EW corrections at high energies

Lorenzo Mai

UNIVERSITY OF SUSSEX

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Artwork by C.Z.

The need for precision

- Despite all the successful SM predictions, still (too many) open questions (e.g. DM & DE, hierarchy problem, …)

- No BSM at colliders so far, but much more data will be collected by HL-LHC & lot of proposal for future colliders

- *Precise* theoretical predictions are mandatory for

  - Direct NP searches: needs for accurate SM backgrounds (e.g. Dark Matter searches based on large MET signatures require extreme control on $Z(\nu\bar{\nu})$ background, Lindert ***et al*** 1705.04664***; 2017***)

  - Indirect NP searches: look for BSM effects in deviations from SM (e.g. $p_{\mathrm{T,H}}$ distributions in Higgs production via gluon fusion, Lindert ***et al*** 1801.08226***; 2018***)

- *Precision* is fundamental to unlock the potential of experimental data

Perturbative expansion

- Higher order corrections: perturbation theory with expansion parameters $\alpha \simeq 0.01$ and $\alpha_{\mathrm{S}} \simeq 0.1$:

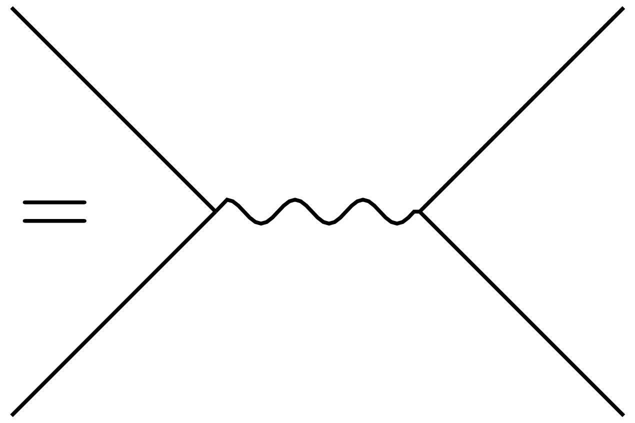
$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_{\mathrm{S}}\, \mathrm{d}\sigma_{\mathrm{NLO\ QCD}} + \alpha_{\mathrm{EW}} \mathrm{d}\sigma_{\mathrm{NLO\ EW}}$$
$$+ \alpha_{\mathrm{S}}^2\, \mathrm{d}\sigma_{\mathrm{NNLO\ QCD}} + \alpha_{\mathrm{EW}}^2\, \mathrm{d}\sigma_{\mathrm{NNLO\ EW}} + \alpha_S \alpha_{\mathrm{EW}} \mathrm{d}\sigma_{\mathrm{NNLO\ QCDxEW}} + \ldots$$

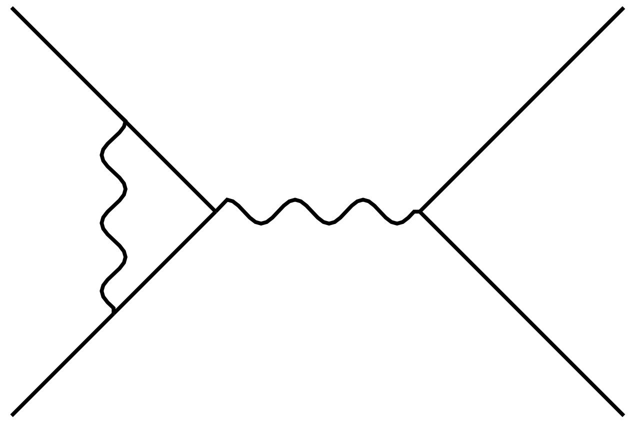
- KLN theorem: IR divergencies cancel between real and virtual contributions

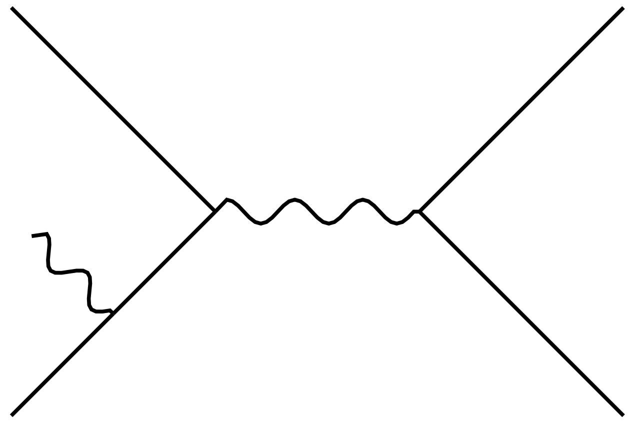
- At NLO:

$$\sigma|_{\mathrm{NLO}} = \frac{1}{2s}\int d\Phi_n \left[\underbrace{|\mathcal{M}_{\mathrm{LO}}|^2}_{B} + \underbrace{2\,\mathrm{Re}\left\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\right\}}_{V}\right] + \frac{1}{2s}\int d\Phi_{n+1}\, \underbrace{|\mathcal{M}_{\mathrm{NLO,R}}|^2}_{R}$$

where, e.g.

$\mathcal{M}_{\mathrm{B}} =$ 

$\mathcal{M}_{\mathrm{NLO,V}} =$ 

$\mathcal{M}_{\mathrm{NLO,R}} =$ 

Sudakov logarithms

- In the energy range above the EW scale ($\sqrt{s} \gg M_W$), Sudakov logs represent the leading contribution of EW radiative corrections

