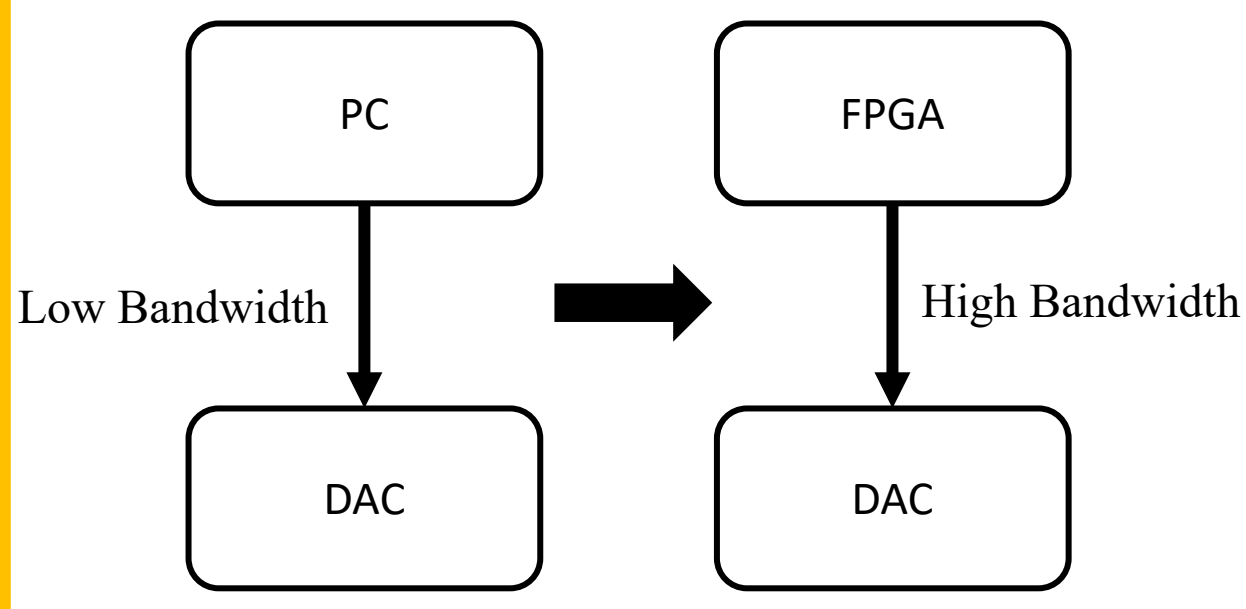


Motivation



The nuclear pulse and noise signals generated based on Monte Carlo are repeatable, pulse amplitude can be adjusted, and have good testing ability for the designed DAQ system. The software-based method for emulator has the bottleneck of data bandwidth under the requirements of multi-channel, high speed and high precision nuclear signal generator. For a nuclear signal generator system with 16-bit DAC, 16-channel and 500MHz clock frequency, the data bandwidth reaches 134Gb/s, which has exceeded the range that the personal computer can bear. Therefore, FPGA-based hardware emulation has advantage for high-bandwidth systems. This work aims to establish a method that can generate nuclear signals in real time in FPGA.

Method

➤ Noise power spectral density

In order to obtain the noise power spectral density of the high-purity germanium preamplifier, curve fitting is performed on the equivalent noise charge(ENC) after the noise has been filtered by first-order CR-RC, so as to obtain constant values a, b, and c in the noise power spectrum parameters.

ENC_a : equivalent noise charge for a noise
 ENC_b : equivalent noise charge for b noise
 ENC_c : equivalent noise charge for c noise
 C_f : feedback capacitor
 q_e : electron charge

a: a noise constant (relative to detector and preamplifier)
 b: b noise constant (relative to detector and preamplifier)
 c: c noise constant (relative to detector and preamplifier)

