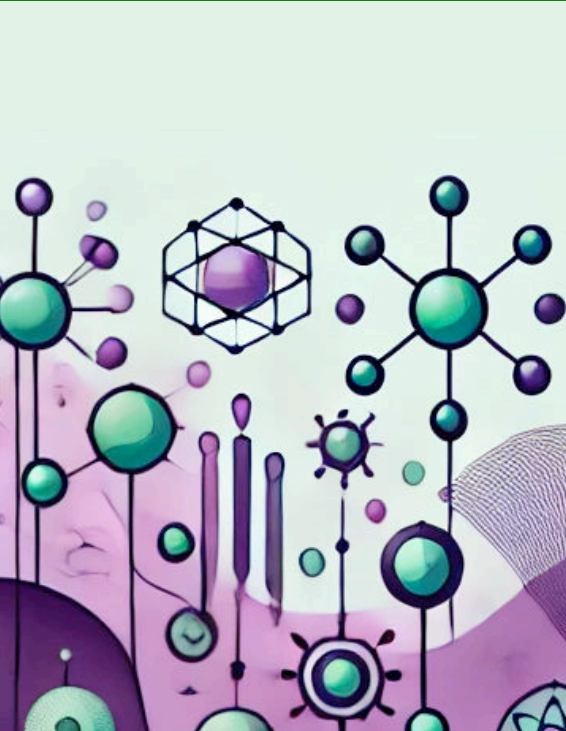


Evgeny Epelbaum, RUB

“Democratizing Models“ Workshop, RUB, 25.-26. Nov 2024

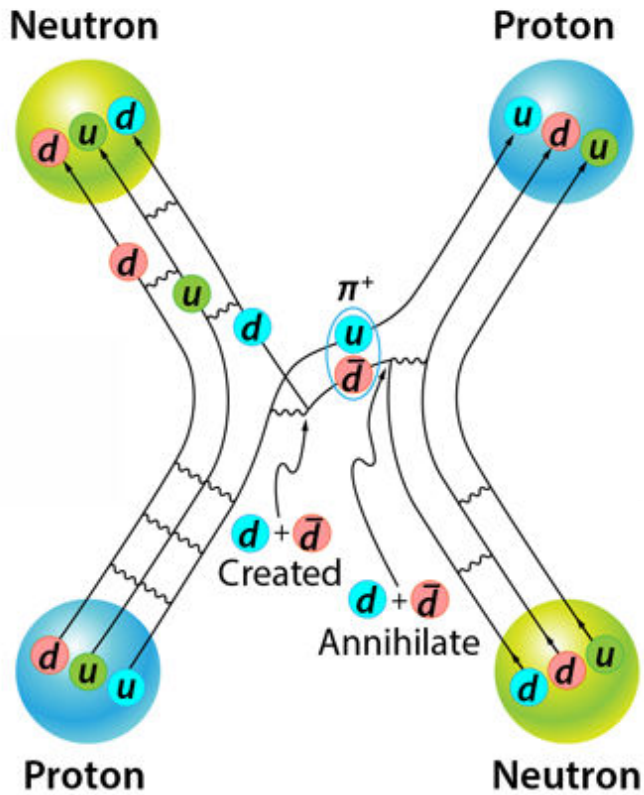
# Nuclear Physics: From Models to Theory



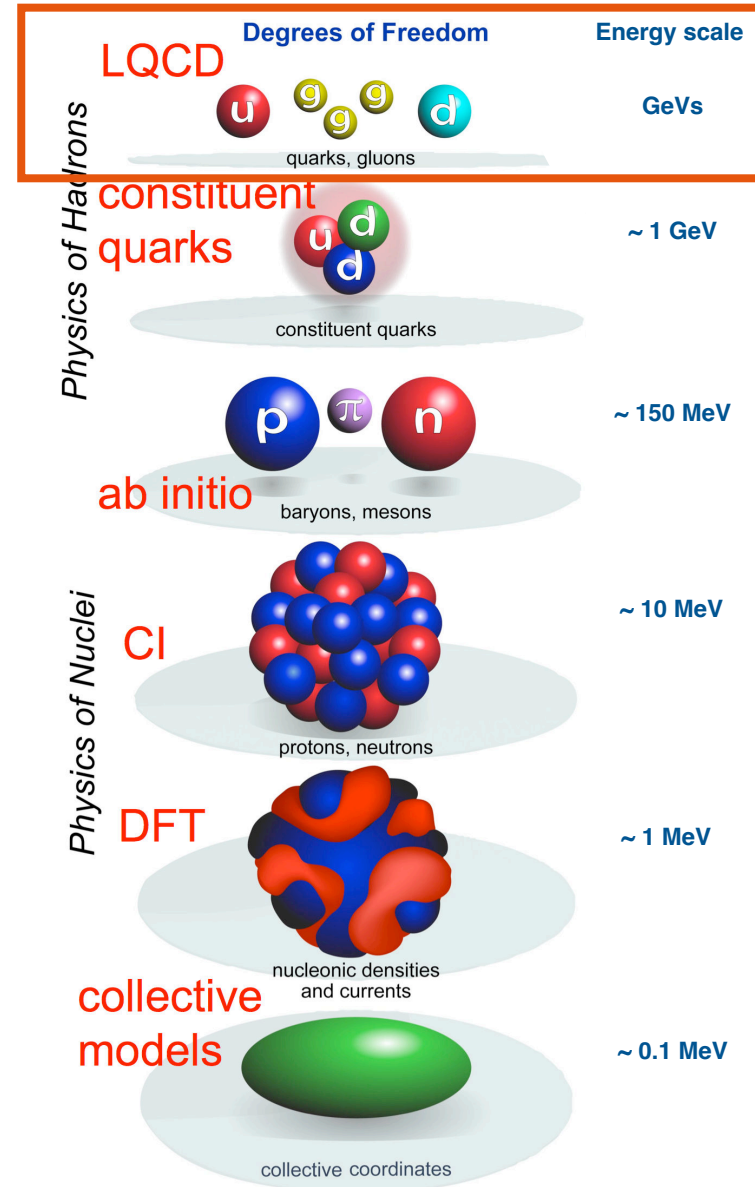
“Democracy is not a fragile flower; still it needs cultivating.”  
—Ronald Reagan