- EW Sudakov logarithms from N^{n}LO EW corrections

$$\alpha^n \log^k \frac{s}{M_W^2}, \qquad 1 \leq k \leq 2n$$

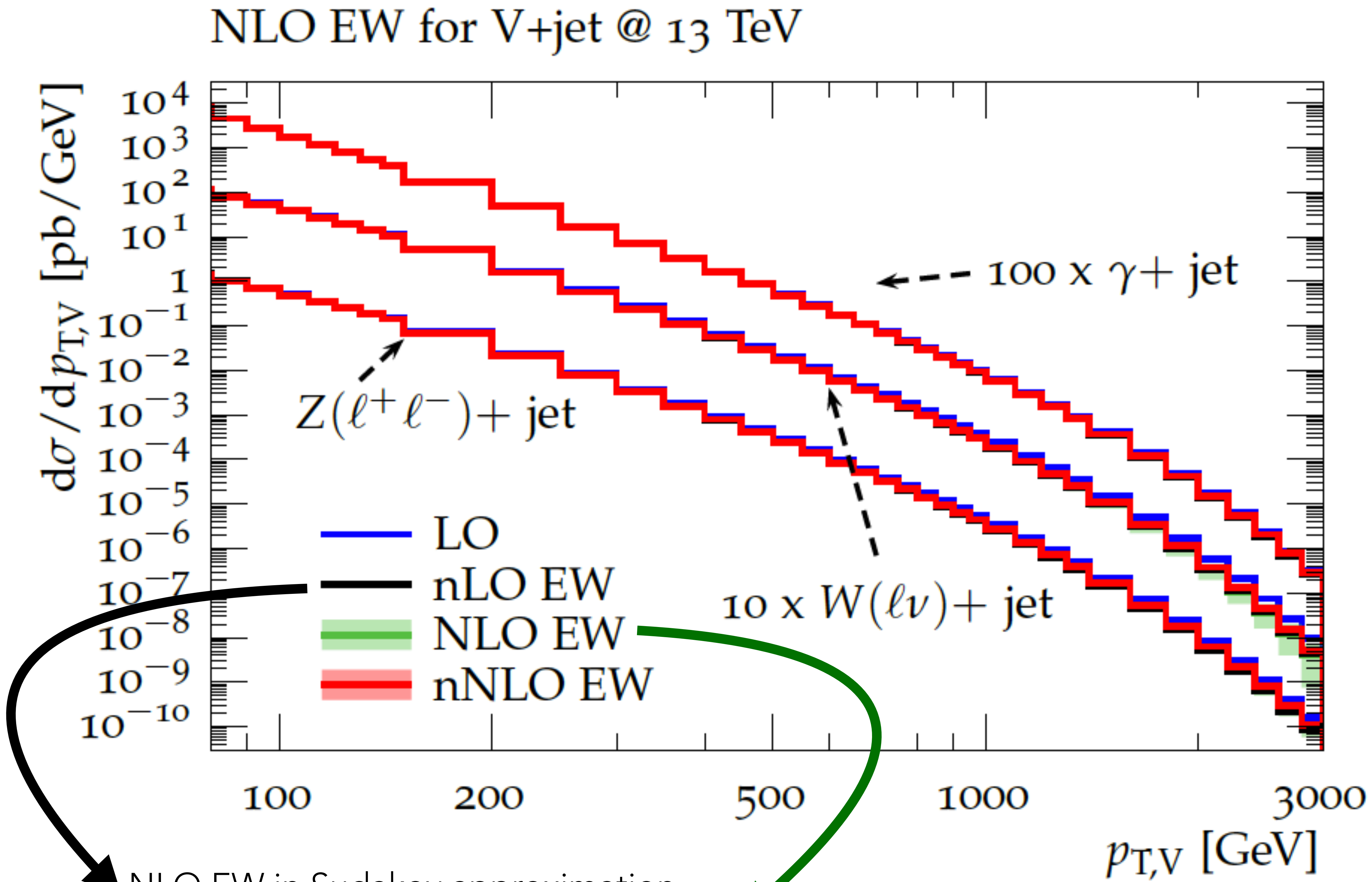
- At NLO

Double logs: $$L(s) = \frac{\alpha}{4\pi} \log^2 \frac{s}{M_W^2},$$

Single logs: $$l(s) = \frac{\alpha}{4\pi} \log \frac{s}{M_W^2}$$

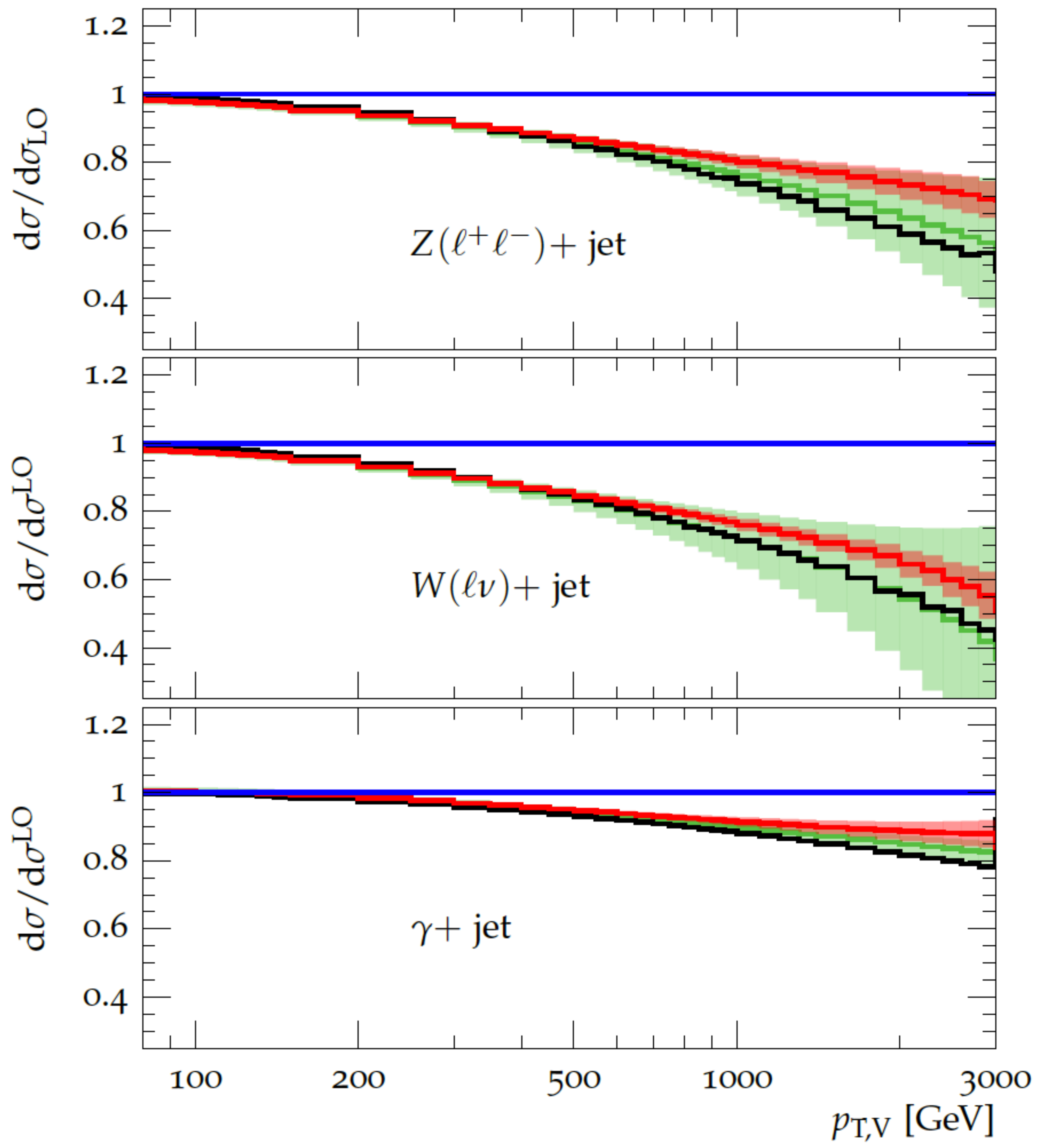
Sudakov logarithms

- These logarithmic corrections enhance tails of kinematic distributions with correction factors of several tens percent



NLO EW in Sudakov approximation

Full NLO EW



$pp \to V + j$ @ 13 TeV (Lindert *et al* 1705.04664; *2017)*

Framework: DP algorithm

- $n \to 0$ process

$$\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n) \to 0$$

with not mass suppressed Born matrix-element, i.e $\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d$

- DP algorithm based on logarithmic approximation (LA):

→ Hierarchy scales

$$\mu^2 = s \sim (p_k + p_l)^2 \gg m_\mathrm{t}^2, M_\mathrm{H}^2 > M_\mathrm{Z,W}^2 \gg m_f^2 \gg \lambda^2$$

→ At one-loop keep only double and singular logarithmic corrections

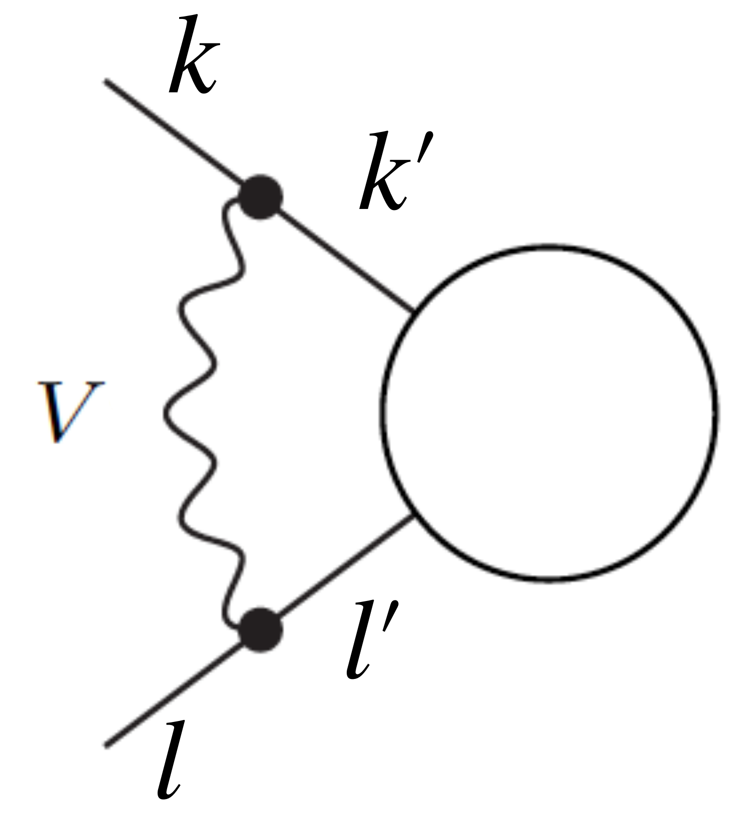
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d L \qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d l$$

neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ contributions

(Denner and Pozzorini 0010201*; 2002)*

Double Logs (DL)

