More Synergies from Beauty, Top, Z and Drell-Yan in the SMEFT

2304.12837

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SMEFT approach to new physics

- No direct measurement of NP at the LHC $\rightarrow$ NP very weakly coupled or **very heavy**

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  $\implies$ largely model independent framework

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- SMEFT is constructed from all SM fields with the full SM symmetry
  $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{\mathsf{SMEFT}} = \mathcal{L}_{\mathsf{SM}} + \sum_{d=5}^{\infty} \sum_{i} \frac{C_i^{(d)}}{\Lambda^{d-4}} O_i^{(d)}$$

$O_n^{(j)}$: Local operators, IR-sensitive (SM-fields and symmetries)

$C_n^{(j)}$: Wilson coefficients, UV-sensitive (effective couplings)

Minimal Flavor Violation

- Dimension 6 operators: Warsaw Basis: 59 operators $\rightarrow$ 2499 free parameters

See also e.g. Bruggisser et al. [arXiv:2212.02532] for MFV in SMEFT

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- Flavor patterns reduce degrees of freedom and impose **correlations** among different sectors
- MFV: Impose a $U(3)^5$ symmetry $\mathcal{G}_F = U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{l_L} \times U(3)_{e_R}$

  [JHEP 05 (2021), 257]
- The SM Yukawa matrices are treated as spurions

$$q_L: \ (3,1,1,1,1), \qquad u_R: \ (1,3,1,1,1), \qquad d_R: \ (1,1,3,1,1),$$
$$l_L: \ (1,1,1,3,1), \qquad e_R: \ (1,1,1,1,3),$$

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$\implies$ Expand the quark bilinears

$$\bar{q}_L q_L: a_1 \mathbb{1} + a_2 Y_u Y_u^\dagger + a_3 Y_d Y_d^\dagger + ... \qquad \bar{u}_R u_R: b_1 \mathbb{1} + b_2 Y_u^\dagger Y_u + ...$$
$$\bar{q}_L u_R: (c_1 \mathbb{1} + c_2 Y_u Y_u^\dagger + ...) Y_u \qquad \bar{d}_R d_R: e_1 \mathbb{1} + e_2 Y_d^\dagger Y_d + ...$$

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Minimal Flavor Violation in SMEFT

- Rotating to the mass basis and retaining only y_t yields:

$$C\,\bar{q}_L q_L \supset \left[\bar{u}_L \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_1 + a_2\,y_t^2 \end{pmatrix} u_L + \bar{d}_L \begin{pmatrix} a_1 + a_2\,|V_{td}|^2\,y_t^2 & a_2\,V_{td}^*V_{ts}\,y_t^2 & a_2\,V_{td}^*V_{tb}\,y_t^2 \\ a_2\,V_{ts}^*V_{td}\,y_t^2 & a_1 + a_2\,|V_{ts}|^2\,y_t^2 & a_2\,V_{ts}^*V_{tb}\,y_t^2 \\ a_2\,V_{tb}^*V_{td}\,y_t^2 & a_2\,V_{tb}^*V_{ts}\,y_t^2 & a_1 + a_2\,|V_{tb}|^2\,y_t^2 \end{pmatrix} d_L \right]$$

$$t\bar{t} \qquad\qquad \text{Drell-Yan} \qquad\qquad b \to s$$

- Imposes **correlations** among flavor entries and allows for **down-type FCNCs**

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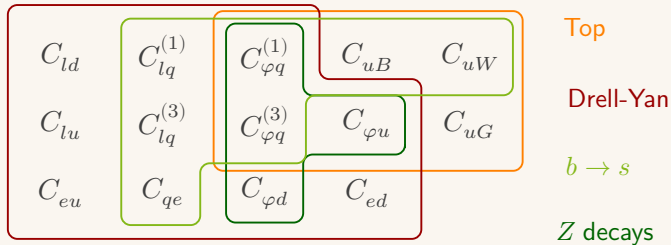
$t\bar{t}$ Drell-Yan $b \to s$

- Imposes **correlations** among flavor entries and allows for **down-type FCNCs**
- Parametrization:

$$\tilde{C}_{q\bar{q}} = \frac{v^2}{\Lambda^2}\,a_1 \qquad \gamma_a = \sum_{n\geq 1} y_t^{2n}\,a_{2n}/a_1$$

- γ_a probes the MFV coefficients $\to$ constrained by combining at least two sectors
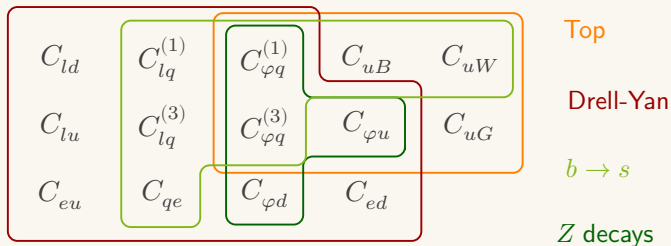
$$u_L^i\,\bar{u}_L^i \sim \tilde{C}_i \qquad\qquad d_L^i\,\bar{d}_L^i \sim \tilde{C}_i(1 + \gamma_A|V_{ti}|^2) \qquad\qquad \bar{u}_L^i\,d_L^j \sim \tilde{C}_i\,V_{ij}$$

$$t_L\,\bar{t}_L \sim \tilde{C}_i(1 + \gamma_a) \qquad\qquad b_L\,\bar{s}_L \sim \tilde{C}_i\,\gamma_a\,V_{ts}^*V_{tb} \qquad\qquad \bar{t}_L\,d_L^j \sim \tilde{C}_i\,(1 + \gamma_A)V_{tj}$$

