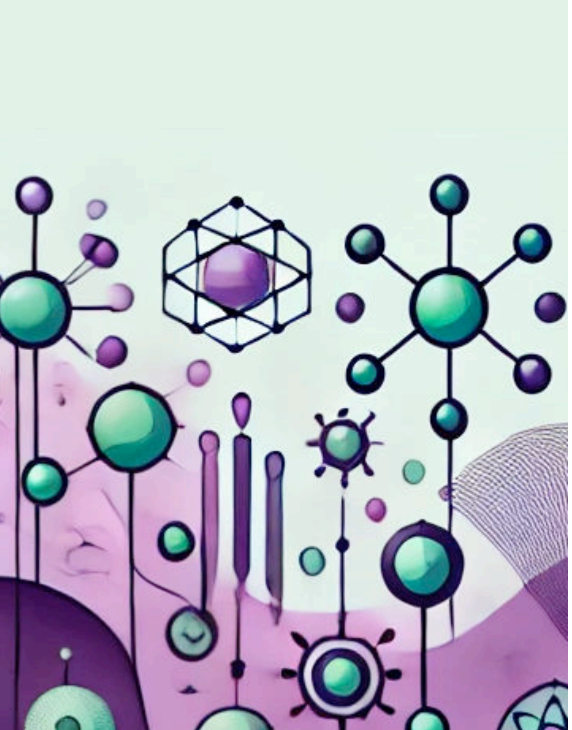
Evgeny Epelbaum, RUB

“Democratizing Models“ Workshop, RUB, 25.-26. Nov 2024

# Nuclear Physics: From Models to Theory



“Democracy is not a fragile flower; still it needs cultivating.”
—Ronald Reagan

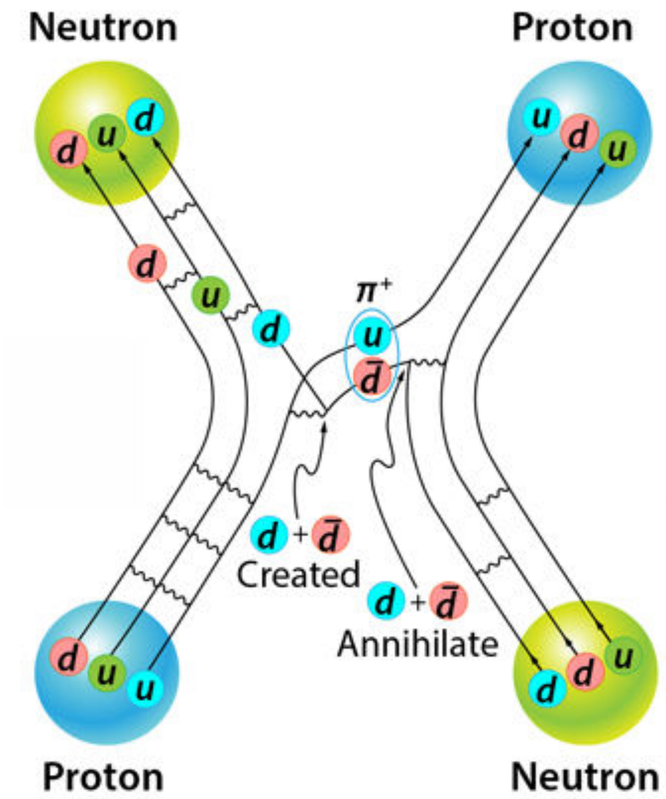
European Research Council

Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen

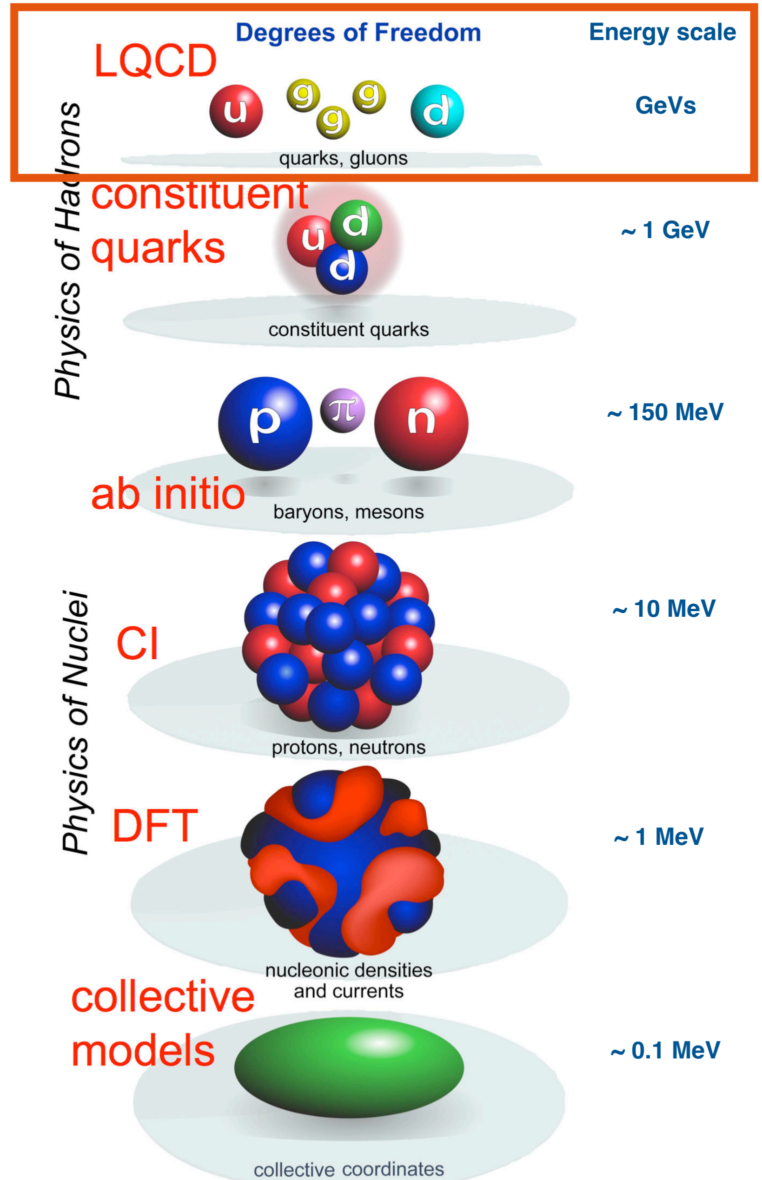
ERATO Exploratory Research for Advanced Technology

# Degrees of freedom

## Nuclear force viewed at a high resolution



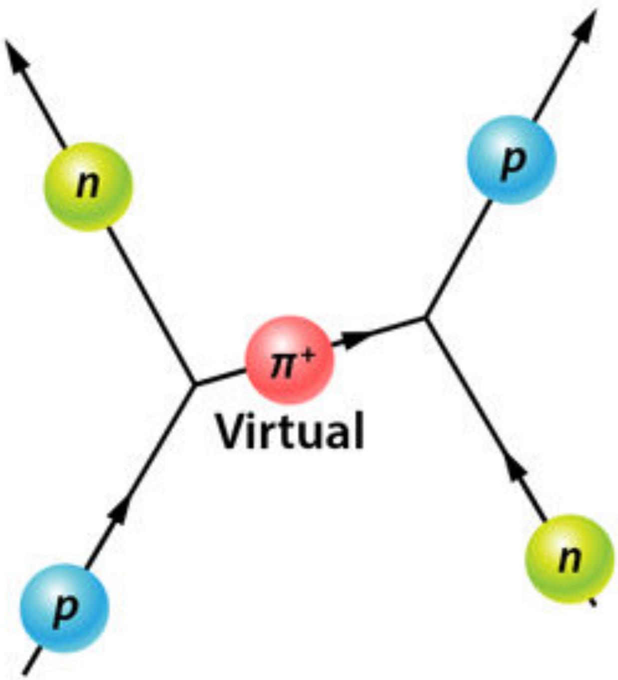

(from physics.aps.org)