- DL originate from triangle diagrams where two external legs exchange a *soft and collinear* (SC) gauge boson V



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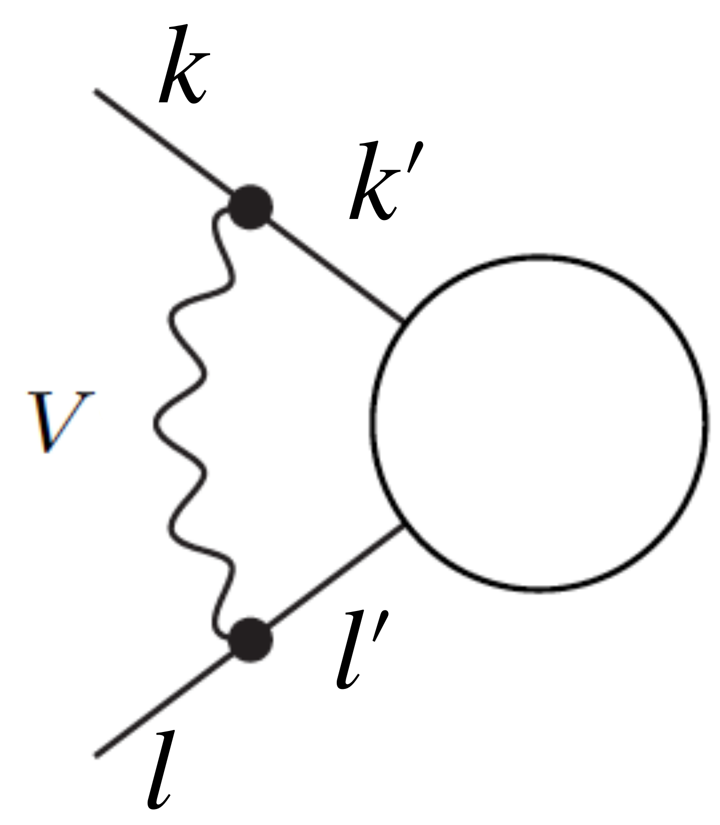

- DL originate from triangle diagrams where two external legs exchange a *soft and collinear* (SC) gauge boson V

- In the *Eikonal approximation*, the loop integral reduces to the scalar three-point function C_0, which <u>*factorises*</u>

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_{k}\sum_{l<k}\sum_{V}\sum_{k',l'} \frac{\alpha}{4\pi} I^{V}_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2\frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{M_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

with $r_{kl} = (p_k + p_l)^2$

- Consequence of C_0 <u>*factorisation*</u>*:* DL are <u>*universal,*</u> i.e. process independent

Double Logs: LSC, SSC, S-SSC

- DL can be split into

→ Leading soft-collinear (LSC): angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_{k}\sum_{k'}\delta^{\mathrm{LSC}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}}\log^2\left(\frac{s}{M_V^2}\right)$$

→ Sub-leading soft-collinear (SSC) and S-SSC: angular dependent, double sum over external legs

$$\delta^{(\mathrm{S-})\mathrm{SSC}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} = \sum_{k}\sum_{l<k}\sum_{k',l'}\delta^{(\mathrm{S-})\mathrm{SSC}}_{kk'll'}\quad\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC}}_{kk'll'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}}\log\left(\frac{s}{M_V^2}\right)\log\left(\frac{|r_{kl}|}{s}\right), \qquad \delta^{\text{S-SSC}}_{kk'll'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}}\log^2\left(\frac{|r_{kl}|}{s}\right)$$

Single Logs (SL)

- SL have a triple origin

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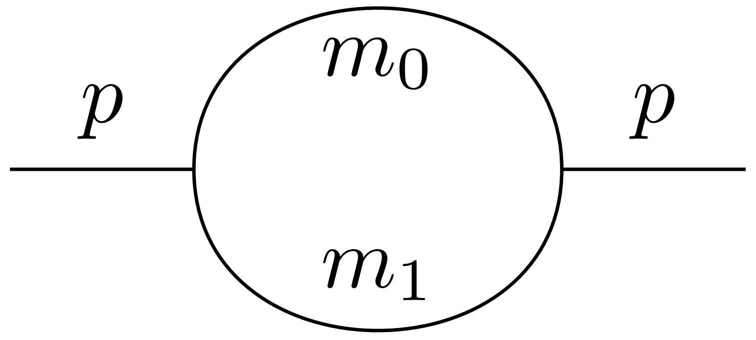
- SL have a triple origin: Renormalisation

$$
\begin{aligned}
\mu^2_{i,0} &= \mu^2_i + \delta\mu^2_i \\
\varphi_{i,0} &= \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right)\varphi_j \\
g_{i,0} &= g_i + \delta g_i = \left(1 + \delta Z_{g_i}\right) g_i
\end{aligned}
$$

- In LA, the finite part of 2-point functions entering in CTs has a SL behaviour



$$\propto \log \frac{\mu^2}{M^2}\Bigg|_{\mu^2=s}$$

Single Logs (SL): PR

$$\mu_{i,0}^2 = \mu_i^2 + \delta\mu_i^2$$

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$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

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→ Renormalisation of EW dimensionless parameters

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→ WF renormalisation of external fields

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

yields to the *factorised* correction

$$\delta^{\mathrm{WF}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} = \sum_k \sum_{k'} \delta^{\mathrm{WF}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\mathrm{WF}}_{kk'} = \frac{1}{2}\delta Z_{kk'}$$

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Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

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Single Logs (SL): Coll

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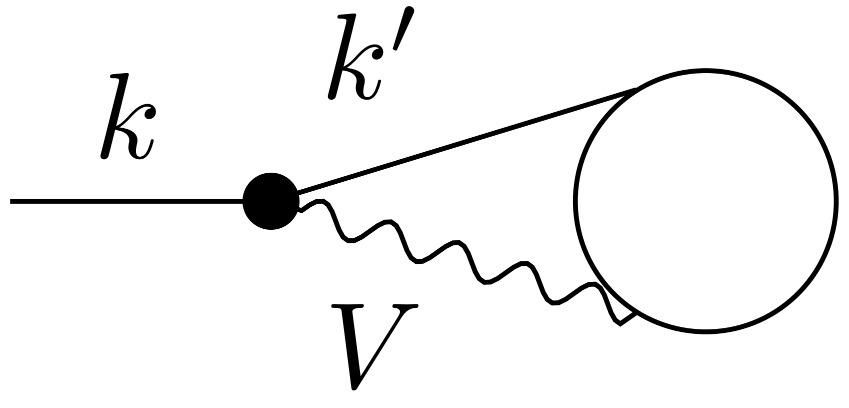
→ External leg emission of a collinear gauge boson



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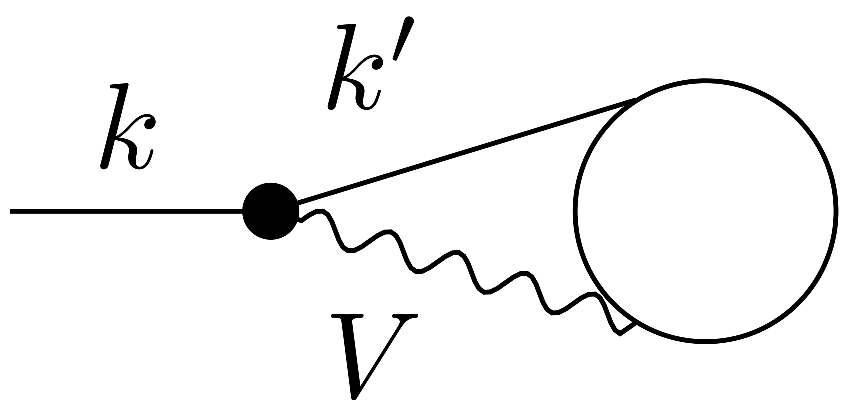

