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Interaction of Radiation With Matte

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CTU
CZECH TECHNICAL UNIVERSITY IN PRAGUE

Welcome to Prague

History of university

The Czech Technical University in Prague (CTU) is one of the oldest technical universities in the world, with its history dating back to the prescript of Emperor Josef I, issued in 1707, by which he agreed with founding an engineering school in Prague by Christian Josef Willenberg.

Faculties of CTU

- Faculty of Civil Engineering
- Faculty of Mechanical Engineering
- Faculty of Electrical Engineering
- Faculty of Nuclear Sciences and Physical Engineering
- Faculty of Architecture
- Faculty of Transportation Sciences
- Faculty of Biomedical Engineering
- Faculty of Information Technology

scientific institutes, including the **Institute of Experimental and Applied Physics**

Faculty of Nuclear Sciences and Physical Engineering (FNSPE)

The faculty was established in 1955, as part of the Charles University, but in 1959 became a new special faculty of the Czech Technical University in Prague. The establishment of the Faculty was connected with the beginning of a new era of the peaceful use of nuclear energy.

Departments at FNSPE

- Department of Mathematics
- Department of Physics
- Department of Humanities and Languages
- Department of Solid State Engineering
- Department of Physical Electronics
- Department of Materials
- Department of Nuclear Chemistry
- Department of Dosimetry and Application of Ionizing Radiation
- Department of Nuclear Reactors
- Department of Software Engineering

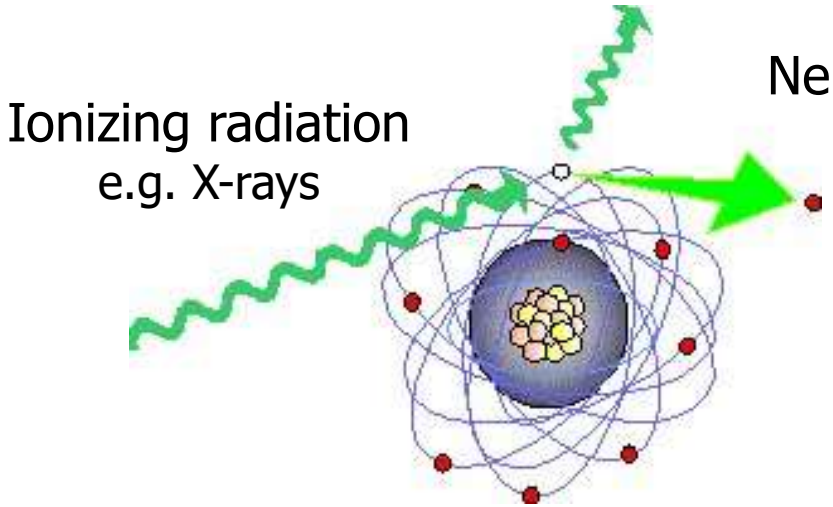
Department of Dosimetry and Application of Ionizing Radiation

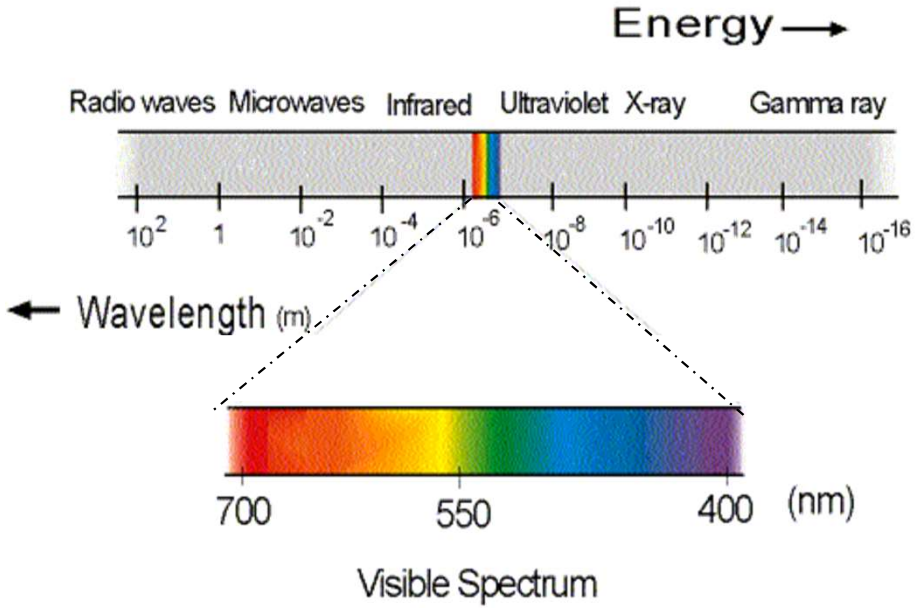
Prof. František Běhounek

1928 – Nobile expedition with airship Italia to north pole

Ionizing radiation

- Ionizing radiation has enough energy per individual photon or particle to ionize atoms or molecules by detaching electrons from them





Ionizing radiation:

$f > 3 \times 10^{15}$ Hz

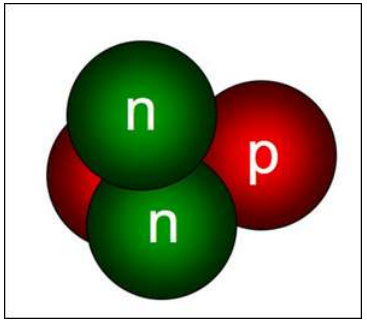
$\lambda < 10^{-7}$ m

$E > \approx 12$ eV

$1 \text{ eV} = 1.602 \times 10^{-19}$ J

Ionizing radiation

Charged (directly ionising) particles – they transfer energy to matter primarily by ionising and exciting their atoms as a result of the Coulomb interaction between particles and electrons bound in atomic shells. Both processes are also primary during interaction with higher molecular structures – they can then also lead to a deeper intervention in the internal structure of the given substance (breaking of chemical bonds, biological effects of ionizing radiation).



Particles (mass > 0) with electric charge

alpha - consist of two protons and two neutrons bound together into a particle identical to a helium nucleus ^{4}He

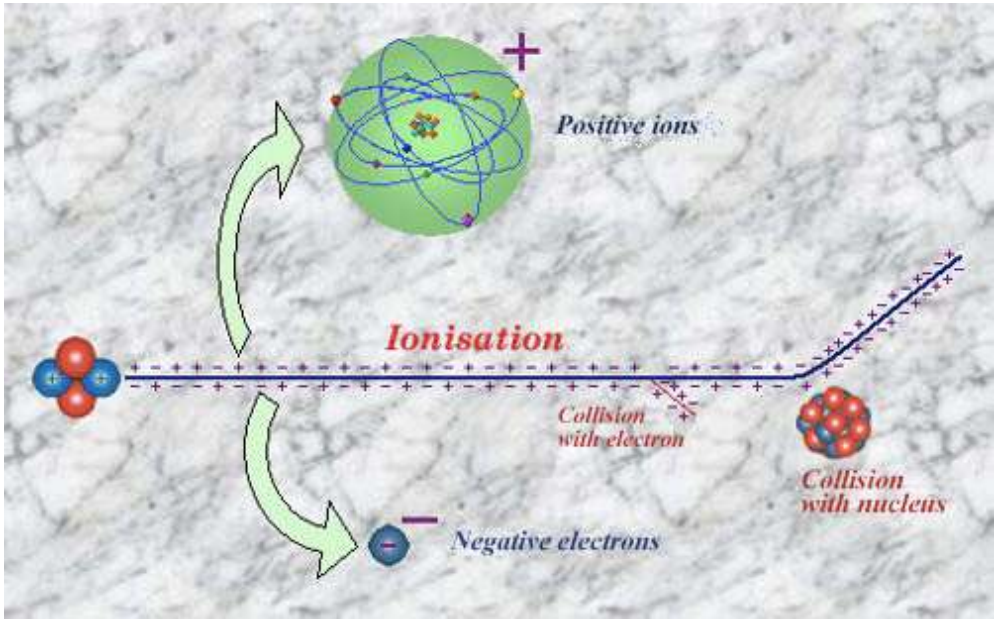
beta - a high-energy (high-speed) electron or positron emitted within a radioactive decay

The most important directly ionising particles: protons, deuterons, α particles, heavy singly or multiply ionized atoms (e.g. fission fragments), electrons and positrons.

Charged (directly ionising) particles

Alpha particle

- produces lot of ionisations (electron-ion pairs) along its pathway in matter
- limited range in a matter
- continuous slowing down
- almost direct trajectory, exception collision with a nucleus



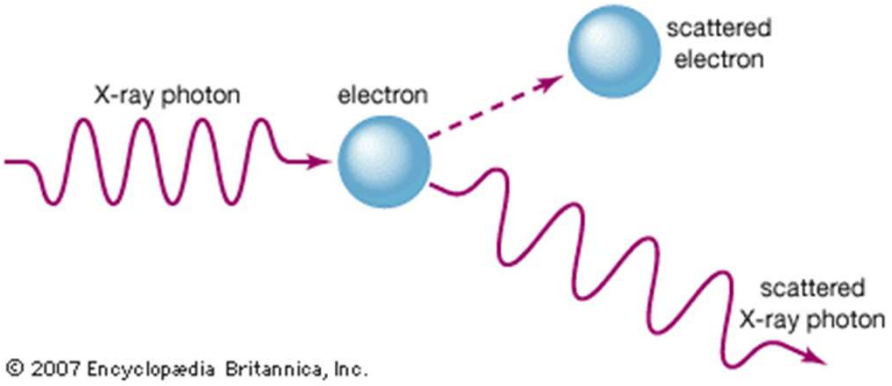
Source: radioactivity.eu.com

Ionizing radiation

Uncharged (indirectly ionising) particles – have no electrical charge – they do not ionise or excite along their path. The result of their interaction is the emission of secondary charged particles that ionise the environment ⇒ these secondary processes can be used for detection and dosimetry of indirectly ionizing particles (detection methods are mainly based on ionization and excitation effects).

The most important indirectly ionising particles: photons and neutrons

Photon (X-rays, gamma radiation) – electromagnetic radiation, energy higher than UV



Gamma rays originate from the nucleus while *X-rays* originate in the electron fields surrounding the nucleus or are machine produced

Neutron (n) - a subatomic particle without electric charge and a mass slightly larger than that of a proton.

Interactions of ionizing radiation

Ionizing radiation can interact with both electron shells and atomic nuclei. The properties of the particle (type, kinetic energy) and the substance in which the interaction takes place (percentage and proton number of the elements of which it is composed) decide which interaction processes are predominant, what is their mutual ratio.

Various types of **ionizing radiation** x **interactions** x **target particles in matter**

fundamental forces of nature

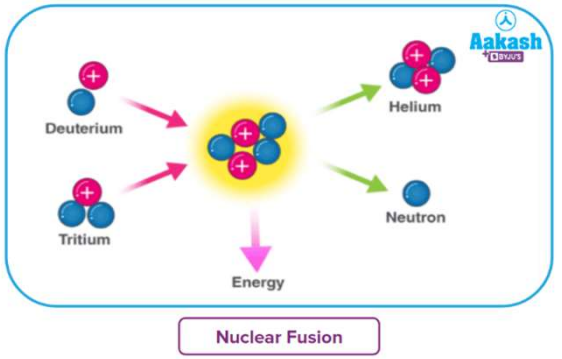
Force	Range of interaction	Relative intensity
Gravity	∞	10^{-40}
Electromagnetism	∞	10^{-2} až 10^{-3}
Strong force	10^{-15} m	1
Weak force	10^{-18} m	10^{-5}

No interaction = no effect on material and no information about radition and material

Sources of ionizing radiation

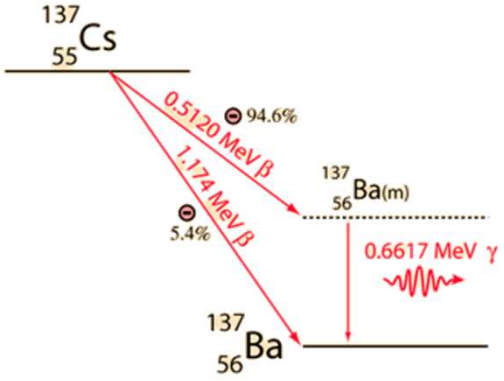
Nuclear reaction - the process in which two nuclei (particles) collide to produce one or more nuclides that are different from the nuclide(s) that began the process

$$^{2}H + {}^{3}H \rightarrow n + {}^{4}He$$



Radioactivity – spontaneous process by which the nucleus of an unstable atom loses energy by emitting radiation

$$^{137}Cs \rightarrow {}^{137}Ba + beta + gamma$$



Particle accelerator - a machine that uses electromagnetic fields to propel charged particles to nearly light speed

Possible views on the interaction of ionizing radiation with matter:

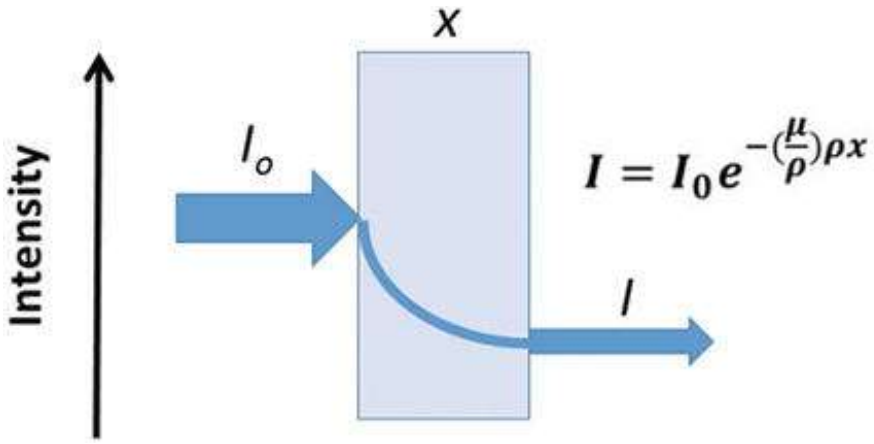
1) the study of the collision of a single bombarding particle with an isolated atom or electron:

$$\sigma = \frac{P}{\Phi}$$

Cross section - effective area that governs the probability of certain interaction. It is a probability P of a selected interaction (absorption, scattering, etc.) for 1 target particle (electron, atom, nucleus) divided by the fluence Φ of incoming radiation.

Unit: m^2 (1 barn =10^{-28} m^2)

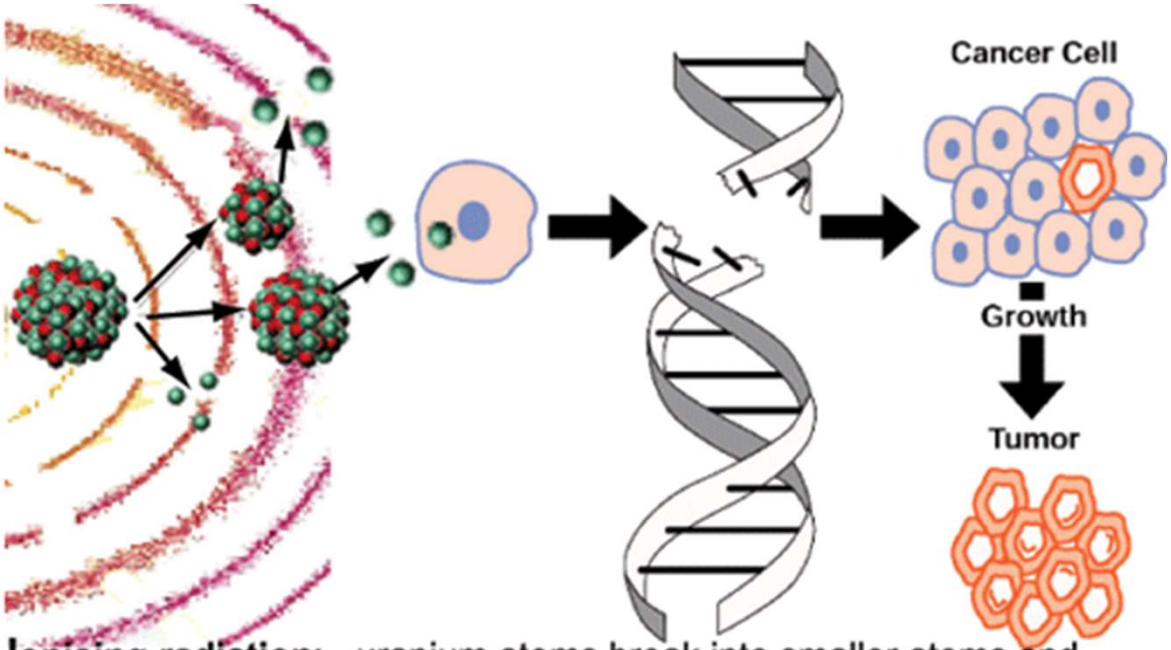
2) the study of changes that occur in a beam of ionizing radiation as it passes through matter



Possible views on the interaction of ionizing radiation with matter:

3) the study of changes induced in matter when a beam of ionizing radiation passes through it.

