Particle Tracking with Space Charge Effect using Graphics Processing Unit

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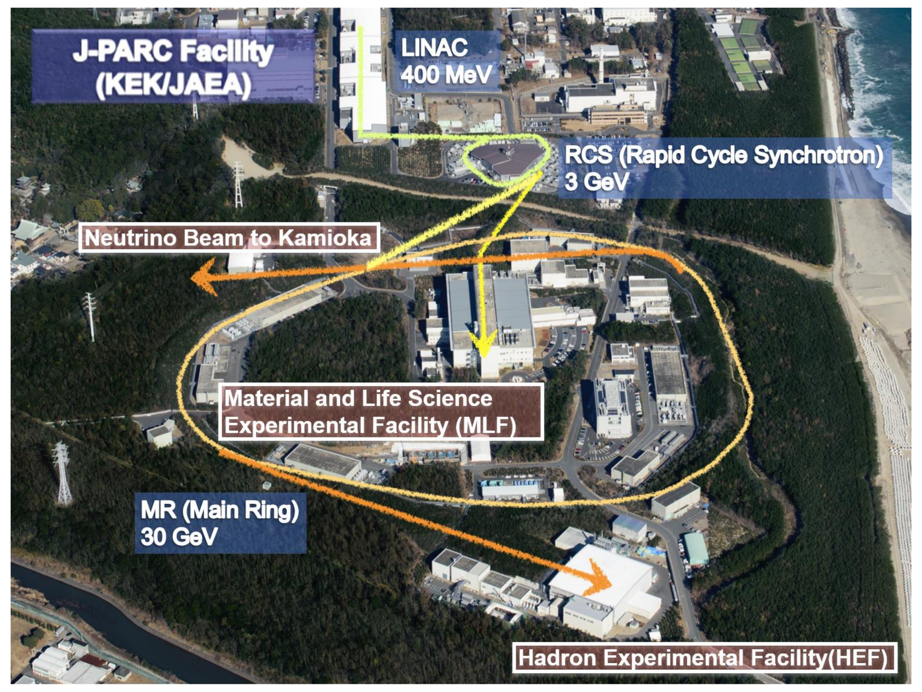
1. MOTIVATION

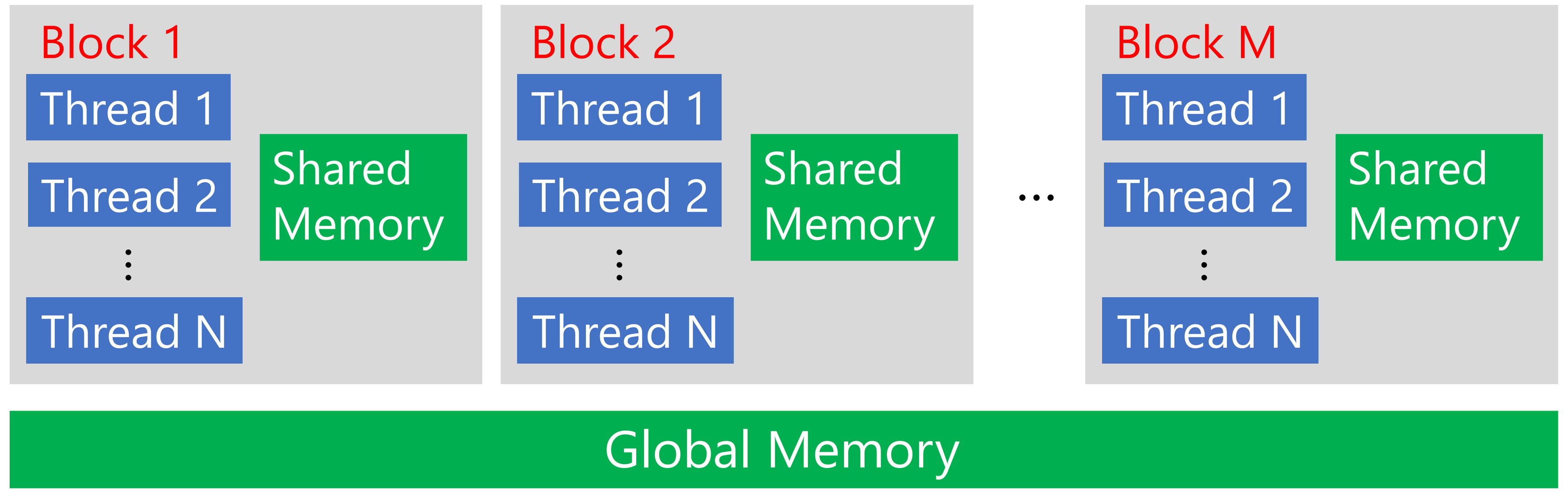
- Understanding space-charge induced beam loss is a main task at high-intensity beam facilities
- Multi-particle tracking simulations are time-consuming