Its evaluation in *Eikonal approximation* leads to the <u>*factorised*</u> contribution

$$\delta^{\text{coll}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} = \sum_k \sum_{k'} \delta^{\text{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\text{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

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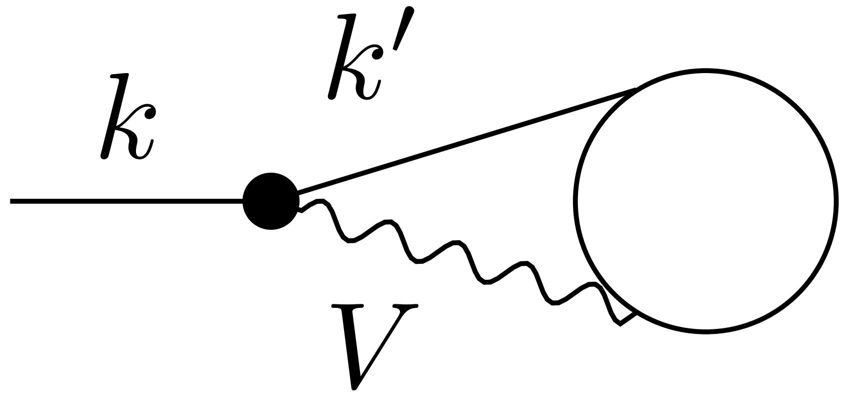

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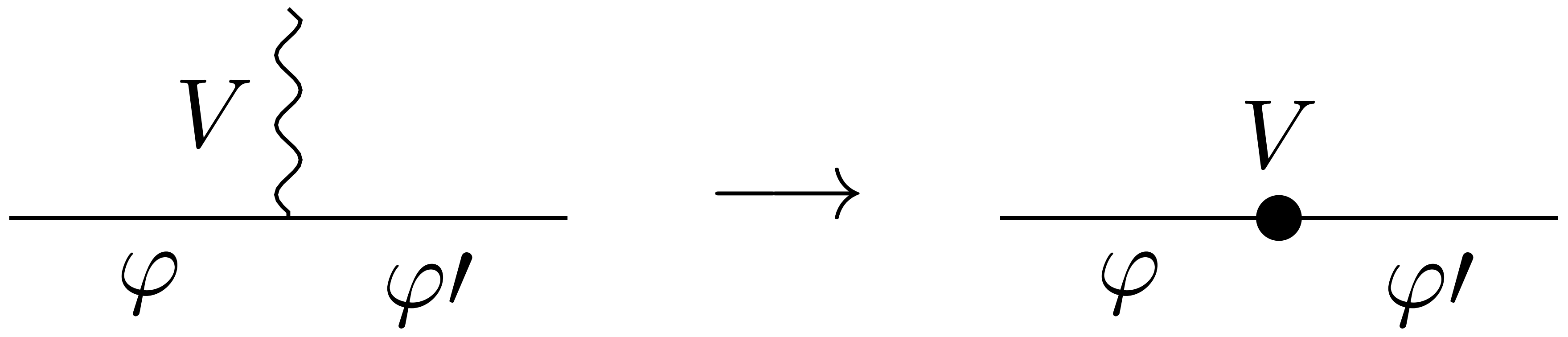
→ Full gauge-invariant SL correction associated to external fields:

$$\delta^{\text{C}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} = \sum_{k}\sum_{k'}\delta^{\text{C}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\text{C}}_{kk'} = \left(\delta^{\text{coll}}_{kk'} + \delta^{\text{WF}}_{kk'}\right)\Big|_{\mu^2=s}$$

Implementation in OpenLoops: how

- Representation of Denner-Pozzorini algorithm via effective CT vertices



$$= ieI^{V}_{\varphi\varphi'}K^{V}_{\mathrm{ew}}$$

reducing one-loop amplitudes to tree-level ones via double CT insertions

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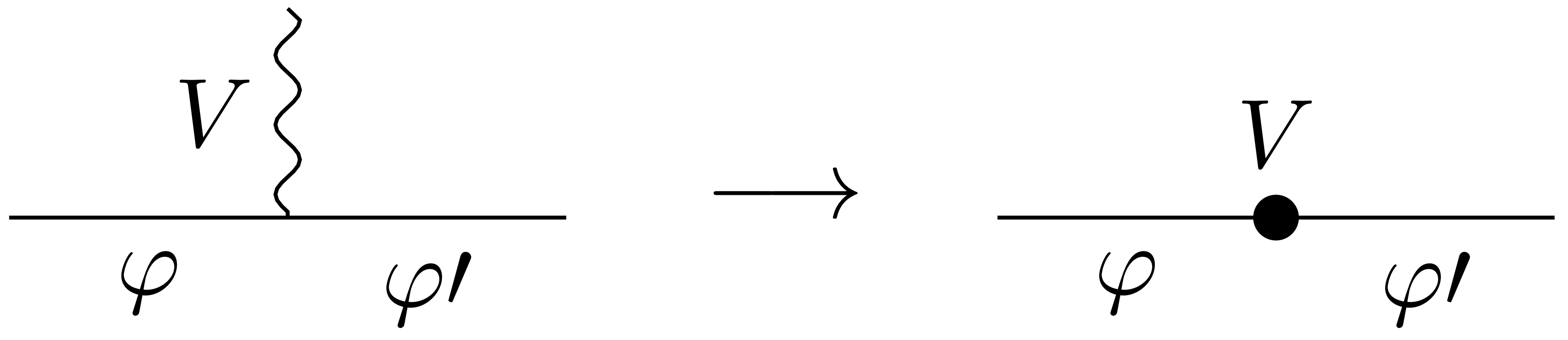

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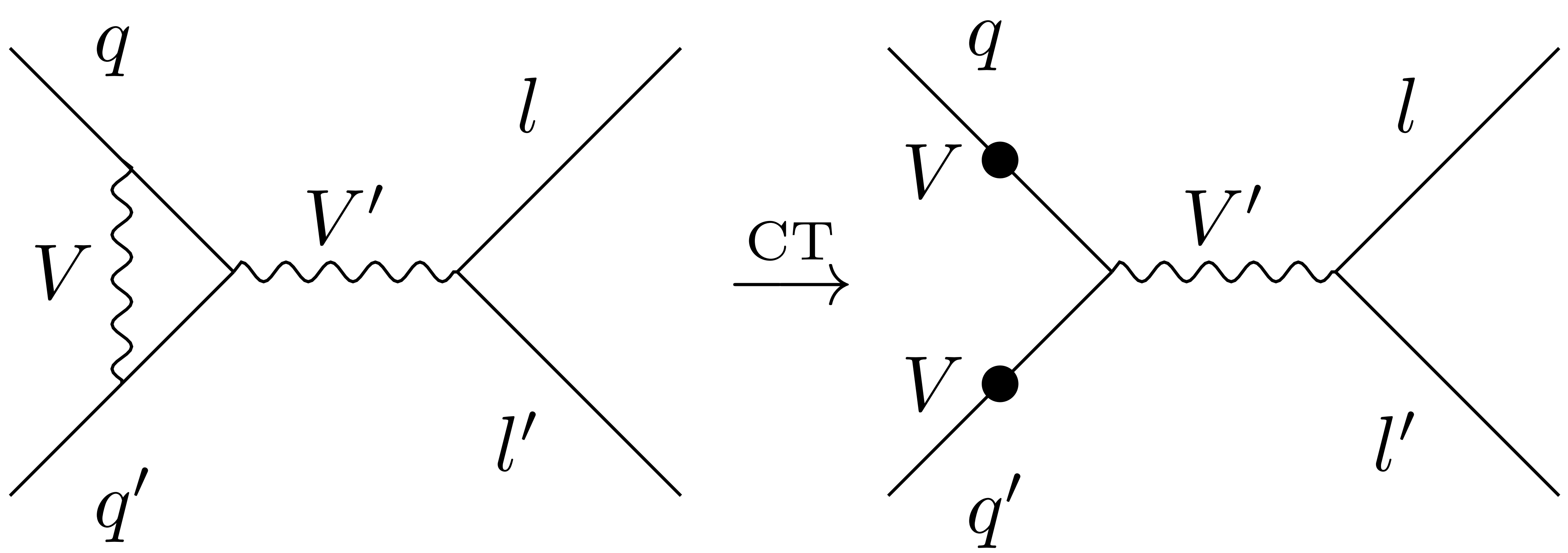
reducing one-loop amplitudes to tree-level ones via double CT insertions