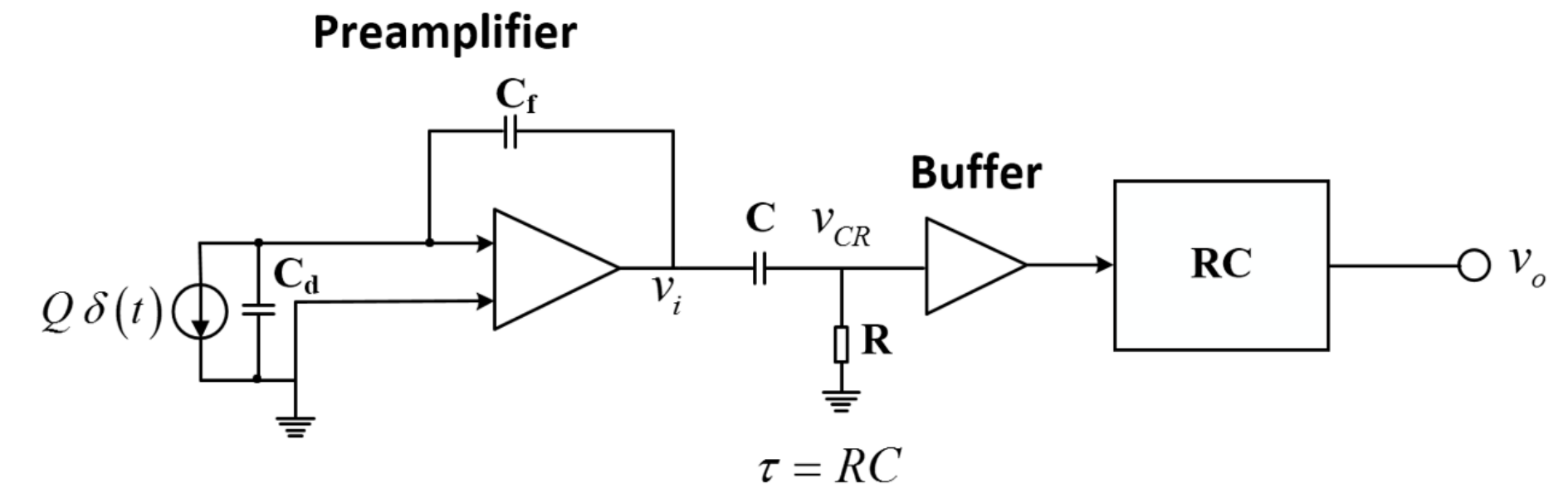


Fig. Simplified diagram of charge sensitive preamplifier.

Table. Equation for preamplifier noise parameter

ENC(CR-RC)	$ENC_a^2 = a^2 \frac{2\pi C_f^2 0.92}{q_e^2 \tau}$	$ENC_b^2 = b^2 \frac{2\pi C_f^2}{q_e^2} 0.92\tau$	$ENC_c^2 = c^2 \frac{C_f^2}{q_e^2} 3.69$
spectral density	$d\bar{v}_a^2 = 2\pi a^2 df$	$d\bar{v}_b^2 = \frac{b^2}{2\pi f^2} df$	$d\bar{v}_c^2 = \frac{c^2}{2\pi f} df$

➤ Gaussian white noise generator(a noise)

Tausworth method is chosen to generate pseudo-random numbers because of its very good statistical properties, long periodicity, and simple circuit structure. On the basis of Tausworth method, the Box-Muller method is used to generate high-quality Gaussian white noise.

$$\begin{cases} x_0 = \sqrt{-2 \ln(u_0)} \times \sin(2\pi u_1) \\ x_1 = \sqrt{-2 \ln(u_0)} \times \cos(2\pi u_1) \end{cases}$$

x_0, x_1 : Gaussian white noise samples
 u_0, u_1 : Evenly distributed samples produced by the Tausworth method

