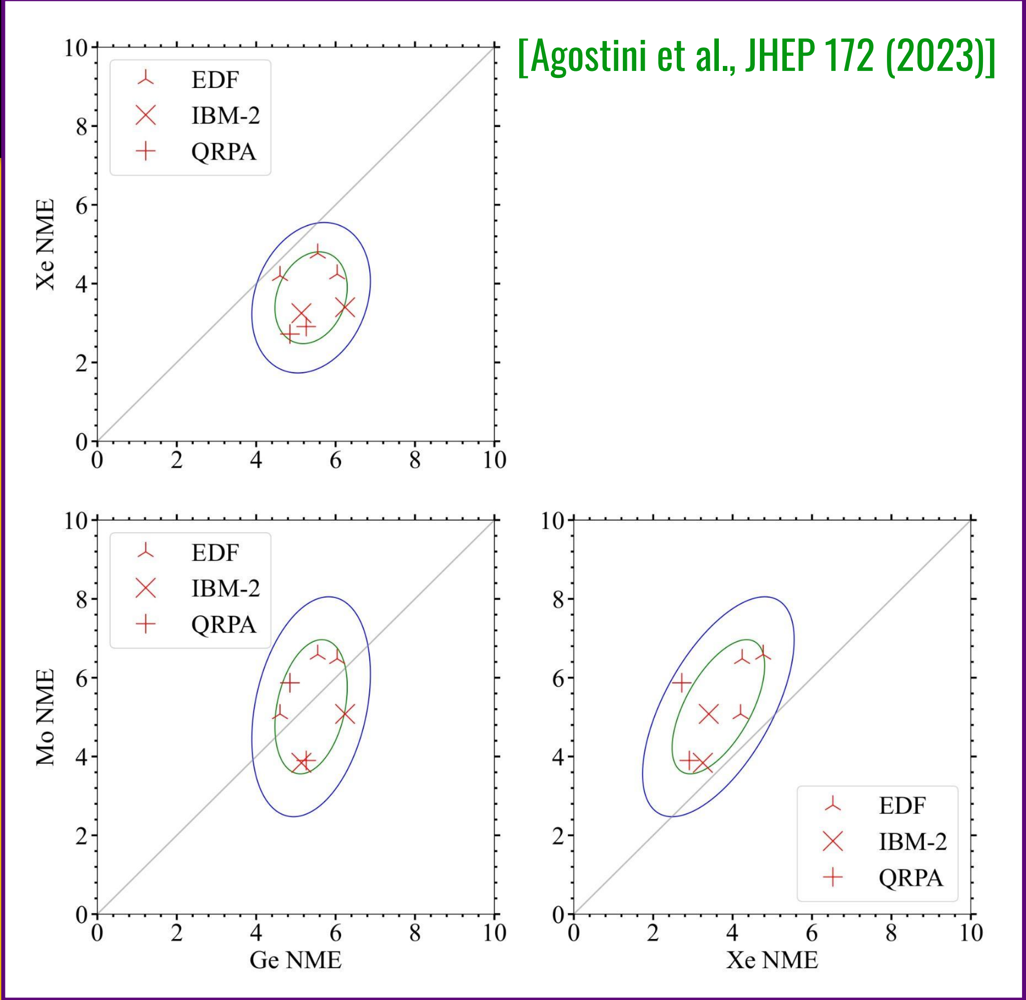
[Agostini et al., 2202.01787 (2022)]

NME Priors

- Multinormal fit on three methods
  - Seven independent estimates

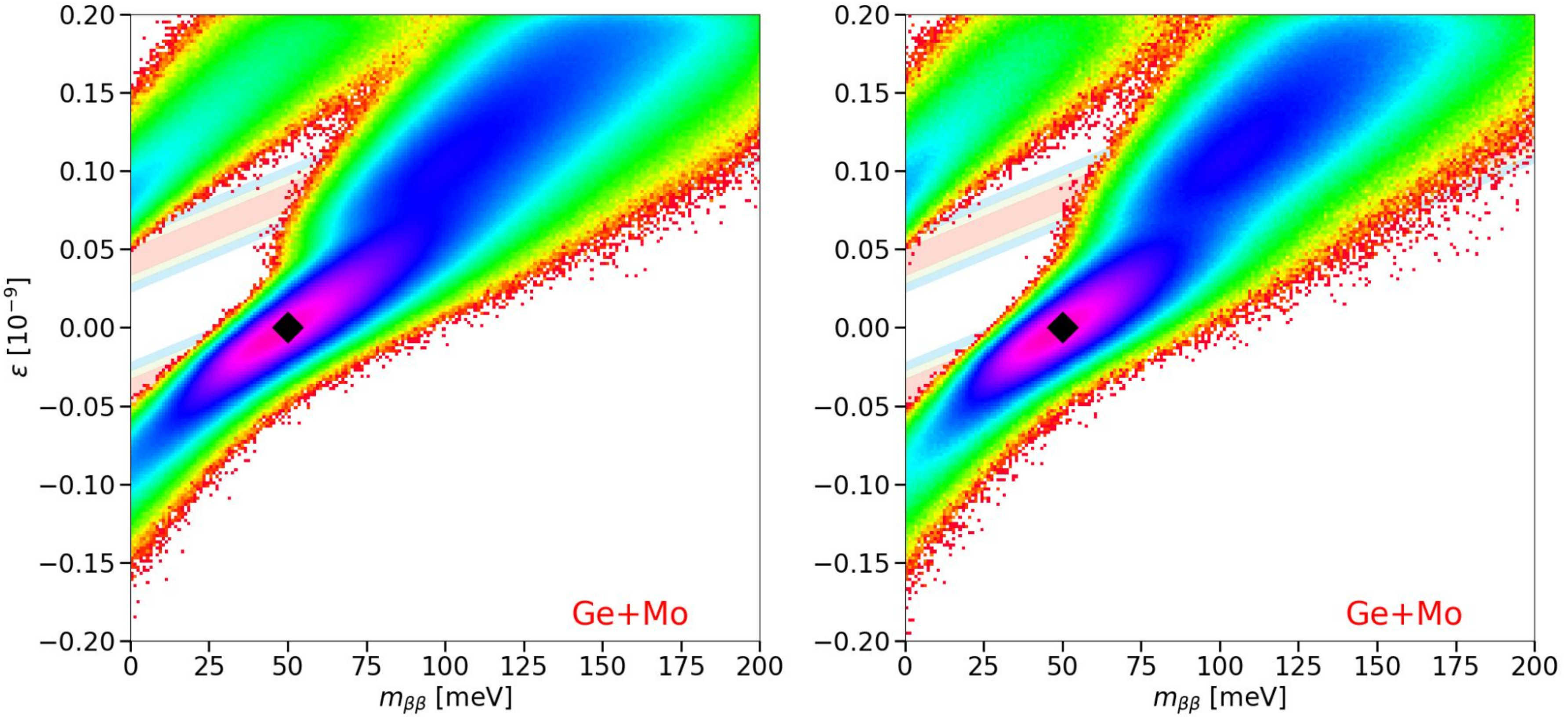
- Placeholder for concrete estimates of theoretical uncertainties

$$\boldsymbol{\mu} = \begin{pmatrix} 5.383 \\ 5.263 \\ 3.641 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 0.361 & 0.200 & 0.102 \\ 0.200 & 1.260 & 0.527 \\ 0.102 & 0.527 & 0.590 \end{pmatrix}$$