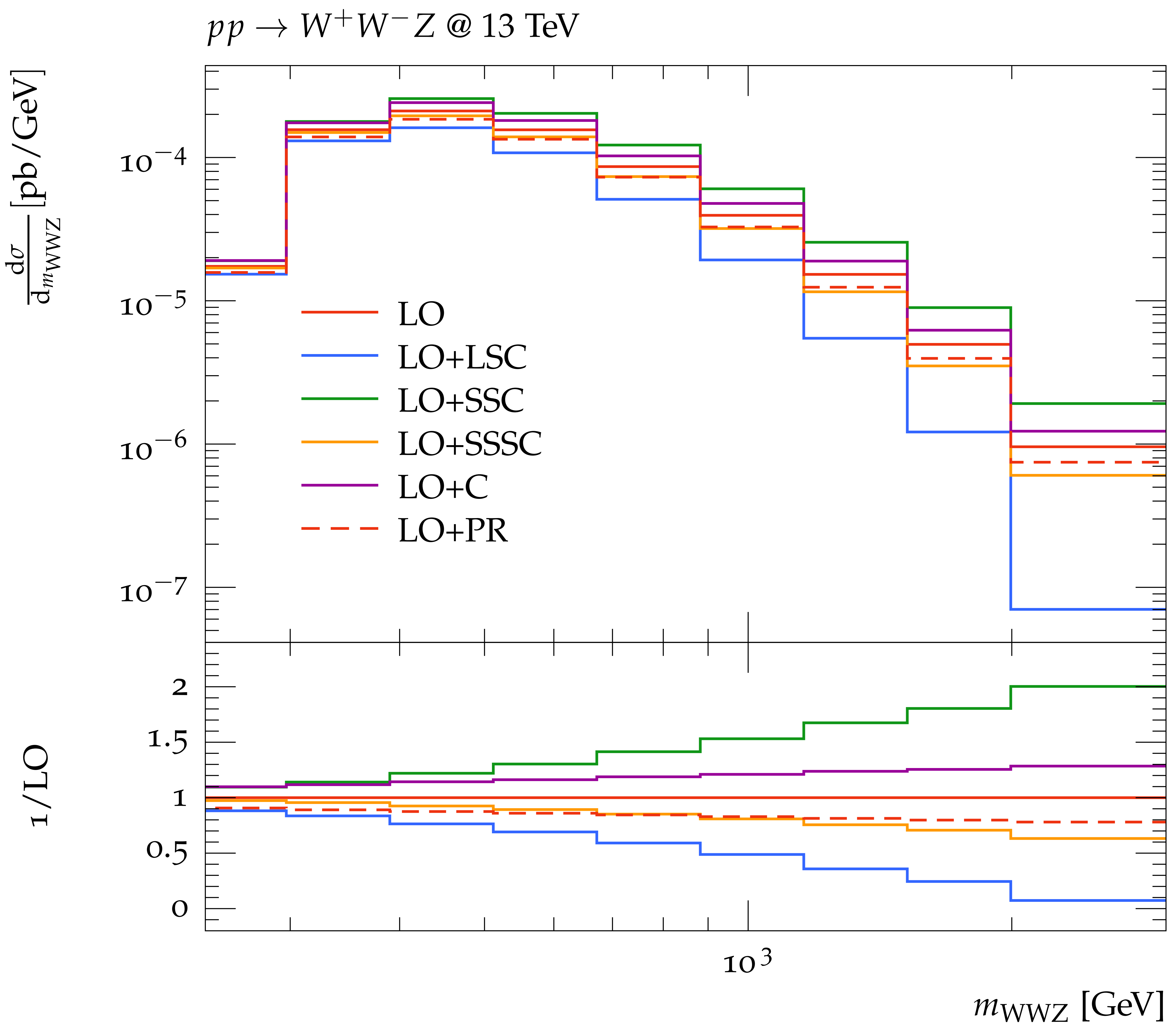
Eg.: Drell-Yann

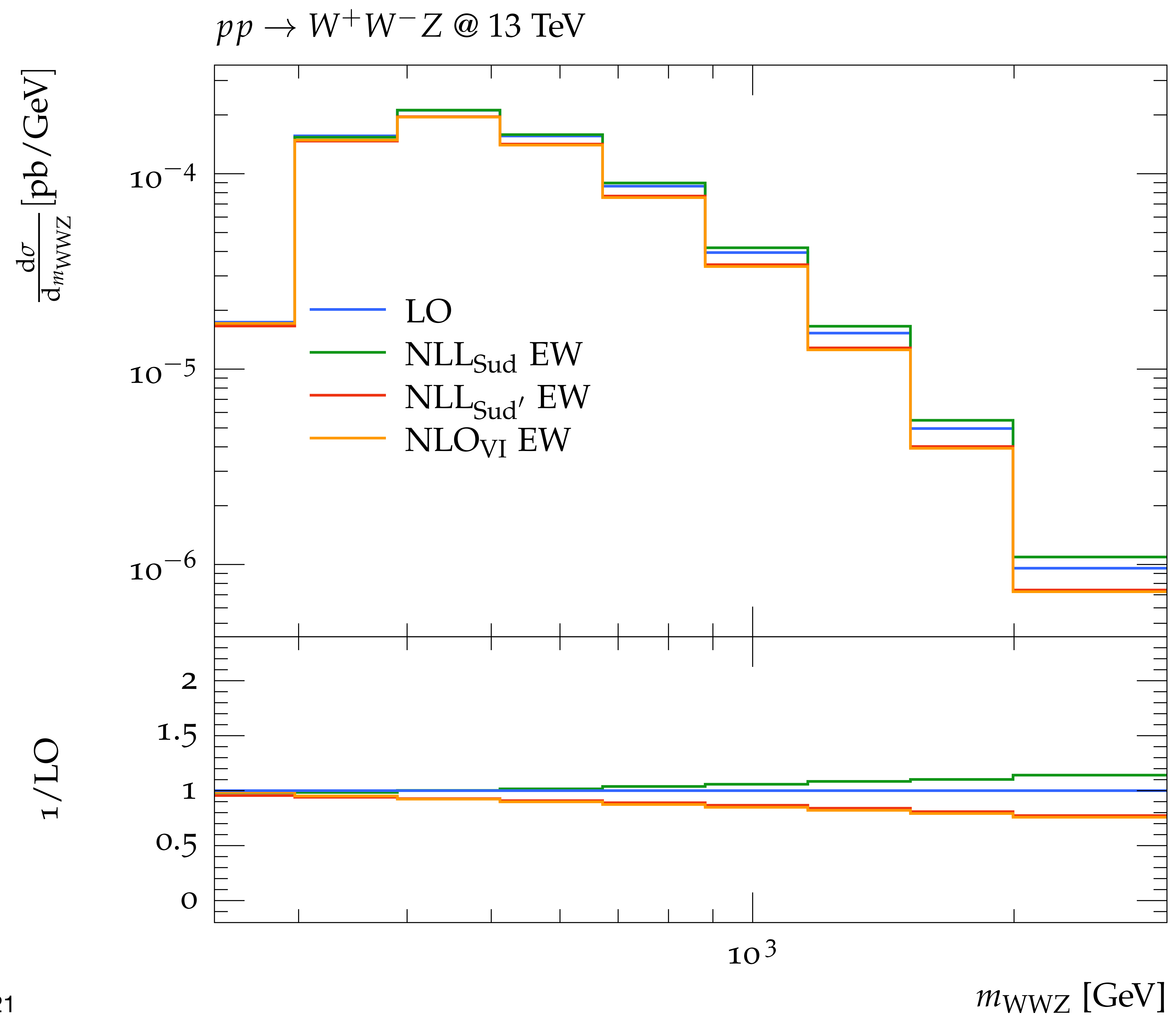


$$\Rightarrow \mathcal{M} \sim e^2 \sum_{V=A,Z,W^{\pm}} I^{V}_{q}\, I^{V}_{q'} \left(K^{V}_{\mathrm{ew}}\right)^2 \mathcal{M}_0$$