# Degrees of freedom

## Nuclear force viewed at a high resolution



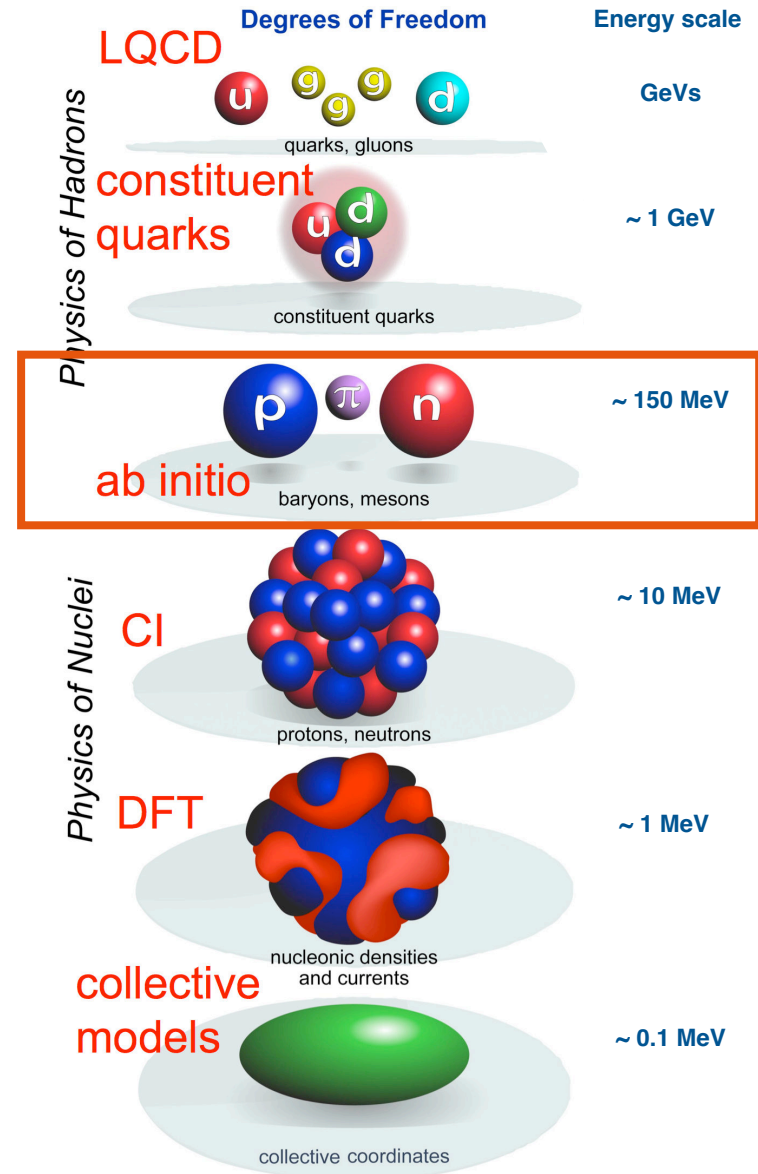
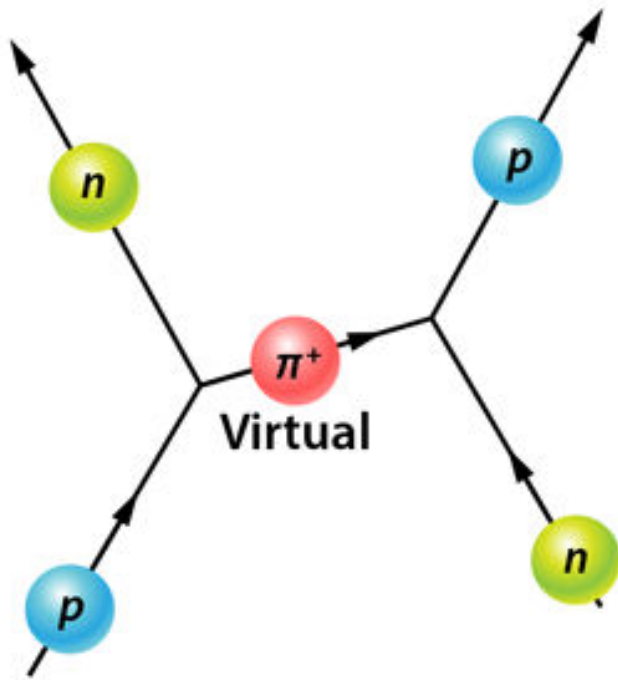
(from physics.aps.org)



(from 2007 NSAC Long-Range Plan)