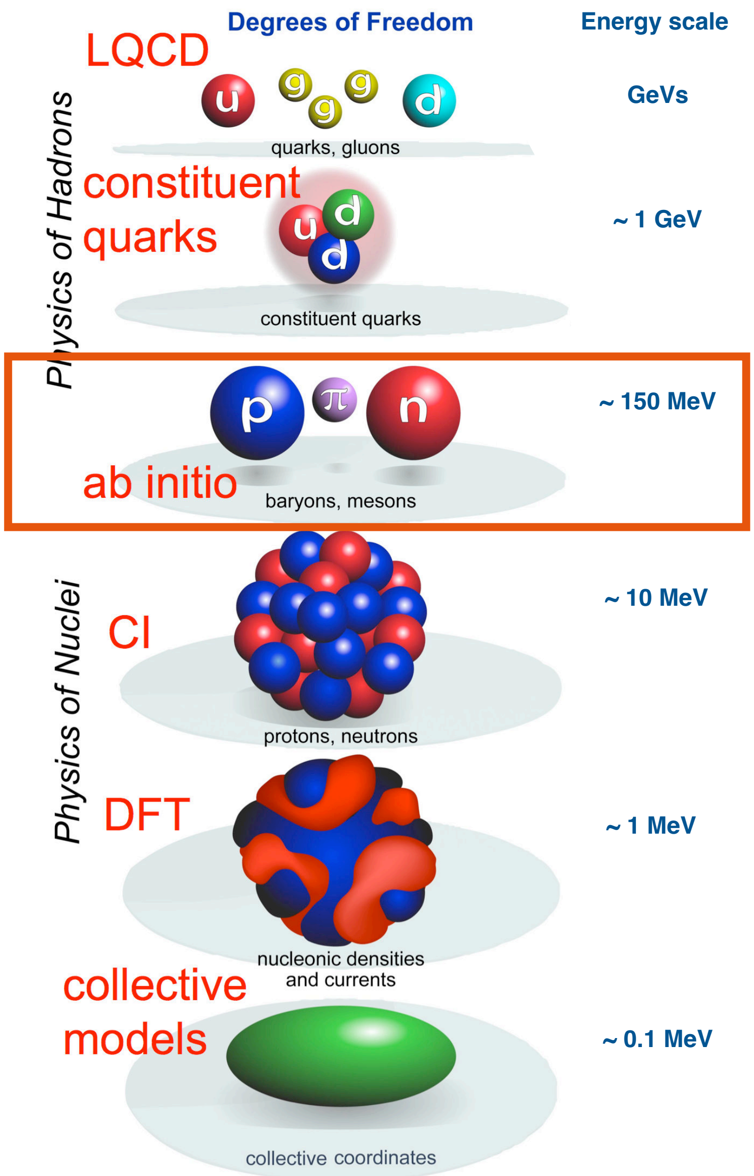



(from 2007 NSAC Long-Range Plan)

# Degrees of freedom

Nuclear force viewed at a
low resolution




(from 2007 NSAC Long-Range Plan)

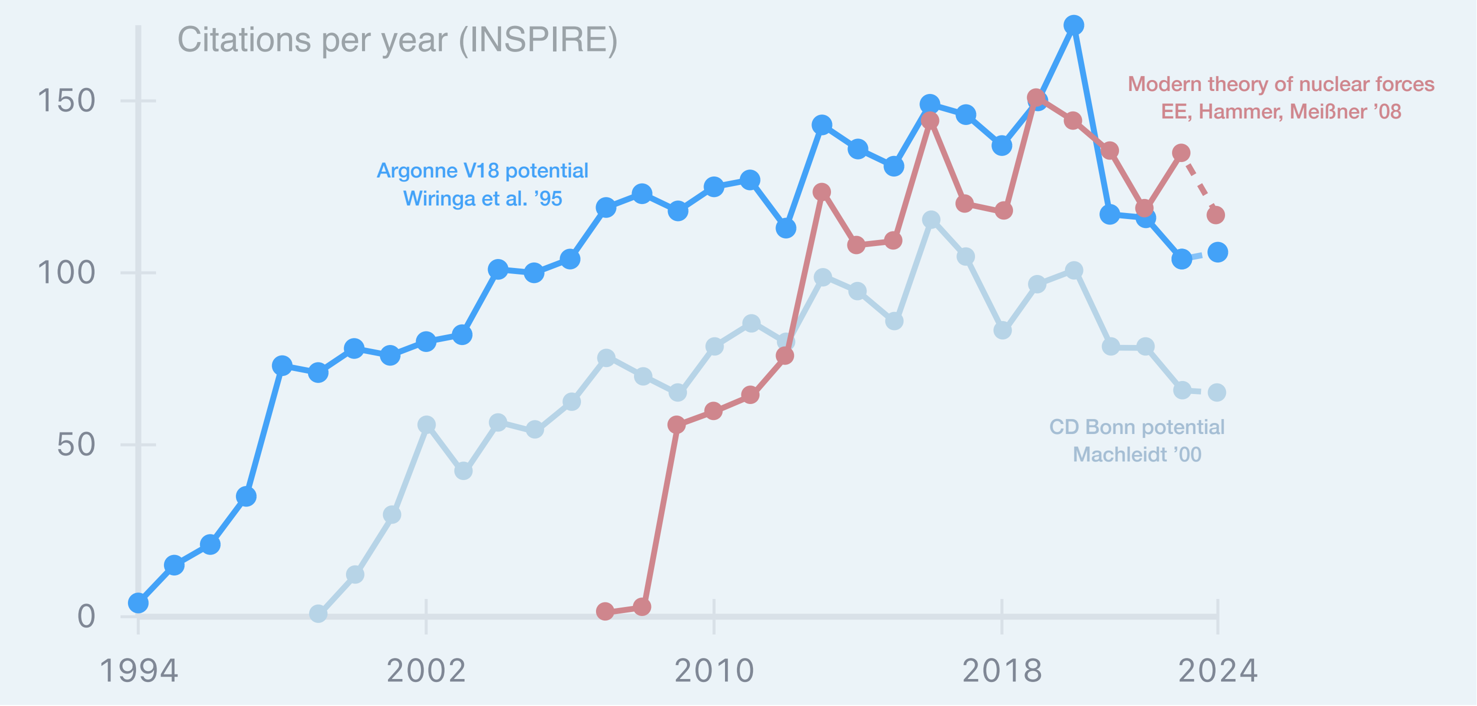
# Chiral EFT: A revolution in nuclear physics