Results:

Results: $pp \to W^+W^-Z$





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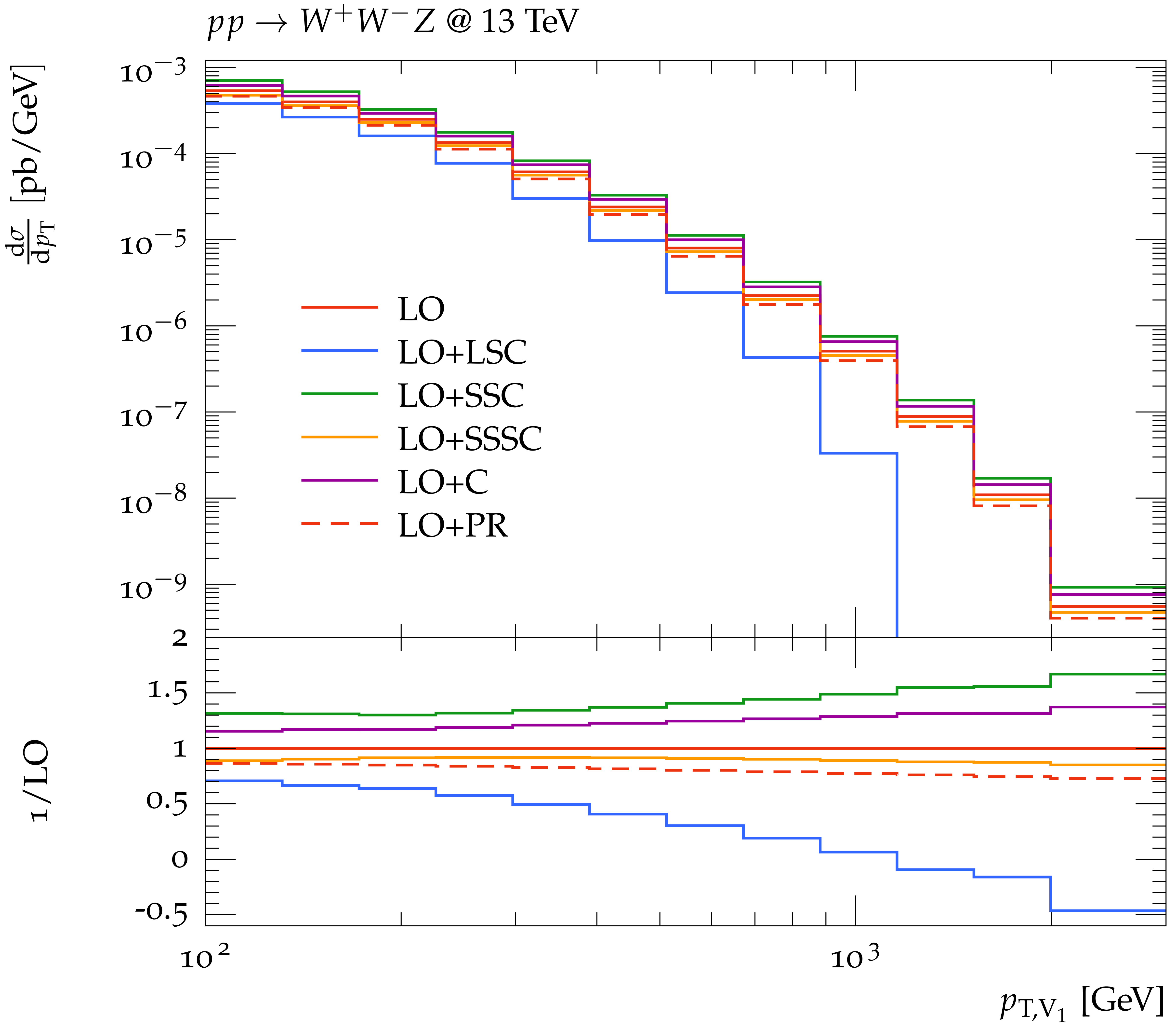