- excitation and ionization of atoms
- chemical changes
- increase in kinetic energy of the particles of a substance
- biological effect in living organisms



Ionising radiation: - uranium atoms break into smaller atoms and particles, which enter a human cell, strike the nucleus, and damage the DNA, causing it to divide in an uncontrolled way - **cancer**

Interaction of heavy charged particle

Interaction of heavy charged particle

Dominant mechanism of energy loss – Coulomb collisions with electrons in atoms leading to ionisation and excitation of atoms.

Positive ions with Z = 1, 2 (protons, deuterons, α particles, etc.)

Collision losses completely prevail at energies significantly lower than the rest energy of the moving particle (however, this is close to or exceeds 1 GeV, i.e. in most cases that we deal with in dosimetry or radiation applications, the condition is met – usually energies are up to the order of tens MeV).

Two more processes:

excitation of bremsstrahlung radiation (at high energies – significant e.g. for large accelerators), scattering on atomic nuclei (it needs to be considered even at low energies – but the probability is significantly lower than the probability of collisions with orbital electrons).

Energy losses along the path of a charged particle – expressed by linear stopping power S =(-dE/dx), or mass stopping power (energy loss related to a unit of the particle path expressed in length or mass units).

$$S = -\frac{dE}{dx}$$

Linear stopping power

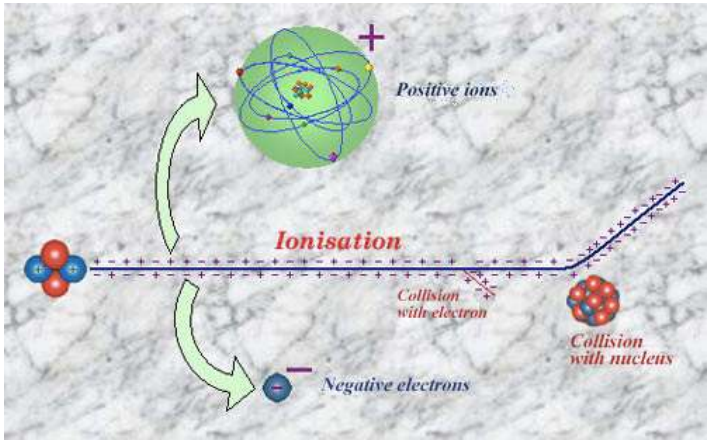
$$S_m = \frac{S}{\rho}$$

Mass stopping power

Interaction of heavy charged particle

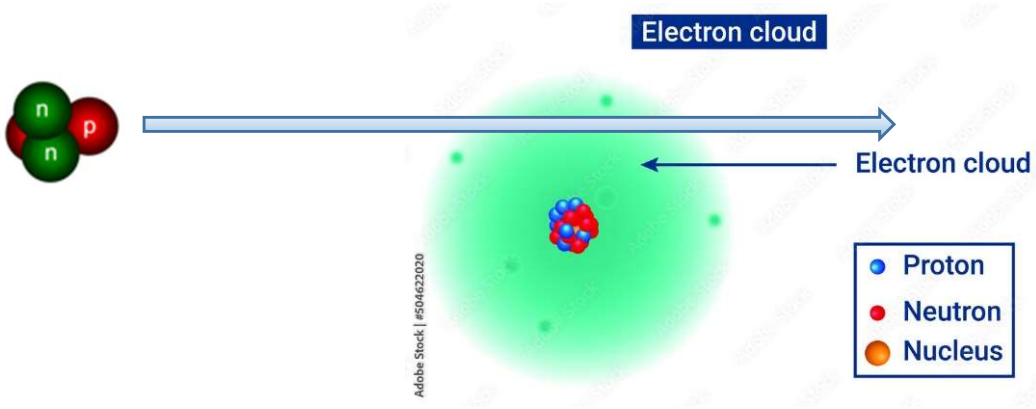
Dominant mechanism of energy loss – Coulomb collisions with electrons in atoms leading to ionisation and excitation of atoms.

Why?



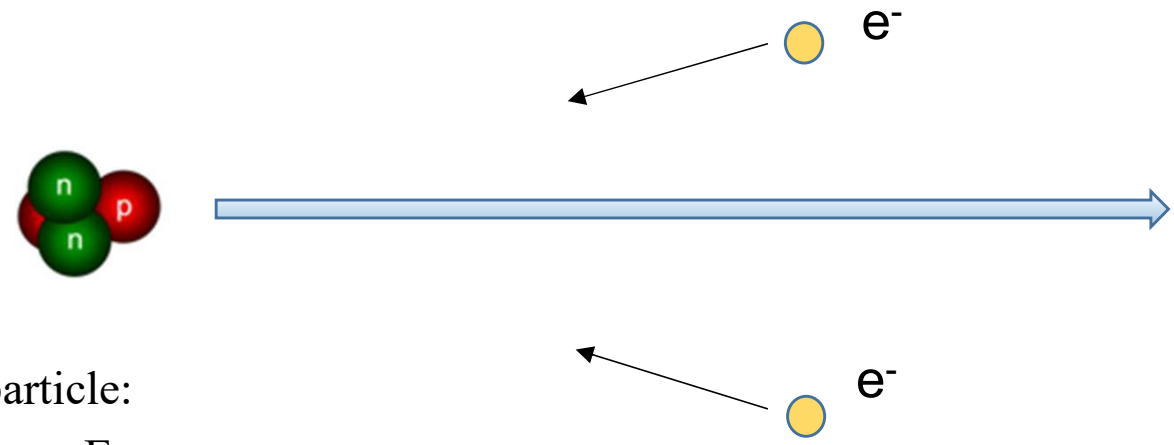
Interaction of heavy charged particles with:

- electrons (mostly weakly bound in the atom compared to the energy of the alpha particle)
- nucleus



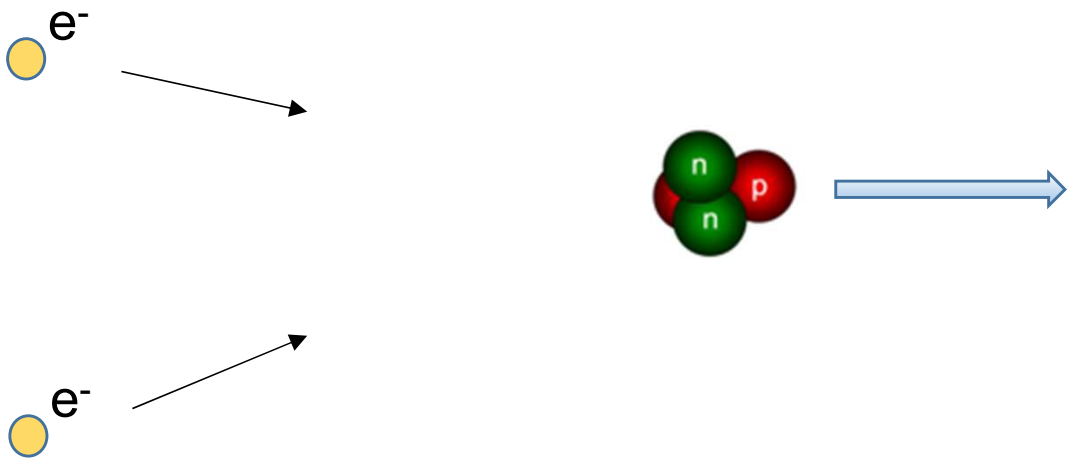
Energy transfer to electrons

Alpha particle in a cloud of free electrons



Alfa particle:

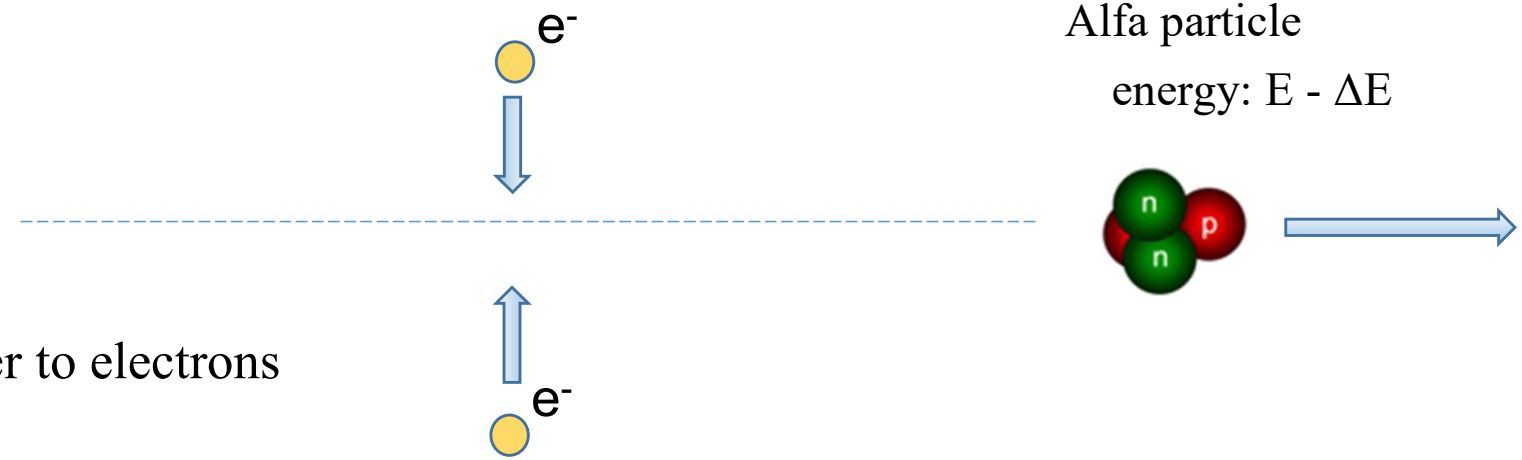
energy: E



Energy transfer to electrons

Effects on alpha particle and electrons

Alfa particle:
energy: E

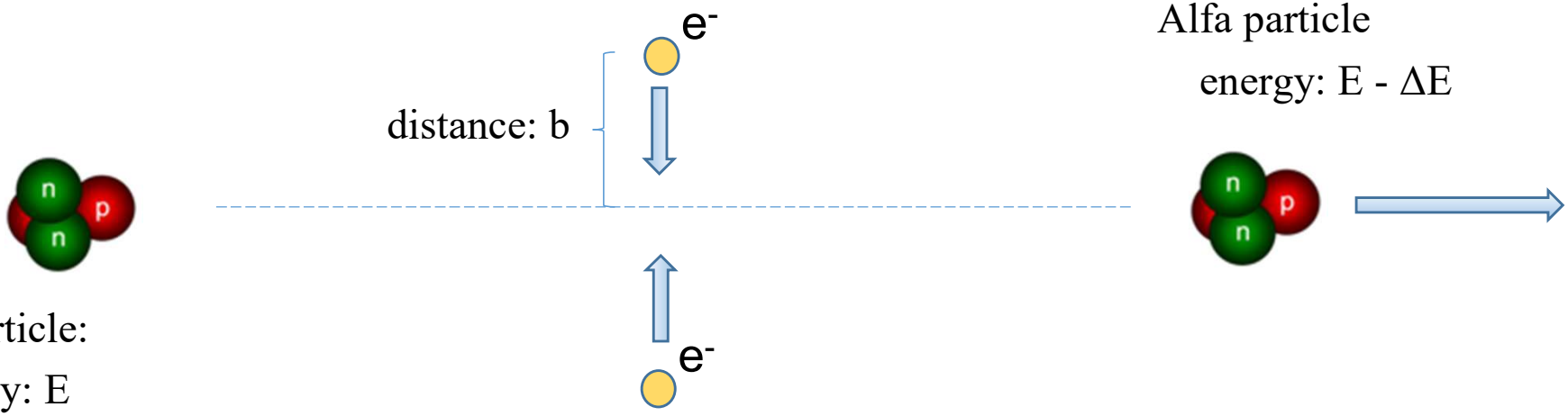


Alfa particle
energy: E - ΔE

ΔE – energy transfer to electrons

Energy transfer to electrons

Effects on alpha particle and electrons



Alfa particle:
energy: E
velocity: v

ΔE – energy transfer to electrons

$$\Delta E \sim \frac{z^2}{m_e b^2 v^2}$$

z – charge of a particle (2 for alpha)
m_e – mass of target particle (electron 9.1 x 10^{-31} kg)
b – distance of an electron from the track of a charged particle
V – velocity of charged particle

analogously for interaction with atomic nuclei

Stopping power

Stopping power (-dE/dx or -dT/dx), where E or T is the kinec energy of charged particle

- described with Bloch equation

- charged particles lose their energy dominantly in interactions with electron, interactions with nuclei are almost negligible

In addition to the general Bloch relation, several simpler versions under different boundary conditions have been derived. The limiting case of the Bloch relation for $Ze_0^2/h\nu \rightarrow 0$ – Bethe–Møller relation for relativistic particles

$$-\frac{dT}{dx} = \frac{e_0^4 z^2}{4\pi\varepsilon_0^2 m_{e0} v^2} NZ \left\{ \ln \frac{2m_{e0}v^2}{(1-\beta^2)I} - \beta^2 \right\}$$

ε_0 – permittivity of vacuum, $\beta = v/c$, v – velocity of a particle, c – velocity of light, N – number of atoms in the medium unit volume, Z – proton number of atoms of matter, I – mean excitation potential of atoms of matter, m_{e0} – electron rest mass, e_0 – electron charge, z – charge of a moving particle, E_{max} – maximum energy loss during one collision with an electron.

Stopping power

For the non-relativistic region, the Bloch relation simplifies to the Bethe formula

Due to the considerable rest energy of heavy charged particles, for most practical cases (i.e. for protons up to approx. 10 MeV, for α particles up to approx. 40 MeV) it is sufficient to consider this non-relativistic relationship.

$$S_{col} = -\frac{dT}{dx} = \frac{e_0^4 z^2}{4\pi\varepsilon_0^2 m_{e0} v^2} NZ \ln\frac{2m_{e0}v^2}{I}$$

N – number of atoms in the medium unit volume,
Z – proton number of atoms of matter,
I – mean excitation potential of atoms of matter
NZ – electron density (number of electrons in the medium unit volume)
ρ – density of medium (mass/volume)

$$-\frac{dT}{dx} \approx \frac{Z}{A}\,\rho\,\ln\frac{1}{I}$$

Stopping power is:

- directly proportional to density of a medium

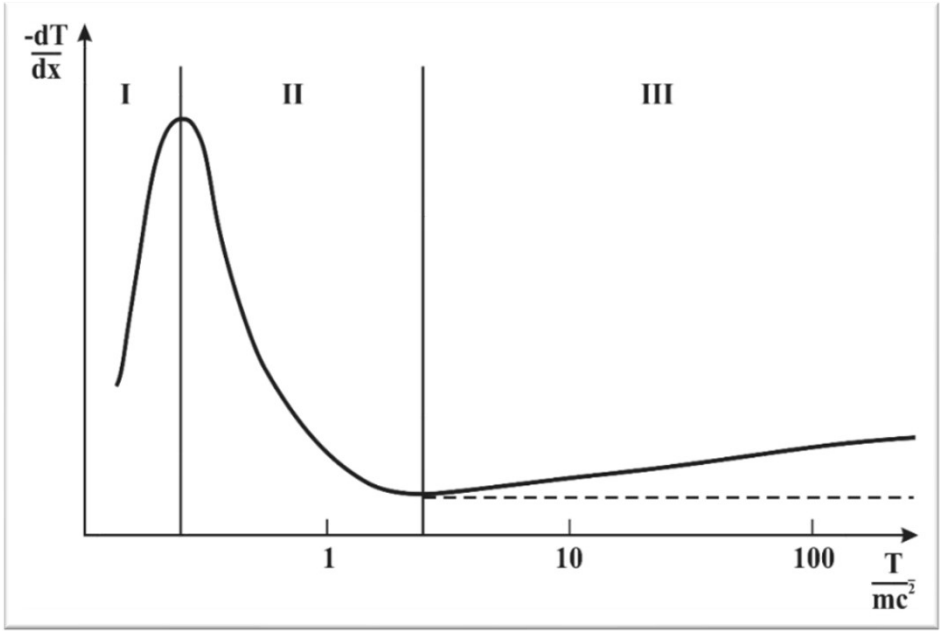
- higher for low Z materials

$$I \approx Z$$

$$\frac{Z}{A}(^1H) = 1 \qquad \frac{Z}{A}(^{12}C, 14N, 16O) \qquad \frac{Z}{A}(Pb) \approx 0.4$$

$$-\frac{1}{}$$

Stopping power



A graphical representation:

I. The region of low energies – the Bethe relation does not apply (the slow particle does not have a full charge on average, it can capture and lose electrons again), elastic collisions between the particle and the atom as a whole also apply.