1979: Weinberg, *Phenomenological Lagrangians*, Physica A96 (1979)

1984: Gasser, Leutwyler, *ChPT to one loop*, Annals Phys. 158 (1984)

1991: Jenkins, Manohar, *Baryon ChPT using a heavy fermion Lagrangian*, Phys. Lett. B255 (1991)

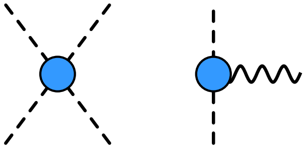
1990: Weinberg, *Nuclear forces from chiral Lagrangians,* Phys. Lett. B251 (1990)



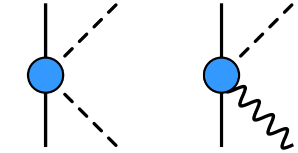

- (In principle) **model-independent** QFT-based approach with a clear relationship to QCD/Standard Model
- **Systematically improvable** (by going to higher orders in the EFT expansion)
- **Quantifiable accuracy** (truncation uncertainty)

# Chiral symmetry ↔ Nuclear architecture

## GB dynamics
Weinberg, Gasser, Leutwyler, ...



## πN dynamics
Bernard-Kaiser-Meißner et al.



← Chiral Perturbation Theory →

$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \ldots \frac{1}{3}$$

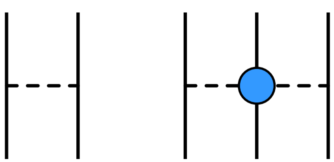
**Effective Lagrangian:**

$$\mathcal{L}_\pi = \frac{F^2}{4} \mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots ,$$

$$\mathcal{L}_{NN} = -\frac{1}{2} C_S (\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

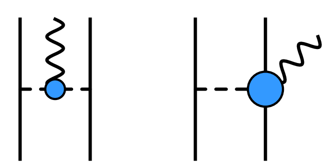
## Nuclear forces
Weinberg, van Kolck, Kaiser, EGM, ...



Enhanced ladder graphs are re-summed by solving the many-body Schrödinger equation

$$\left[\left(\sum_{i=1}^{A} \frac{\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right]|\Psi\rangle = E|\Psi\rangle$$

## Nuclear currents
Park et al, Bochum-Bonn, JLab-Pisa



# Chiral symmetry ↔ Nuclear architecture

Two challenges:

— derivation/construction of nuclear interactions

— solution of the quantum mechanical A-body problem

# The QM A-body problem

2N: Rewrite to the integral Lippmann-Schwinger eq.: $t = V_{2N} + V_{2N} G_0 t$ — easy to solve in p-space.

3N: Faddeev equations, e.g. for elastic Nd scattering:

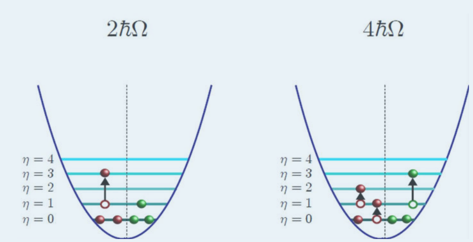
$$\underbrace{T\phi}_{\text{asymptotic Nd state}} = tP\phi + (1+tG_0)\underbrace{V_{3N}^{(1)}}_{\text{symmetric under exchange of nucleons 2,3}}(1+P)\phi + tPG_0T\phi + (1+tG_0)V^{(1)}(1+\underbrace{P}_{P_{12}P_{23}+P_{13}P_{23}})G_0T\phi$$

Solved iteratively in partial-waves (for fixed $J$, $T \sim 10^5 \times 10^5$), few minutes on 1 CPU.