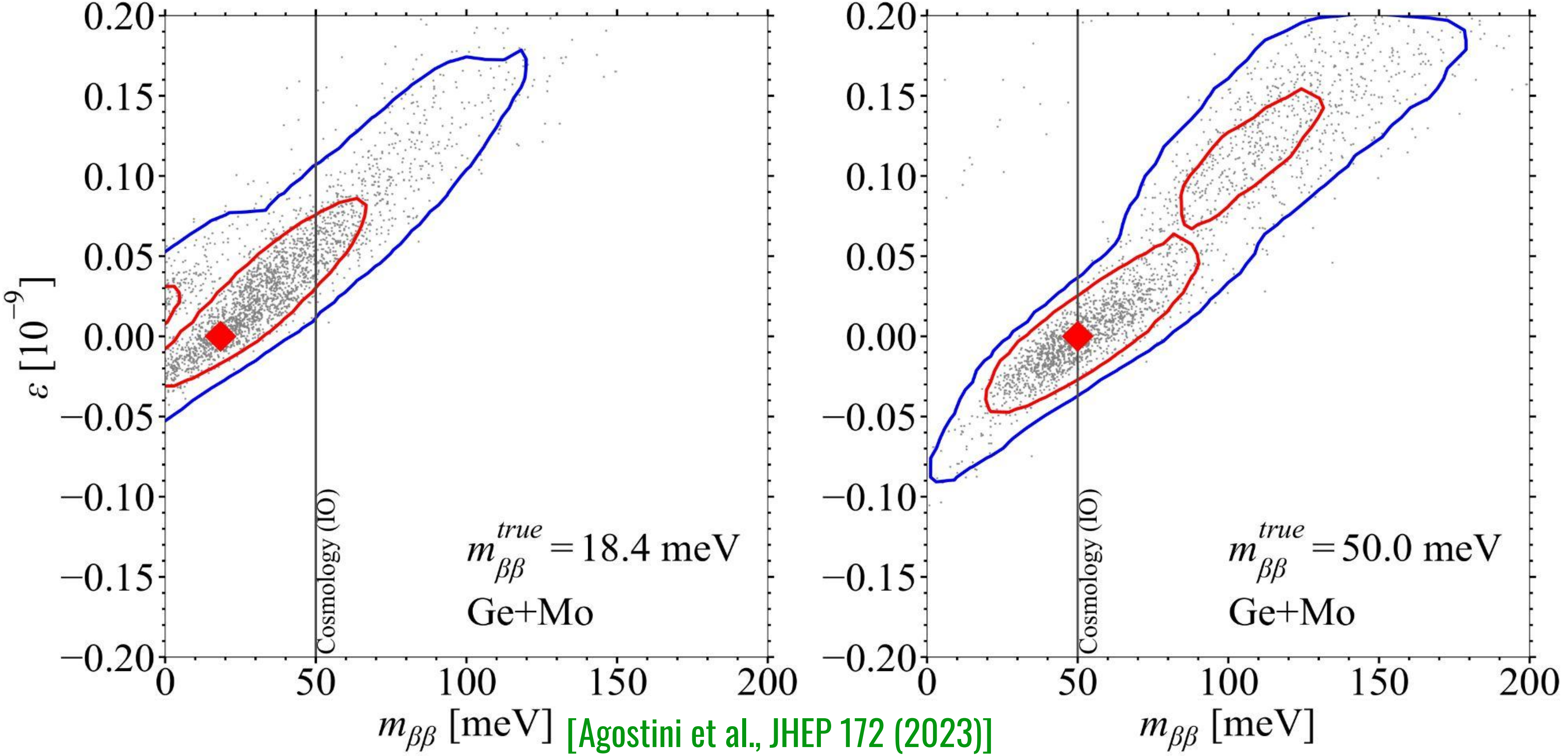


[Agostini et al., JHEP 172 (2023)]