II. The region in which the $1/v^2$ dependence in the Bethe (or Bloch) relation prevails.

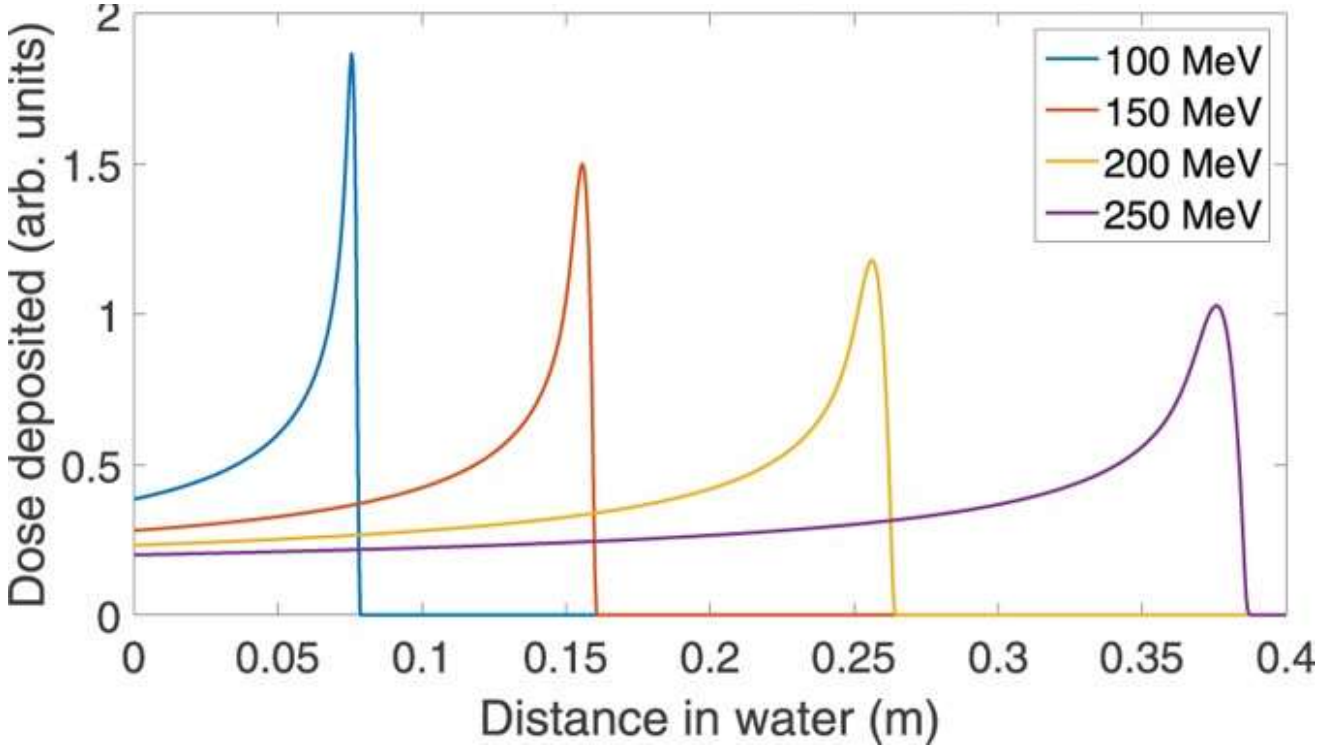
III. The region of higher energies – the velocity of the particle already increases more slowly with energy – the logarithmic term in the Bloch relation begins to dominate.

IV. At ultrarelativistic energies (out of the picture), the velocity of the particle almost does not change with increasing energy – there is a polarization of the environment between distant electrons and ionizing particles (the so-called density effect) – the linear stopping power is practically constant – this is, however, a state that is reached only on large accelerators.

The first approach to this curve – the experimental measurement of linear ionisation (i.e. the density of formed ions) along the path of the α particle – it was carried out by William Henry Bragg – resulting in the so-called Bragg curve.

Bragg curve

Bragg curve for protons of various energies in water – energies in the order of hundreds of MeV are used in proton therapy.



R – range of heavy charged particles

$$R_{CSDA} = \int_0^{E_0} \frac{dE}{S(E)}$$

CSDA = Continuous slowing down approximation

Alpha particle in water

Typical stopping power: 100 keV μm^{-1}

Energy loss of an alpha particle is transferred to electrons in H_2O molecules and cause ionization of molecules

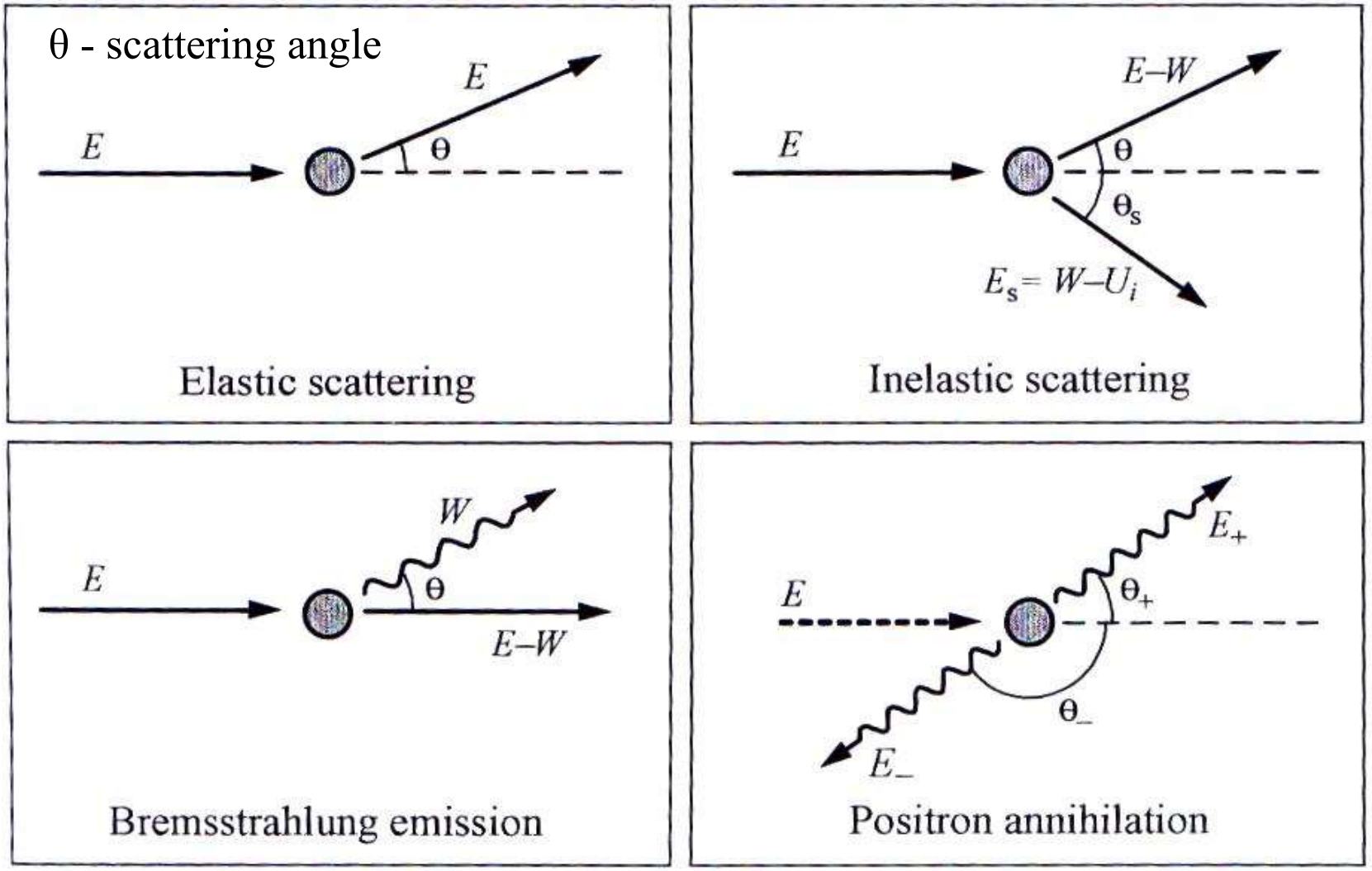
Mean energy required for one ionisation: 25 eV

Number of ionisations: 4 000 per 1 μm

A typical human cell is 25 μm in diameter
- high risk of DNA damage

Interaction of electrons (and positrons)

Interaction of electrons (and positrons)



Interaction of electrons with a matter

The principles of electron interactions with matter are largely analogous to those of heavy charged particles – i.e. ionisation and excitation are again one of the most important processes. But at higher energies, the production of bremsstrahlung radiation is also significant. For positrons, the production of photon radiation is also applied during the annihilation of electron-positron pairs. Various scattering processes are also important.

The relation for the linear stopping power of electrons in matter – derived by Hans Bethe. In the non-relativistic field of energies:

$$-\frac{dT}{dx}=\frac{e_0^4}{4\pi\varepsilon_0^2 m_{e_0} v^2} NZ\left\{\ln\frac{m_{e_0} v^2 T}{2I^2\left(1-\beta^2\right)}-\left(2\sqrt{1-\beta^2}-1+\beta^2\right)\ln 2+1-\beta^2+\frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right\}$$

For the relativistic range of energies (important – electrons must be taken as relativistic particles from about 5 keV):

$$-\frac{dT}{dx}=\frac{e_0^4}{4\pi\varepsilon_0^2 m_{e_0} v^2} NZ\ln\left(\frac{m_{e_0} v^2}{2I}\sqrt{\frac{e}{2}}\right)$$

T – kinetic energy of a moving electron.

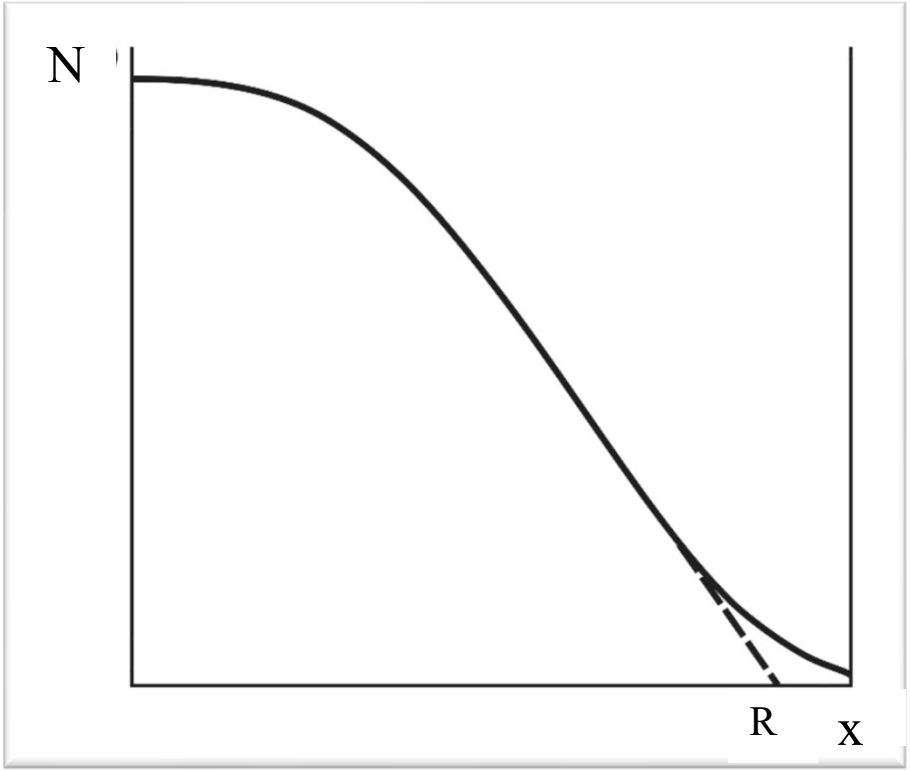
Electron range

Electron range in material – has a large variance due to the fact that the electron undergoes a significant number of collisions with the electrons of the stopping material in its trajectory. The interaction can again be described from the laws of conservation of energy and momentum. But the mass of the electron is small ⇒ the probability of scattering by a large angle is significantly greater than for a heavy charged particle (it can occur in a collision with an electron, moreover, electron needs not to pass so close to the atomic nucleus to be deviated observably from its original direction). In other words – the path of the electron is not straight, but has frequent curves – in addition, during a single collision, the electron can lose significantly more energy (up to half of the original energy). The dependence of the detected number of electrons on the thickness of the irradiated layer therefore looks different than for heavy charged particles. The extrapolated range R_e is much more important here than the mean range – essentially it gives the maximum depth of penetration of a particle into the layer.

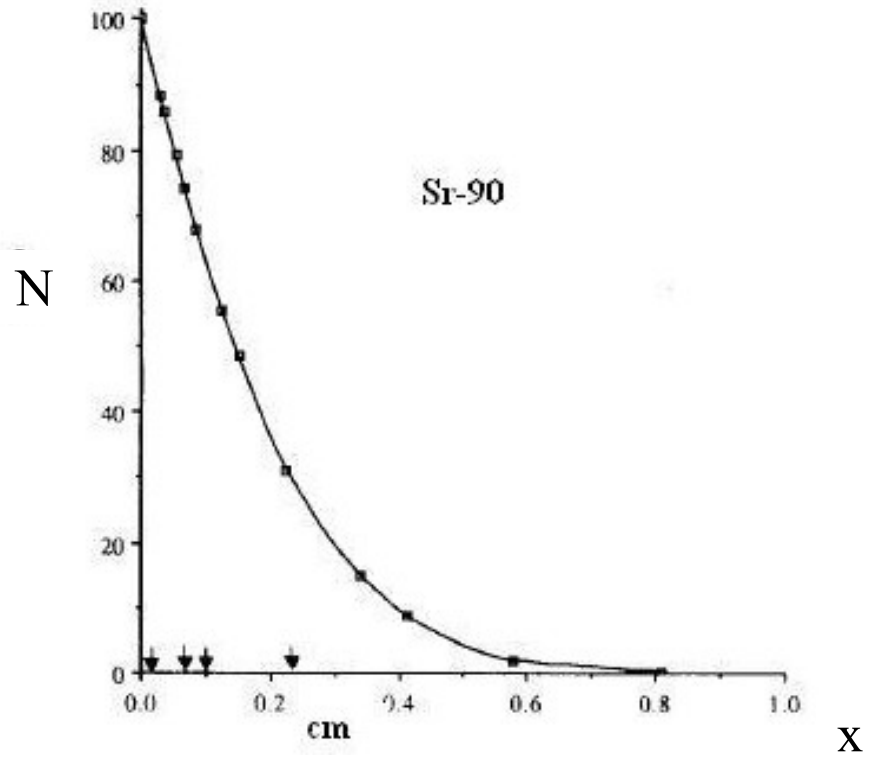
Electron range

Number of electrons (N) versus depth in a material (x)

Monoenergetic electrons



β radiation (in water)



For β radiation with a continuous spectrum, an approximately exponential decay of the $e^{-\mu x}$ type is obtained

Comparison of electrons and heavy charged particles

Comparing ionisation losses for electrons and heavy charged particles: it can be noted that the terms ahead of the logarithm in the Bethe relation for the electron

$$-\frac{dT}{dx}=\frac{e_0^4}{4\pi\varepsilon_0^2 m_{e_0} v^2} NZ\left\{\ln\frac{m_{e_0} v^2 T}{2I^2\left(1-\beta^2\right)}-\left(2\sqrt{1-\beta^2}-1+\beta^2\right)\ln 2+1-\beta^2+\frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right\}$$

and for a heavy charged particle carrying one elementary charge (Z=1)

$$-\frac{dT}{dx}=\frac{e_0^4 Z^2}{4\pi\varepsilon_0^2 m_{e0} v^2} NZ\ln\frac{2m_{e0} v^2}{I}$$

are identical. The differences caused by the calculation of the logarithmic terms are still only about 5% at $1/(1-\beta^2)=10 \Rightarrow$ for low-energy particles (in the non-relativistic region) at the same energy, the ionization loss is inversely proportional to the square of their velocity and therefore directly proportional to the mass.