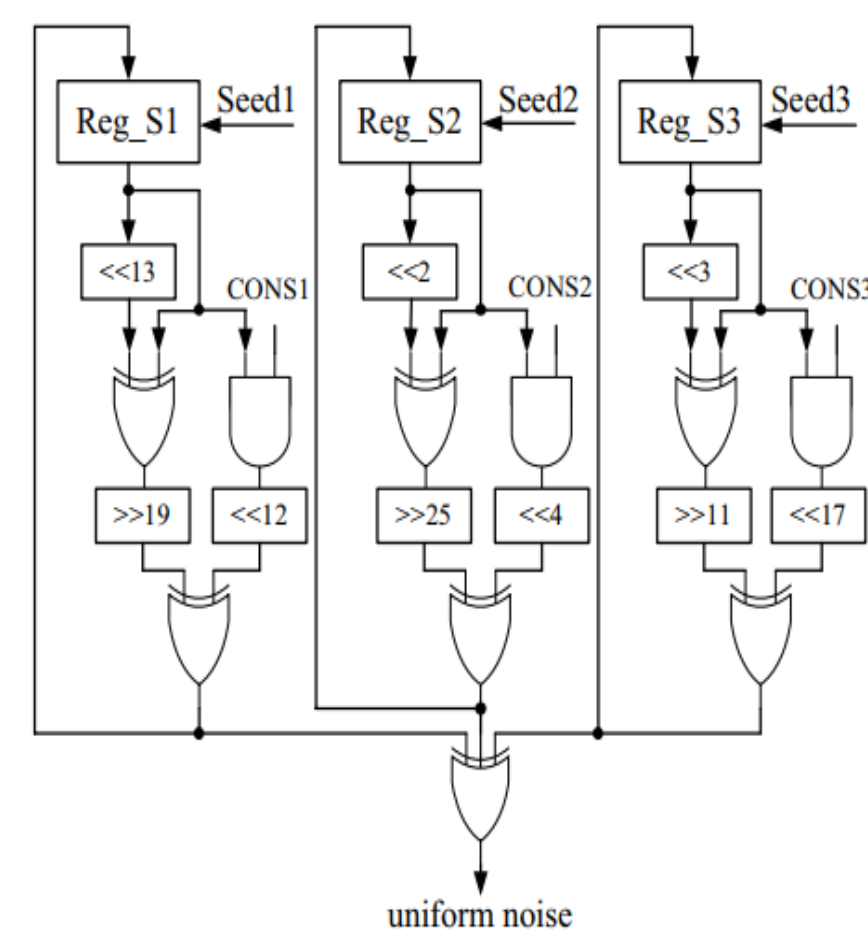


Fig. Structure of Tausworth random number generator

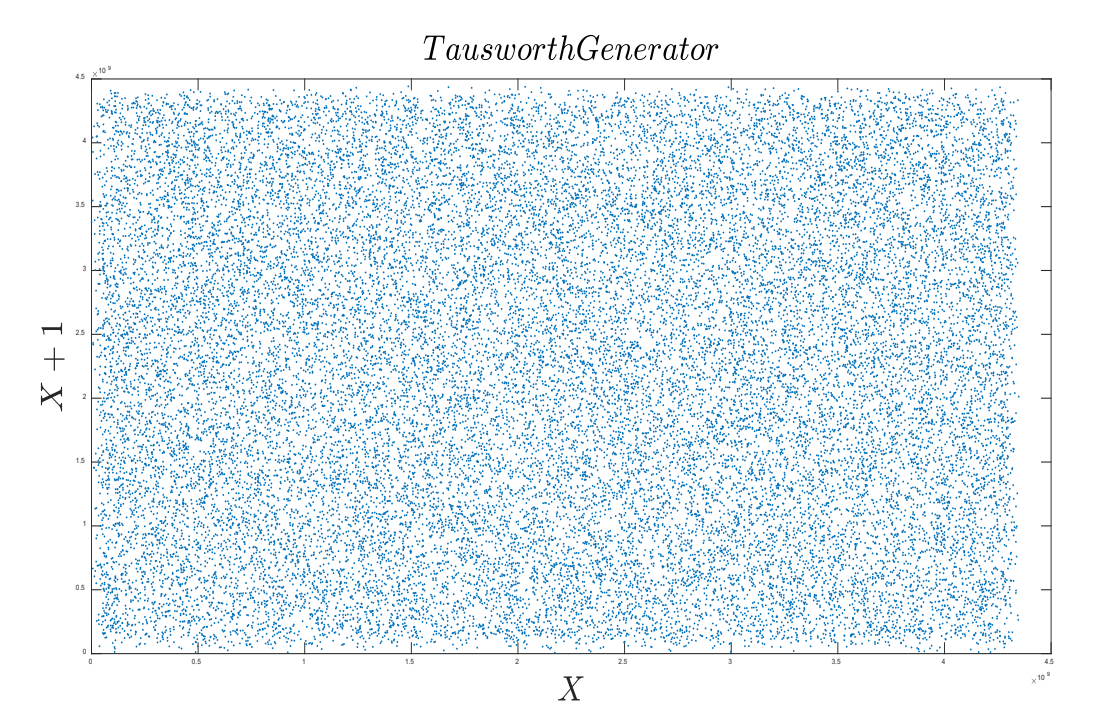


Fig. Correlation of successive samples which are generated through 32-bit Tausworth generator