Uncorrelated vs. Correlated

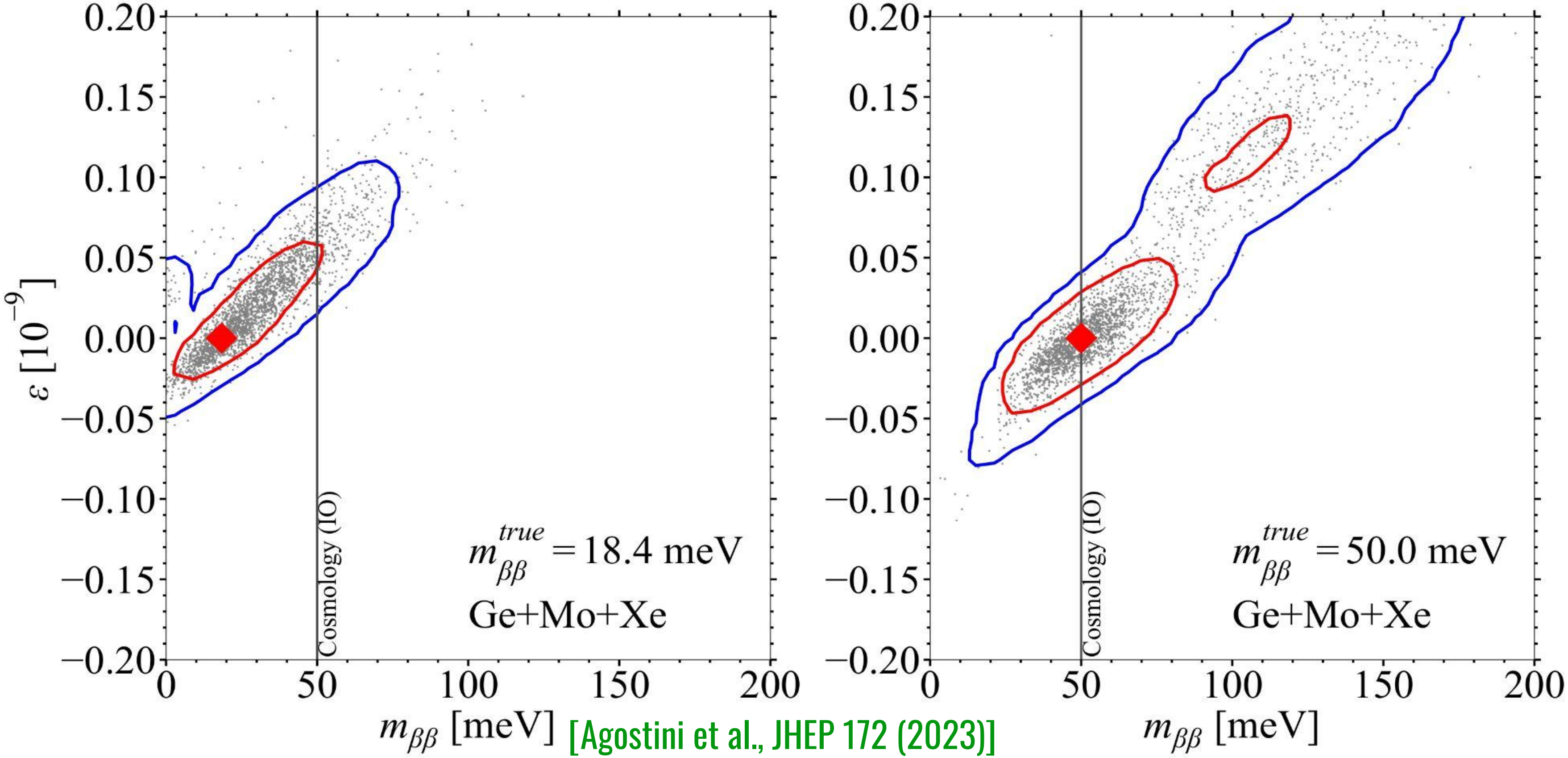


Ge+Mo, Current NMEs



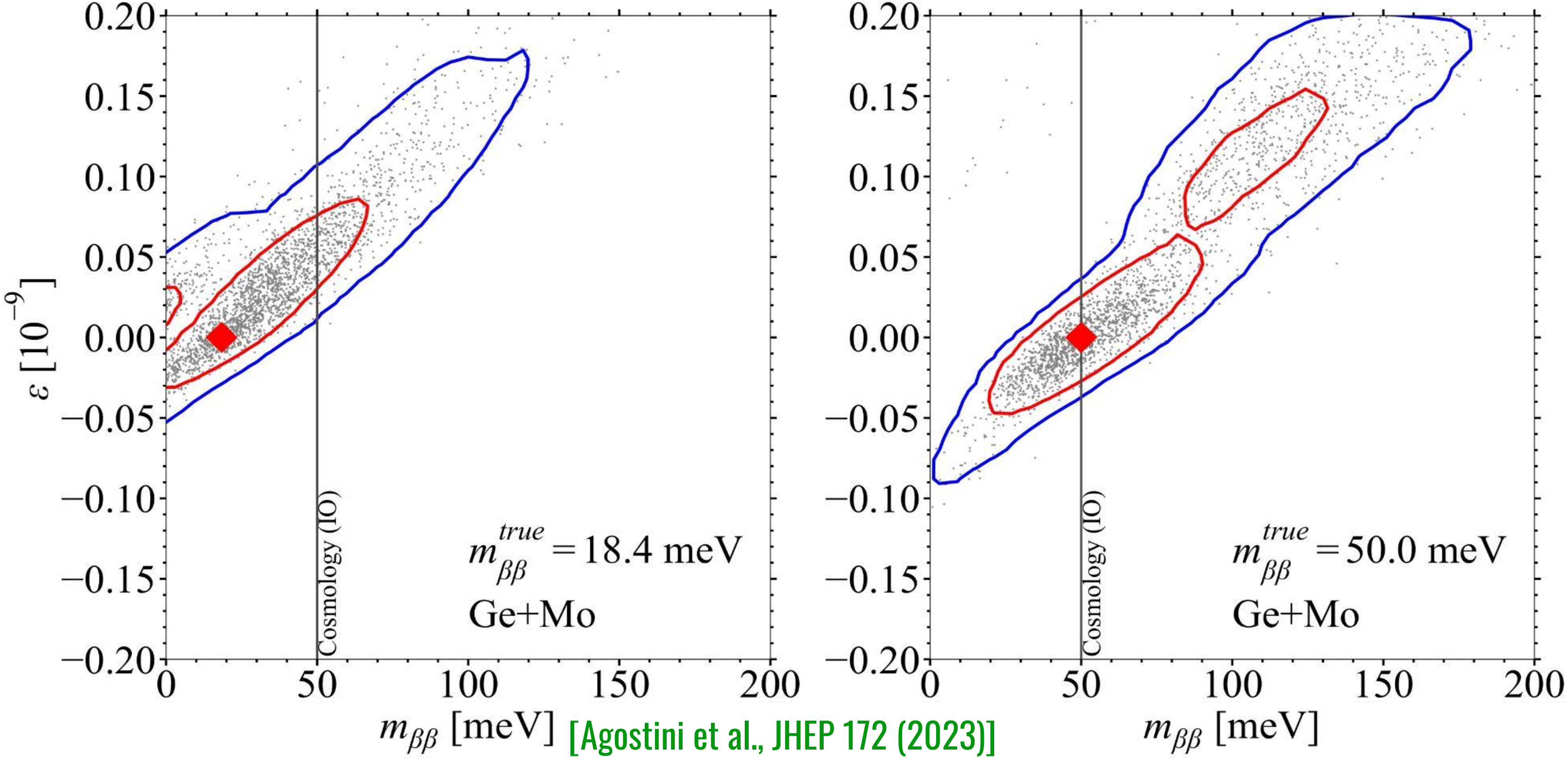
[Agostini et al., JHEP 172 (2023)]

Ge+Mo+Xe, Current NMEs



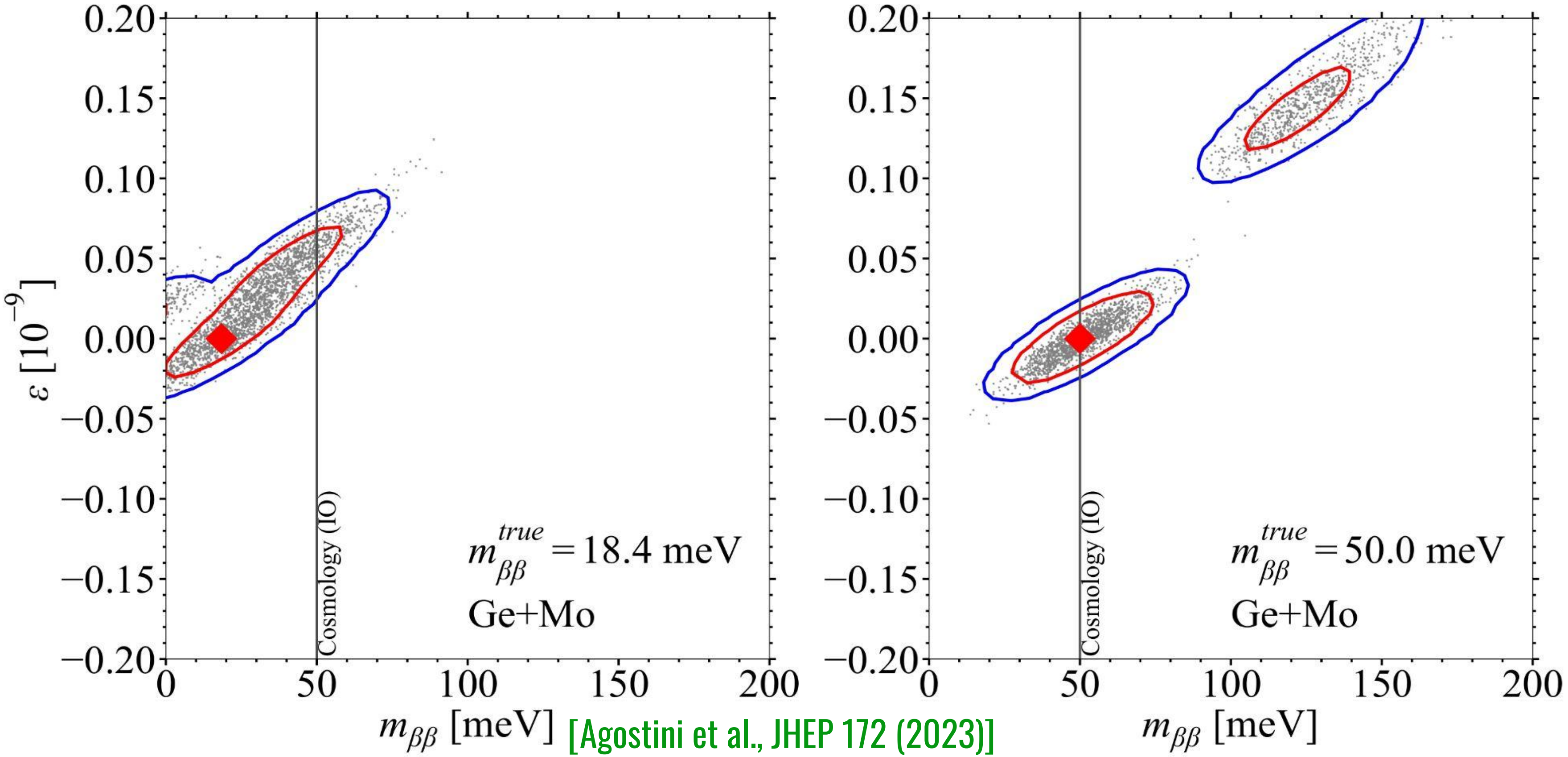
[Agostini et al., JHEP 172 (2023)]