Consequence: the loss of a low-energy proton per unit path is proportionally to the mass by about 2000 times greater than the loss of an electron of the same energy.

At high energies: velocities approach the speed of light and expressions ahead of the logarithm become approximately constant ⇒ logarithmic terms have a decisive influence ⇒ energy losses of ultrarelativistic particles change little with mass.

However, the relations for ionisation losses do not express well the real losses at higher energies – a serious competing mechanism – bremsstrahlung production.

Production of bremsstrahlung

Electron energy losses by bremsstrahlung

If a charged particle is subject to changes in velocity (e.g. during interaction with the electric field of atomic nuclei or shell electrons) ⇒ energy is emitted in the form of electromagnetic radiation. This means that bremsstrahlung photons are emitted when the velocity of charged particles changes in the electric field of atomic nuclei and shell electrons. These radiative energy losses are proportional to the square of the acceleration $(dv/dt)^2$. The forces of the Coulomb interaction corresponding to the change in particle velocity are proportional only to its charge. Thus, for particles with the same charge:

$$\frac{dv}{dt} \approx \frac{1}{M}$$

M – mass of a moving charged particle. Therefore:

$$\left(-\frac{dT}{dx}\right)_{rad} \approx \frac{1}{M^2}$$

Hence the explanation why radiation losses are important especially for particles with a small mass (electrons and positrons). The derivation of effective cross sections for the excitation of bremsstrahlung by electrons can be divided into two problems:

a) excitation of bremsstrahlung in the field of an atomic nucleus
b) excitation in the field of shell electrons.

The resulting relations are complicated by the fact that the effect of partial shielding of nuclei by shell electrons must be taken into account during the calculation. The general relation was derived by Fritz Sauter.

Production of bremsstrahlung

Non-relativistic relation for $T \ll m_e c^2$:

$$\left(-\frac{dT}{dx}\right)_{rad} = \frac{16}{3}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 NT\frac{Z^2 e_0^2}{137 m_{e0}^2 c^4}$$

In the range of relativistic energies, when it is still possible to neglect the shielding of the nucleus by shell electrons, i.e. ($m_e c^2 \ll T \ll 137\, m_e c^2 Z^{-1/3}$):

$$\left(-\frac{dT}{dx}\right)_{rad} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{Z^2 e_0^4}{137 m_{e0} c^4} NT\left(4\ln\frac{27}{m_{e0}c^2} - \frac{4}{3}\right)$$

In the region of extremely high energies, when full shielding of the nucleus by shell electrons can be considered, i.e. ($T \gg 137\, m_e c^2 Z^{-1/3}$):

$$\left(-\frac{dT}{dx}\right)_{rad} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{Z^2 e_0^4}{137 m_{e0}^2 c^4} NT\left(4\ln\frac{183}{Z^{1/3}} + \frac{2}{9}\right)$$

This means that, to a first approximation, the radiative losses of an electron are directly proportional to the square of the proton number of the substance, the concentration of N atoms, and the kinetic energy T of the electrons:

$$\left(-\frac{dT}{dx}\right)_{rad} \approx Z^2 NT$$

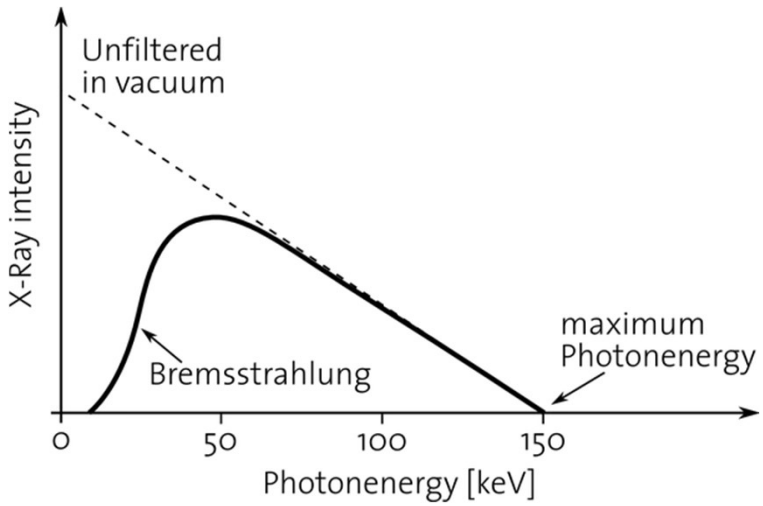
Mass and charge are in the ratio $e^2/m^2 \Rightarrow$ a particle with a mass equal to M times the mass of an electron and a charge equal to Z times the charge of an electron will lose M^2/Z^2 times less energy than an electron at the same energy in the same substance.

Production of bremsstrahlung

The angular distribution of bremsstrahlung photons depends on the energy of the moving electrons – with increasing energy, the forward direction of its emission is preferred.

Energy spectrum – continuous and does not depend on the direction of emission from the path.

At high electron energies – formation of electron-positron pairs. The effective cross-section of the process is small, however, when electrons from cosmic rays or accelerators interact, the effect contributes to the formation of electron-photon showers.



For an electron velocity in a substance greater than the velocity of light in the same substance – Cherenkov radiation excitation. Radiation is in the visible range, energy losses are small.

The scattering processes – the most important is the elastic scattering of electrons on the atoms of the substance – they change their direction of flight, but practically do not lose energy.

A consequence of this variety of interaction processes – ionisation losses can be considered as main proces only in the region of very low energies – otherwise the range determined on the basis of ionisation losses is much larger than the actual range.

For positrons – the same interaction processes as for electrons. In addition, there is a certain (small) probability of their annihilation during a collision with an electron in flight and the certainty that they will annihilate after slowing down $\Rightarrow$ the creation of usually two, exceptionally more photons.

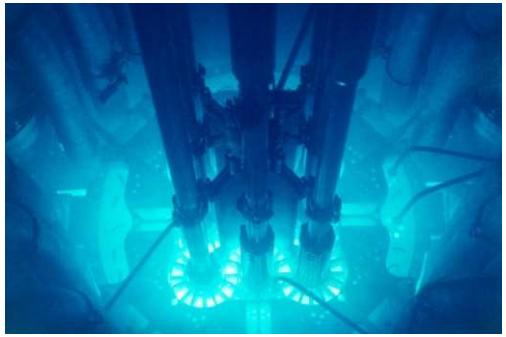
Production of Cherenkov Radiation

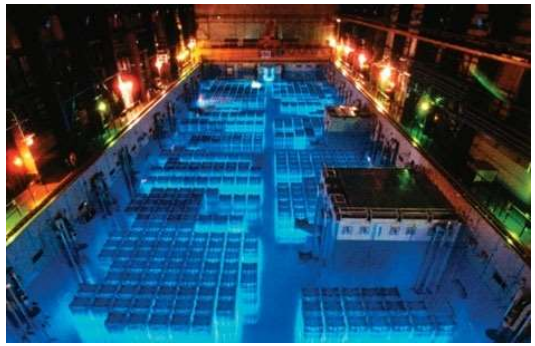
Cherenkov Radiation: 1934 Pavel A. Cherenkov studied the luminescence of uranium salts under the action of radiation γ of radium – discovered a new radiation that cannot be explained by the excitation mechanism of fluorescence radiation.

Theoretical clarification 1937 Igor J. Tamm, Ilja M. Frank based on classical electrodynamics: this is the radiation loss of a charged particle when moving through a medium with a refractive index $n > 1$. If a particle moving through a given medium at a velocity greater than the velocity of light in this medium is a source of electromagnetic radiation emitted at a precisely defined angle. It can be explained as the coherent radiation of dipoles formed by the electric field of a moving particle. If a particle moves faster than the velocity of light in a given medium, the polarization is delayed behind the moving particle, the dipoles are mainly oriented in the direction of the particle's movement $\Rightarrow$ the waves emitted by individual dipoles reach the same phase in a certain direction, the emission of electromagnetic radiation is observed.

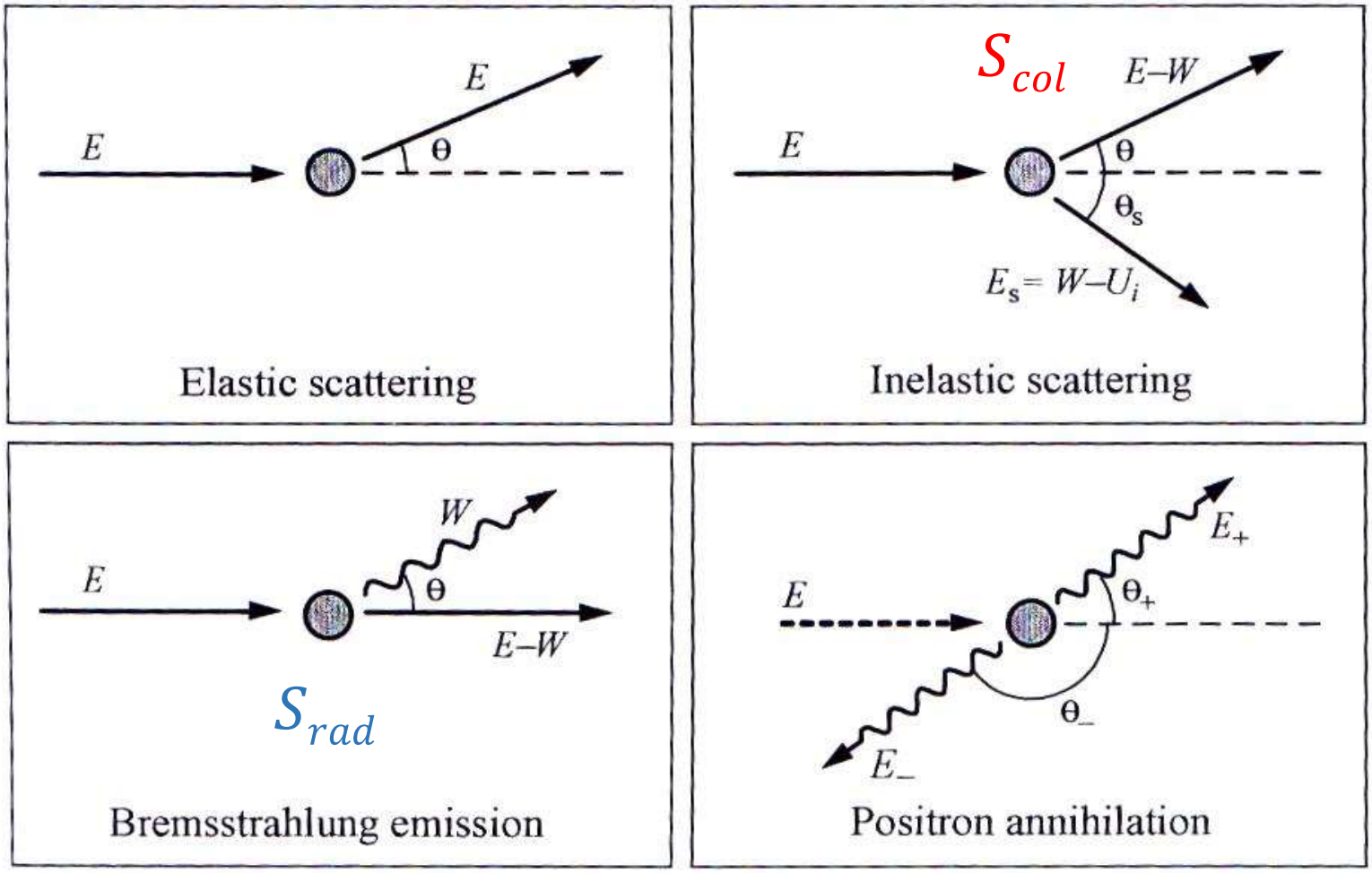
These considerations apply to arbitrary particles, Cherenkov radiation can be produced by any charged particles at the corresponding velocities (and thus energies).

Use – Cherenkov detectors – e.g. for threshold discrimination between lighter and heavier particles.



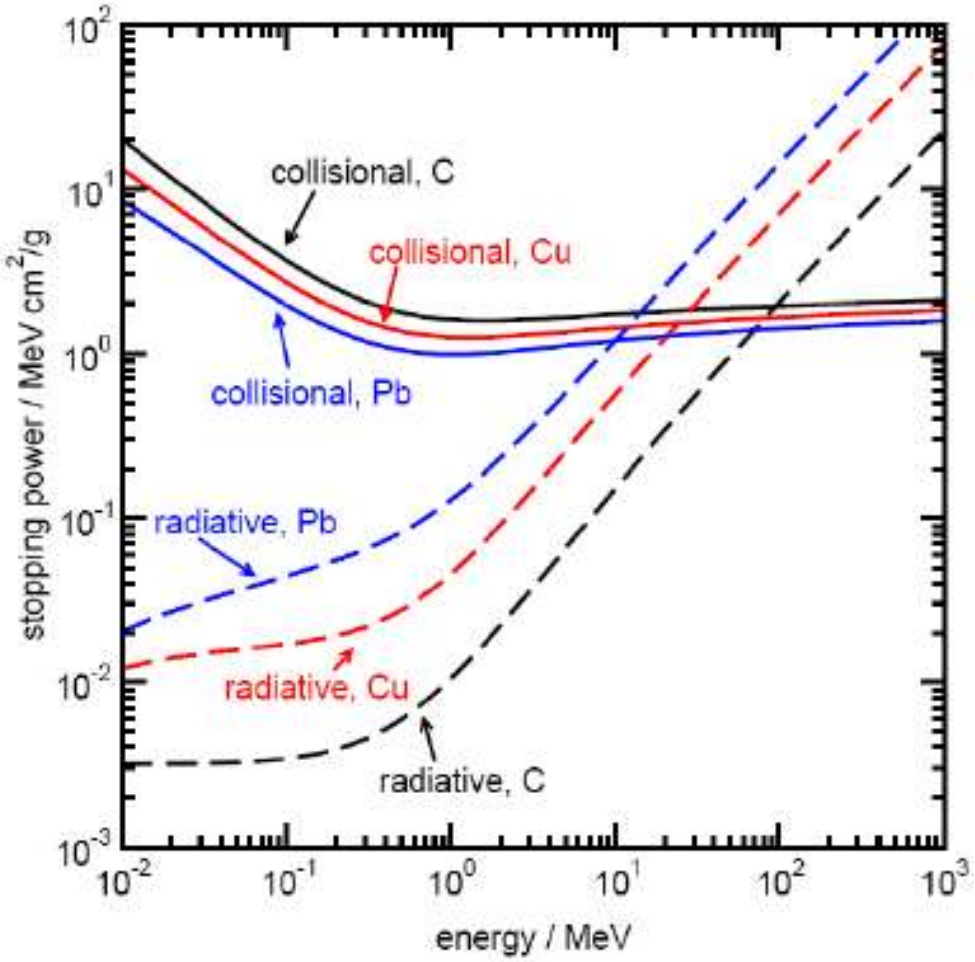


Interaction of electrons (and positrons)



$$S = -\frac{dT}{dx} = S_{col} + S_{rad}$$

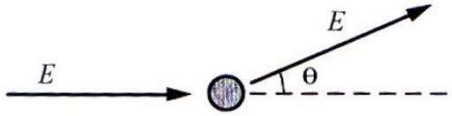
Stopping power of electrons: collisional and radiative