➤ Parametric model for b and c noise

$$x(n) = -\sum_{k=1}^p a_k x(n-k) + \sum_{k=0}^q b_k u(n-k)$$

Find the z-transform on both sides of the above equation, and assume $b_0 = 1$:

$$H(z) = \frac{1 + \sum_{k=1}^q b_k z^{-k}}{1 + \sum_{k=1}^p a_k z^{-k}} = \frac{B(z)}{A(z)}, \quad P_x(z) = \sigma^2 \frac{|1 + \sum_{k=1}^q b_k z^{-k}|^2}{|1 + \sum_{k=1}^p a_k z^{-k}|^2}$$

Ideal transfer function of b and c noise:

$$H_b(s) = \frac{b}{s}, \quad P_b(s) = |H_b(s)|^2 = \frac{b^2}{s^2} \quad \xrightarrow{\text{Bilinear transformation}} \quad P_b(z) = \left(\frac{bT_s}{2}\right)^2 \frac{1+2z^{-1}+z^{-2}}{1-2z^{-1}+z^{-2}} = P_{xb}(z)$$

$$H_c(s) = \frac{c}{\sqrt{s}}, \quad P_c(s) = |H_c(s)|^2 = \frac{c^2}{s} \quad \xrightarrow{\text{Bilinear transformation}} \quad P_c(z) = \frac{c^2 T_s}{2} \frac{1+z^{-1}}{1-z^{-1}} = P_{xc}(z)$$

According to the principle of equal coefficients of corresponding items:

$$\begin{aligned} x_b(n) &= x_b(n-1) + \left(\frac{bT_s}{2}\right)^2 \{w(n) + w(n-1)\} \\ x_c(n) &= 0.5x_c(n-1) + 0.125x_c(n-2) + 0.063x_c(n-3) + 0.036x_c(n-4) - 0.026x_c(n-5) - 0.007x_c(n-6) - 0.005x_c(n-7) \\ &\quad + \frac{c^2 T_s}{2} \{w(n) + 0.5w(n-1) - 0.125w(n-2) + 0.063w(n-3) - 0.036w(n-4) - 0.026w(n-5) + 0.007w(n-6) - 0.005w(n-7)\} \end{aligned}$$