Ge+Mo, Current NMEs



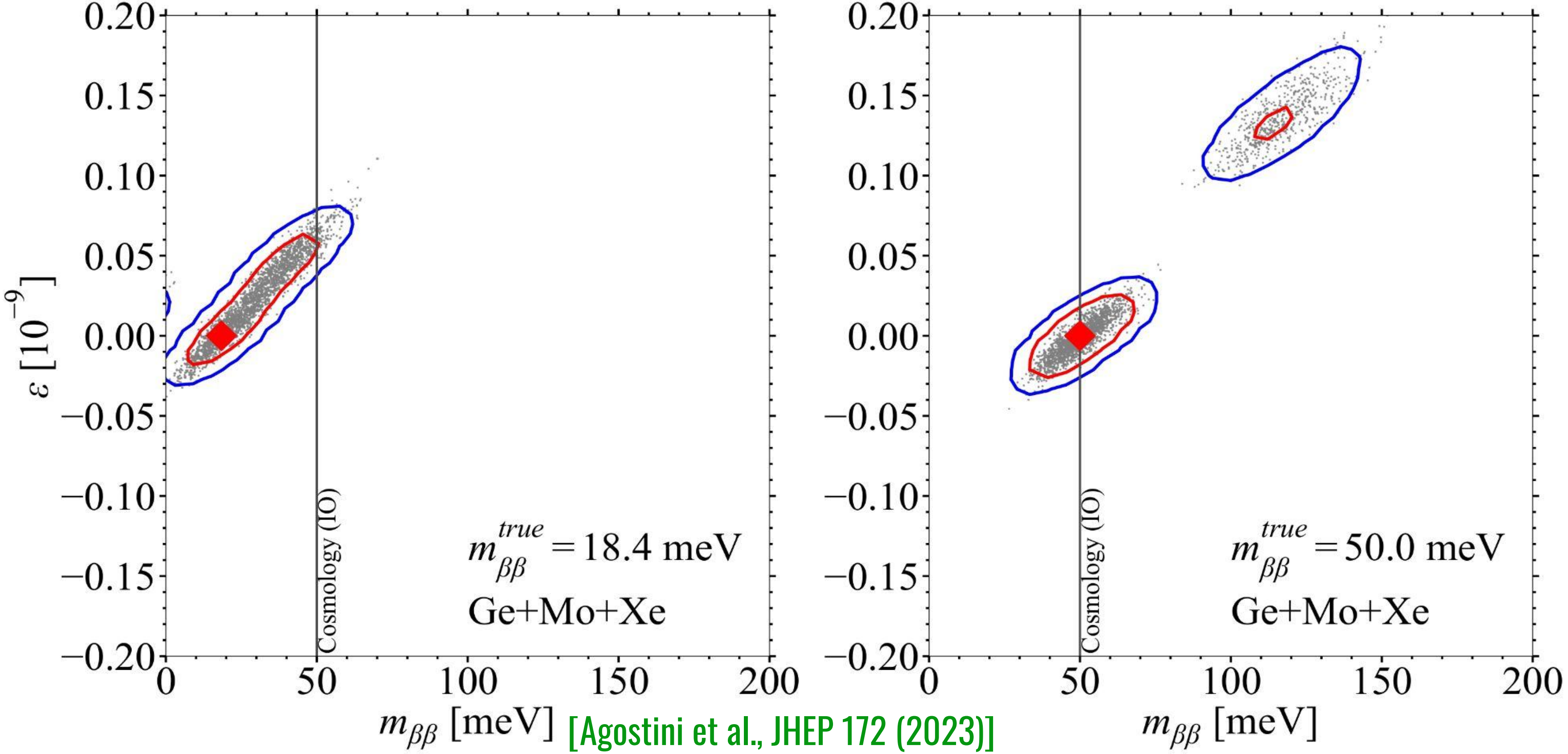
[Agostini et al., JHEP 172 (2023)]

Ge+Mo, Future NMEs



[Agostini et al., JHEP 172 (2023)]

Ge+Mo+Xe, Future NMEs

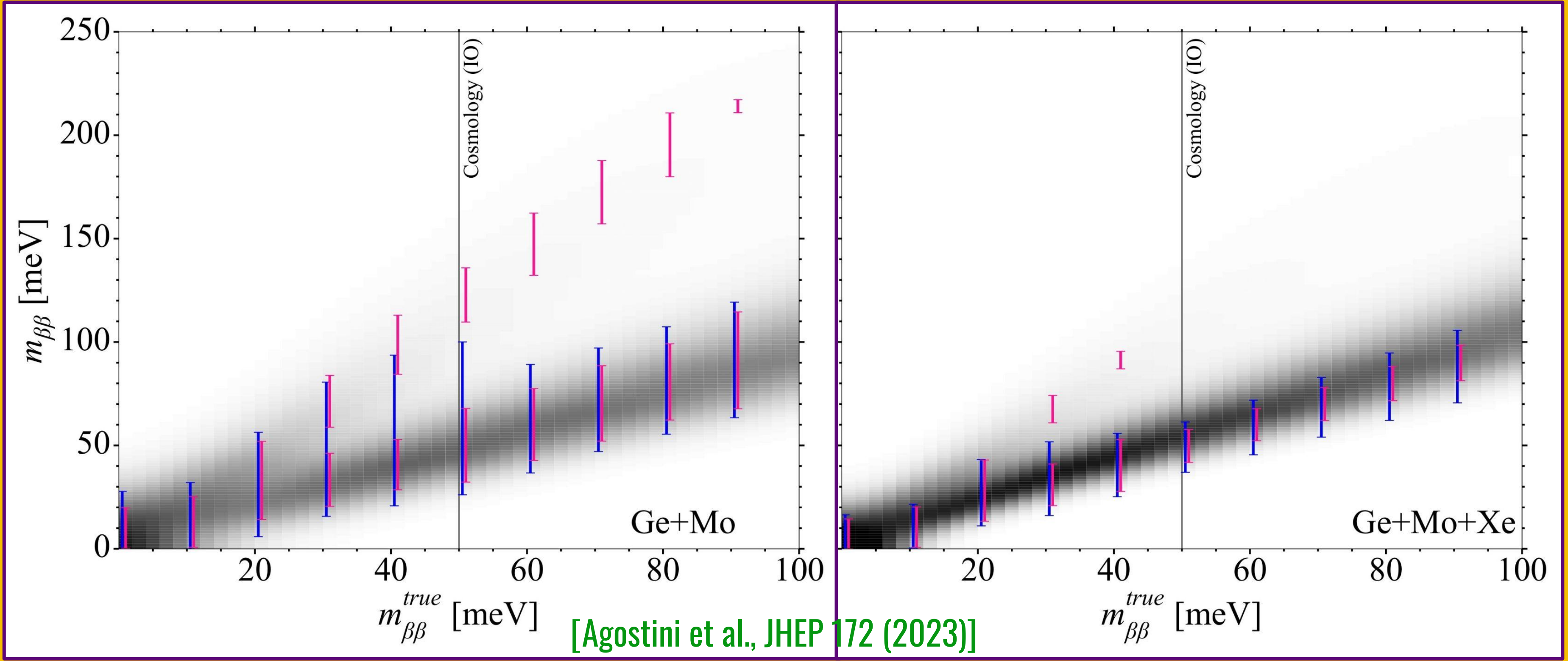


[Agostini et al., JHEP 172 (2023)]