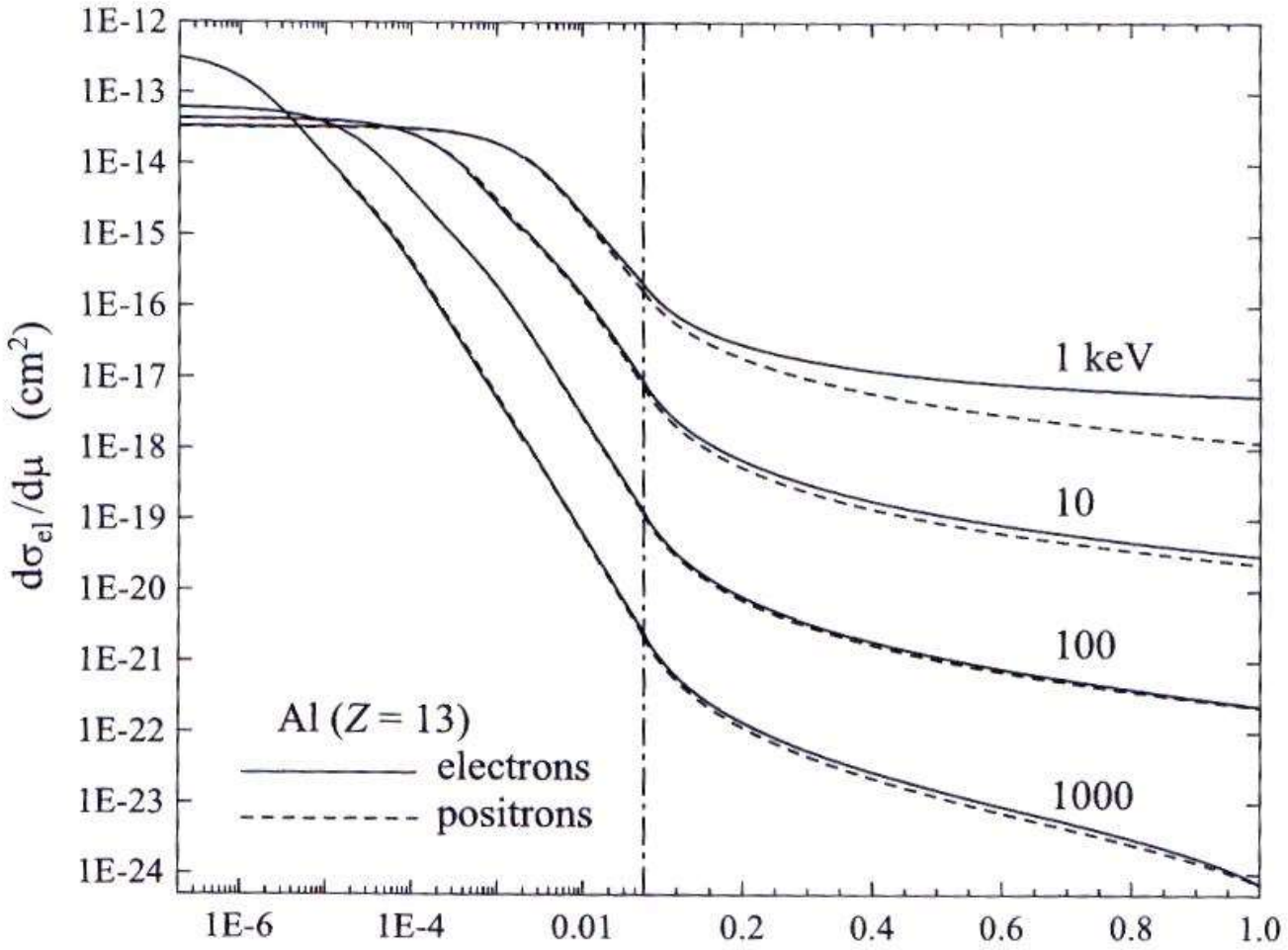
$$S = -\frac{dT}{dx} = S_{col} + S_{rad}$$

$$\frac{S_{rad}}{S_{col}} \approx \frac{T \cdot Z}{1600\ m_e c^2}$$

Electron scattering cross section



Electron and positrons in alluminum



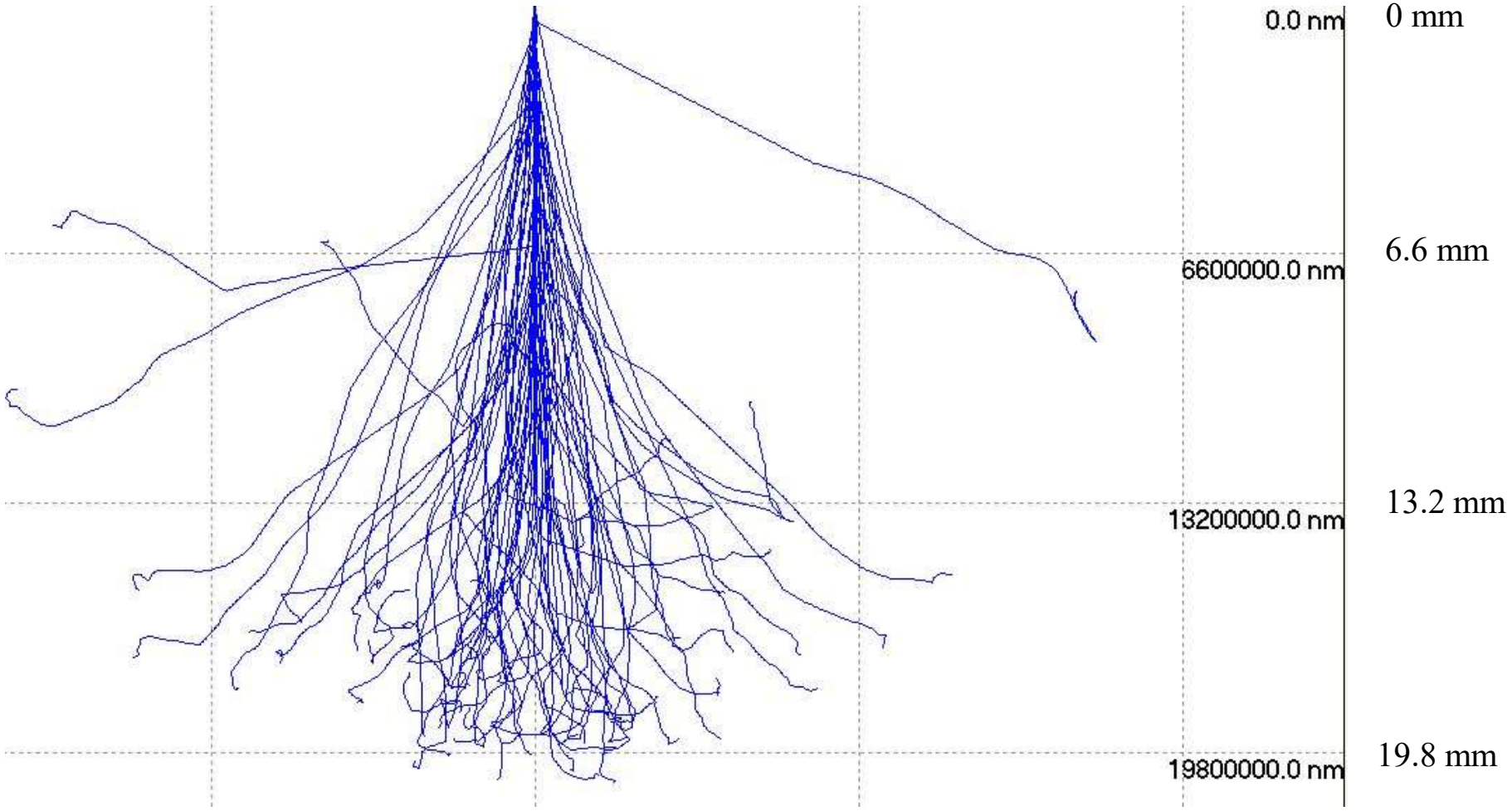
forward scattering

back scattering

$$\mu=(1 - \cos\theta)/2$$

Electron beam in alluminum

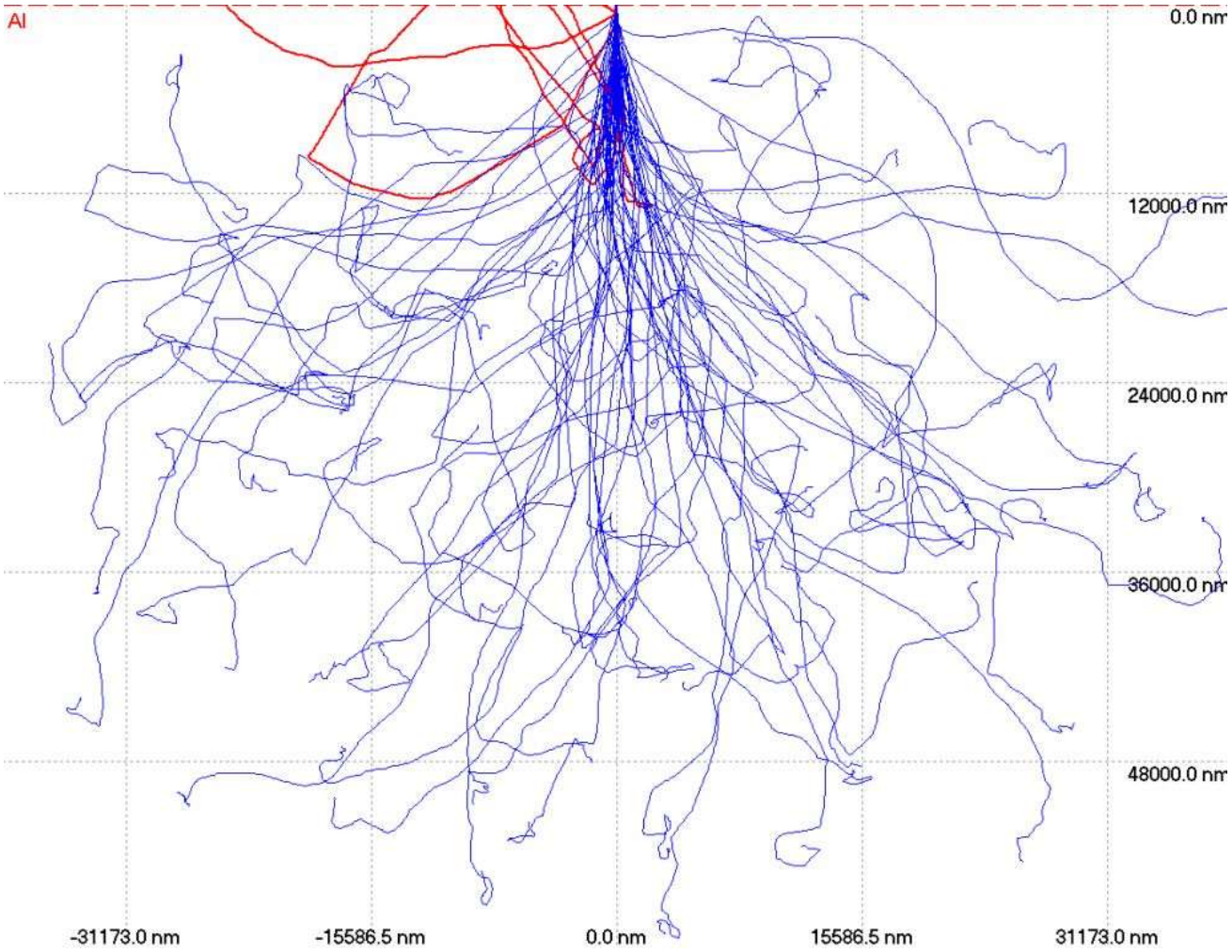
10 MeV electrons in alluminum



Monte Carlo simulation - CASINO code

Electron beam in alluminum

100 keV electrons in alluminum



0 μm

12 μm

24 μm

36 μm

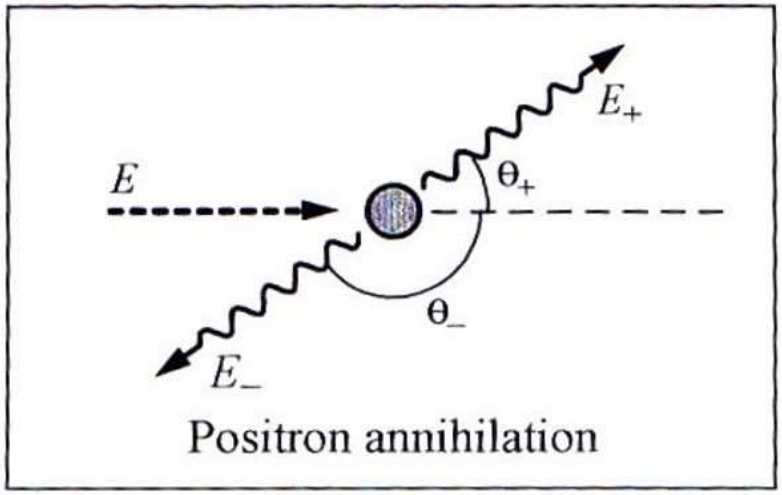
48 μm

Monte Carlo simulation - CASINO code

Electron - positron anihilation

Electron–positron annihilation occurs when an electron (e^-) and a positron (e^+, the electron's antiparticle) collide. At low energies, the result of the collision is the annihilation of the electron and positron, and the creation of energetic photons:

$$e^- + e^+ \rightarrow \gamma + \gamma$$



Positron annihilation

The most probable is the creation of two photons. In the most common case, two gamma photons are created, each with energy equal to the rest energy of the electron or positron (0.511 MeV)

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Interaction of photons (X-rays and gamma radiation)

Interaction of photons

The wave character of electromagnetic radiation in the X and γ energy range is only slightly manifested during interaction with matter – as a rule, the interaction can be viewed as the collision of a particle (photon) with atoms, their nuclei or electrons.

There are many possible ways of interaction:

Interaction type ⇒ / Interaction with ⇓	Absorption	Coherent scattering	Incoherent scattering	Multiparticle effects
electron shells in atoms	**photoelectric effect**	Thomson and Rayleigh scattering	**Compton effect**	two-photon Compton effect
atomic nuclei	photonuclear reactions	elastic nuclear scattering	nuclear resonant scattering	
elmag. field around charged particles	**electron-positron pair production**	Delbrück scattering		
field of nuclear forces in atomic nuclei	photoproduction of mesons			

The dominant interactions for certain material and photon energy range

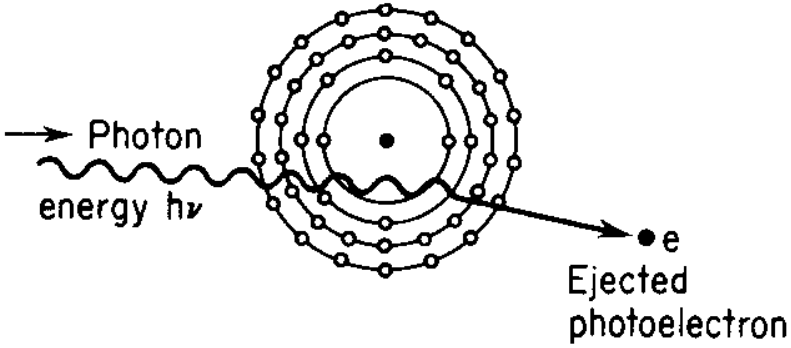
Other important interactions

Photoelectric effect (photoeffect)

A photon is absorbed in the electron shell of an atom and transfers all its energy to an electron at some energy level of an atom of the absorbing medium – an electron is emitted from the atom with a kinetic energy T given by Einstein's equation

$$T = h\nu - E_v$$

$h\nu$ – the energy of the incident photon, E_v the binding energy of the electron in the corresponding energy level.



Photon energy: $h\nu$

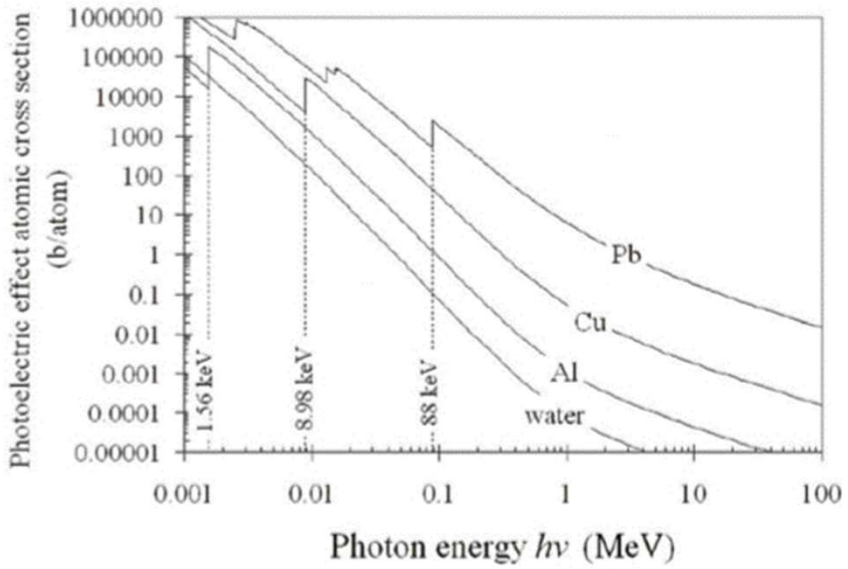
h – Planck constant ($6.62607015 \times 10^{-34}$ J s)

ν – frequency

On a free electron, the laws of conservation of energy and linear momentum cannot be fulfilled at the same time – with a bound electron, part of the momentum is transferred to the atom on which the process took place (the energy of this process is negligible, because the atom has a huge mass) ⇒ **the photoeffect takes place only on bound electrons**. At the same time, it follows from Einstein's relation that the photoeffect is not possible on the given shell if $h\nu < E_v$. If $h\nu > E_v$ for the K shell, about 80% of the photoeffect occurs on the K shell. The ratio of the partial effective cross sections on the individual shells is then approx.

$$\tau_K : \tau_L : \tau_M = 20 : 4 : 1$$

Cross section of photoeffect



If we proceed from higher energies to lower ones – always at the absorption edge of the respective shell there will be a sharp drop in the effective cross-section, because the process on the given shell ceases to be applied. Obviously, the photoeffect applies more significantly, the greater the binding energy of the electron is. Related to this, it is steep dependence on Z – the cross section increases rapidly with increasing Z (roughly ~ Z^5).