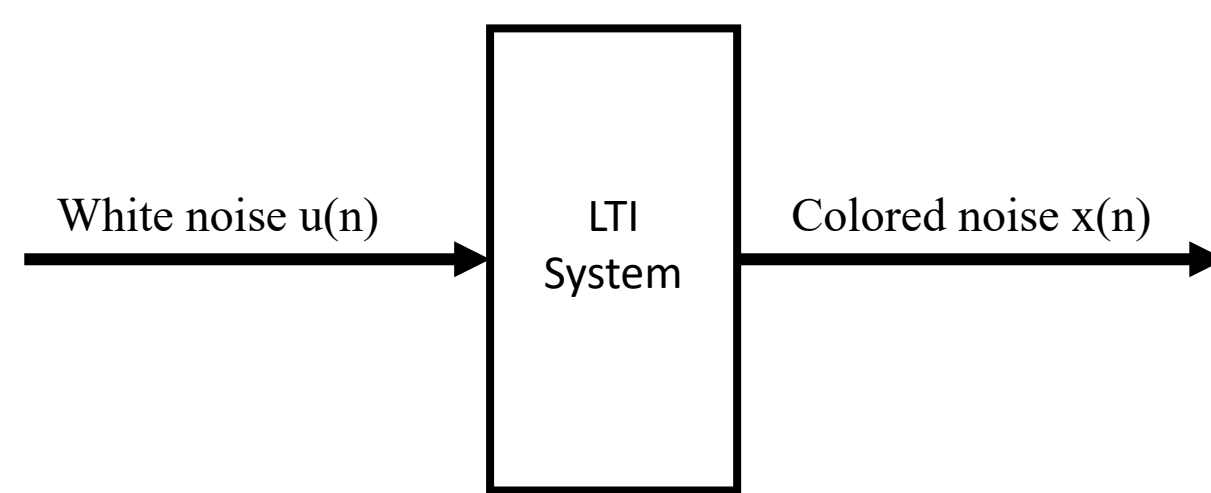


Fig. Structure of noise model based on spectrum decomposition theorem

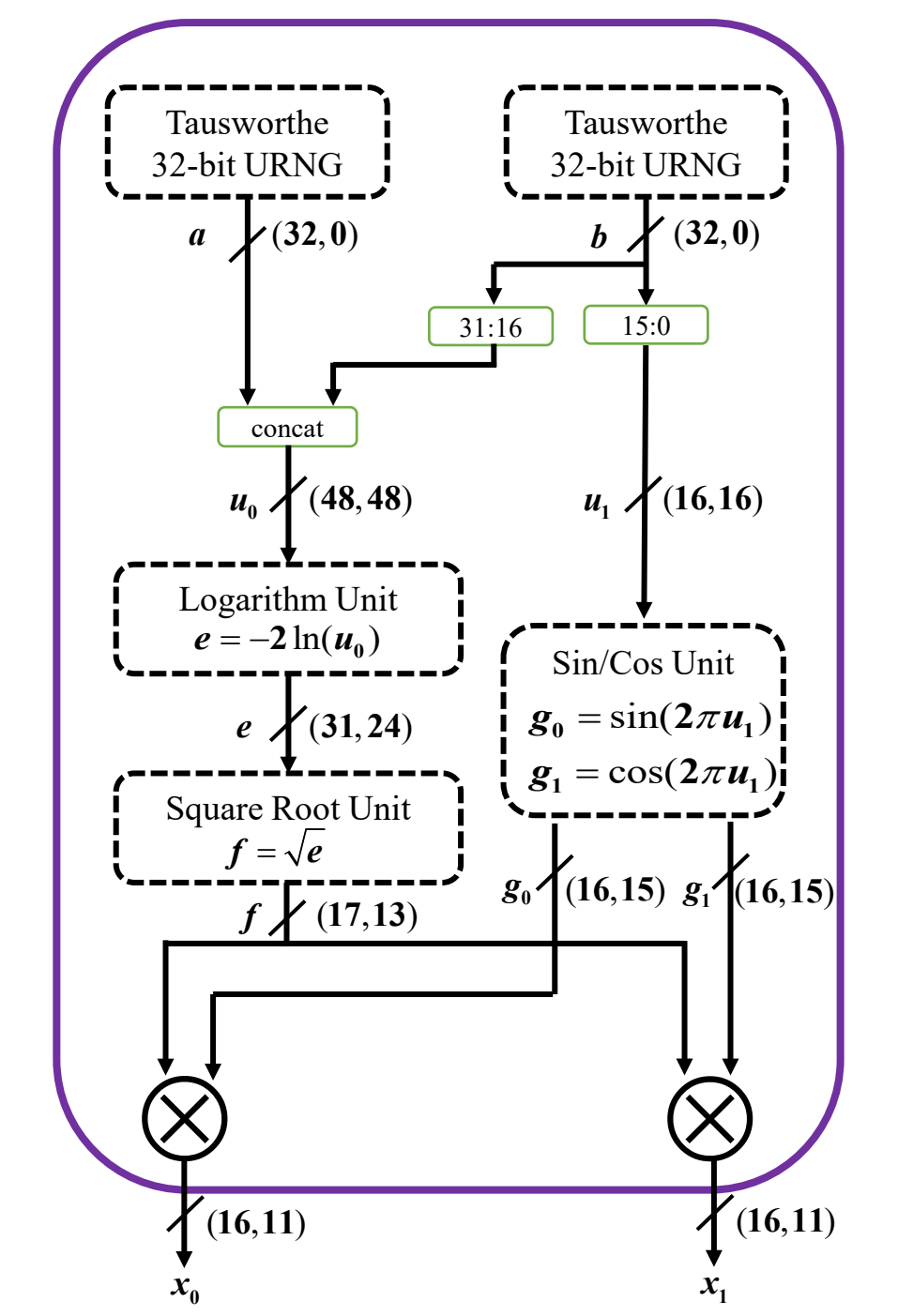


Fig. White noise generator with Box-muller method

Experiment

The Canberra Intelligent Preamplifier for High-Purity Germanium(HPGe) Detectors is used to get the noise parameters a and b and c. After the noise is filtered by CR-RC, a 16bit, 125Mhz ADC is used for data acquisition, and experimental parameters are obtained by curve fitting.