$m_{\beta\beta}$ - Marginalized Statistics

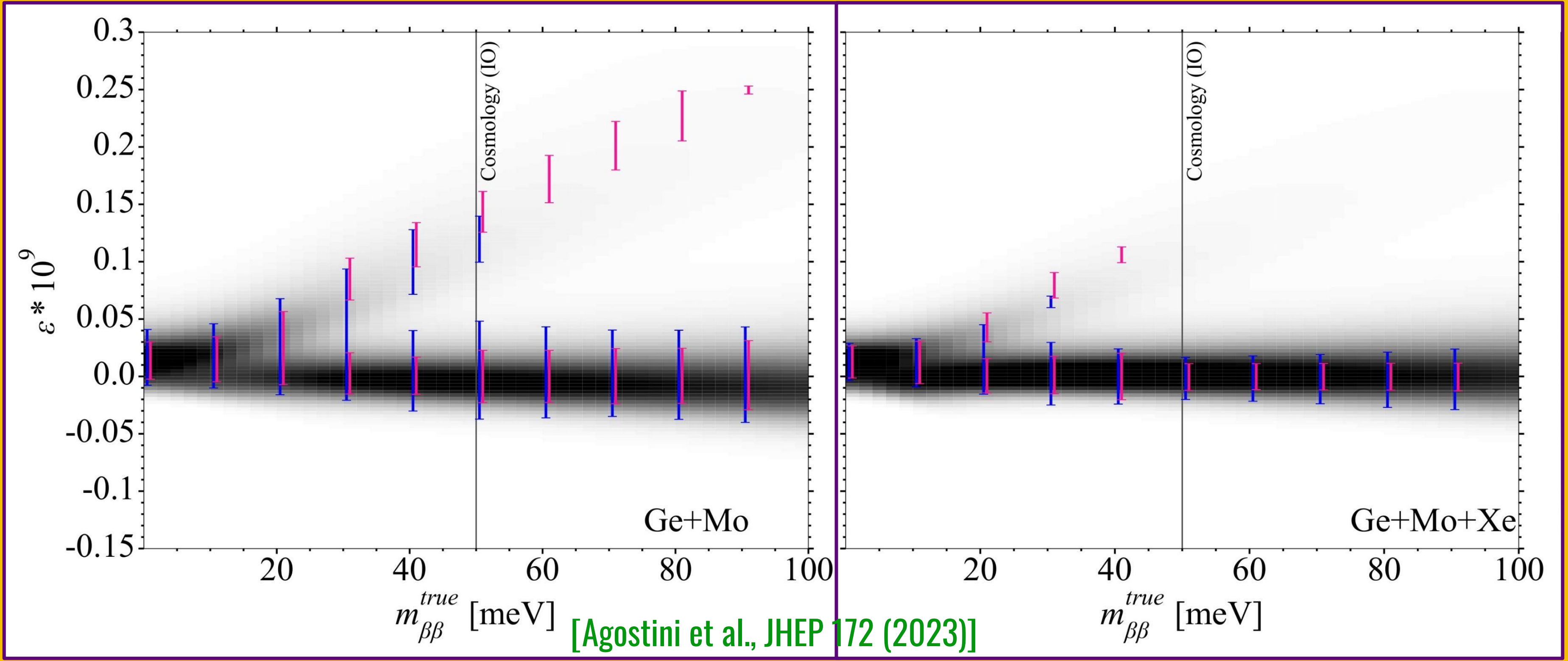
Two isotopes

Three isotopes



[Agostini et al., JHEP 172 (2023)]

ε - Marginalized Statistics



Two isotopes

Three isotopes

[Agostini et al., JHEP 172 (2023)]

Impact

- Multi-isotope observations put multi-mechanism constraints within reach!

- Two-isotope combinations can inform joint estimates on $\{m_{\beta\beta}, \varepsilon\}$, excluding null values.
  - A third isotope can suppress anomalous secondary peak, contingent on its slope.
  - Extra peaks also excluded by cosmology in many scenarios. [3]

- These results hold even when taking account of NME uncertainties and correlations.
  - NME correlations indeed improve inference, but more weakly than a third isotope.
  - Improving NME uncertainties of crucial value.

THANK YOU FOR LISTENING!

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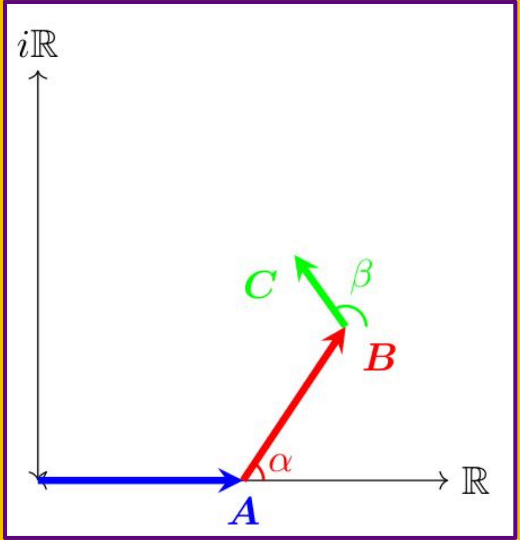
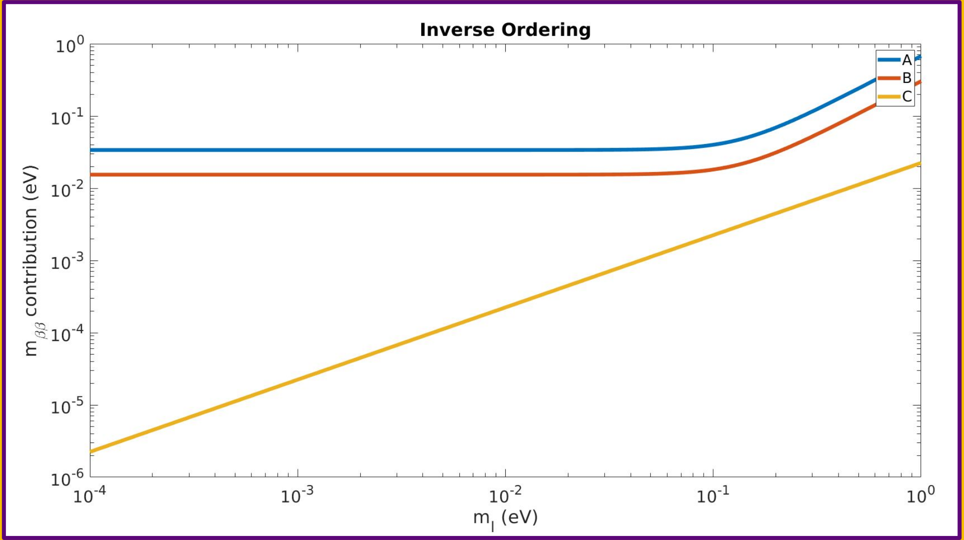
Backup I: Neutrinos and $0\nu\beta\beta$

- $m_{\beta\beta}$ dependent on neutrino masses and two Majorana phases α and β:

$$m_{\beta\beta} = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{i\alpha} + s_{13}^2 m_3 e^{i\beta} \right| = \left| A + Be^{i\alpha} + Ce^{i\beta} \right|$$

- With IO hierarchy, small m_3 causes C << A, B.





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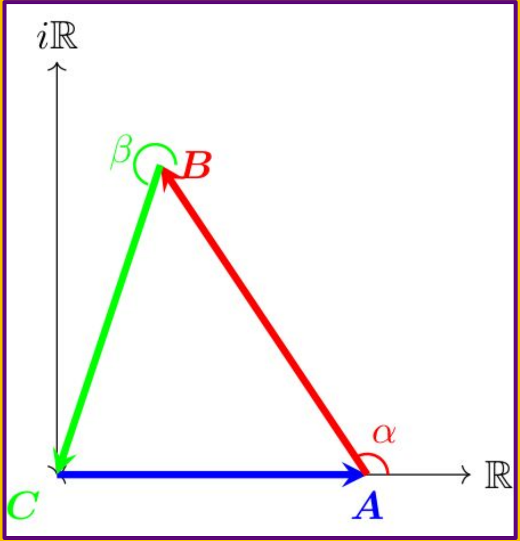
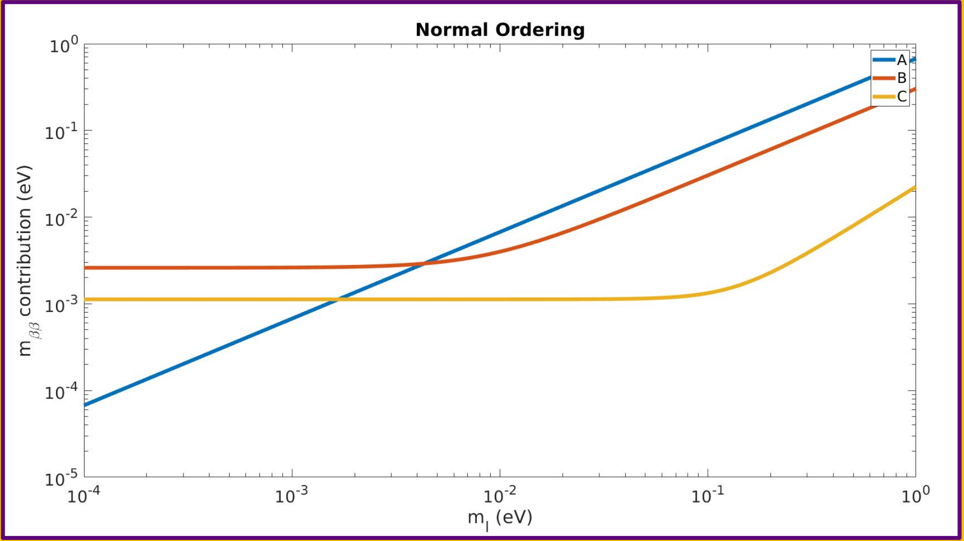
- With NO hierarchy, possible to tune parameters and drive $m_{\beta\beta} \rightarrow 0$.