# Degrees of freedom

Nuclear force viewed at a  
low resolution



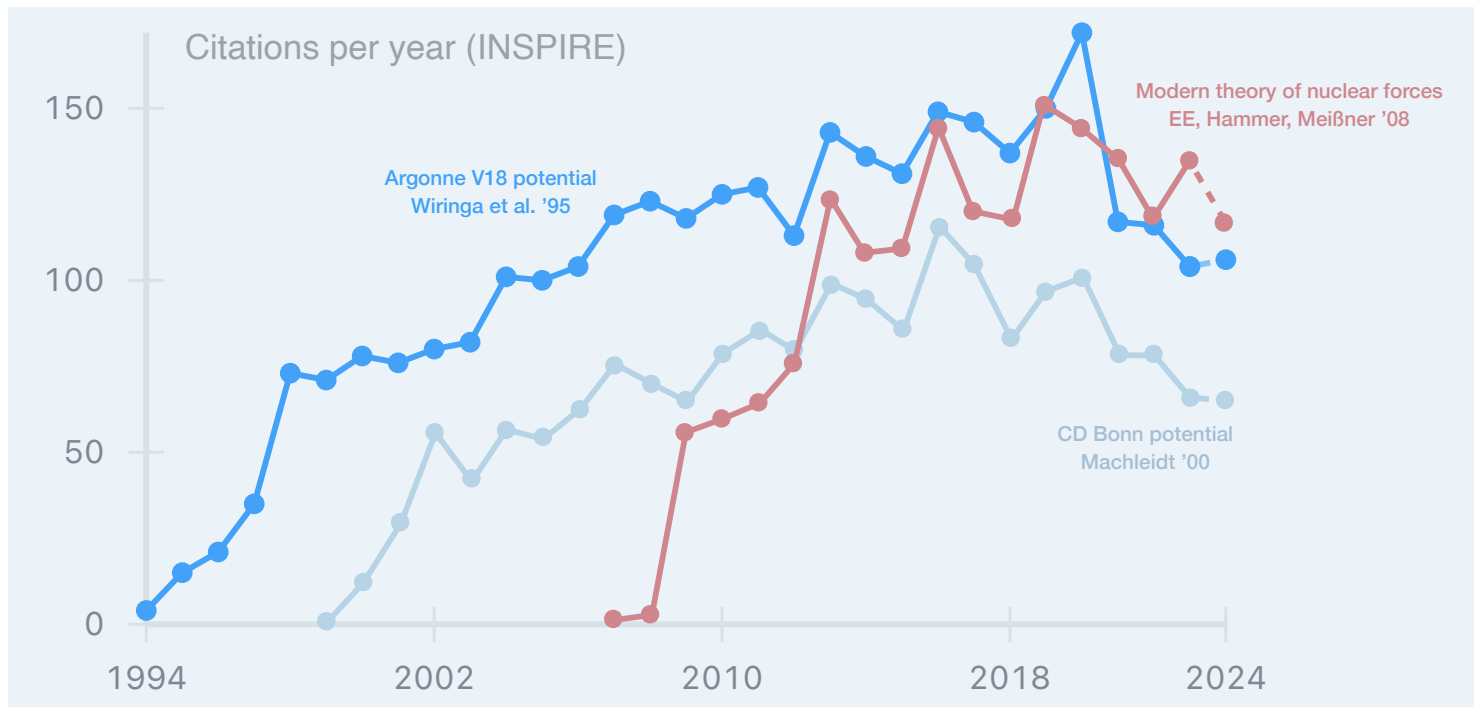
# Chiral EFT: A revolution in nuclear physics

1979: Weinberg, *Phenomenological Lagrangians*, Physica A96 (1979)

1984: Gasser, Leutwyler, *ChPT to one loop*, Annals Phys. 158 (1984)

1991: Jenkins, Manohar, *Baryon ChPT using a heavy fermion Lagrangian*, Phys. Lett. B255 (1991)

1990: Weinberg, *Nuclear forces from chiral Lagrangians*, Phys. Lett. B251 (1990)

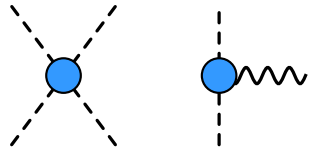


- (In principle) **model-independent** QFT-based approach with a clear relationship to QCD/Standard Model
- **Systematically improvable** (by going to higher orders in the EFT expansion)
- **Quantifiable accuracy** (truncation uncertainty)

# Chiral symmetry $\leftrightarrow$ Nuclear architecture

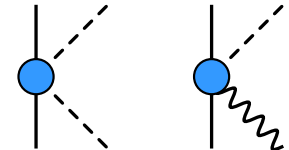
## GB dynamics

Weinberg, Gasser, Leutwyler, ...



## $\pi$ N dynamics

Bernard-Kaiser-Meißner et al.



Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \dots \frac{1}{3}$$

Effective Lagrangian:

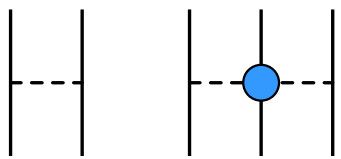
$$\mathcal{L}_\pi = \frac{F^2}{4} \text{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \dots,$$

$$\mathcal{L}_{\pi N} = \bar{N}(i v \cdot D + g_A u \cdot S)N + \dots,$$

$$\mathcal{L}_{NN} = -\frac{1}{2} C_S (\bar{N}N)^2 + 2C_T (\bar{N}S N)^2 + \dots$$

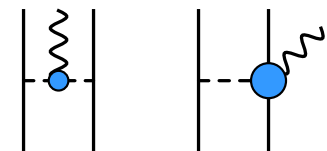
## Nuclear forces

Weinberg, van Kolck, Kaiser, EGM, ...



## Nuclear currents

Park et al, Bochum-Bonn, JLab-Pisa



Enhanced ladder graphs are re-summed  
by solving the many-body Schrödinger equation

$$\left[ \left( \sum_{i=1}^A \frac{\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + V_{2N} + V_{3N} + V_{4N} + \dots \right] |\Psi\rangle = E|\Psi\rangle$$

# Chiral symmetry $\leftrightarrow$ Nuclear architecture

**Two challenges:**

- **derivation/construction of nuclear interactions**
- **solution of the quantum mechanical A-body problem**



# The QM A-body problem

2N: Rewrite to the integral **Lippmann-Schwinger eq.**:  $t = V_{2N} + V_{2N}G_0t$  — easy to solve in p-space.

3N: **Faddeev equations**, e.g. for elastic Nd scattering:

$$T\phi = \underbrace{tP\phi}_{\text{asymptotic Nd state}} + (1 + tG_0)\underbrace{V_{3N}^{(1)}}_{\text{symmetric under exchange of nucleons 2,3}}(1 + P)\phi + tPG_0T\phi + (1 + tG_0)V^{(1)}\underbrace{(1 + P)G_0T\phi}_{P_{12}P_{23} + P_{13}P_{23}}$$

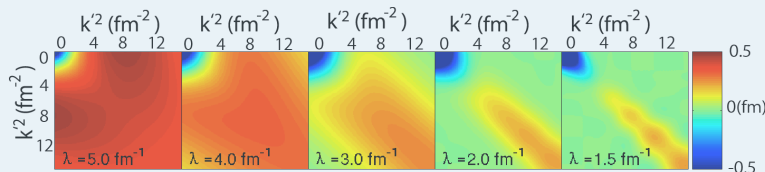
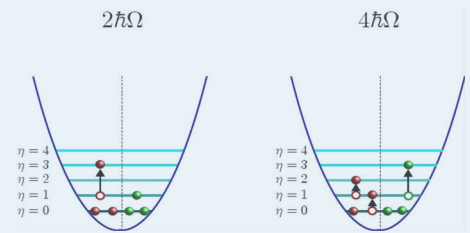
Solved iteratively in partial-waves (for fixed  $J, T \sim 10^5 \times 10^5$ ), few minutes on 1 CPU.