Minimal Flavor Violation

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$$t_L\bar{t}_L \sim \tilde{C}_i(1 + \gamma_a) \qquad b_L\bar{s}_L \sim \tilde{C}_i\gamma_a\,V_{ts}^* V_{tb} \qquad \bar{t}_L\,d_L^j \sim \tilde{C}_i\,(1 + \gamma_A)V_{tj}$$
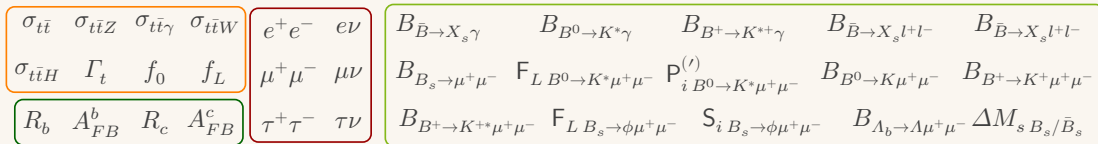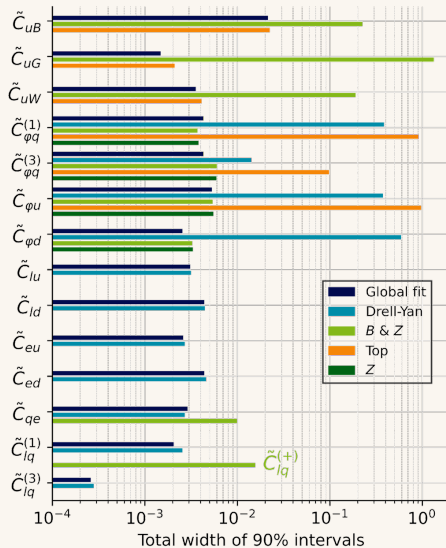
Parameters and Observables

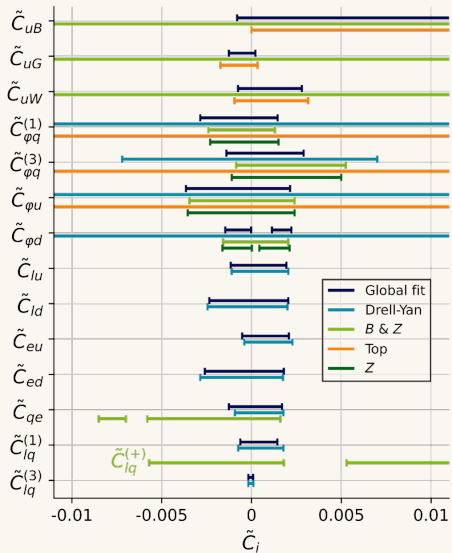14 Wilson coefficients and $\gamma_{a/b} \rightarrow$ 16 degrees of freedom

Parameters and Observables

$$C_{ld} \quad C_{lq}^{(1)} \quad C_{\varphi q}^{(1)} \quad C_{uB} \quad C_{uW}$$

$$C_{lu} \quad C_{lq}^{(3)} \quad C_{\varphi q}^{(3)} \quad C_{\varphi u} \quad C_{uG}$$

$$C_{eu} \quad C_{qe} \quad C_{\varphi d} \quad C_{ed}$$

Top

Drell-Yan

$b \to s$

Z decays

14 Wilson coefficients and $\gamma_{a/b} \to$ 16 degrees of freedom

$\sigma_{t\bar{t}}$	$\sigma_{t\bar{t}Z}$ $\sigma_{t\bar{t}\gamma}$ $\sigma_{t\bar{t}W}$		
$\sigma_{t\bar{t}H}$	Γ_t f_0 f_L		
R_b	A_{FB}^b R_c A_{FB}^c		

e^+e^-	$e\nu$
$\mu^+\mu^-$	$\mu\nu$
$\tau^+\tau^-$	$\tau\nu$

$$B_{\bar{B}\to X_s\gamma} \quad B_{B^0\to K^*\gamma} \quad B_{B^+\to K^{*+}\gamma} \quad B_{\bar{B}\to X_s l^+ l^-} \quad B_{\bar{B}\to X_s l^+ l^-}$$

$$B_{B_s\to\mu^+\mu^-} \quad F_{L\,B^0\to K^*\mu^+\mu^-} \quad P_{i\,B^0\to K^*\mu^+\mu^-}^{(\prime)} \quad B_{B^0\to K\mu^+\mu^-} \quad B_{B^+\to K^+\mu^+\mu^-}$$

$$B_{B^+\to K^{*+}\mu^+\mu^-} \quad F_{L\,B_s\to\phi\mu^+\mu^-} \quad S_{i\,B_s\to\phi\mu^+\mu^-} \quad B_{\Lambda_b\to\Lambda\mu^+\mu^-} \quad \Delta M_{s\,B_s/\bar{B}_s}$$