Backup II: Least-Informative Priors

- **Information theory:** we want posterior to arise from experiment, not our prior biases.
- Information gain from prior to posterior captured in the Kullback-Leibler divergence.
- Define Least-Informative Prior (LIP) as prior which maximises expected information gain.

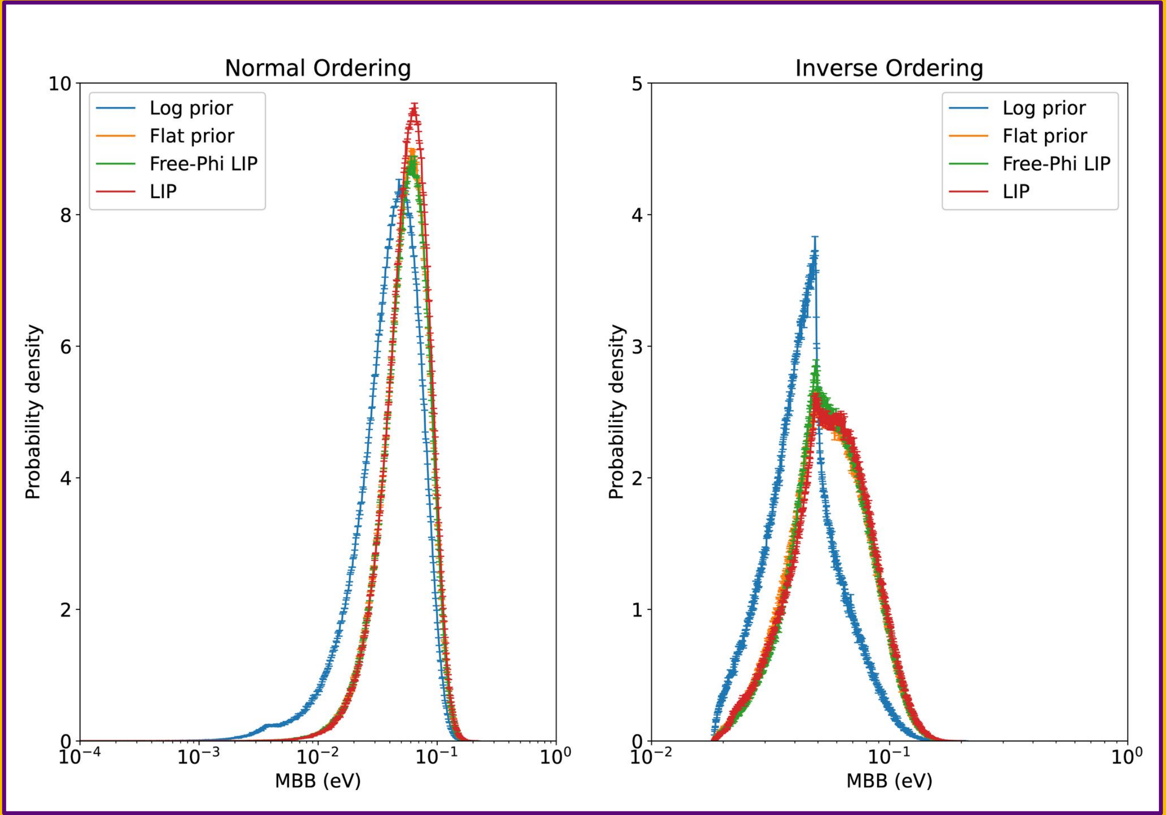
$$\boxed{\langle D_{KL}(p|\pi)\rangle_{data} = \int dX \int d\theta \; p(\theta|X) \log\left(\frac{p(\theta|X)}{\pi(\theta)}\right)}$$

- No prior is bias-free, but the LIP can serve as a reference with which to compare others.

Aside: Least-Informative Priors

- Assess choice of priors on $m_{\beta\beta}$ by comparing to LIPs.
- LEGEND-200 (n=1) modelled by Poisson likelihood.
- LIPs nearly reproduce posteriors for flat priors.

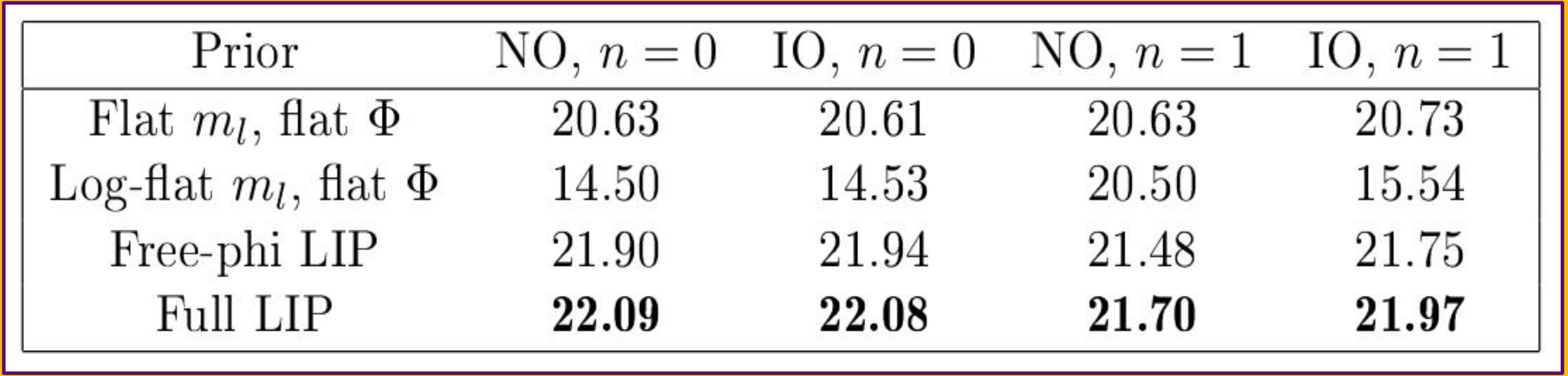
NO: $m_{\beta\beta}$~58 meV   IO: $m_{\beta\beta}$~45 meV



[Deppisch, **GVG** Phys. Rev. D 104, 055040 (2021)]

Aside: Least-Informative Priors

- Maximal information gain of LIP confirmed by computing Kullback-Leibler divergence.
- Further confirms sufficiency of flat prior for our purposes.