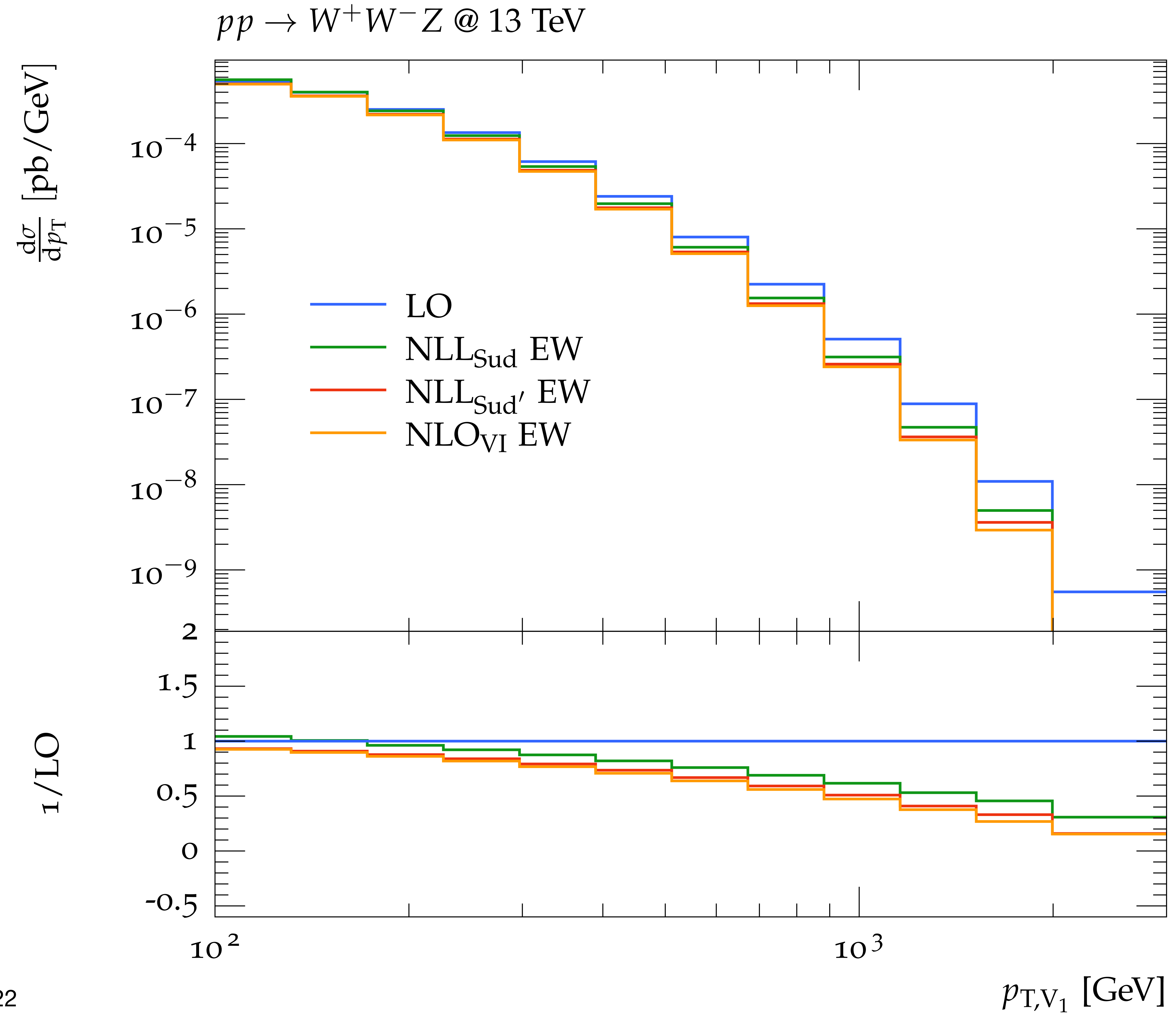

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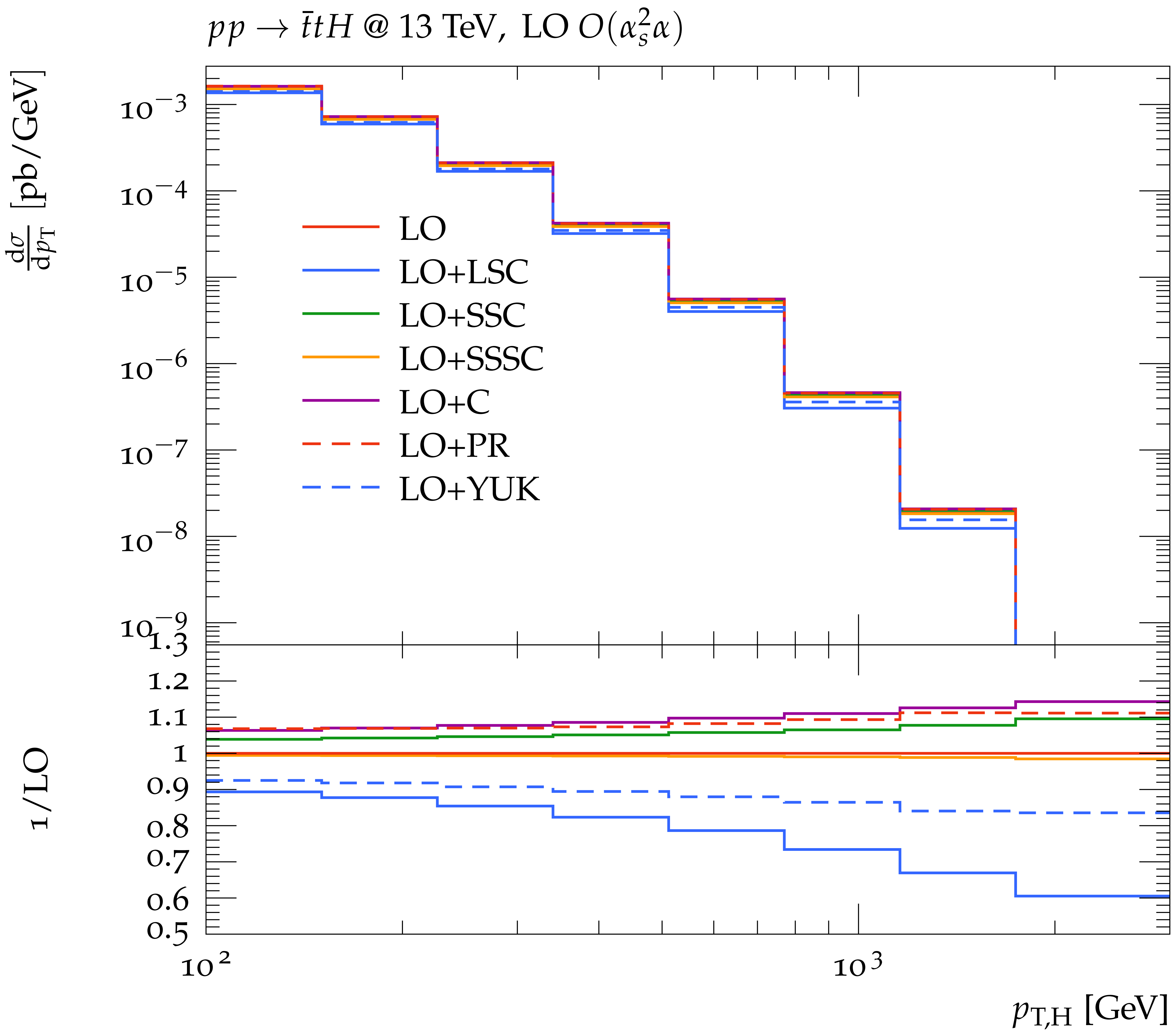