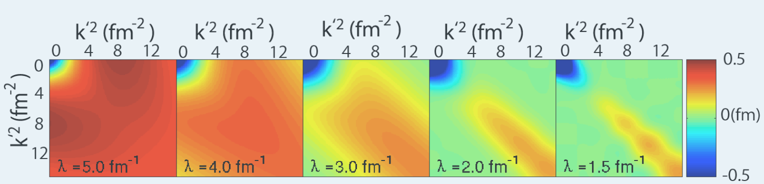
4N: Yakubovsky equations. Take several hours on JURECA@FZJ to solve for bound state; only 3 groups can do scattering (with large restrictions)

>4N: So far only (mainly) possible for bound states. E.g., the No-Core-Shell-Model:

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle, \quad |\Psi_i\rangle = \sum_{n=0}^{\infty} A_n^i |\Phi_n\rangle, \quad |\Phi_n\rangle = \underbrace{[a_\alpha^\dagger \ldots a_\zeta^\dagger]_n|0\rangle}_{n\,=\,1,2,\ldots,10^{12}\text{ or more}}$$



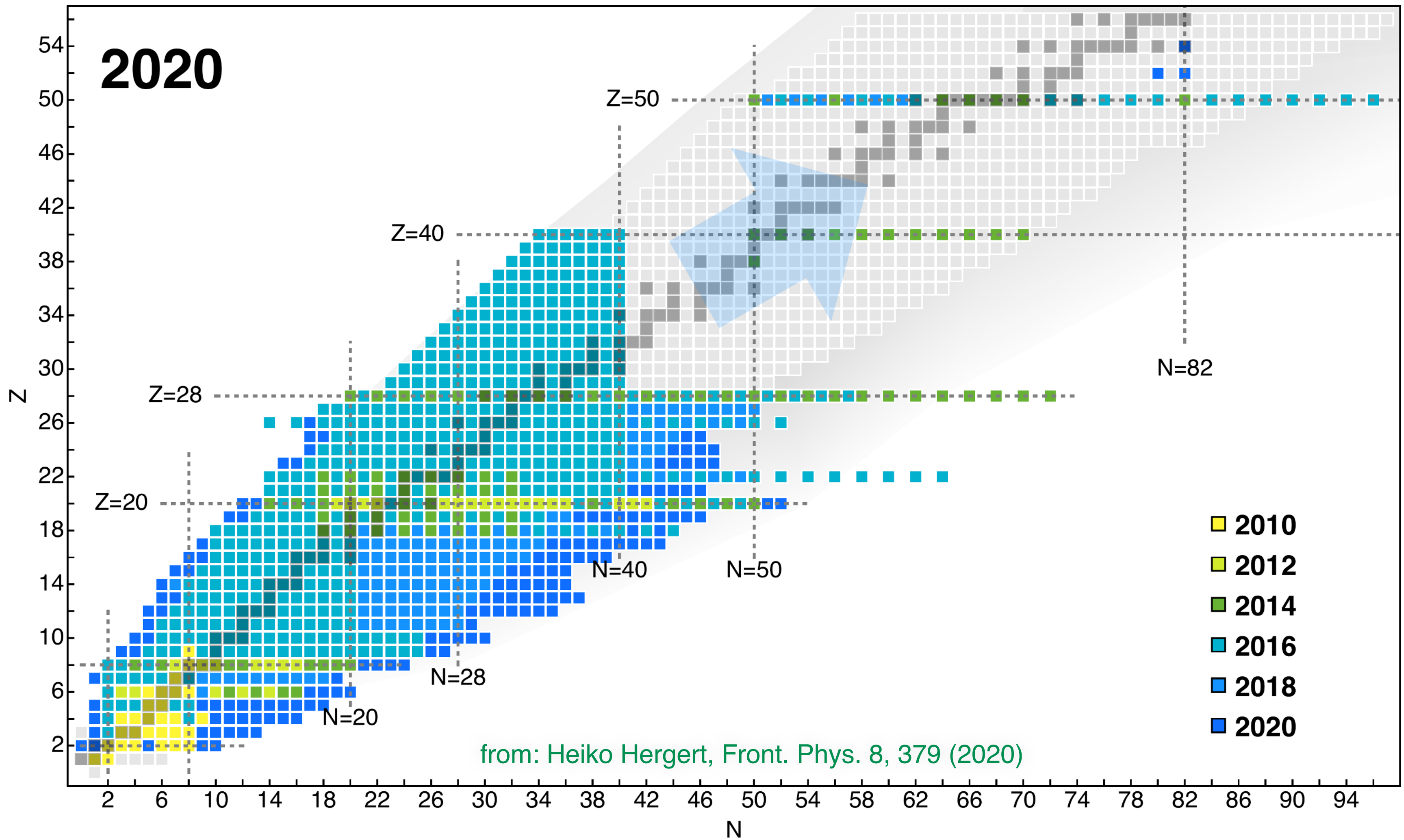

⇒ sparse matrix $H_{mn} = \langle\Phi_m|H|\Phi_n\rangle$ diagonalization (Lánczos), extrapolation in $N_{max}$, pre-diagonalization of $H$ (SRG):




$$H(s) = U(s)HU^\dagger(s) \;\Rightarrow\; \frac{dH_s}{ds} = \underbrace{[\eta_s, H_s]}_{\text{e.g., } \eta_s = [T_{\text{kin}}, H_s]}, \quad H_{s\to 0} = H$$