Synergies from Beauty, Top, Z and Drell-Yan
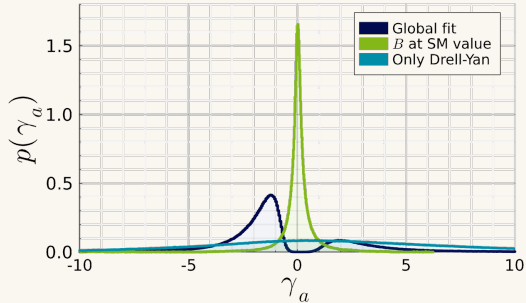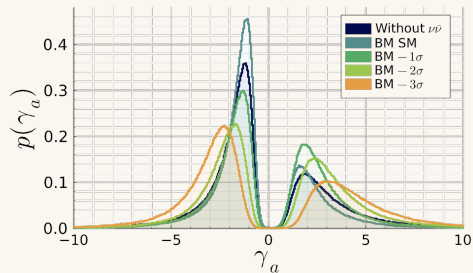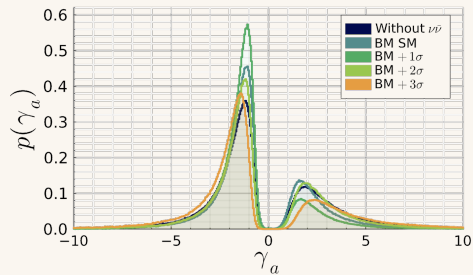
Probing the MFV parameters

$$\gamma_a = \sum_{n \geq 1} y_t^{2n} \frac{a_{2n}}{a_1} \quad \text{left-handed doublets}$$

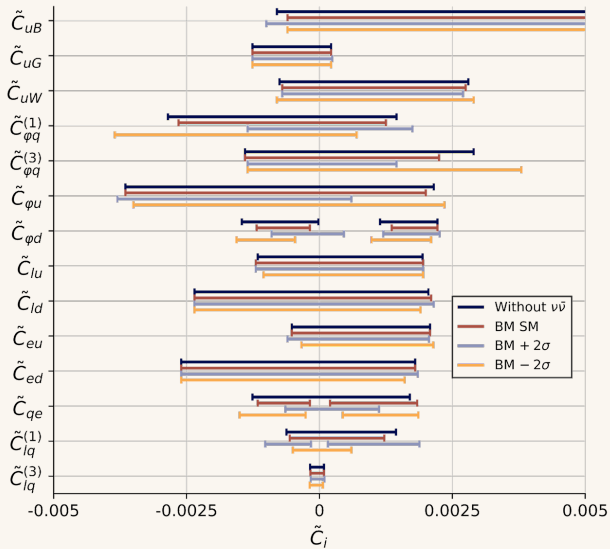$$\gamma_b = \sum_{n \geq 1} y_t^{2n} \frac{b_{2n}}{b_1} \quad \text{right-handed up-type quarks}$$
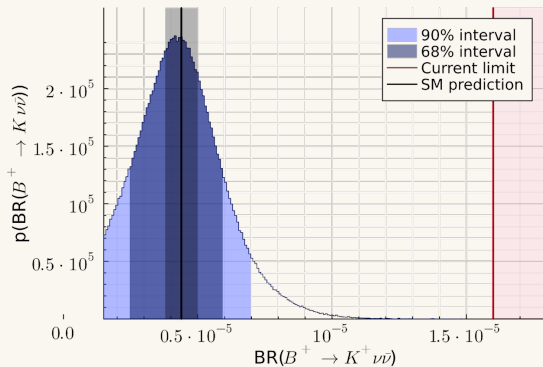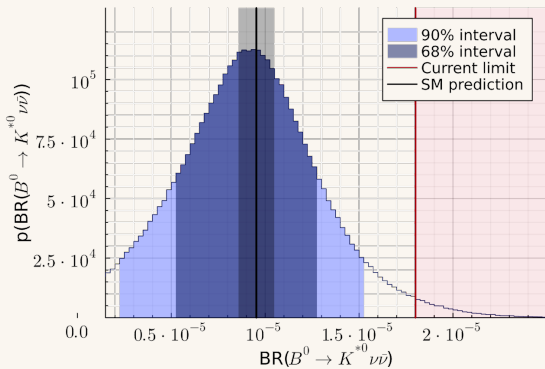
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Impact of $B(B^{0/+} \to K^{*0/+}\nu\bar{\nu})$
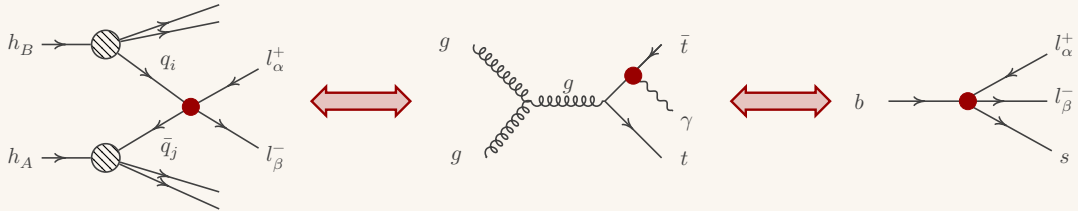
Predictions of dineutrino branching ratios