4N: **Yakubovsky equations**. Take several hours on JURECA@FZJ to solve for bound state; only 3 groups can do scattering (with large restrictions)

>4N: So far only (mainly) possible for bound states. E.g., **the No-Core-Shell-Model**:

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle, \quad |\Psi_i\rangle = \sum_{n=0}^{\infty} A_n^i |\Phi_n\rangle, \quad |\Phi_n\rangle = \underbrace{[a_\alpha^\dagger \dots a_\zeta^\dagger]_n |0\rangle}_{n = 1, 2, \dots, 10^{12} \text{ or more}}$$

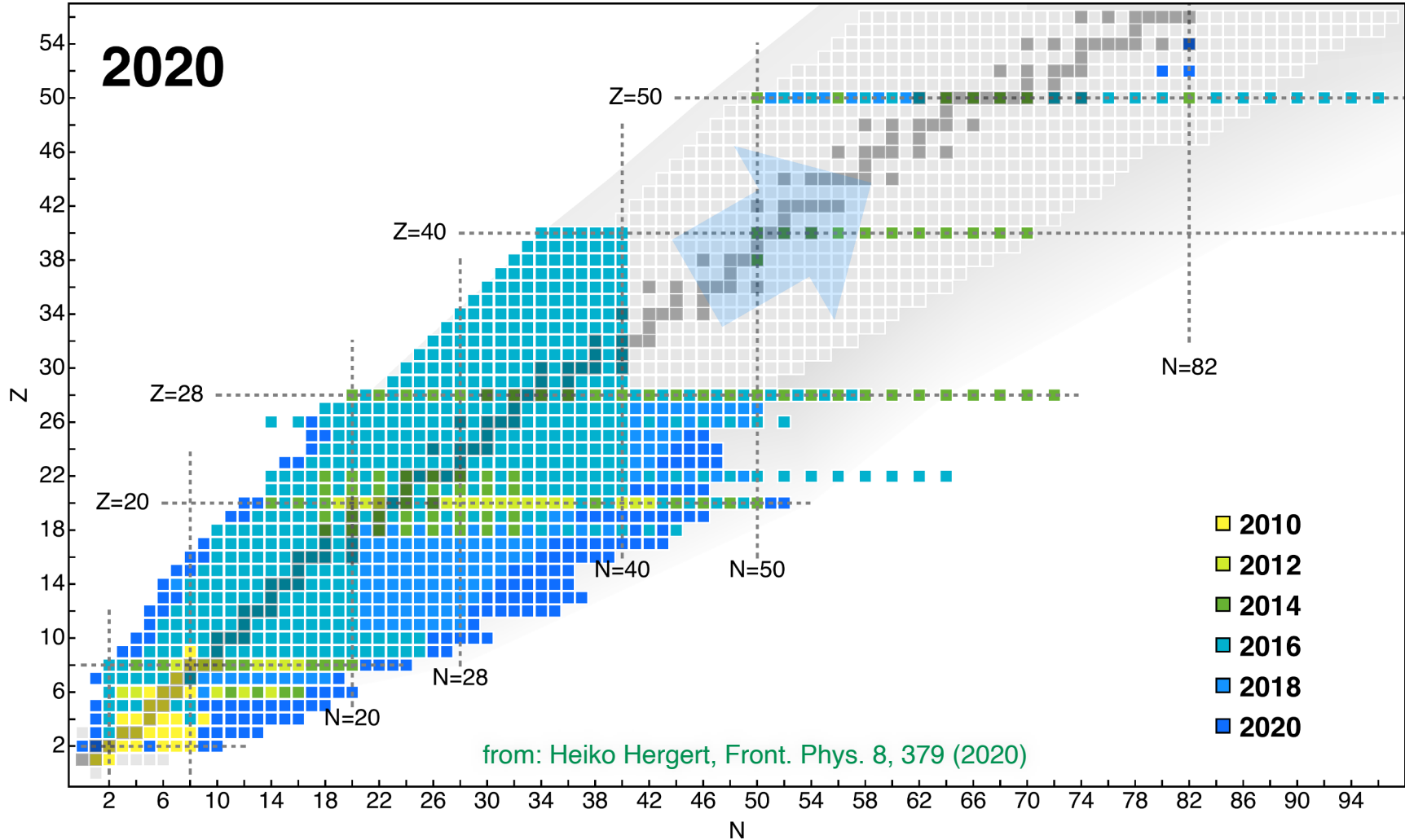
⇒ sparse matrix  $H_{mn} = \langle \Phi_m | H | \Phi_n \rangle$  diagonalization (Lánczos), extrapolation in  $N_{\max}$ , pre-diagonalization of  $H$  (SRG):



$$H(s) = U(s)HU^\dagger(s) \Rightarrow \frac{dH_s}{ds} = \underbrace{[\eta_s, H_s]}_{\text{e.g., } \eta_s = [T_{\text{kin}}, H_s]}, \quad H_{s \rightarrow 0} = H$$

Other ab-initio methods: **NL-EFT, QMC, CC expansion, IM-SRG, Lorentz IT, Green's functions, ...**

# Ab initio many-body calculations



The main bottleneck in developing nuclear physics into precision and predictive science is the accuracy of the interaction (especially of 3N forces).



# Chiral architecture

Decide on the choice of DoF ( $N$ ,  $N + \pi$ ,  $N + \Delta + \pi$ ) in  $\mathcal{L}_{\text{eff}}$



Decide on the treatment of relativistic effects



Decide on the employed power counting scheme



DimReg + Cutoff violate symmetries  $\Rightarrow$  Gradient Flow method Krebs, EE '24

Decide on the regularization scheme

sometimes unjustified or uncontrolled approximations are invoked



Extensive symbolic calculations have to be performed beyond N<sup>2</sup>LO (Bochum)



Derive nuclear forces/currents or use existing results  
(caution: off-shell consistency!)



Computationally heavy beyond 2N

Determine LECs from experimental data  
(Choice of data, frequentist vs Bayesian)



Error analysis (BUQEYE, ...)