Other ab-initio methods: NL-EFT, QMC, CC expansion, IM-SRG, Lorentz IT, Green's functions, …

# *Ab initio* many-body calculations



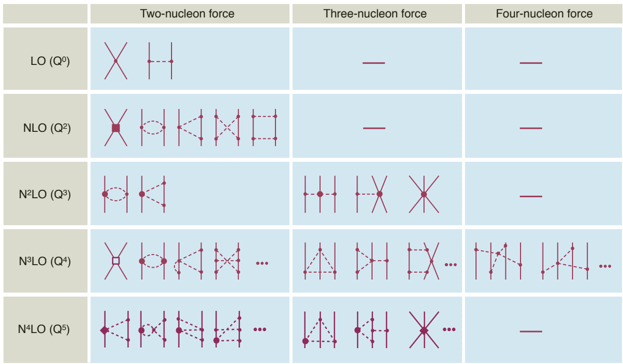

The main bottleneck in developing nuclear physics into precision and predictive science is the accuracy of the interaction (especially of 3N forces).

# Chiral architecture

Decide on the choice of DoF ($N$, $N+\pi$, $N+\Delta+\pi$) in $\mathscr{L}_{\rm eff}$

Decide on the treatment of relativistic effects

Decide on the employed power counting scheme




DimReg + Cutoff violate symmetries ⇒ **Gradient Flow method** Krebs, EE '24

Decide on the regularization scheme

**Extensive symbolic calculations have to be performed beyond N²LO** (Bochum)

sometimes unjustified or uncontrolled approximations are invoked

Derive nuclear forces/currents or use existing results
**(caution: off-shell consistency!)**