68% credible intervals:

$$5.3 \cdot 10^{-6} \leq B(B^0 \to K^{*0}\nu\bar{\nu}) \leq 12.8 \cdot 10^{-6} \qquad 2.5 \cdot 10^{-6} \leq B(B^+ \to K^+\nu\bar{\nu}) \leq 5.9 \cdot 10^{-6}$$

$$\frac{B(B^+ \to K^+\nu\bar{\nu})}{B(B^0 \to K^{*0}\nu\bar{\nu})} = 0.46 \pm 0.08 \, ,$$

Conclusion

- Flavor pattern allow to connect different sectors and exploit correlations
- Synergies arise in the global fit due to the flavor-link established by MFV
- The second order in the MFV expanion is tested by combining beauty and collider measurements
- Dineutrino branching ratios can provide crucial information in the search for BSM physics
- We predicted the dineutrino ratios to be centred around the SM prediction, and within the reach of Belle II

Supplementary Slides

Dineutrino benchmarks, limits and SM predictions

- Experimental upper limits [Phys. Rev. D 96, 091101 (2017)]

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{exp}} < 1.8 \cdot 10^{-5} \qquad B(B^+ \to K^+\nu\bar{\nu})_{\text{exp}} < 1.6 \cdot 10^{-5}$$

- SM prediction [arXiv:1810.08132]

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{SM}} = (9.53 \pm 0.95) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{SM}} = (4.39 \pm 0.60) \cdot 10^{-6}$$
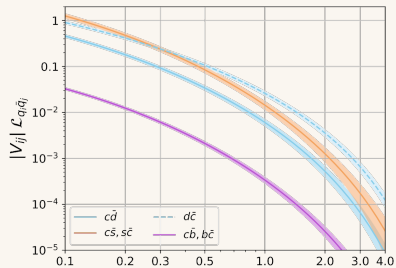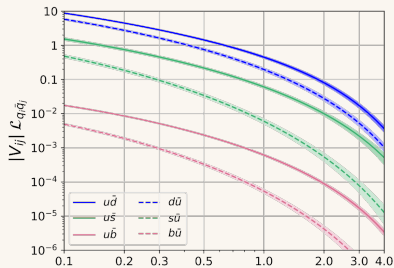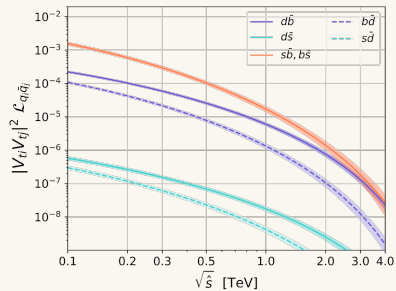
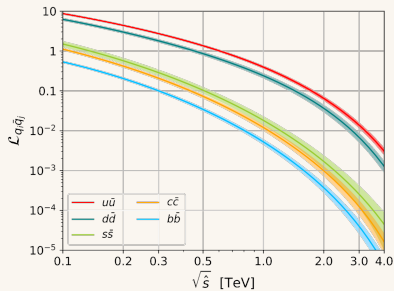- Benchmark measurements

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM SM}} = (9.5 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM SM}} = (4.4 \pm 1.3) \cdot 10^{-6}$$
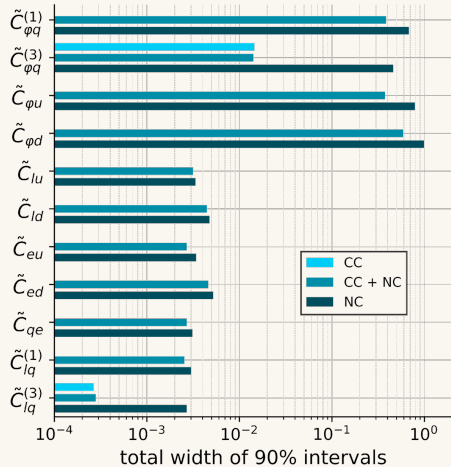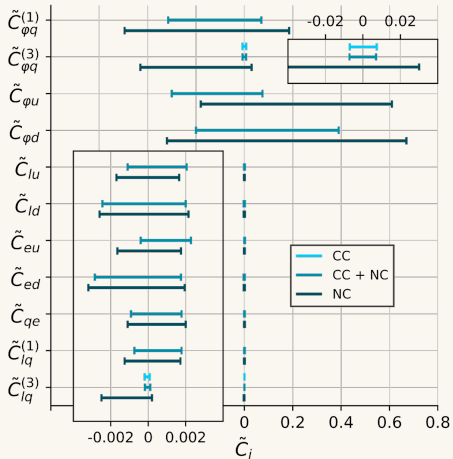
$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM}+2\sigma} = (14.5 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM}+2\sigma} = (7.0 \pm 1.3) \cdot 10^{-6}$$