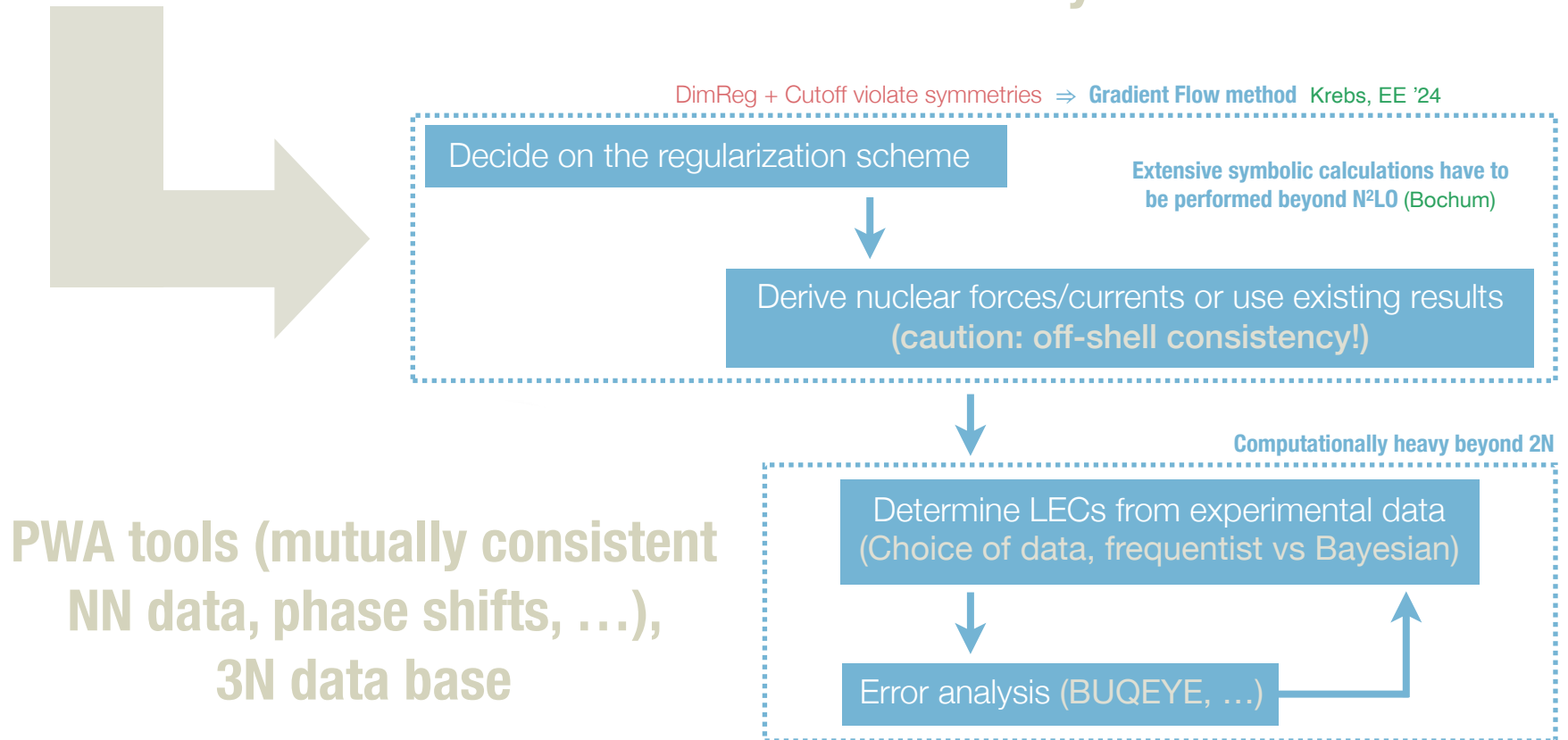


	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO ( $Q^2$ )		—	—
NLO ( $Q^2$ )		—	—
N <sup>2</sup> LO ( $Q^2$ )			—
N <sup>3</sup> LO ( $Q^2$ )			
N <sup>4</sup> LO ( $Q^2$ )			—



# Chiral architecture

A digital platform to provide off-shell consistent symbolic expressions (highly complex beyond N<sup>2</sup>LO) for 2N-, 3N- and 4N- forces and currents (electroweak, scalar), along with the documentation and appropriate  $\pi$ N LECs would be a valuable service to the community





# The landscape of chiral NN interactions

## Different regularizations (cutoff choices)

- fully nonlocal Entem, Machleidt, Nosyk 2017 (**Idaho**); Ekström et al. 2013-18 (**GO**): NNLO<sub>opt</sub>, NNLO<sub>sat</sub>, NNLO- $\Delta$
- semi-local EE, Krebs, Meißner 2015; Reinert, EE, Krebs 2018 (**LENPIC**)
- local Gezerlis et al. 2013; Piarulli et al., 2016 (**Norfolk models**); Saha, Entem, Machleidt, Nosyk 2023
- local, nonlocal + lattice Lee, Elhatisari, EE, Lähde, Meißner, Krebs et al. (**Nuclear Lattice EFT**)

## Highest available EFT order

- $N^4\text{LO}^+$ : **Low-Energy Nuclear Physics International Collaboration (LENPIC)** Idaho
- $N^3\text{LO}$ : **Norfolk, NLEFT**
- $N^2\text{LO}$ : **Gothenburg-Oak Ridge (GO)**

our  $N^4\text{LO}^+$  NN potentials are the only chiral EFT interactions on the market that provide a statistically satisfactory description of NN data below  $\pi$ -production threshold (i.e., qualify as a PWA)

## Degrees of freedom in the effective Lagrangian

- $\pi, N$ : **LENPIC, Idaho, GO, NLEFT**
- $\pi, N, \Delta$ : **Norfolk, GO**

## Strategy in the determination of LECs

- $\pi N$  from the Roy-Steiner analysis (not fitted),  $NN$  LECs from two-nucleon data  
**LENPIC, Idaho, Norfolk, NLEFT**
- LECs determined from a global fit to  $\pi N, NN, \text{nuclei, EoS}$  **GO**



# PWA of NN data

NN-OnLine



https://nn-online.org

24 November 2024

info@nn-online.org

Home

About NN-OnLine

Past, present, and future

NN interaction

YN interaction

nNN coupling constants

Publications

Code

Obituary  
J.J. de Swart

Physics in Nijmegen

## Welcome on NN-OnLine

NN-OnLine is devoted to the work on the baryon-baryon interaction of current and former members of the [Theoretical High Energy Physics Group](#) of the [Radboud University Nijmegen](#), the Netherlands. The nucleon-nucleon (NN) interaction is most visibly present on this site, but you will also find information about activities on the hyperon-nucleon (YN) interaction, antinucleon-nucleon (NN) interaction, and pion-nucleon (nN) interaction.