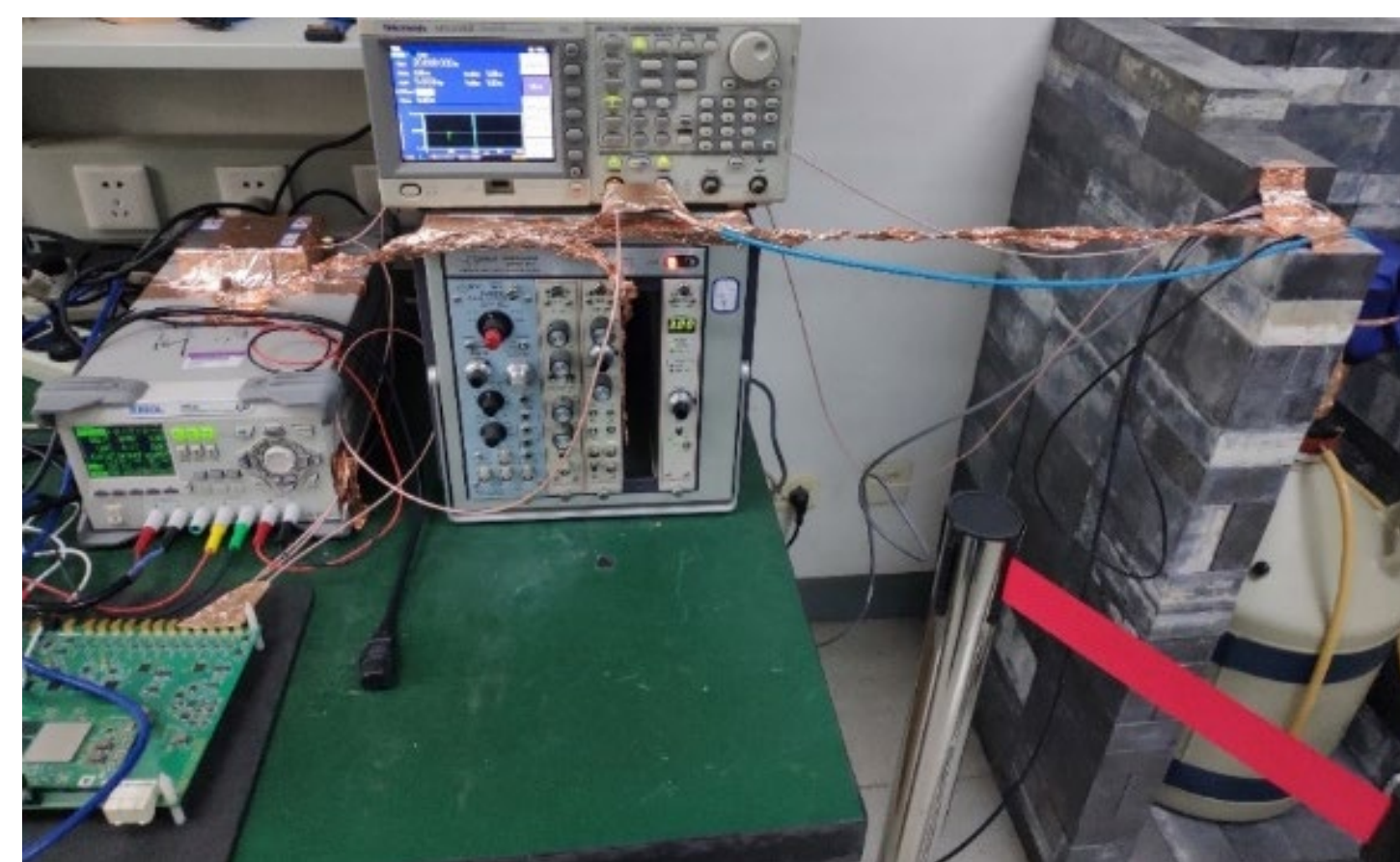


Fig. The Broad Energy Germanium Detectors (BEGe).