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM}-2\sigma} = (4.6 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM}-2\sigma} = (1.8 \pm 1.3) \cdot 10^{-6}$$
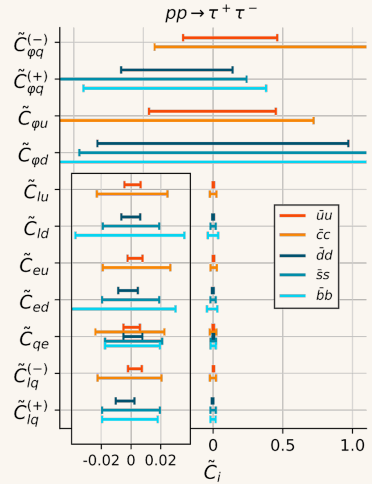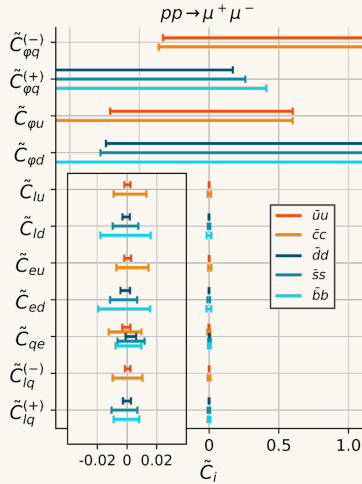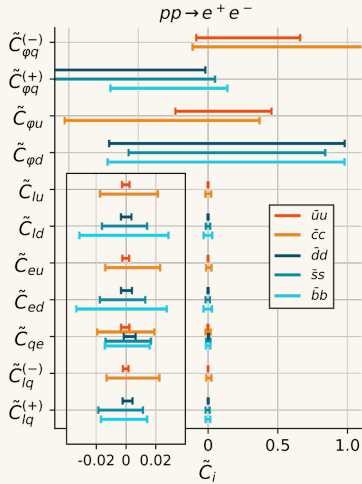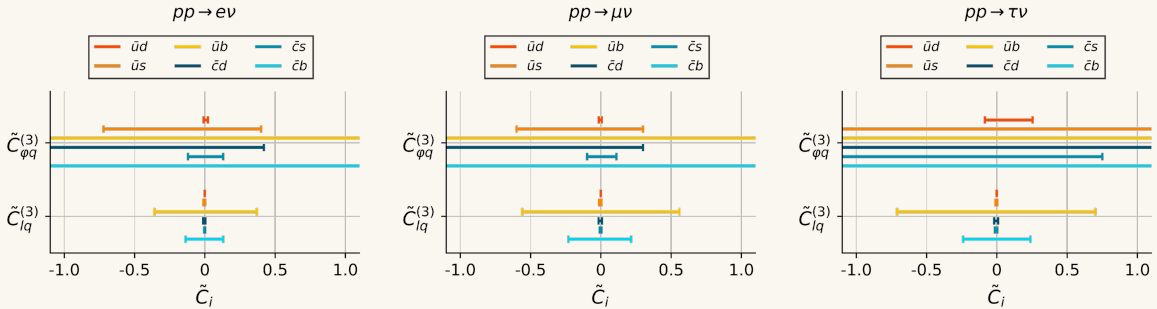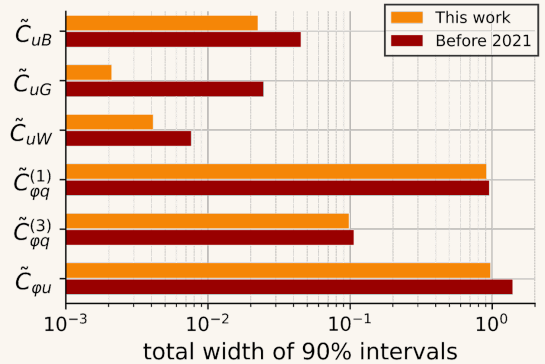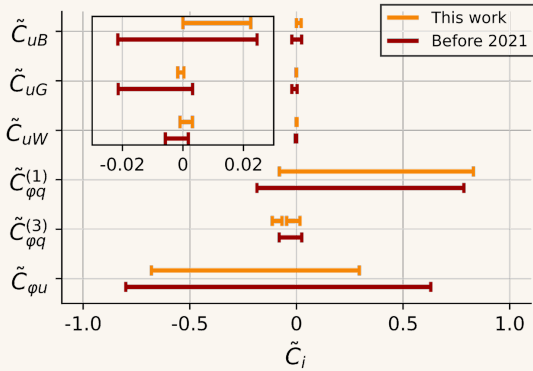
Parton-Parton Luminosities
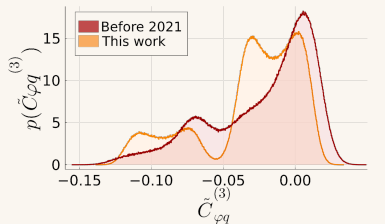
MFV Drell-Yan Fits