The relation for the cross section of the photoeffect related to one atom (for $h\nu >> E_v$, but T still in the non-relativistic region):

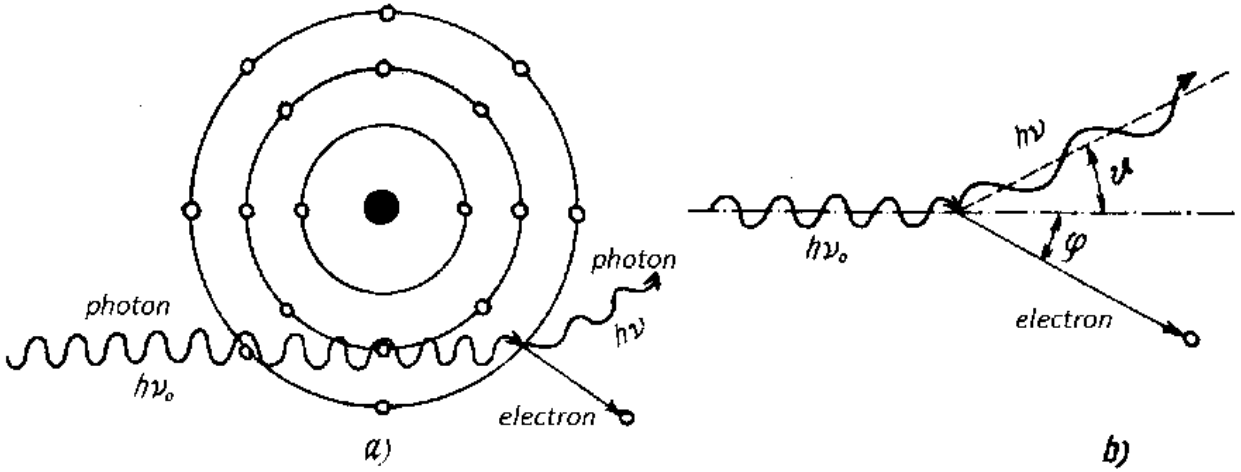
$$\tau_K = \sigma_0 h\sqrt{2}\,\frac{Z^5}{137^4}\left(\frac{m_{e0}c^2}{h\nu}\right)^{\frac{7}{2}}$$

where

$$\sigma_0 = \frac{1}{6\pi}\left(\frac{e_0^{\,2}}{\varepsilon_0 m_{e0}c^2}\right)^2 = 6{,}651.10^{-29}\ \mathrm{m}^2$$

is the scattering cross section of low energy photons on a free electron at rest (Thomson scattering).

Compton effect



Incoherent scattering of a photon on a free electron (or so weakly bound that the effect of the binding can be neglected). I.e. the photon transfers part of its energy to the electron, and the scattered photon continues with a wavelength greater than that of the primary photon.

Compton described the interaction as an elastic collision of two free particles – from the law of conservation of energy and linear momentum:

$$h\nu = h\nu' + m_{e0}c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\vartheta + \frac{m_{e0}\beta c}{\sqrt{1-\beta^2}}\cos\varphi$$

$$0 = \frac{h\nu'}{c}\sin\vartheta + \frac{m_{e0}\beta c}{\sqrt{1-\beta^2}}\sin\varphi$$

ν – frequency of incident photon, ν‘ – frequency of scattered photon at angle ϑ, β = v/c, v – velocity of recoiled electron at angle φ.

Compton effect

From the laws of conservation of energy and momentum, the relations between the energy of the incident photon and the scattered photon or of the reflected electron depending on the angle of scattering and the angle of reflection of the electron can be derived:

For the scattered photon energy:

$$h\nu' = \frac{h\nu}{1 + (h\nu/m_{e0}c^2)(1 - \cos\vartheta)}$$

With increasing scattering angle, the energy of the scattered photon decreases from hν for $\vartheta = 0$ to $h\nu/1 + 2(h\nu/m_{e0}c^2)$ for $\vartheta = \pi$. This means that the energy of the reflected electron lies between 0 and $h\nu - [h\nu/1 + 2(h\nu/m_{e0}c^2)]$.

For the kinetic energy of the recoiled electron depending on the angle of recoil φ:

$$T = h\nu - h\nu' = m_{e0}c^2 \frac{2(h\nu)^2\cos^2\varphi}{(h\nu + m_{e0}c^2)^2 - (h\nu)^2\cos^2\varphi}$$

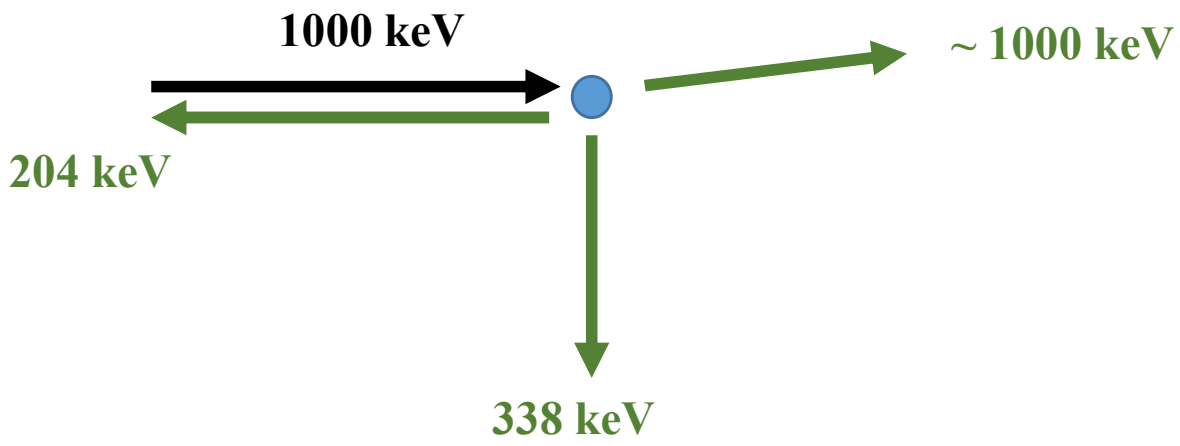
Compton effect – gama radiation

scattered photon energy:

$$h\nu' = \frac{h\nu}{1 + (h\nu / m_{e0}c^2)(1 - \cos\vartheta)}$$

Energy of scattered photon depends of scattering angle.

Example for incident photon energy: 1000 keV (1 MeV)



$m_{e0}c^2 = 511$ keV

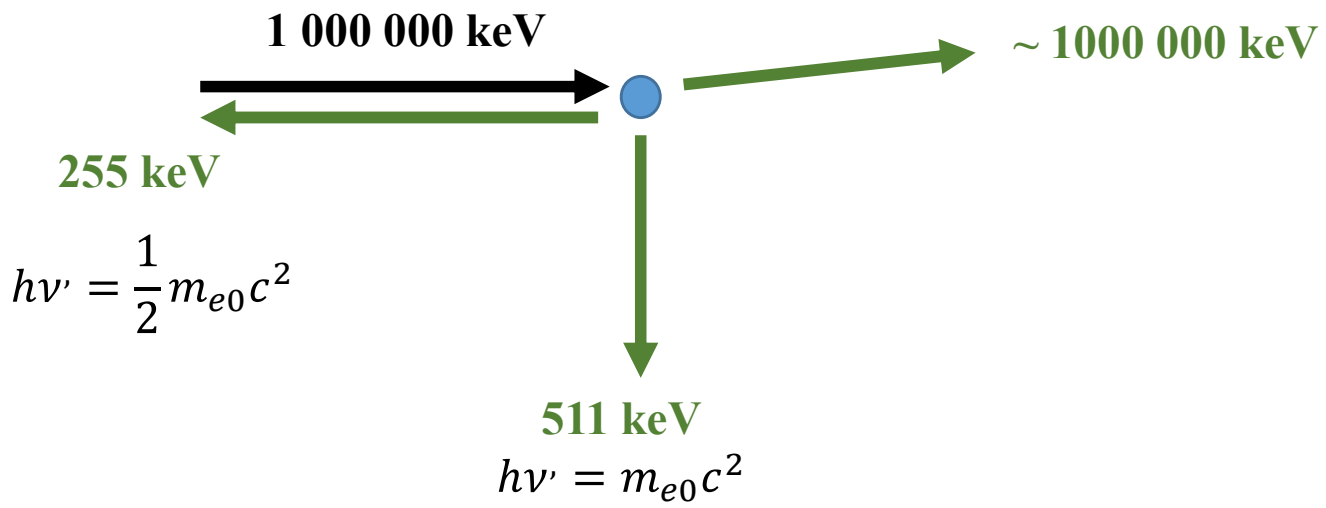
Compton effect – high energy photon

scattered photon energy:

$$h\nu' = \frac{h\nu}{1 + (h\nu / m_{e0}c^2)(1 - \cos\vartheta)}$$

Energy of scattered photon depends of scattering angle.

Example for incident photon energy: 1 000 000 keV = 1 GeV $\quad h\nu \gg m_{e0}c^2$



$m_{e0}c^2 = 511$ keV

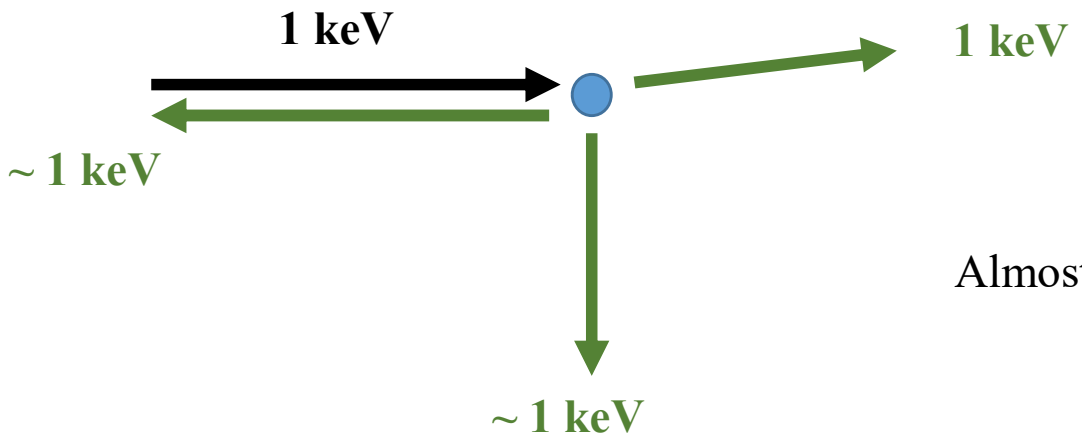
Compton effect – low energy photon

scattered photon energy:

$$h\nu' = \frac{h\nu}{1 + (h\nu/m_{e0}c^2)(1 - \cos\vartheta)}$$

Example for incident photon energy: 1 keV

$h\nu \ll m_{e0}c^2$

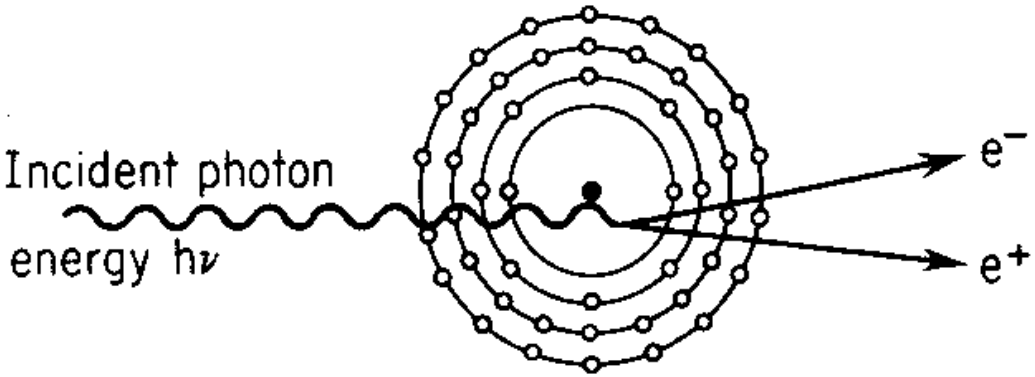


Almost coherent scattering

$m_{e0}c^2 = 511$ keV

Production of a particle-antiparticle pair

The energy of the photon must be higher than the rest energy of the resulting pair ⇒ outside the field of high-energy particle physics, only the production of an electron-positron pair is of practical importance ⇒ the threshold must correspond to the rest energy of these particles ($2m_0c^2$ = 1.022 MeV). Other pairs of particles-antiparticles can be formed only in the range of hundreds of MeV or more.



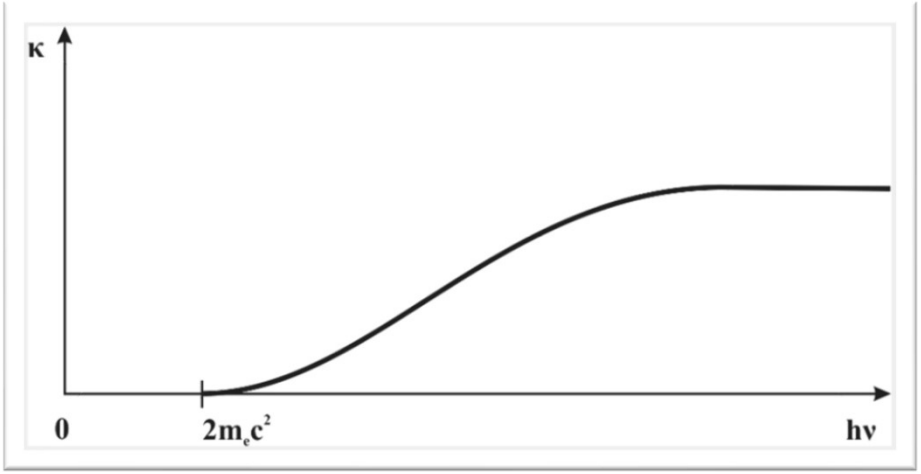
The process cannot occur without the presence of a third particle (most often the atomic nucleus), which must take over part of the linear momentum (the sum of the momentum of the electron-positron pair < the momentum of the primary photon) ⇒ this particle takes over part of the energy at the same time – however, this is in the case of the formation of pairs in the electromagnetic field of the nucleus negligible (strictly speaking, it leads to the fact that the threshold of the process is slightly higher than $2m_0c^2$ by the energy of the recoiled nucleus). The calculation is possible from the laws of conservation of energy and momentum, but essentially the energy balance of the process in the field of a nucleus can be written by the equation:

$$h\nu = 2m_{e0}c^2 + T_{e^-} + T_{e^+}$$

T_{e-}, T_{e+} – kinetic energies of the resulting electron and positron. The energy imparted to the third particle is significant for pair production in the field of an electron, but this is a much less frequent process.

Production of electron-positron pair

Pair production theory: H.A. Bethe and W.H. Heitler in connection with the theory of the formation of bremsstrahlung - the positron considered as an unoccupied state in an infinite number of negative energy states attributed to electrons. The resulting relation for the effective cross-section is complicated and depends on the degree of shielding of the nuclei by the shell electrons.