An Example : J-PARC (Japan Proton Accelerator Research Complex)

- **Up to 30 GeV**
- **> 3×10[13] per bunch**
- **Neutrino, Hadron, Material and Life Science**

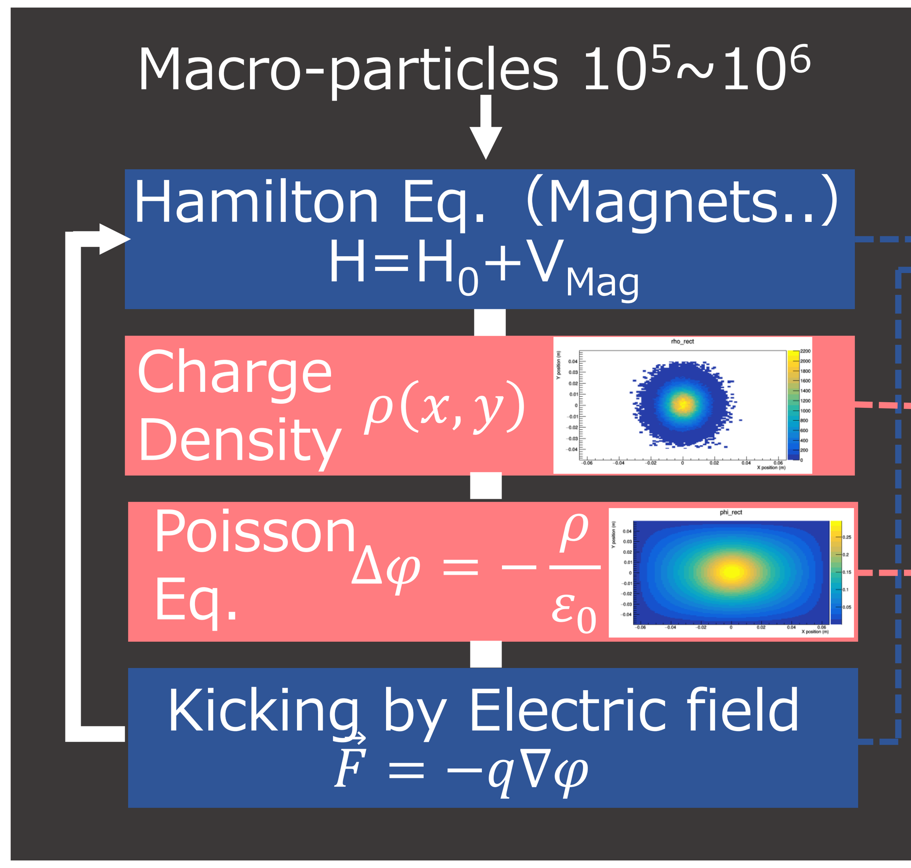
2. GRAPHIC PROCESSING UNIT (GPU)



- Each operation can be assigned to each thread.
- **Execution of each thread can be parallelly done.**
- Threads in a common block can access the shared memory.
- Shared memory is limited (12288 double words) but **very fast**