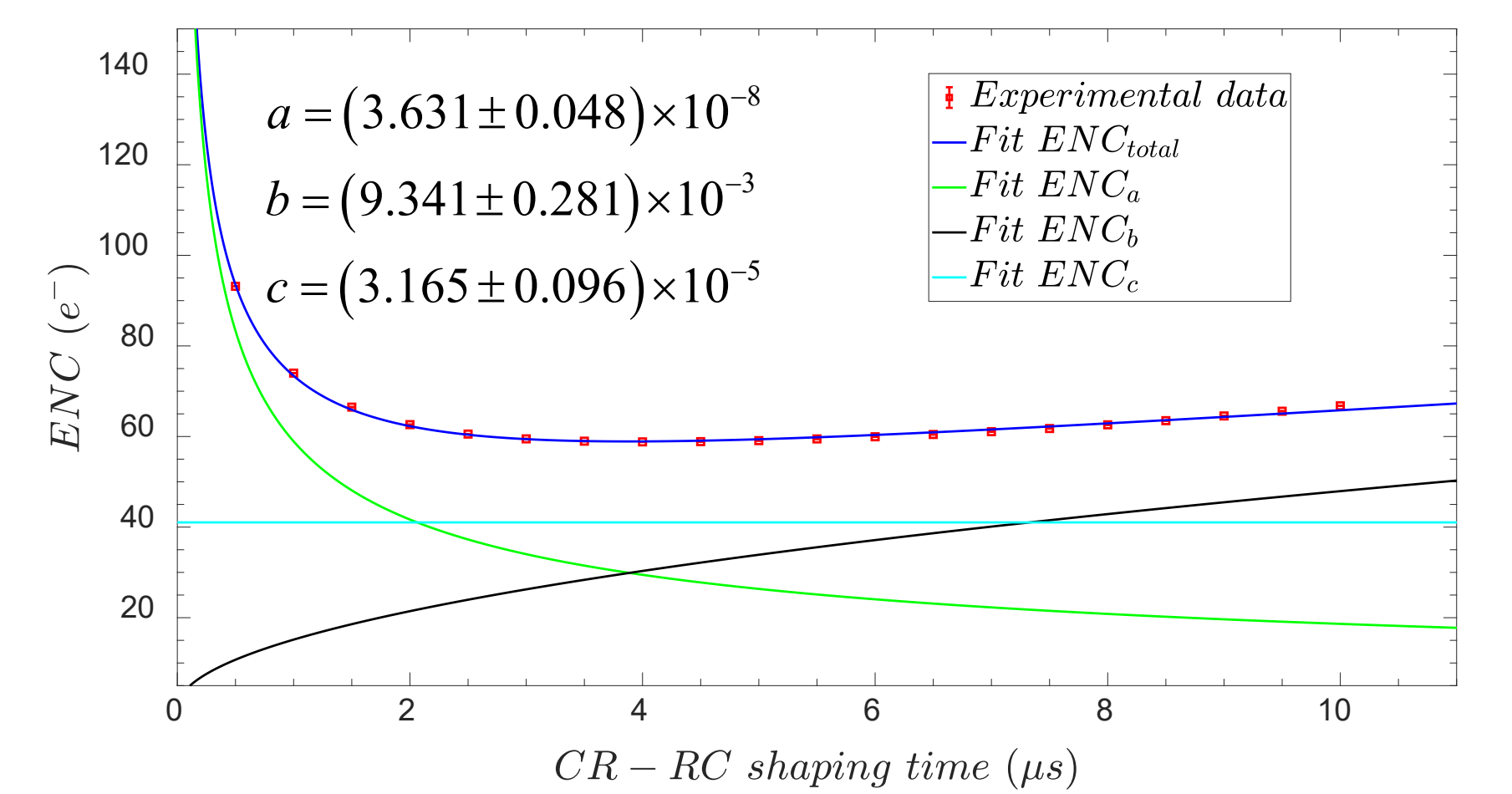


Fig. ENC with different CR-RC shaping time.

Result

The noise samples are generated by the parameter estimation model, the sampling frequency is 1Hz, and the power spectral density of the noise a, b, and c is obtained by the spectral estimation method.

The noise power spectral density obtained by the simulation is consistent with the theoretical power spectral density in trend, but is quite different from the theoretical power spectral density at low frequency and high frequency.