Flavor-Specific NC Drell-Yan Fits

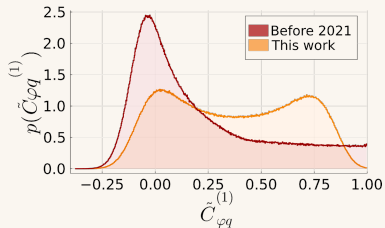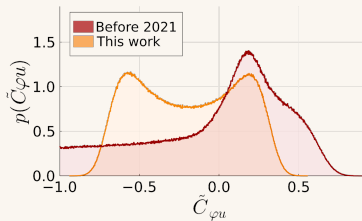# Flavor-Specific CC Drell-Yan Fits

Top Fits

Top Fit Posterior Probability Distributions

SMEFT Operators in Warsaw basis

$$O_{uG} = \left(\bar{q}_L \sigma^{\mu\nu} T^A u_R\right) \tilde{\varphi} G_{\mu\nu}^A \,,$$

$$O_{uB} = \left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \tilde{\varphi} B_{\mu\nu} \,,$$

$$O_{lq}^{(1)} = \left(\bar{l}_L \gamma_\mu l_L\right) \left(\bar{q}_L \gamma^\mu q_L\right) \,,$$

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$$O_{\varphi q}^{(1)} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right) \left(\bar{q}_L \gamma^\mu q_L\right) \,,$$

$$O_{\varphi u} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right) \left(\bar{u}_R \gamma^\mu u_R\right) \,,$$

$$O_{uW} = \left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \tau^I \tilde{\varphi} W_{\mu\nu}^I \,,$$

$$O_{qe} = \left(\bar{q}_L \gamma_\mu q_L\right) \left(\bar{e}_R \gamma^\mu e_R\right) \,,$$

$$O_{lq}^{(3)} = \left(\bar{l}_L \gamma_\mu \tau^I l_L\right) \left(\bar{q}_L \gamma^\mu \tau^I q_L\right) \,,$$

$$O_{ed} = \left(\bar{e}_R \gamma_\mu e_R\right) \left(\bar{d}_R \gamma^\mu d_R\right) \,,$$

$$O_{ld} = \left(\bar{l}_L \gamma_\mu l_L\right) \left(\bar{d}_R \gamma^\mu d_R\right) \,,$$

$$O_{\varphi q}^{(3)} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi\right) \left(\bar{q}_L \tau^I \gamma^\mu q_L\right) \,,$$

$$O_{\varphi d} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right) \left(\bar{d}_R \gamma^\mu d_R\right) \,,$$

Weak Effective Theory

Effective Lagrangian for $b \to sll$

$$\mathcal{L}_{\mathsf{WET}}^{bs} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad Q_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$Q_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell) \qquad Q_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$$Q_L = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu)$$

Weak Effective Theory for Meson mixing

Effective Lagrangian for $B_s \bar{B}_s$

$$\mathcal{L}^{\mathsf{mix}}_{\mathsf{WET}} = \frac{G_F^2 m_W^2}{16\pi^2} \left| V_{tb} V_{ts}^* \right|^2 \sum_i Q_i^{\mathsf{mix}}(\mu) C_i^{\mathsf{mix}}(\mu) \, ,$$

$$Q_{V,LL}^{\mathsf{mix}} = \left(\bar{s}_L \gamma_\mu b_L \right) \left(\bar{s}_L \gamma^\mu b_L \right)$$

Running and Matching

- Match a potential UV theory onto the SMEFT at the matching scale Λ

- Apply the SMEFT RGE to evolve the Wilson coefficients to lower energy scales

- Compute top and collider observables at the scale μ_t

- Match the SMEFT onto the WET at the scale μ_W

- Use the WET RGE to compute the Wilson coefficients at the scale μ_b of flavor-observables

EFT cross section and Simulation Chain

$$\mathcal{M} = \mathcal{M}^{\mathsf{SM}} + \frac{1}{\varLambda^2}\sum_i C_i \mathcal{M}_i^{\mathsf{BSM}} \xrightarrow{\sigma \propto |\mathcal{M}|^2} \sigma = \sigma^{\mathsf{SM}} + \frac{1}{\varLambda^2}\sum_i C_i \sigma_i^{\mathsf{int}} + \frac{1}{\varLambda^4}\sum_{i \le j} C_i C_j \sigma_{ij}^{\mathsf{BSM}}$$

model definition	SMEFTSIM [arXiv:2012.11343]
parton level Monte Carlo	MADGRAPH5 [arXiv:1405.0301]
hadronisation and parton shower	PYTHIA8 [arXiv:1410.3012]
detector simulation	DELPHES3 [arXiv:1307.6346]