Nvidia provides a parallel computing platform called **CUDA**

3. PARTICLE TRACKING SIMULATION with SPACE-CHARGE EFFECT

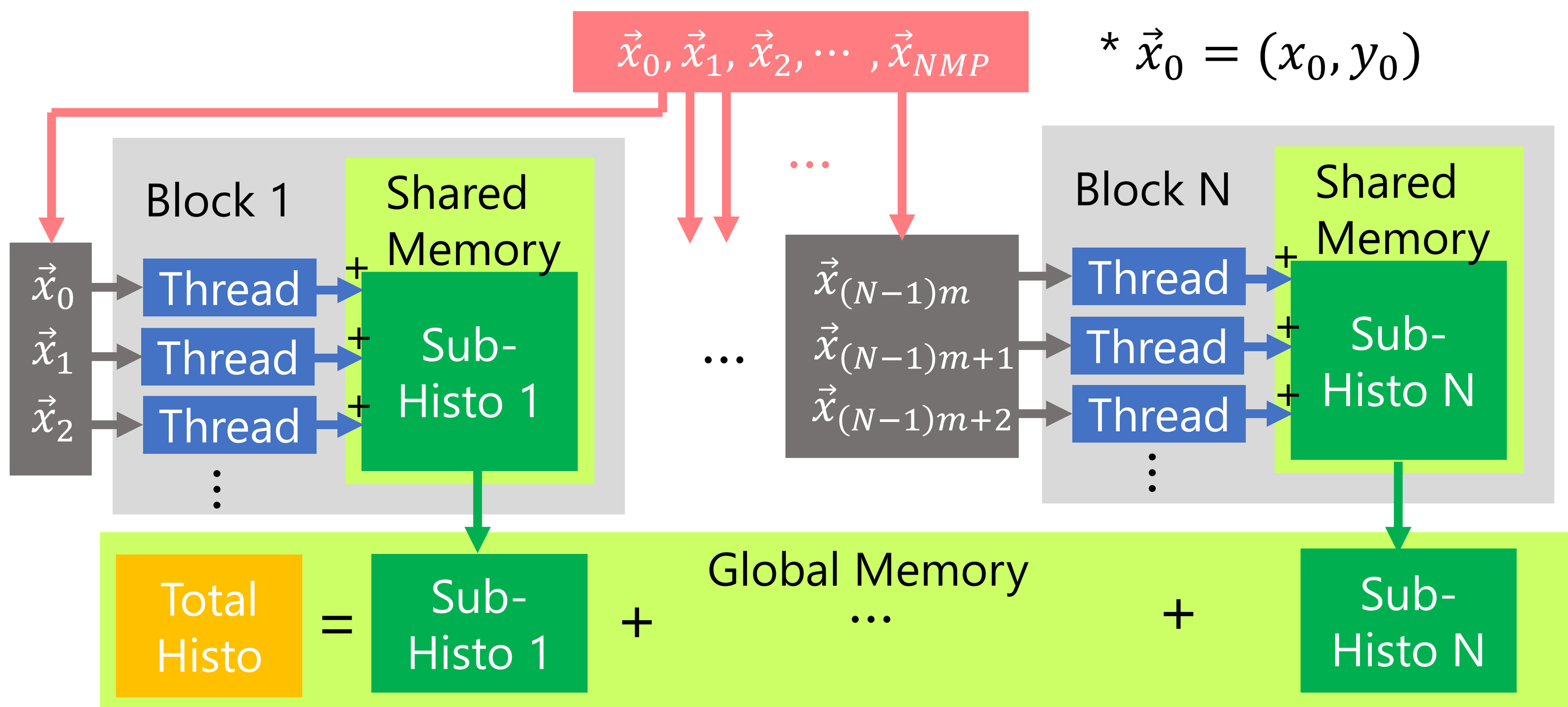


- Can be calculated in parallel for each particle
- Each calculation is assigned to a "Thread"

- Cannot be done in parallel
- The results involves charge densities which requires histogram creations

4. CHARGE DENSITY (HISTOGRAM CREATION)

1. Divide (macro) particles into N groups
2. Assign each group to each block and each particle to each thread
3. Sub-histogram in each shared memory be made in parallel.
4. Copy sub-histograms to the global memory and make summation



* Only 2D charge distributions (~100×100) can be made in this way because of the shared memory capacity (12K FP64s for each)