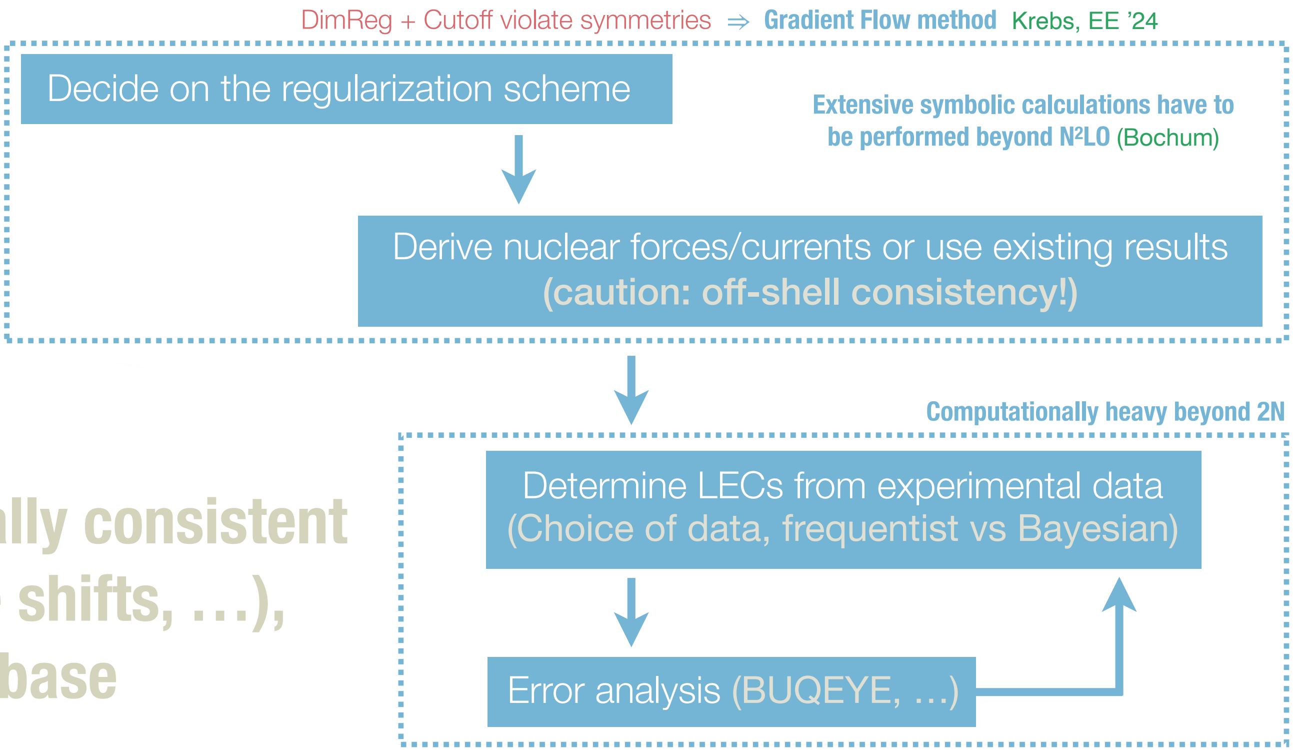
**Computationally heavy beyond 2N**

Determine LECs from experimental data
(Choice of data, frequentist vs Bayesian)

Error analysis (BUQEYE, …)

# Chiral architecture

A digital platform to provide off-shell consistent symbolic expressions (highly complex beyond $N^2LO$) for 2N-, 3N- and 4N-forces and currents (electroweak, scalar), along with the documentation and appropriate $\pi$N LECs would be a valuable service to the community




PWA tools (mutually consistent NN data, phase shifts, …), 3N data base

# The landscape of chiral NN interactions

## Different regularizations (cutoff choices)

— fully nonlocal Entem, Machleidt, Nosyk 2017 (Idaho); Ekström et al. 2013-18 (GO): $\text{NNLO}_{\text{opt}}$, $\text{NNLO}_{\text{sat}}$, NNLO-Δ

— semi-local EE, Krebs, Meißner 2015; Reinert, EE, Krebs 2018 (LENPIC)

— local Gezerlis et al. 2013; Piarulli et al., 2016 (Norfolk models); Saha, Entem, Machleidt, Nosyk 2023

— local, nonlocal + lattice Lee, Elhatisari, EE, Lähde, Meißner, Krebs et al. (Nuclear Lattice EFT)

## Highest available EFT order

— $\text{N}^4\text{LO}^+$: Low-Energy Nuclear Physics International Collaboration (LENPIC), Idaho

— $\text{N}^3\text{LO}$: Norfolk, NLEFT

— $\text{N}^2\text{LO}$: Gothenburg-Oak Ridge (GO)

our $\text{N}^4\text{LO}^+$ NN potentials are the only chiral EFT interactions on the market that provide a statistically satisfactory description of NN data below $\pi$-production threshold (i.e., qualify as a PWA)