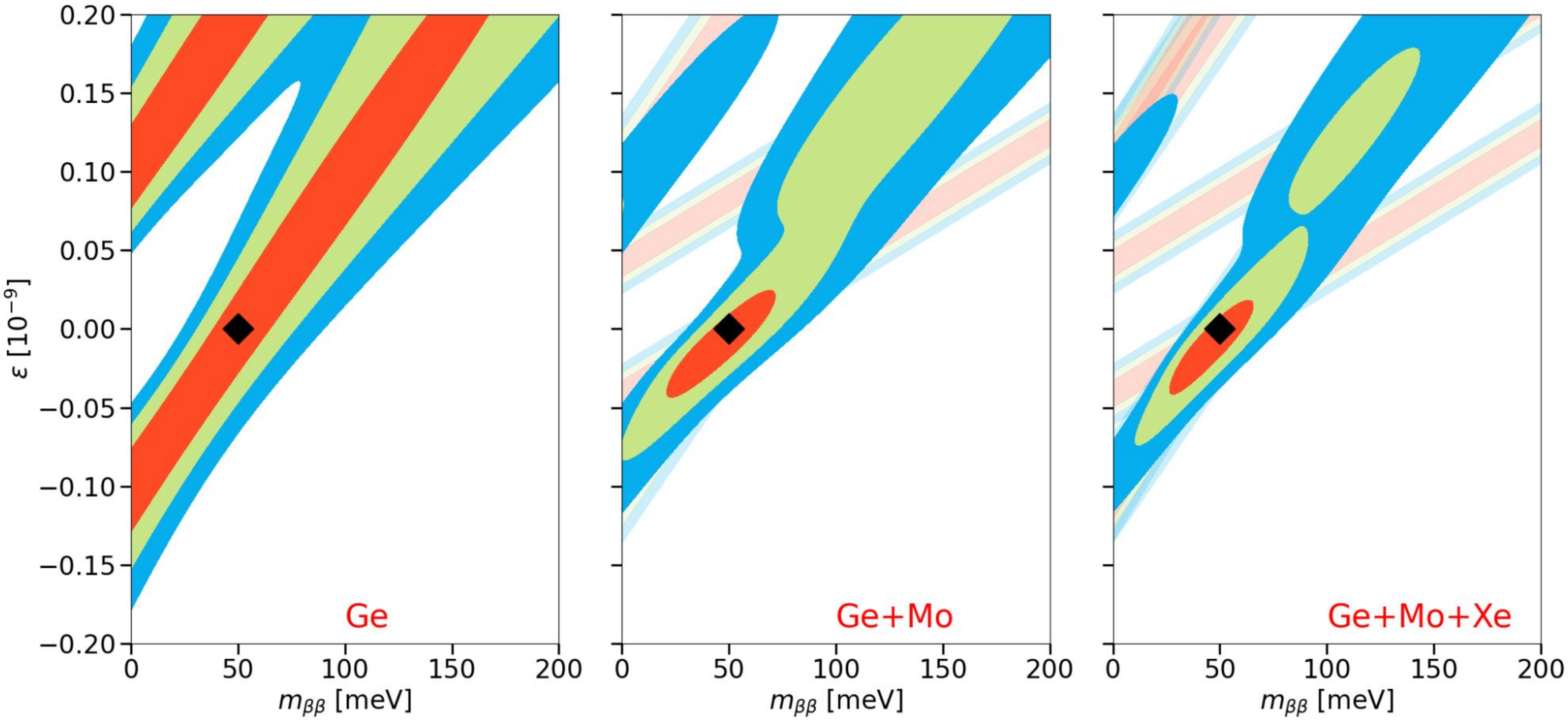


Prior	NO, $n = 0$	IO, $n = 0$	NO, $n = 1$	IO, $n = 1$
Flat m_l, flat Φ	20.63	20.61	20.63	20.73
Log-flat m_l, flat Φ	14.50	14.53	20.50	15.54
Free-phi LIP	21.90	21.94	21.48	21.75
Full LIP	**22.09**	**22.08**	**21.70**	**21.97**

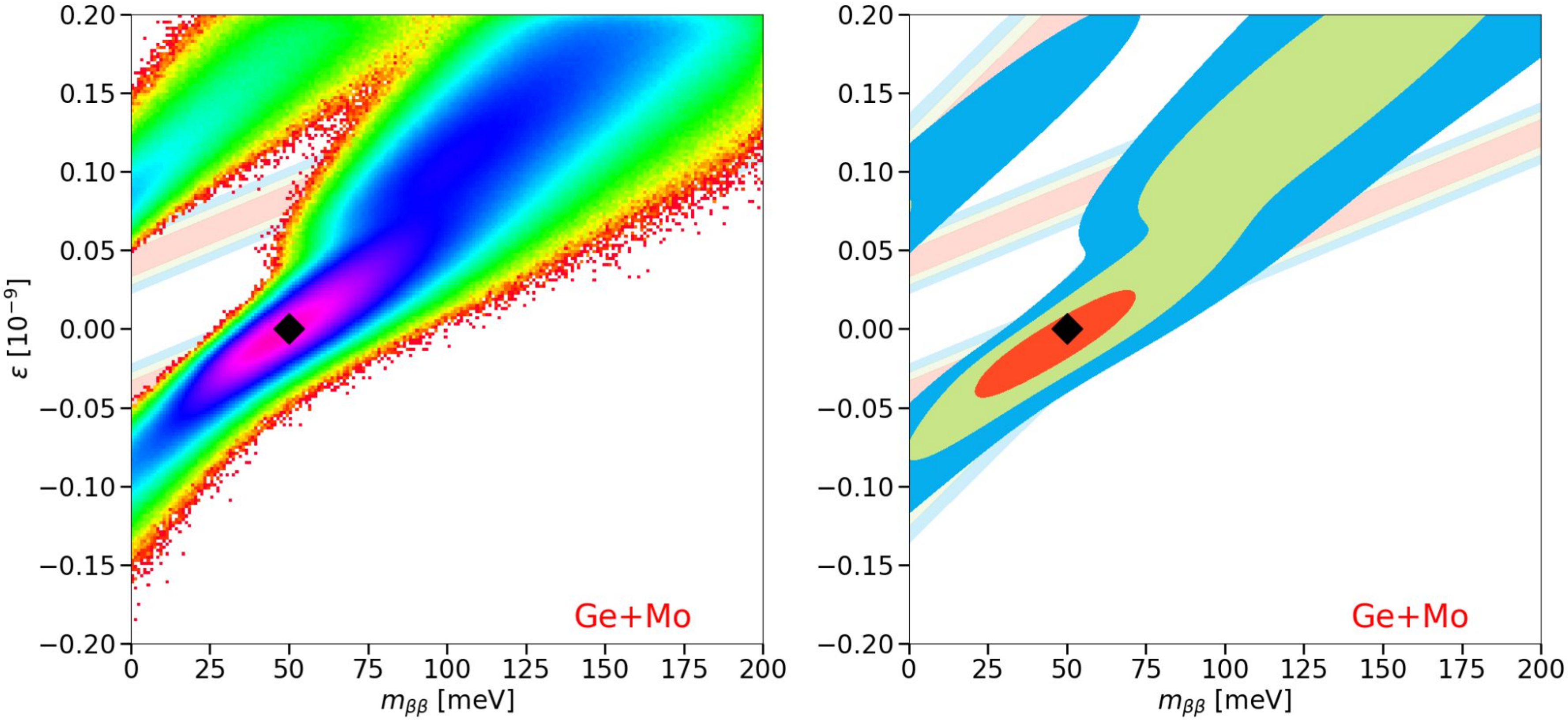
[Deppisch, **GVG** Phys. Rev. D 104, 055040 (2021)]