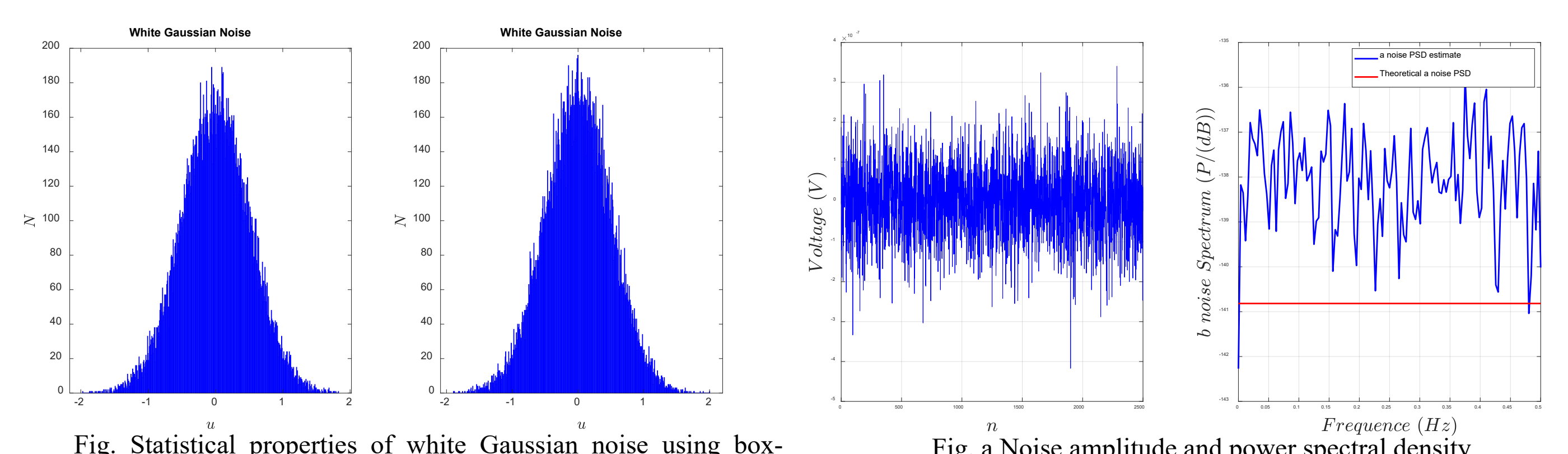


Fig. Statistical properties of white Gaussian noise using box-muller method.

Fig. a Noise amplitude and power spectral density

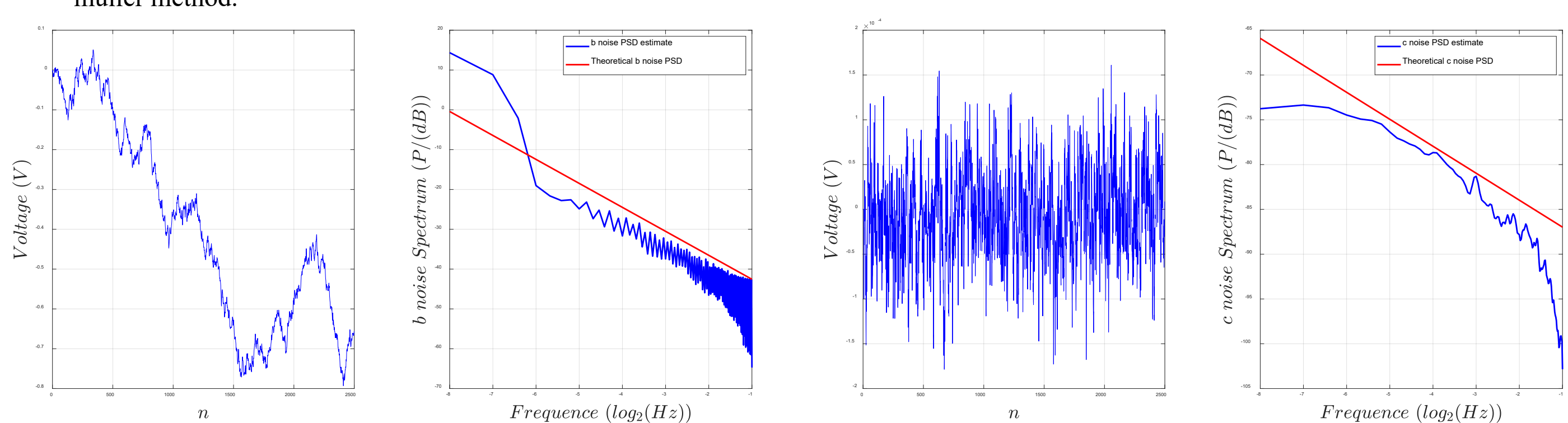


Fig. b Noise amplitude and power spectral density

Fig. c Noise amplitude and power spectral density