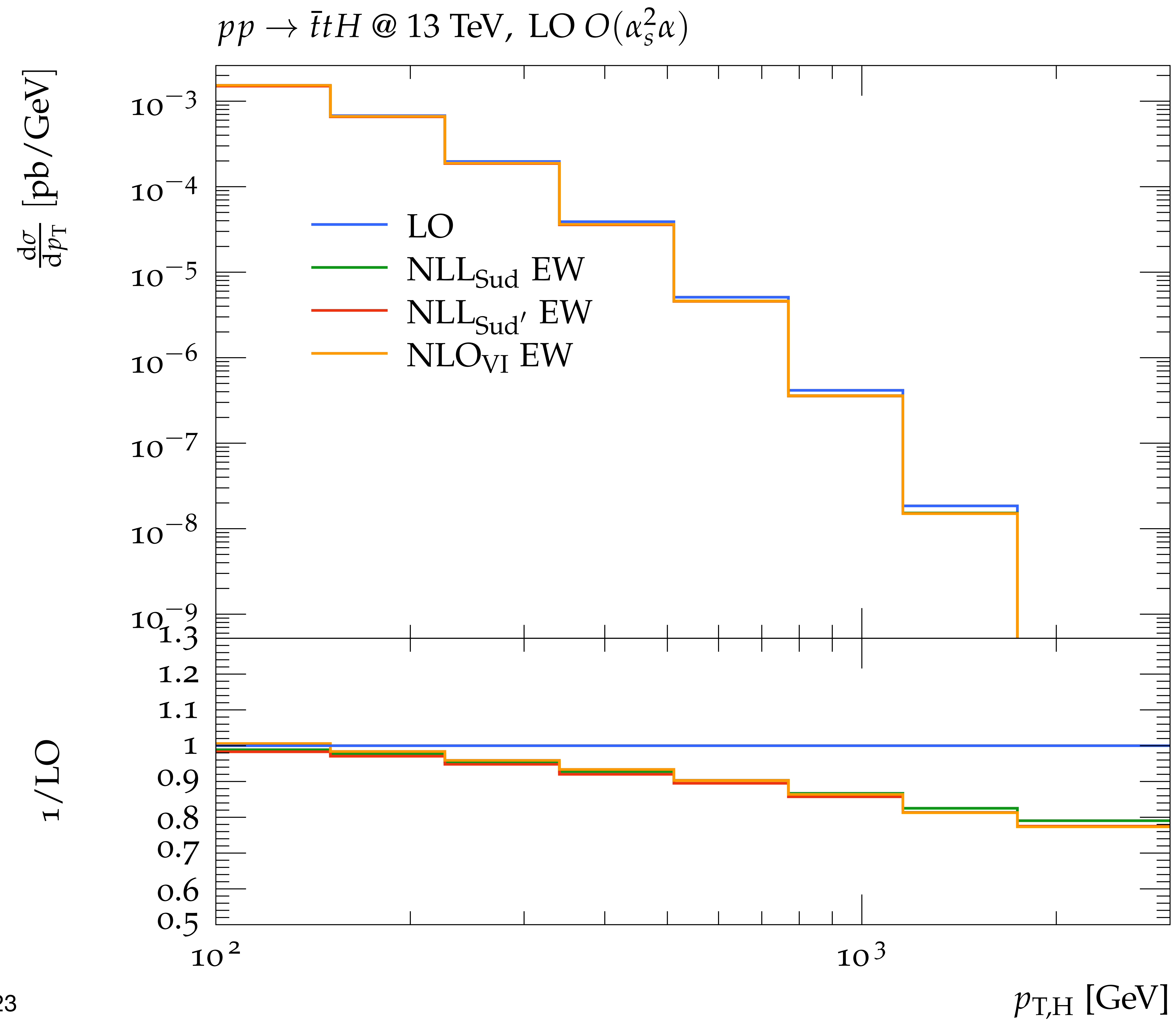

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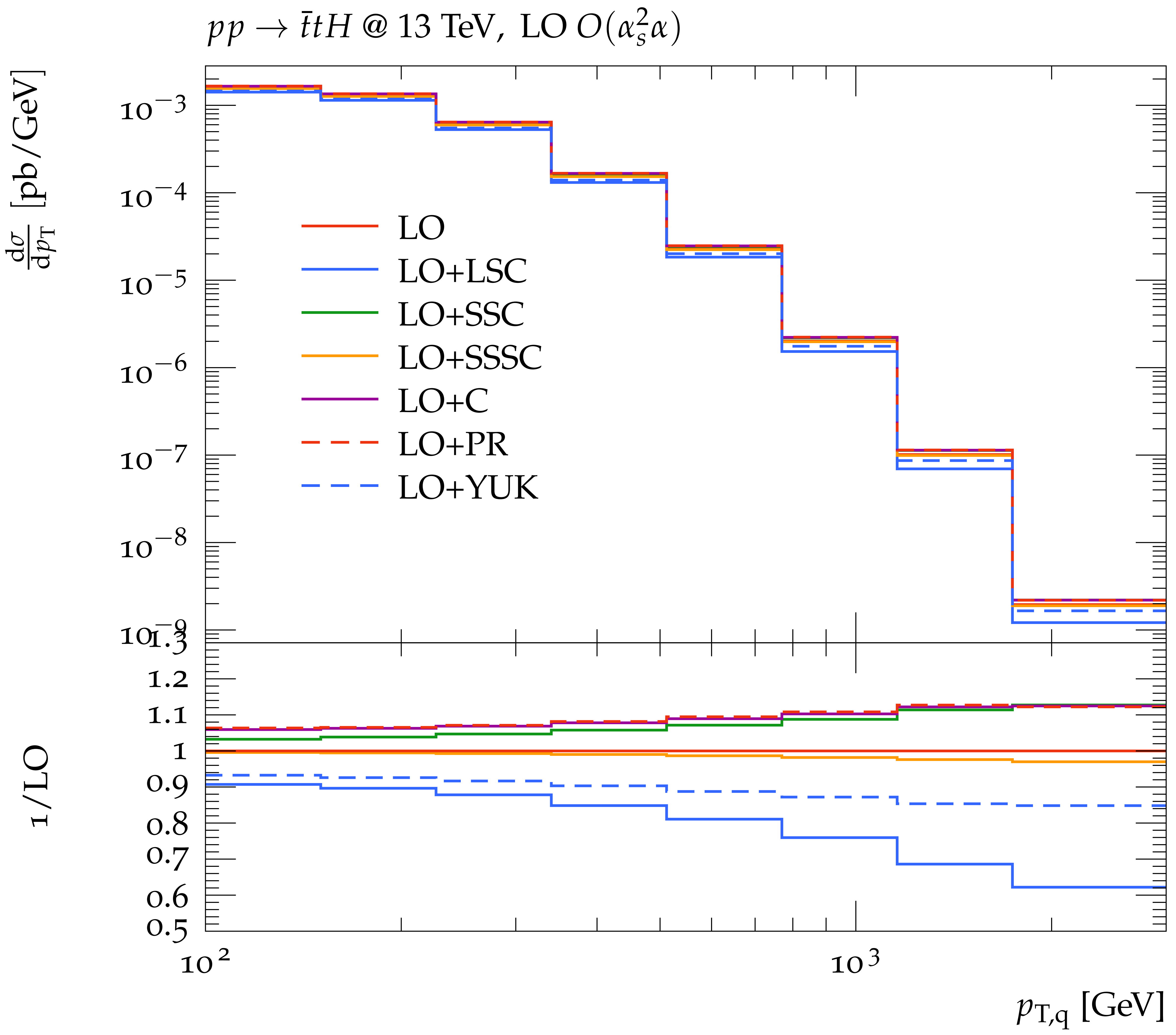

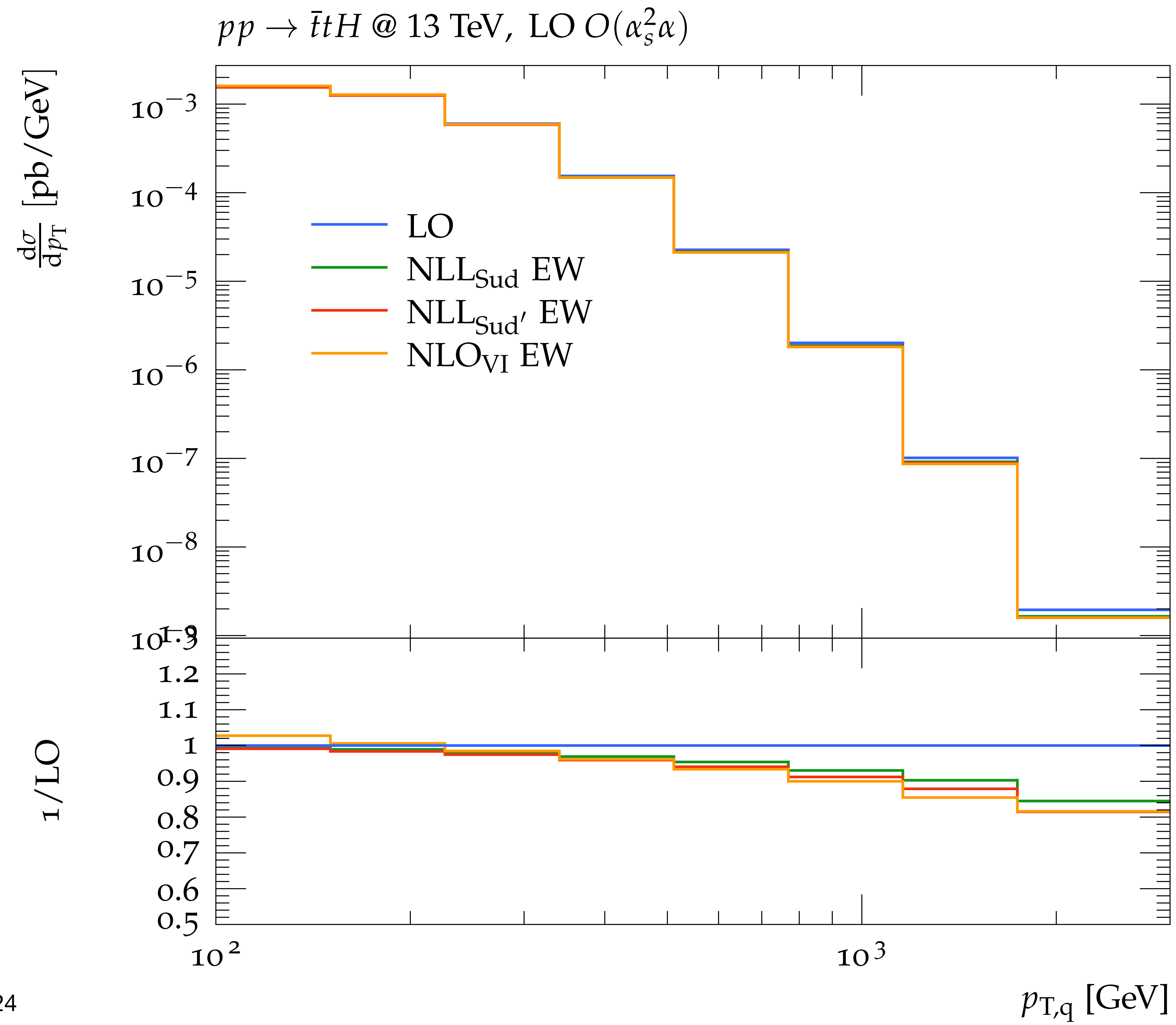

Conclusions and outlook

- *Precise* theoretical predictions at higher orders are nowadays mandatory for comparison with data and represent one of the best way for progression in particle physics

- In the EW sector, radiative corrections at high energies are dominated by *S*udakov logs which significantly enhance tails of kinematic distributions (> 10%)

- The *universality* of Sudakov logs (*factorisation* property) allows the reduction of complexity in higher order corrections preserving a percent level of accuracy

- Aspects of our implementation in OL:

    ► Model independent

    ► Support EW corrections for resonant processes (novelty)

    ► Suitable for NNLO/two-loop extension