For $m_{e0}c^2 << h\nu_0 << 137 m_{e0}c^2 Z^{-1/3}$ (shielding can be neglected): Cross-section $\kappa \sim Z^2$

$$\kappa = \frac{Z^2}{137} r_e^2 \left[\frac{28}{9} \ln\left(\frac{2h\nu}{m_{e0}c^2}\right) - \frac{218}{27}\right] \quad \text{(A)}$$

For $h\nu_0 >> 137\, m_{e0}c^2 Z^{-1/3}$ (shielding of the nucleus can be considered complete):

$$\kappa = \frac{Z^2}{137} r_e^2 \left[\frac{28}{9} \ln\left(183 Z^{-1/3}\right) - \frac{2}{27}\right] \quad \text{(B)}$$

r_e is the classical electron radius. Due to the dependence of the effective cross-section κ on Z^2 – the probability of pair formation is higher for heavy elements. With increasing energy hν – the effective cross-section first grows proportionally to the logarithmic term in equation (A), the dependence smoothly changes to the form (B) ⇒ at high energies hν, κ does not change with hν anymore.

Photonuclear reactions (nuclear photoeffect)

The most common reactions under the action of photons are (γ, n), (γ, p), (γ, α). The necessary condition for the reaction to take place is: $h\nu > \varepsilon_p$, or $h\nu > \varepsilon_n$, $h\nu > \varepsilon_\alpha$, where ε_x is the binding energy of the given particle to the target nucleus.

They were first observed in 1934:

$$\gamma + {}^2H \rightarrow p + n$$

the bombarding photons came from the decay of ^{208}Tl ($h\nu$ = 2.62 MeV). Experimentally, protons of E_p=0.2 MeV were observed and since the mass of a proton is approximately equal to the mass of a neutron, E_n is also approximately 0.2 MeV and thus the binding energy of the deuteron can be determined.

$$\Delta W({}^2H) = 2.62 - 2 \cdot 0.2 = 2.22\ MeV$$

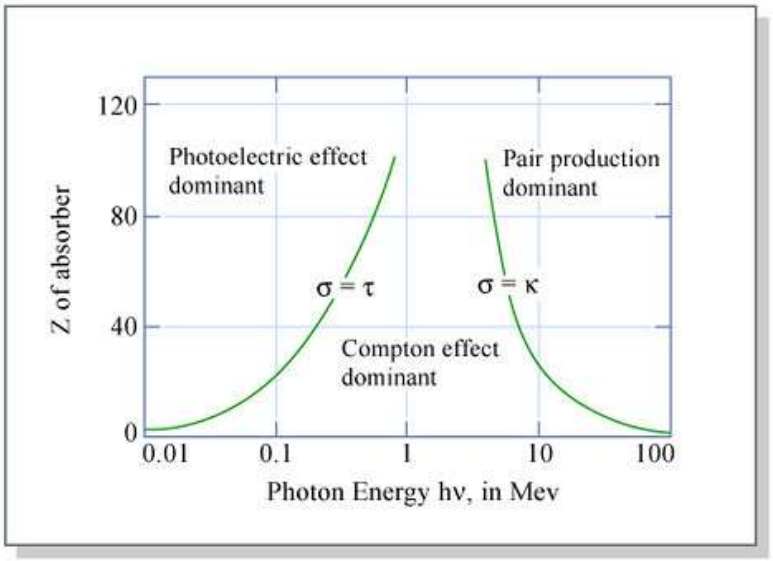
Another such reaction feasible with photons from natural radionuclides is

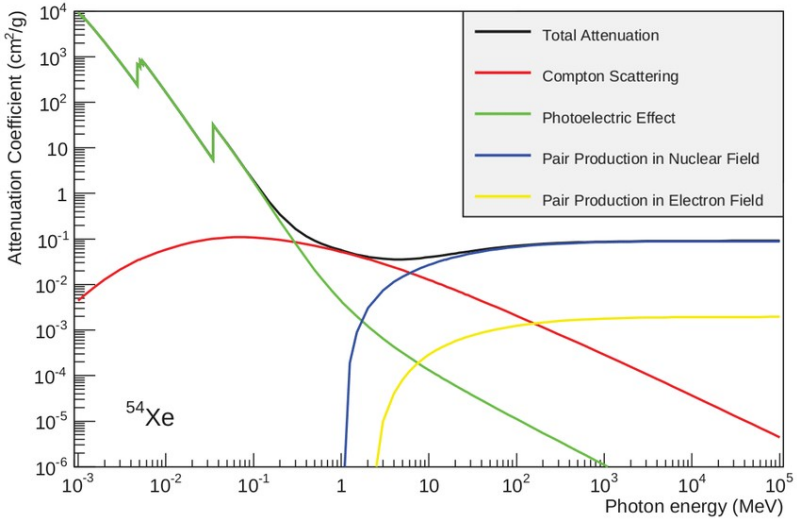
$$\gamma + {}^9Be \rightarrow {}^8Be + n \qquad {}^8Be \rightarrow 2\ {}^4He$$

which is also possible with ^{214}Po photons ($h\nu$ = 1.78 MeV). For all other nuclei near the β-stability line, the binding energy of the nucleon is so high that it cannot be separated by the photons emitted by the radioactive nuclei. Therefore, the research was carried out by bombardment with photons created in the reactions of protons with nuclei or bremsstrahlung from the targets of electron accelerators.

The effective cross-sections of photonuclear processes are relatively small – even in the maximum of their dependence on energy, they are of the order of 10^{-27} – 10^{-29} m^2.

Passage of a beam of photons through matter





The three most important processes are mainly applied, the photoeffect, the Compton effect and the production of pairs – depending on hν and Z, one or the other is dominant. Fig.: on the curves, the effective cross-sections of the respective processes are equal – at low energies, the photoeffect prevails, at medium energies, the Compton effect, at high energies, pair production.

The total cross-section is the sum of the cross-sections of individual processes. Per unit photon path in matter: the linear (or mass) attenuation coefficient (actually the total macroscopic effective photon interaction cross-section):

$$\mu = \frac{\rho N_A}{A_r}(\tau + \sigma + \kappa + \ldots)$$

ρ – substance density

A_r – relative atomic mass,

N_A – Avogadro's number..

Passage of a beam of photons through matter

Idealised assumption: every photon that passes through the interaction is removed from the beam (the so-called narrow beam) – in practice this assumption can be approached by very narrow collimation of the beam (Fig. a). Then the photon flux density reduction in the beam will be given as

$$dJ = -\mu J dx$$

for an elementary layer of thickness dx at the location in which the photon flux density is J. For a layer of finite thickness x is obtained

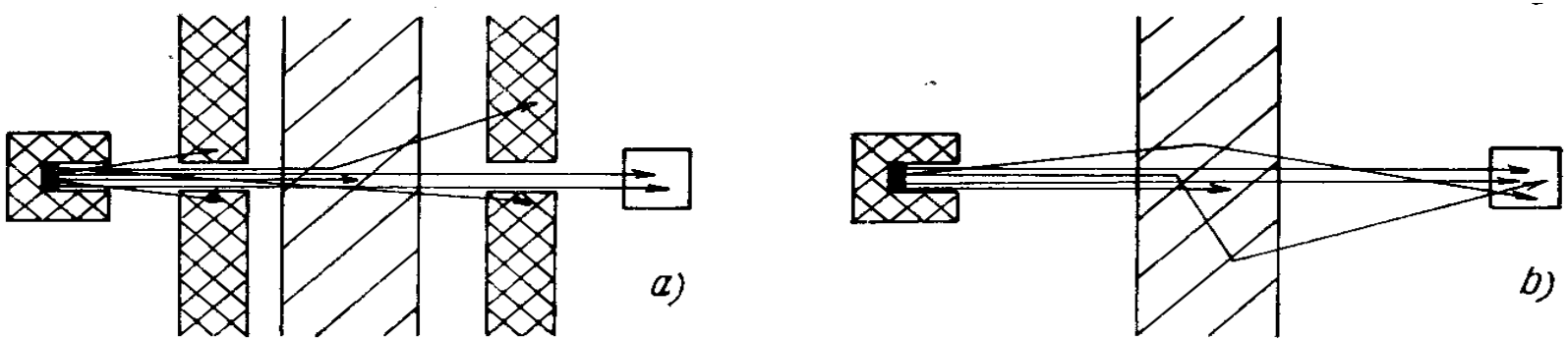
$$J = J_0 e^{-\mu x}$$

by integrating from 0 to x. J_0 – the original photon flux density in the unattenuated beam.

In the more general case – inhomogeneous irradiated layer and non-monoenergetic photons – the relation is modified to

$$J = \int_0^{h\nu_{max}} J_0\,(h\nu) e^{-\int_0^x \mu(x,h\nu)} d(h\nu)$$

In real beams, there may also be scattered photons or annihilation photons from the production of pairs at the point of detection – a wide beam of radiation (Fig. b).



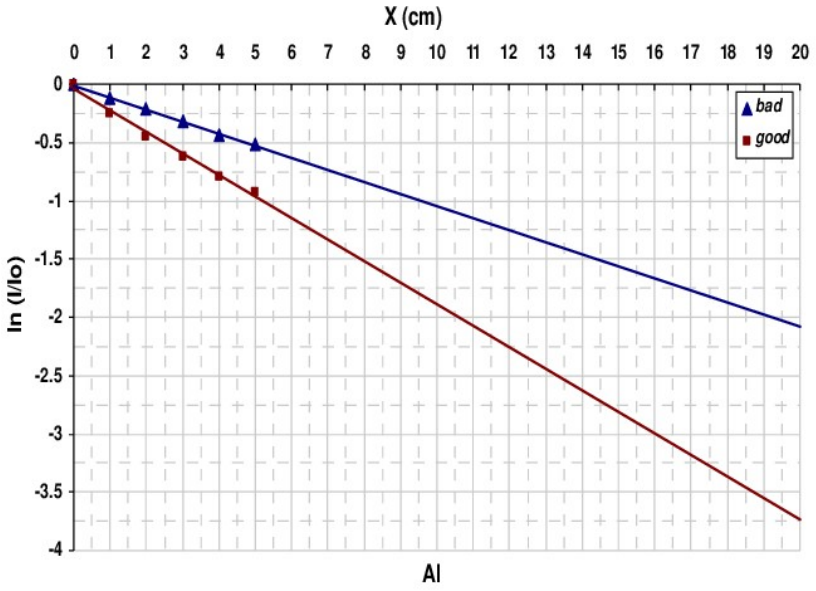
Passage of a beam of photons through matter

Correction of exponential dependence: a more or less empirical quantity, the so-called build-up factor, is introduced:

$$B = \frac{\int_0^{h\nu_{max}} f(h\nu)J(h\nu)d(h\nu)}{\int_0^{h\nu_{max}} f(h\nu)J_n(h\nu)d(h\nu)}$$

J(hν) – the total flux of photons with energy between hν and hν + dhν at the detector, J_n(hν) – the flux of unscattered photons in the same energy interval, f(hν) – the function converting the photon flux into a quantity, which is the subject of interest (dose, exposure, energy flux, etc.). The build-up factor B is therefore equal to the ratio of the investigated effect caused by all photons to the effect caused only by unscattered photons

$$J = J_0 B e^{-\mu x}$$



B depends on the energy of primary photons, the geometry of the radiation beam, the material and the thickness of the attenuating layer, the type of the observed response – the calculation was made only for a few idealised geometric arrangements from the transport theory or the Monte Carlo method – the results are tabulated or described by an empirical formula.

The attenuation curves will then have the character shown in Fig. (quantified for ^{60}Co and aluminum).

Mean free path of photons in material

Material	Photon energy	Linear attenuation coefficient [cm^{-1}]	mean free path [cm]
Lead (Pb)	1 MeV	0.806	1.24
Water (H_20)	1 MeV	0.071	14.1
Lead (Pb)	0.1 MeV	63	0.016
Water (H_20)	0.1 MeV	0.171	5.85

μ – linear attenuation coefficient

mean free path λ is the average distance over which a moving particle travels before substantially changing its direction or energy

$$\lambda = \frac{1}{\mu}$$

Interaction of neutrons

Sources of neutrons

1) radionuclide sources – nuclear reactions (α,n), (γ,n), spontaneous fission

(α,n)

^{226}Ra and ^{210}Po were historically used as an α source, today almost exclusively ^{241}Am, ^{238}Pu, ^{239}Pu. Target material: most often ^{9}Be

(γ,n)

^{2}H (γ,n) ^{1}H Q = -2.226 MeV

^{9}Be (γ,n) ^{8}Be Q = -1.666 MeV

spontaneous fission

^{252}Cf - radioactive conversion α: 97%, spontaneous fission: 3%

2) neutron generators – nuclear reactions (d,n), (p,n), (α,n)

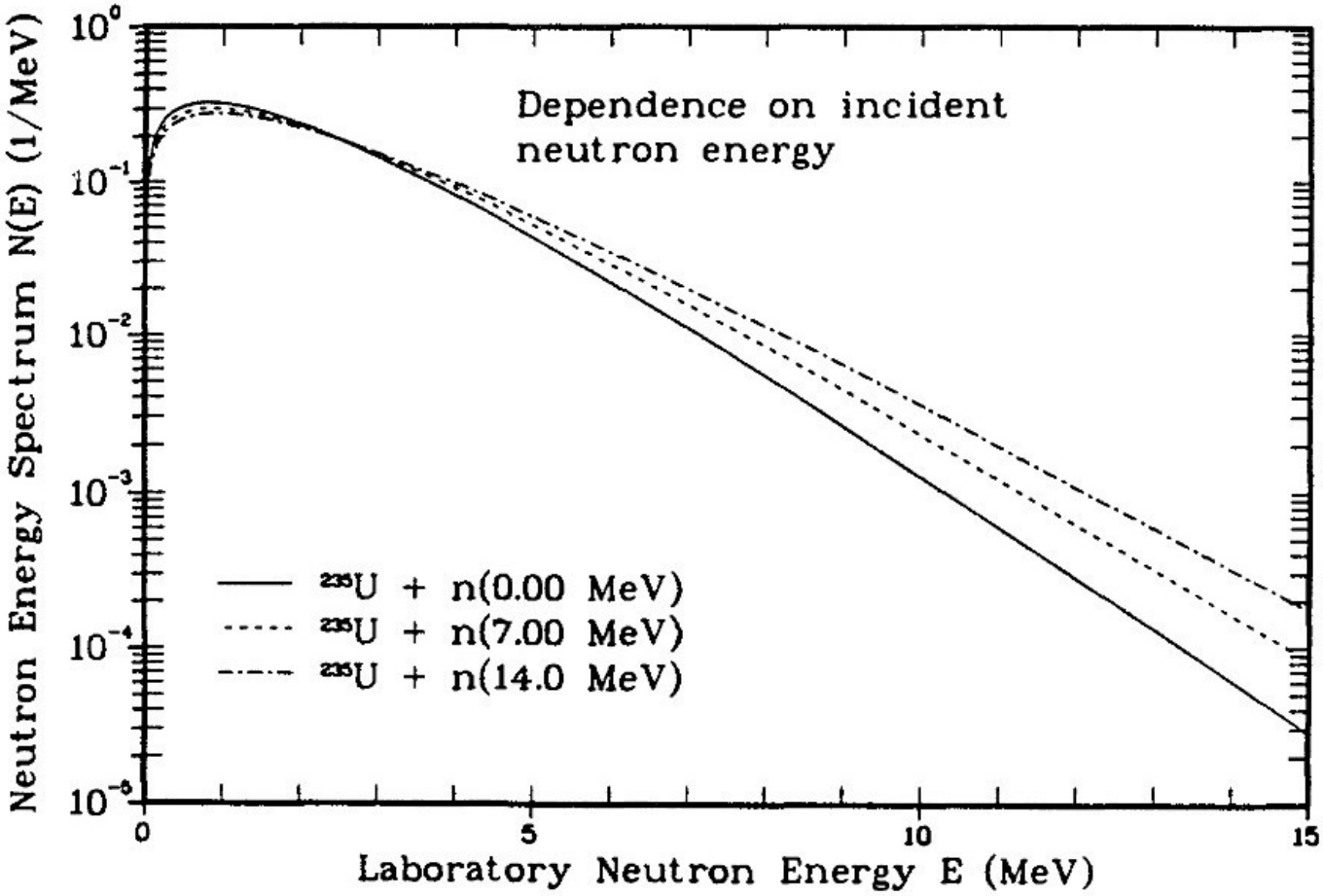
projectiles: d, p, exceptionally also α

targets: most often ^{2}H, ^{3}H, ^{7}Li, ^{9}Be

3) fragmentation sources – fragmentation of nuclei by high-energy protons

4) nuclear reactors – forced nuclear fission

Fission of uranium - Spectrum of neutrons



David G. Madland

Dependence of the prompt fission neutron spectrum on the kinetic energy of the incident neutron for the fission of ^{235}U

Interaction of neutrons

The character of the interaction is diametrically different from the interaction of charged particles and photons:

The electromagnetic interaction of the neutron as an electrically neutral particle is determined only by the existence of its magnetic moment – it is very weak and can be neglected in practice.