## News

### NN-OnLine is alive and kicking!

There is no way to deny that NN-OnLine has been slumbering a bit recent years. But it is still alive and kicking! In 2024 we will celebrate our 30<sup>th</sup> year of existence and you will definitely notice that!



— Data Analysis Center —  
Institute for Nuclear Studies  
THE GEORGE WASHINGTON UNIVERSITY  
WASHINGTON, D.C.

INS DAC Home  
Nucleon Nucleon  
INS Home  
SAID Start

Analysis Options  
Data Base  
Observables  
Partial Wave Amplitudes  
Compare Different Analyses  
Compare Fits to Data

## Nucleon Nucleon [Overview]

### Data Base

The data base contains on the order of 37K data points from elastic pp and np scattering.

### Observables

Relations for the observables in terms of several amplitude types are given in, for example, Bystricky, Lehar and Winternitz, Journal de Physique 39, 1 (1978).

### Partial Wave Amplitudes

Partial wave amplitudes are denoted by  $^{2S+1}L_J$  (unmixed waves) with S:spin, L:angular momentum and J:total spin. Mixed states,  $^{2S+1}(J-1)_J$ - $^{2S+1}(J+1)_J$  are denoted by  $c_j$ .

No regular updates,  
restricted functionality

OPEP + e.m. interactions  
phenomenology  
(No systematic errors!)

The screenshot shows the NN-OnLine website interface. On the left, there is a navigation menu with 'home', 'publications', and 'database'. The main content area features a photograph of three people in a meeting, followed by the text '2013 GRANADA DATABASE' and the names 'Rodrigo Navarro-Perez, Enrique Ruiz Arriola, and José Enrique Amaro Soriano'. Below this is the affiliation: 'Department of Atomic, Molecular and Nuclear Physics, Institute of Theoretical and Computational Physics, University of Granada'. On the right, there is a 'Phase Shifts' plot showing Neutron-Proton Phaseshift. SPIN=0. The plot has a y-axis from 0 to 170 and an x-axis from 0 to 300. Data points are shown as red circles with error bars, and a red curve represents the fit. A 'COMPUTING...' message is visible above the plot. Below the plot is a table of parameters:
 

R00	-1.8125	radius=10
l=00	-0.046875	1.062; 202425/0.5
R01	-1.8181250095367432	3.0764; 537075/0.5
l=01	-0.46500000834465027	75.0763; 48071/0.5
lmax=100.0		110.0195; 778995/0.5
		220.0153; 3863/0.5
		330.0148; 31525/0.5
		440.0144; 2513770.5
		550.0142; 68550000000004/0.5
		660.0135; 86135/0.5
		770.0133; 0.669464999

 At the bottom right, there is a control panel for the search. It includes a 'Search NN provider' button, a 'Channellet' dropdown set to 'pp', an 'Observable' dropdown set to 'all', and an 'Energy (MeV)' range set to '0 < E < 350'. There are checkboxes for 'Write to file: ppdata.txt' and 'Output format: separate data'. Below this are 'Order by: energy' and checkboxes for 'Include star (\*) data' and 'Include excluded data'. A 'Storing Data' button is also visible at the bottom of the plot area.

# PWA of NN data using chiral EFT

About 3000 proton-proton + 5000 neutron-proton data below 350 MeV.

Selection of mutually consistent data achieved via the  $3\sigma$ -criterion:

Nijmegen '94, Granada '13, Bochum '21 data bases.

Longest-range interaction in all PWAs:  $1\gamma + 2\gamma + 1\pi$

(complicated treatment of e.m. interactions beyond Coulomb...)

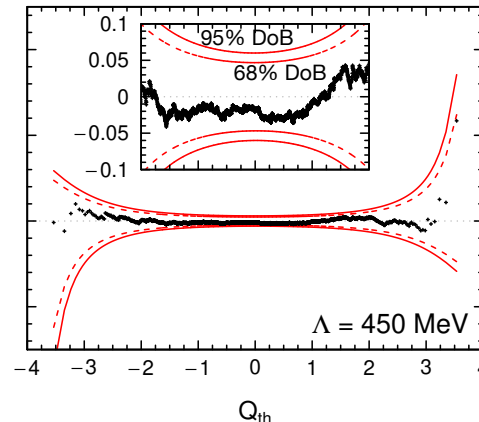
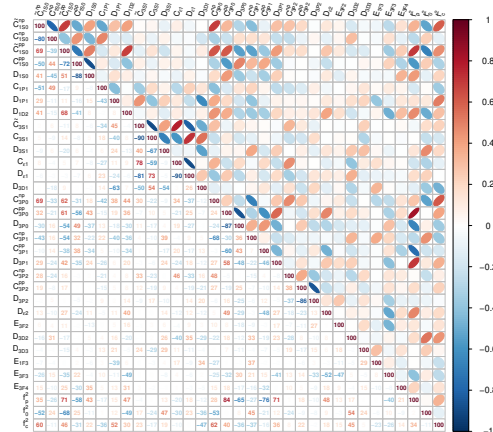
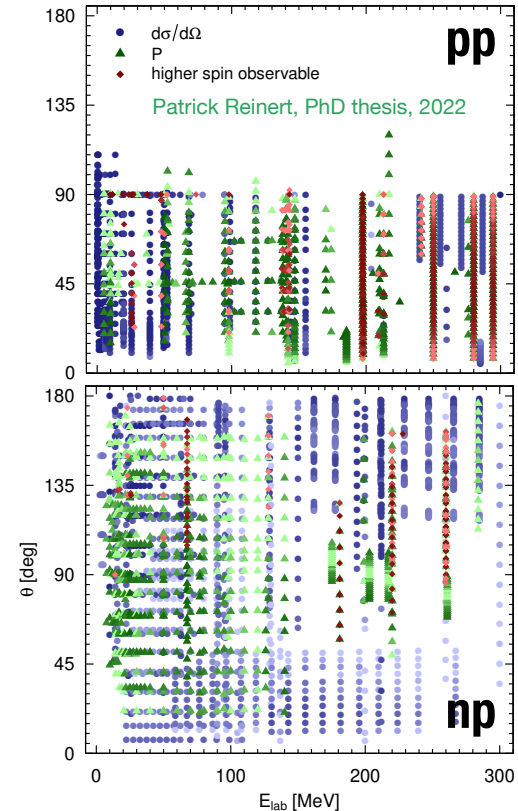
Shorter-range interactions:

Nijmegen: Phenomenological boundary conditions

Granada: Coarse graining ( $\delta$ -shells)

Bochum: Chiral EFT Reinert, Krebs, EE, EPJA (18); PRL 126 (21)

$E_{\text{lab}}$ bin	CD Bonn	Nijm I	Nijm II	Reid 93	Bochum $N^4\text{LO}^+$
0-300 MeV	1.042	1.061	1.070	1.078	1.013

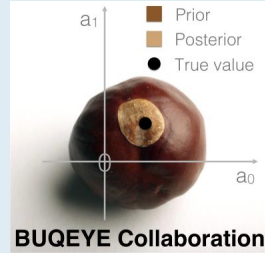


← Tail-sensitive normality test  
Aldor-Noiman '13

# Truncation uncertainty

Bayesian approach to truncation error estimation Furnstahl, Phillips, Klco, Melendez, EE, ...

$$X(p) = \underbrace{X^{\text{LO}}}_{\text{observable}} + \underbrace{\Delta X^{\text{NLO}} + \Delta X^{\text{N}^2\text{LO}} + \dots + \Delta X^{\text{N}^k\text{LO}}}_{\text{known from explicit calculations}} + \underbrace{\Delta X^{\text{N}^{k+1}\text{LO}} + \dots}_{\delta X^{(k)} \text{ to be estimated}}$$



Idea: Bayesian inference of the size of higher-order contributions from the known low-order ones

$$= X^{\text{LO}} (1 + c_2 Q^2 + c_3 Q^3 + \dots + c_k Q^k + c_{k+1} Q^{k+1} + \dots)$$

⇒ compute  $p(\delta X^{(k)} | \{c_2, \dots, c_k\}, \text{model})$  ⇒ truncation error (DoB intervals)

Assumptions (model): Same expansion for  $X$  as for the Hamiltonian  $H$ ; expansion parameter  $Q$ ;  $\forall c_i$  obey the same probability distribution

BUQEYE Software (Python, Jupiter): correlated observables, diagnostics of EFT models, emulators

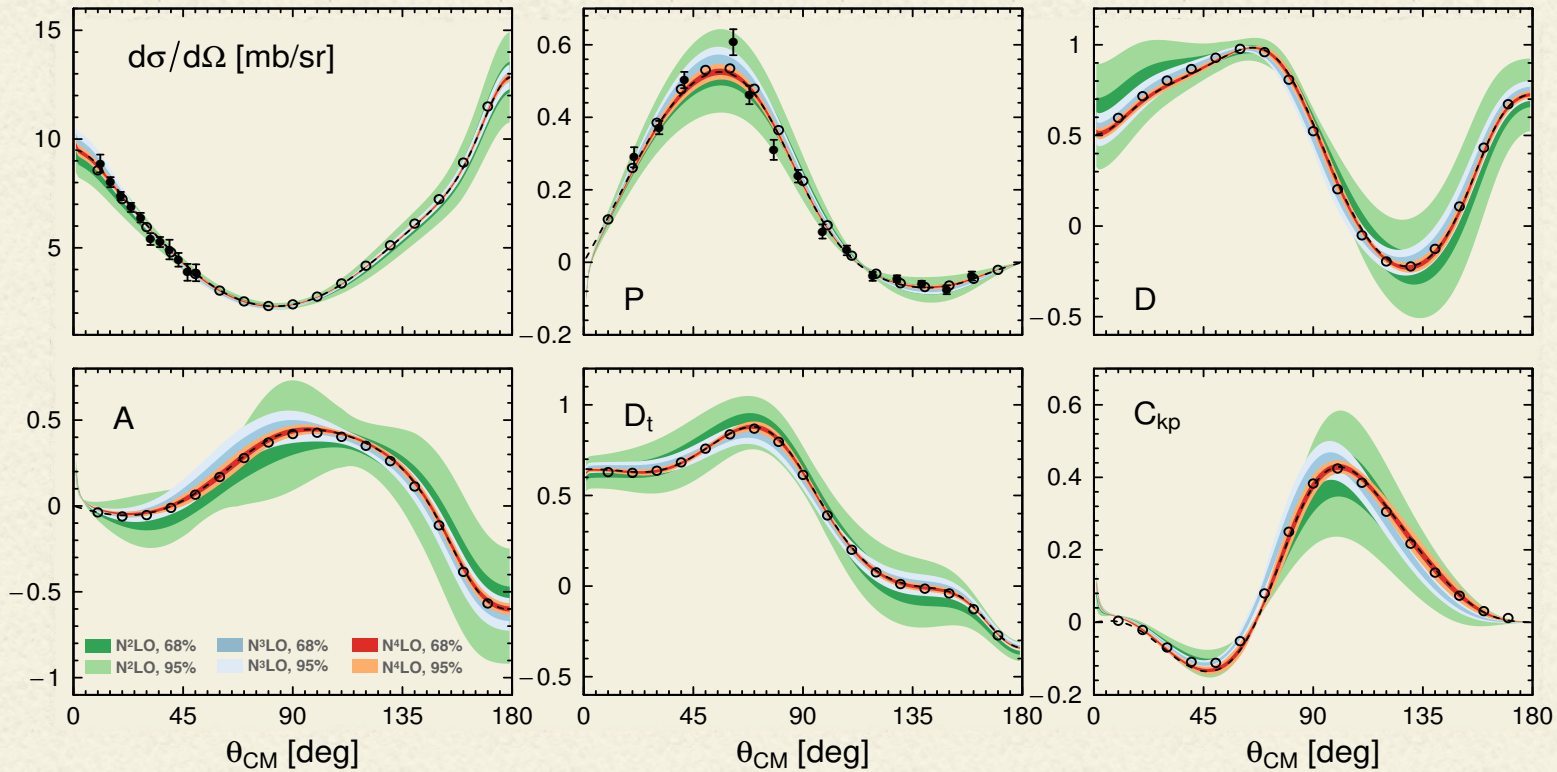
Alternative: Explicit marginalization over higher orders (samples of potentials) S. Heihoff, EE, in progress