Top quark measurements included in the fit

Process	Observable	SMEFT operators	Experiment
$t\bar{t}$	$\dfrac{\mathrm{d}\sigma}{\mathrm{dm}(t\bar{t})}$	$\tilde{C}_{uG}$	CMS
$t\bar{t}Z$	$\dfrac{\mathrm{d}\sigma}{\mathrm{dp_T}(Z)}$	$\tilde{C}_{uG}\ \tilde{C}_{uZ}\ \tilde{C}_{\varphi u}\ \tilde{C}_{\varphi q}^{-}$	ATLAS
$t\bar{t}\gamma$	$\dfrac{\mathrm{d}\sigma}{\mathrm{dp_T}(\gamma)}$	$\tilde{C}_{uG}\ \tilde{C}_{u\gamma}$	ATLAS
$t\bar{t}W$	$\sigma_{t\bar{t}W}$	$\tilde{C}_{uG}$	ATLAS
$t\bar{t}H$	$\sigma_{t\bar{t}H} \times B_{\gamma\gamma}$	$\tilde{C}_{uG}$	ATLAS
$t \to Wb$	$f_0,\ f_L$	$\tilde{C}_{uW}$	ATLAS
$t \to Wb$	Γ_t	$\tilde{C}_{uW}\ \tilde{C}_{\varphi q}^{3}$	ATLAS

Drell-Yan measurements included in the fit

Process	Observable	Experiment	$\sqrt{s}$	Int. luminosity
$pp \to e^+e^-$	Events, 68 bins	CMS	13 TeV	137 fb^{-1}
$pp \to \mu^+\mu^-$	Events, 36 bins	CMS	13 TeV	140 fb^{-1}
$pp \to \tau^+\tau^-$	Events, 17 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to e\nu$	Events, 40 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to \mu\nu$	Events, 35 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to \tau\nu$	Events, 10 bins	ATLAS	13 TeV	139 fb^{-1}

Flavor measurements included in the fit

Process	Observable	Experiment	q^2 bin [GeV^2]
$\bar{B} \to X_s \gamma$	$B_{E_\gamma > 1.6 \text{ GeV}}$	HFLAV	/
$B^0 \to K^* \gamma$	B	HFLAV	/
$B^+ \to K^{*+} \gamma$	B	HFLAV	/
$\bar{B} \to X_s \ell^+ \ell^-$	B	BaBar	$[1,6]$
	A_{FB}	Belle	$[1,6]$
$B_s \to \mu^+ \mu^-$	B	CMS	/
$B^0 \to K^* \mu^+ \mu^-$	F_L, P_1, P_2, P_3, P_4', P_5', P_6', P_8'	LHCb	$[1.1,6]$
$B^0 \to K \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[1,6]$
$B^+ \to K^+ \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$ LHCb	$[1,6]$	
$B^+ \to K^{+*} \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[1,6]$
$B_s \to \phi \mu^+ \mu^-$	F_L, S_3, S_4, S_7	LHCb	$[1,6]$
$\Lambda_b \to \Lambda \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[15,20]$
$B_s - \bar{B}_s$ mixing	ΔM_s	HFLAV	/

Tree-level Matching

$$\Delta C_9^{\text{tree}} = \frac{\pi}{\alpha}\,\gamma_a\,\left[\tilde{C}_{lq}^+ + \tilde{C}_{qe} + \left(-1 + 4\sin^2\theta_w\right)\tilde{C}_{\varphi q}^+\right]$$
$$= \gamma_a \cdot \left(430.511\,\left(\tilde{C}_{qe} + \tilde{C}_{lq}^+\right) - 45.858\tilde{C}_{\varphi q}^+\right)\,,$$

$$\Delta C_{10}^{\text{tree}} = \frac{\pi}{\alpha}\,\gamma_a\,\left[-\tilde{C}_{lq}^+ + \tilde{C}_{qe} + \tilde{C}_{\varphi q}^+\right]$$
$$= \gamma_a \cdot 430.511\left(\tilde{C}_{\varphi q}^+ + \tilde{C}_{qe} - \tilde{C}_{lq}^+\right)\,,$$

$$\Delta C_L^{\text{tree}} = \frac{\pi}{\alpha}\,\gamma_a\,\left[\tilde{C}_{lq}^- + \tilde{C}_{\varphi q}^+\right]$$
$$= \gamma_a \cdot 430.511\left(\tilde{C}_{\varphi q}^+ + \tilde{C}_{lq}^-\right)$$