The interaction of neutrons with matter is almost exclusively the result of the action of nuclear forces.

Neutrons are not affected by the electron shell of an atom – they only collide with atomic nuclei. Similar to photons, they belong to indirectly ionizing particles, i.e. ionization and excitation are caused by secondary charged particles, but the interaction mechanisms are significantly different from photons.

Basic interaction processes:

- elastic scattering (n, n),
- inelastic scattering (n, n'γ),
- nuclear reactions with the emission of a charged particle or more neutrons (n, p), (n, α), (n, pn), (n, 2n) and others,
- radiation capture (n, γ),
- nuclear fission (n, f).

The significance of individual interaction processes depends on the neutron energy and on the elemental and isotopic composition of the material in which the interaction occurs.

Elastic scattering of neutrons on atomic nuclei

The most important interaction process for fast neutrons leads to their slowing down (moderation), most often down to thermal energy.

The mass of the neutron – comparable to the mass of the nucleus ⇒ the nucleus can receive quite a lot of energy in the collision – the lighter the nucleus, the more energy is transferred on average – extreme: in a frontal collision with a hydrogen nucleus, all the kinetic energy is transferred.

Neutron rest energy $m_n c^2 = 939.55$ MeV ⇒ except for the region of high energies (tens of MeV and more), the processes can be solved non-relativistically.

Solving the kinematics of the scattering process – laws of conservation of linear momentum and energy. In the laboratory coordinate system – part of the energy of neutron is transferred to the recoiled nucleus – the neutron changes its direction of movement and energy. In the center of mass coordinate system – the direction of trajectory of the neutron and the nucleus will change – not only the sum of the kinetic energies but also the individual kinetic energies will be preserved. The law of conservation of momentum (in the laboratory coordinate system):

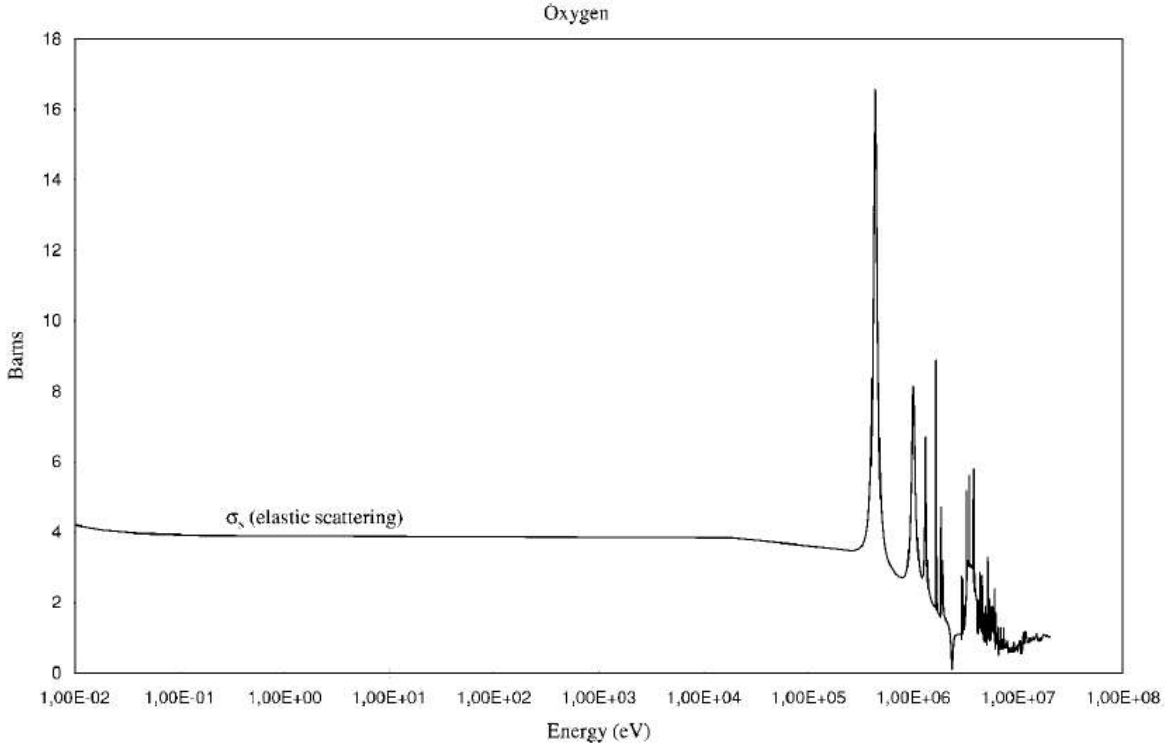
$$p_j^2 = p^2 + p'^2 - 2pp'\cos\vartheta$$

Law of conservation of energy:

$$T = T' + T_j$$

p, T – neutron momentum and energy before scattering, p′, T′ – after scattering, p_j and T_j momentum and kinetic energy of the recoiled nucleus, δ – neutron scattering angle.

Cross section of elastic scattering



Elastic scattering cross section – usually a complex curve with a series of resonances (these occur when the neutron energy is close to one of the neutron-nucleus eigenenergies – their width is related to the lifetime of these states according to the Heisenberg uncertainty principle). The general character of the effective cross section x neutron energy function is decreasing with increasing energy. The usual fate of a neutron – the neutron slows down with elastic scatterings (usually down to thermal energy) and is then absorbed by radiation capture. The effective cross-sections of the other types of interactions, with exceptions (high energy or (n, f) reactions on heavy nuclei) is small.

Inelastic scattering of neutrons on nuclei

During scattering, the target nucleus is excited to a higher energy state. The process can take place either as direct excitation or as absorption and re-emission of a neutron by the nucleus – the scattered neutron is not necessarily the same one that entered the nucleus (so-called exchange scattering).

The effect has threshold – it is determined by the energy difference between the ground state and the lowest excited state of the nucleus. Energy levels of the nucleus – the decisive factor determining the probability of scattering and the probability of emission and the energy of photons emitted during the deexcitation of the nucleus. The energy spectrum differs from nucleus to nucleus and is characteristic for each nucleus.

In general: for light nuclei and highly stable nuclei (with a magic number of nucleons) – the energy levels are distant from each other $\Rightarrow$ the energy threshold of inelastic scattering is high – a smaller number of spectral lines of relatively high energy photons are emitted during deexcitation. In the case of heavy nuclei or at very high neutron kinetic energies (i.e. also the excitation energies of the nucleus after the collision) – energy levels are close to each other, the photon spectrum becomes more complex, the lines may partially overlap, a complex curve is obtained experimentally.

From this it can be concluded: the effective cross-section of inelastic scattering will have an increasing trend with the increasing mass of the nuclei (the number of energy levels to which the nucleus can be excited increases). At kinetic energies lower than 5 –10 MeV (depending on the mass of the nucleus), the effective cross section of the process is negligible compared to the elastic scattering.

Nuclear reactions with neutrons

A very important group of reactions – the neutron is a structural part of the nucleus, it is bound in it by nuclear forces ⇒ when approaching the nucleus, a strong interaction must occur (the same applies to scattering processes, which actually also have the character of nuclear reactions). At the same time, however, the Coulomb potential barrier of the nucleus does not exist for the neutron ⇒ exoergic reactions with neutrons take place even at very low energies.

Types of reactions with neutrons

Radiation capture of neutrons (reaction (n, γ))

$$(A, Z) + n \rightarrow (A+1, Z) + \gamma$$

i.e. the resulting nucleus has more neutrons compared to the original one and is often (but not in all cases) β^- active. These reactions have a high probability especially with slow neutrons (practically always $M(A,Z) + m_n > M(A+1, Z)$) ⇒ they are well usable for neutron detection. **Radiation capture is the most common mechanism of neutron absorption by nuclei.**

Reactions with proton emission ((n, p) reactions)

$$(A, Z) + n \rightarrow (A, Z-1) + p$$

Although there is no Coulomb potential barrier for a neutron entering the nucleus, they most often take place with fast neutrons ($T_n \approx 0.5$ - 10 MeV), since the emitted proton as a charged particle must have sufficient energy to overcome this barrier during emission from the nucleus.

Nuclear reactions with neutrons

A reaction with emission of α particles (reaction (n, α))

$$(A, Z) + n \rightarrow (A-3, Z-2) + {}^{4}He$$

Most reactions are only initiated by fast neutrons so that the emitted α particle must overcome the Coulomb potential barrier of the nucleus – there are a few reactions with energy so high that it is enough to overcome the barrier – they take place with high cross-sections even with thermal neutrons and can be used to detect thermal neutrons.

A reaction with emission of two or more nucleons

At high neutron energies (T_n > 10 MeV), reactions of the type (n, 2n), (n, 3n), (n, pn) and others are possible from the point of view of energy. In general, these reactions have a high threshold, as it is clear that the energy required to separate two nucleons will be higher than the energy required to separate one nucleon.

Fission reaction (i.e. reaction of the type (n, f) – symbolism from fission = fission)

$$(A, Z) + n \rightarrow (A_1, Z_1) + (A_2, Z_2) + kn$$

$$A_1 + A_2 + k = A + 1$$

$$Z_1 + Z_2 = Z$$

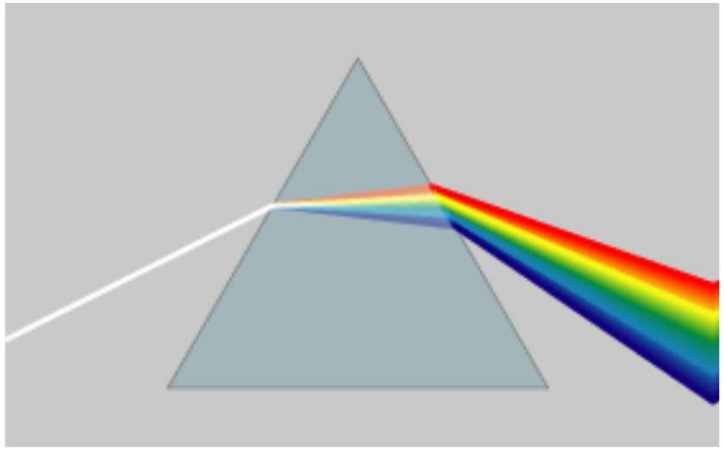
and both resulting nuclei are "comparably" heavy. They occur during irradiation of heavy nuclei (roughly from ${}_{90}Th$ above) with fast neutrons, on ${}^{233}U$, ${}^{235}U$ and some transuranium nuclides also during irradiation with thermal neutrons. Particles of very high energies can also fission the nuclei of lighter elements – practically any particle can be fissioned at high energies (but at very high energies the nuclei tend to be fragmented to smaller parts).

X-ray optics

X-ray optics

X-ray optics is the branch of optics dealing with X-rays, rather than visible light. It deals with focusing and other ways of manipulating the X-ray beams for research techniques such as X-ray diffraction, X-ray crystallography, X-ray fluorescence, small-angle X-ray scattering, X-ray microscopy, X-ray phase-contrast imaging, and X-ray astronomy.

X-rays and visible light are both electromagnetic waves, and propagate in space in the same way, but because of the much higher frequency and photon energy of X-rays they interact with matter very differently. Visible light is easily redirected using lenses and mirrors, but because the real part of the complex refractive index of all materials is very close to 1 for X-rays, they instead tend to initially penetrate and eventually get absorbed in most materials without significant change of direction.

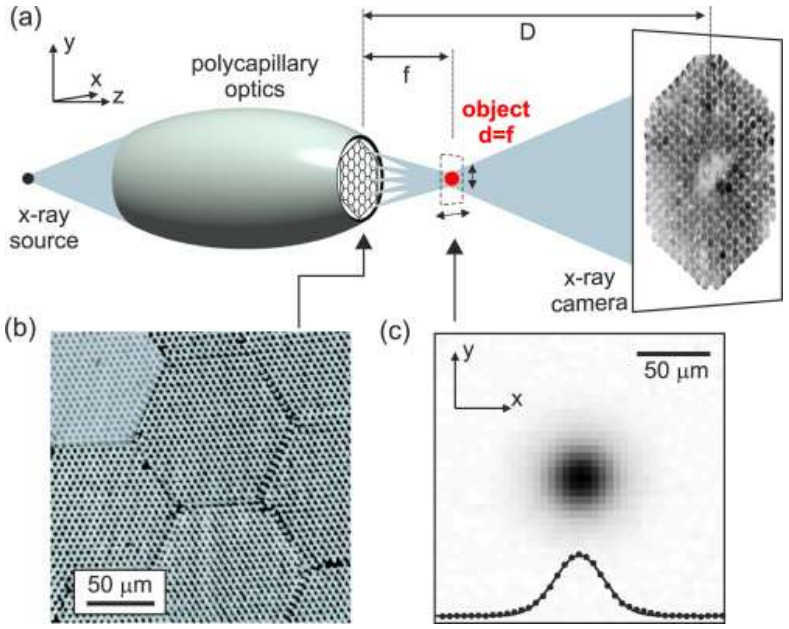


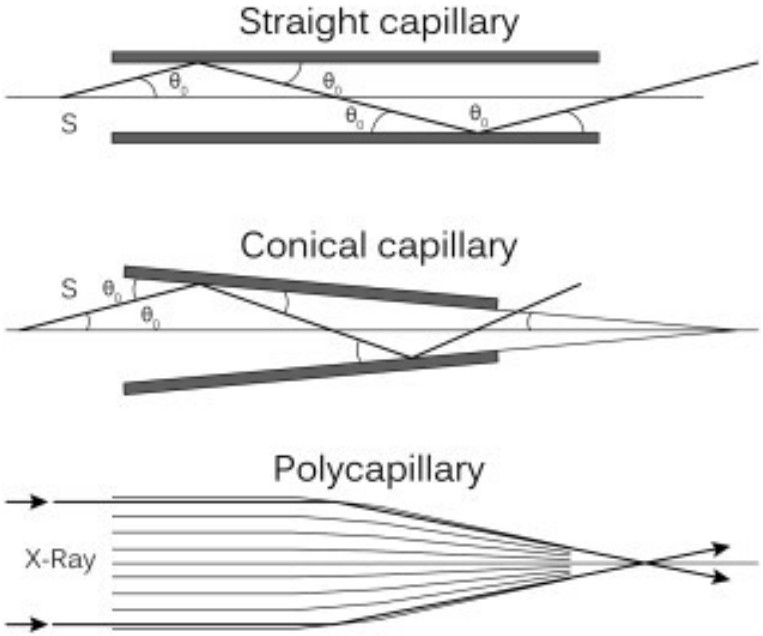
Refraction

Interference

X-ray optics

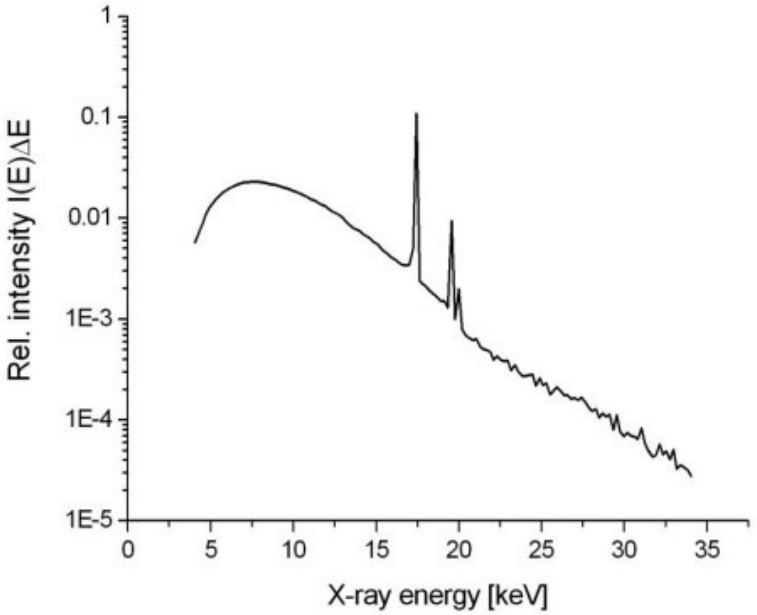
Focusing polycapillary optics : This type of polycapillary optics focuses X-rays to a small spot size or a high intensity on the sample surface. The capillaries are tapered and arranged in a circular or elliptical shape.





X-ray tube with X-ray optics

X-ray tube



X-ray optics

Suitable for low-energy X-rays up to 20 keV

Thank you for your attention