# Truncation uncertainty

Bay

$a_1$  ↑  
 ■ Prior  
 ■ Posterior  
 ● True value



laboration

nes

A

Q;

B

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A

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 BUQEYE Collaboration

Members   Publications   Talks   Software

## What is BUQEYE?

Bayesian Uncertainty Quantification: Errors in Your EFT

**BAND**  
 Bayesian Analysis of Nuclear Dynamics

Check out the new tools and examples in [BAND Framework v0.4!](#)

See the [BAND Manifesto](#) in Journal of Physics G1 (or on [arXiv](#))

# Matching nuclear $\chi$ EFT to lattice QCD

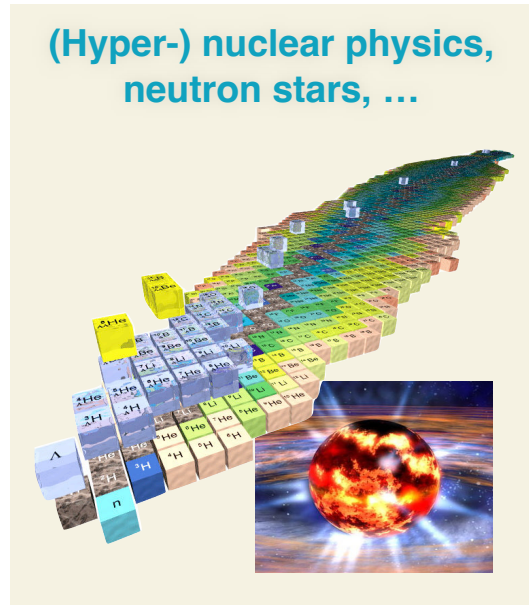
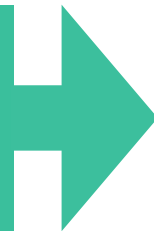
**Lattice-QCD**  
input for few-B systems  
(crucial for  $S \neq 0$ , BSM MEs and  
variable  $m_q$ )



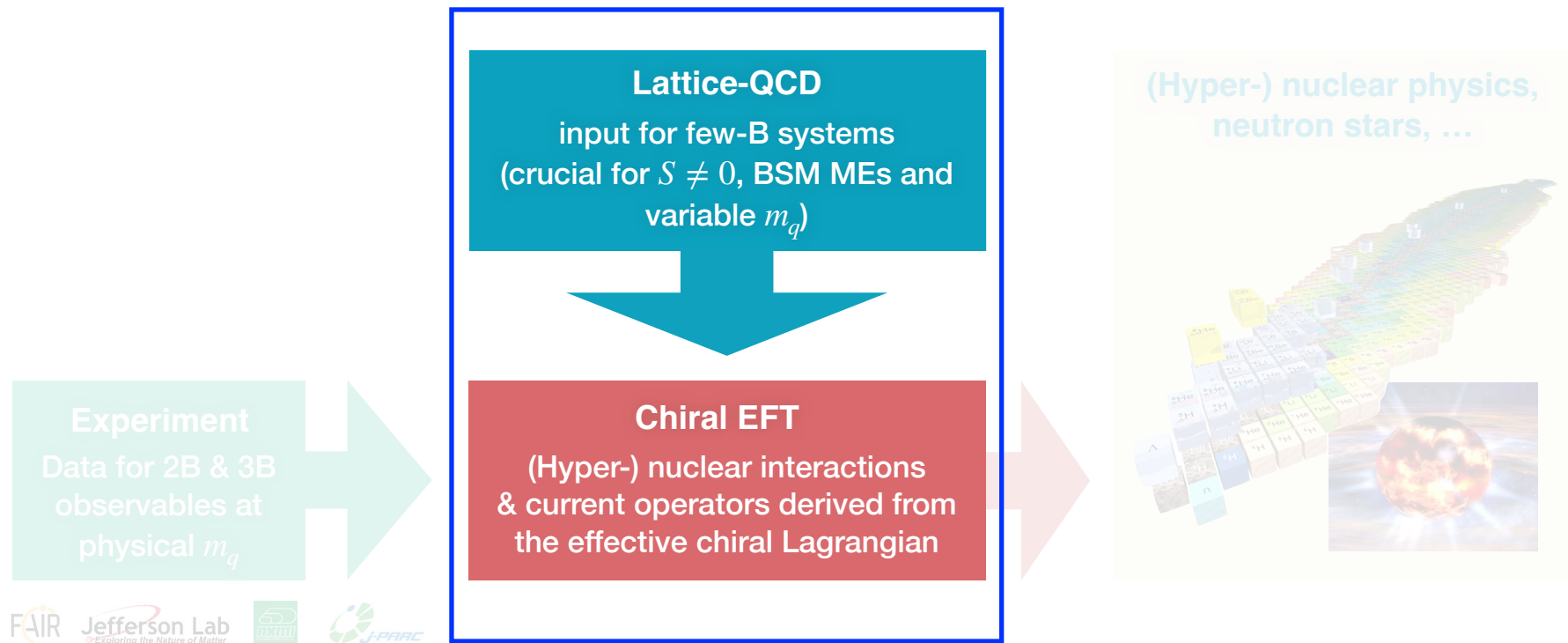
**Chiral EFT**  
(Hyper-) nuclear interactions  
& current operators derived from  
the effective chiral Lagrangian



**Experiment**  
Data for 2B & 3B  
observables at  
physical  $m_q$



# Matching nuclear $\chi$ EFT to lattice QCD



## Finite volume energy spectra as an efficient interface between lattice-QCD and chiral EFT

[Lu Meng](#), EE, JHEP 10 (21); [Lu Meng](#), Baru, EE, Filin, Gasparyan, PRD 109 (24)

- infinite- $V$  extrapolations without Lüscher
- solves the t-channel cut problem
- partial wave mixing included

*known function of FV energies*

$$\det \left[ M_{ln,l'n'}^{(\Gamma,\mathbf{P})} - \delta_{ll'} \delta_{nn'} \cot \delta_l \right] = 0$$

*Lüscher's quantization condition is not valid below the left-hand cut*

# Democratizing chiral EFT: A long-term vision