One-Loop Matching

$$C_7 = -2.351\,\tilde{C}_{uB} + 0.093\,\tilde{C}_{uW} + \gamma_a \cdot \left(-0.095\,\tilde{C}_{\varphi q}^+ + 1.278\,\tilde{C}_{\varphi q}^{(3)}\right) + (1 + \gamma_a) \cdot \left(-0.388\,\tilde{C}_{\varphi q}^{(3)}\right)$$

$$C_8 = -0.664\,\tilde{C}_{uG} + 0.271\,\tilde{C}_{uW} + \gamma_a \cdot \left(0.284\,\tilde{C}_{\varphi q}^+ + 0.667\,\tilde{C}_{\varphi q}^{(3)}\right) + (1 + \gamma_a) \cdot \left(-0.194\,\tilde{C}_{\varphi q}^{(3)}\right)$$

$$C_9 = 2.506\,\tilde{C}_{uB} + 2.137\,\tilde{C}_{uW} + (1 + \gamma_b)\left(0.213\,\tilde{C}_{\varphi u} + 2.003\left(-\tilde{C}_{lu} - \tilde{C}_{eu}\right)\right)$$
$$+ (1 + \gamma_a) \cdot \left(-0.213\,\tilde{C}_{\varphi q}^{(1)} + 4.374\,\tilde{C}_{\varphi q}^{(3)} + 2.003\left(\tilde{C}_{qe} + \tilde{C}_{lq}^{(1)}\right) - 3.163\,\tilde{C}_{lq}^{(3)}\right)$$

$$C_{10} = -7.515\,\tilde{C}_{uW} + (1 + \gamma_b) \cdot \left(2.003\left(-\tilde{C}_{\varphi u} - \tilde{C}_{eu} + \tilde{C}_{lu}\right)\right)$$
$$+ (1 + \gamma_a) \cdot \left(2.003\left(\tilde{C}_{\varphi q}^{(1)} + \tilde{C}_{qe} - \tilde{C}_{lq}^{(1)}\right) - 17.884\,\tilde{C}_{\varphi q}^{(3)} + 3.163\,\tilde{C}_{lq}^{(3)}\right)$$

$$C_L = 12.889\,\tilde{C}_{uW} + (1 + \gamma_a) \cdot \left(2.003\left(\tilde{C}_{\varphi q}^{(1)} + \tilde{C}_{lq}^{(1)}\right) - 22.830\,\tilde{C}_{\varphi q}^{(3)} - 16.275\,\tilde{C}_{lq}^{(3)}\right)$$
$$+ (1 + \gamma_b) \cdot 2.003\left(-\tilde{C}_{\varphi u} - \tilde{C}_{lu}\right)$$

$$C_{V,LL}^{\mathsf{mix}} = -22.023\,\tilde{C}_{uW} + \gamma_a \cdot \left(14.317\,\tilde{C}_{\varphi q}^{(1)} + 11.395\,\tilde{C}_{\varphi q}^{(3)}\right).$$

MFV Spurion Expansion

$$\bar{q}_L q_L :\sim a_1 \mathbb{1} + a_2 Y_u Y_u^\dagger + a_3 Y_d Y_d^\dagger + ... \quad \bar{u}_R u_R :\sim b_1 \mathbb{1} + b_2 Y_u^\dagger Y_u + ...$$

$$\bar{q}_L u_R :\sim (c_1 \mathbb{1} + c_2 Y_u Y_u^\dagger + ...) Y_u \quad\quad \bar{q}_L d_R :\sim (d_1 \mathbb{1} + d_2 Y_u Y_u^\dagger + ...) Y_d$$

$$\bar{d}_R d_R :\sim e_1 \mathbb{1} + e_2 Y_d^\dagger Y_d + ...$$

After rotation to mass basis:

$$\bar{q}_L q_L : \bar{d}_{Li} d_{Lj} \to a_1 \delta_{ij} + a_2 y_t^2 V_{ti}^* V_{tj}$$

$$\bar{u}_{Li} u_{Lj} \to a_1 \delta_{ij} + a_2 y_t^2 \delta_{3i} \delta_{3j}$$

$$\bar{u}_{Li} d_{Lj} \to a_1 V_{ij} + a_2 y_t^2 \delta_{3i} V_{tj}$$

$$\bar{d}_{Li} u_{Lj} \to a_1 V_{ji}^* + a_2 y_t^2 V_{ti}^* \delta_{3j}$$

$$\bar{q}_L u_L : \bar{u}_{Li} u_{Rj} \to (c_1 + c_2 y_t^2) y_t \delta_{3i} \delta_{3j},$$

$$\bar{d}_{Li} u_{Rj} \to (c_1 + c_2 y_t^2) y_t V_{ti}^* \delta_{3j},$$

$$\bar{q}_L d_R : \; \simeq 0$$

$$\bar{u}_R u_R : \bar{u}_{Ri} u_{Rj} \to b_1 \delta_{ij} + b_2 y_t^2 \delta_{3i} \delta_{3j}$$

$$\bar{d}_R d_R : \bar{d}_{Ri} d_{Rj} \to e_1 \delta_{ij}$$