*information units in bits

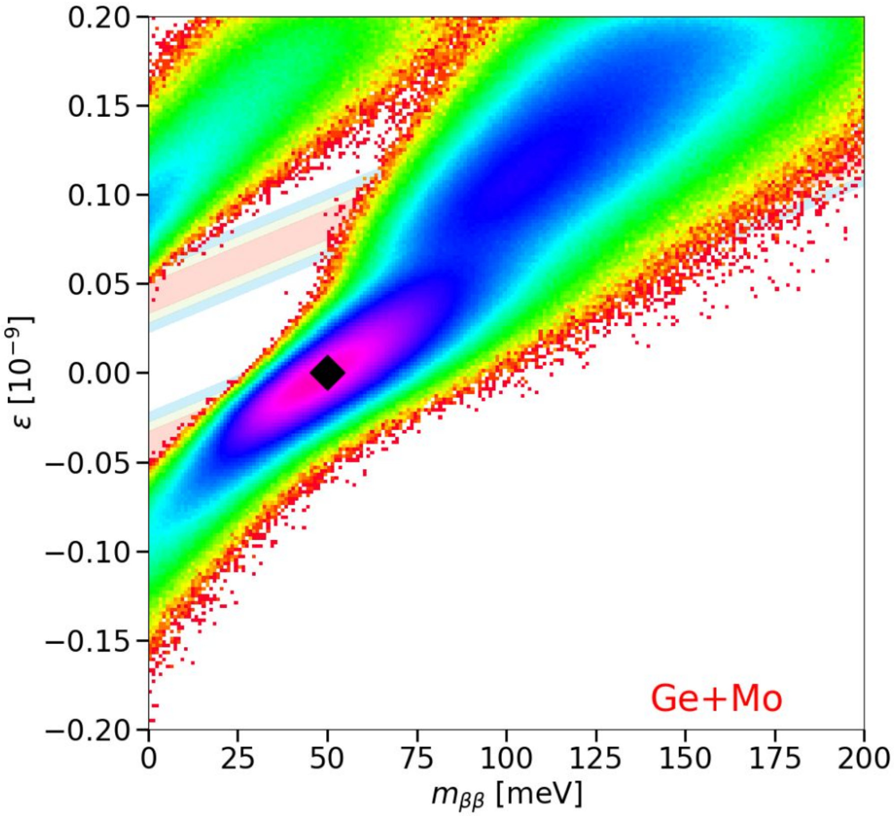
χ^2 Analysis with Uncorrelated NME Uncertainties

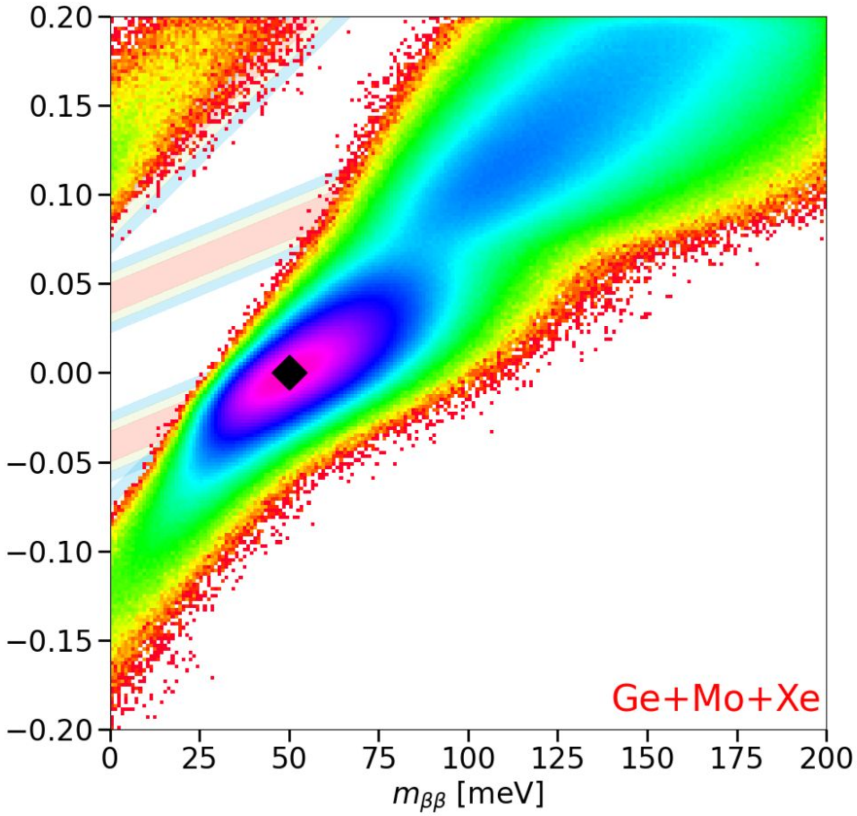


MCMC with Uncorrelated NME Errors

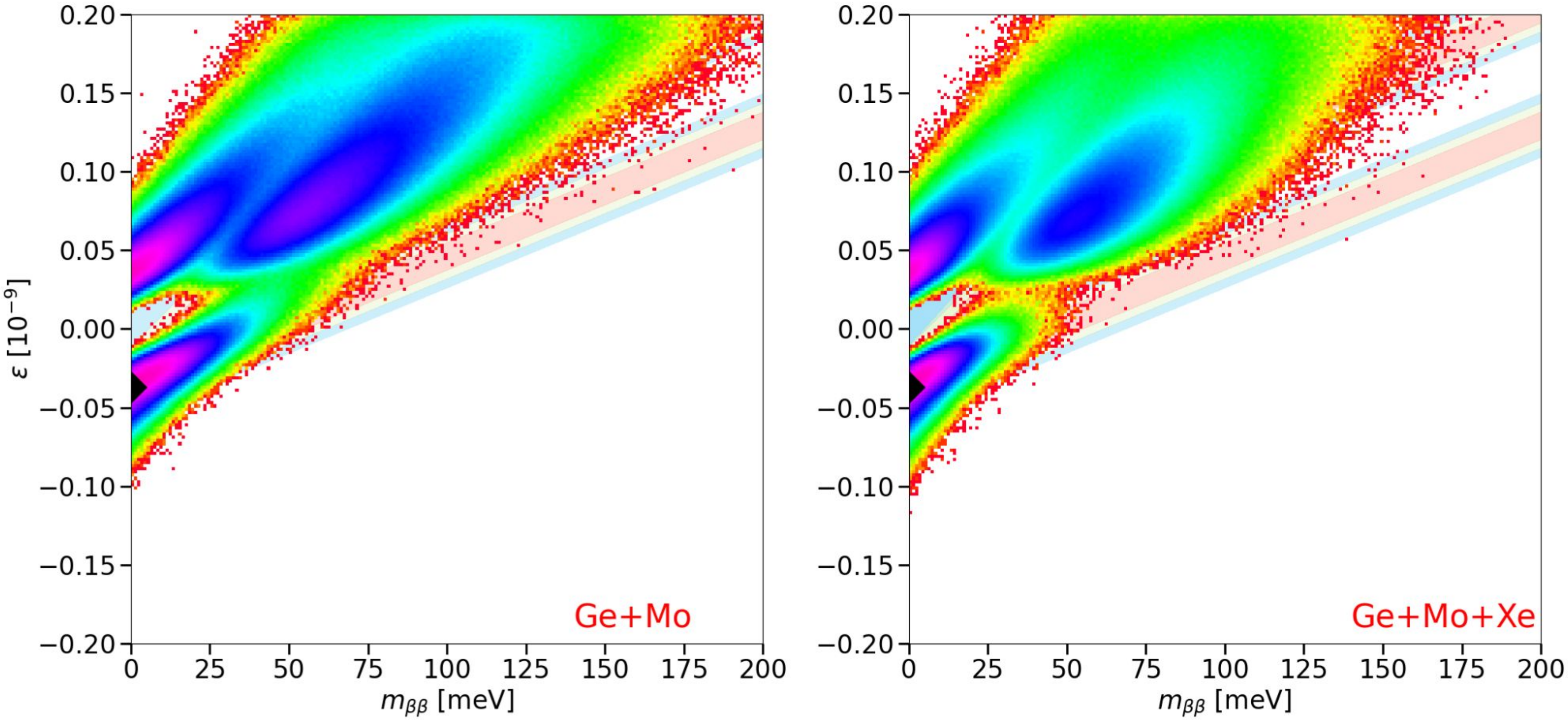


Ge+Mo vs. Ge+Mo+Xe





$\varepsilon \neq 0$ Ge+Mo vs. Ge+Mo+Xe



ε=-0.037 vs. ε=-0.099