→ **Useful for only long bunch shape**

5. POISSON SOLVER

CUDA includes libraries of FFT (cuFFT) and Linear algebra (cuBLAS) → Solving equations using DFT (Discrete Fourier Transform)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y)$$

with boundary conditions (Beam Duct)

$$u(x,0)=u(x,L_y)=u(0,y)=u(L_x,y)=0$$

$$DFT_y(DFT_x(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})_p)_q=DFT_y(DFT_x(f)_p)_q$$

$$-4\left(\frac{1}{\Delta x^2}\sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}\sin^2\frac{q\pi}{2(m+1)}\right)DFT_y(DFT_x(V)_p)_q=DFT_y(DFT_x(F_{i,j})_p)_q$$

$$DFT_y(DFT_x(V)_p)_q=-\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2}\sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}\sin^2\frac{q\pi}{2(m+1)}\right)}\quad p\neq 0 \text{ or } q\neq 0$$

$$DFT_y(DFT_x(V)_0)_0=\sum_{l=0}^{2(m+1)-1}\sum_{l'=0}^{2(m+1)-1}V_{l,l'}=0\quad p=0 \text{ and } q=0$$

($V_{l,l'}$ is odd extension of $u_{i,j}$)

$$V_{l,l'}=-iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2}\sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}\sin^2\frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

Four DFT's !!! (i : inverse)

Supplements

- DFT (Discrete Fourier Transform) $DFT\,(g_l)_p\equiv\sum_{l=0}^{N-1}g_l e^{-i\frac{2\pi pl}{N}}$
- Discretization

$$\frac{u_{i-1,j}-2u_{i,j}+u_{i+1}}{\Delta x^2}+\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{\Delta y^2}=f_{i,j}$$

- V,F : Odd Extensions of u,f → Boundary Condition

$$V=\begin{pmatrix}0&0&0&\cdots&0&0&0&0&\cdots&0\\0&u_{1,1}&u_{1,2}&\cdots&u_{1,m}&0&-u_{1,m}&-u_{1,m-1}&\cdots&-u_{1,1}\\0&u_{2,1}&u_{2,2}&\cdots&u_{2,m}&0&-u_{2,m}&-u_{2,m-1}&\cdots&-u_{2,1}\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots&\ddots&\vdots\\0&u_{m,1}&u_{m,2}&\cdots&u_{m,m}&0&-u_{m,m}&-u_{m,m-1}&\cdots&-u_{m,1}\\0&0&0&\cdots&0&0&0&0&\cdots&0\\0&-u_{m,1}&-u_{m,2}&\cdots&-u_{m,m}&0&u_{m,m}&u_{m,m-1}&\cdots&u_{m,1}\\0&-u_{m-1,1}&-u_{m-1,2}&\cdots&-u_{m-1,m}&0&\vdots&\vdots&\cdots&\vdots\\\vdots&\vdots&\vdots&\cdots&\ddots&\vdots&\vdots&\vdots&\cdots&\vdots\\0&-u_{1,1}&-u_{1,2}&\cdots&-u_{1,m}&0&u_{1,m}&u_{1,m-1}&\cdots&u_{1,1}\end{pmatrix}$$

6. EXAMPLE (J-PARC MAIN RING)

- **Accelerator Component File**

Component Name	Length	Shape	Δx	Δy	Δϕ
DRIFT_INSA_001_1_D	0.26785				
SPCHDRIFT_INSA_001_1_D	0.26785				
TQFS_INSA_001_U	0.0		−0.00559733	−0.00029039	−0.00003500
APRQFS00101_INSA_001_U	0.0	124C	−0.00559733	−0.00029039	−0.00003500
QFS00101_INSA_001_A	0.66215	124C	−0.00559733	−0.00029039	−0.00003500
SPCHQFS00101_INSA_001_A	0.66215	124C	−0.00559733	−0.00029039	−0.00003500

⋮

Implemented Components: Thick Bend. Mag. / Thick Quad. Mag. / Thick Sextuple Mag. / Thick Bend. Mag. / Thin Multipole Mag. / Cavity / Drift / Aperture / Space Charge

Shape (Cross Section): beam loss criteria, boundary condition for Poisson-eq: Circle (C) / Rectangle (Default) / 45° Tilt Rectangle (H)

- **Simulation Condition for J-PARC Main Ring**

Circumference : 1567.5 m
Simulated Components : 5826/turn (2109 for Space Charge)
Number of Turns : 20000 turns
Number of Macro particles ; 200000
GPU : single **TESLA-V100 (Amazon Web Service)**

7 TFLOP (Double Precision)
16 GB for Global Memory
$3-4/hour (Amazon Web Service)

→ Approximately **10000** seconds

Approx. 10 times faster than our previous code using Intel Xeon E5-2667 V2 (3.3GHz)