## Degrees of freedom in the effective Lagrangian

— $\pi, N$: LENPIC, Idaho, GO, NLEFT

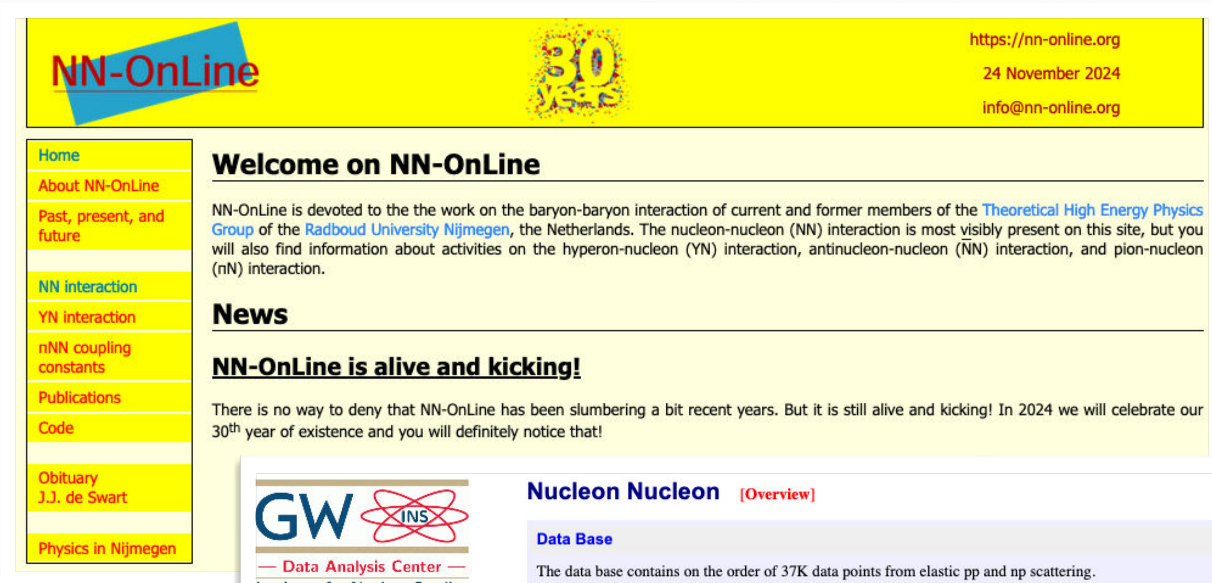
— $\pi, N, \Delta$: Norfolk, GO

## Strategy in the determination of LECs

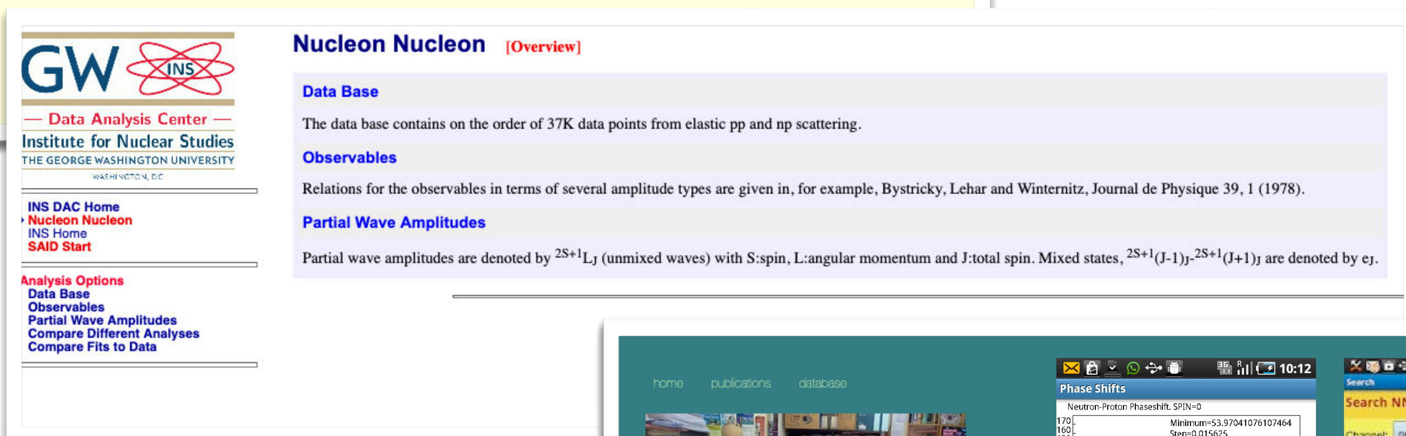
— $\pi N$ from the Roy-Steiner analysis (not fitted), $NN$ LECs from two-nucleon data
LENPIC, Idaho, Norfolk, NLEFT

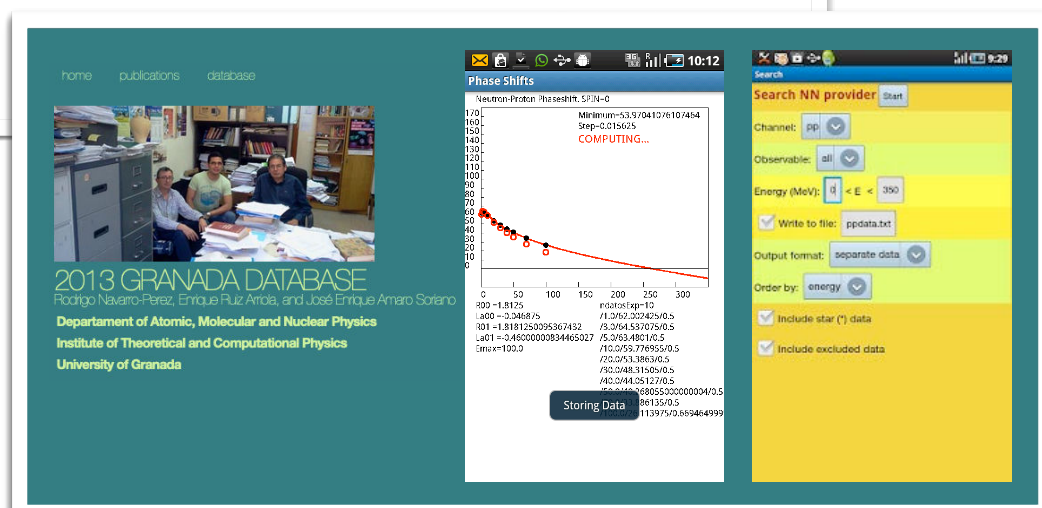
— LECs determined from a global fit to $\pi N$, $NN$, nuclei, EoS GO

# PWA of NN data




No regular updates, restricted functionality




OPEP + e.m. interactions phenomenology (No systematic errors!)

# PWA of NN data using chiral EFT

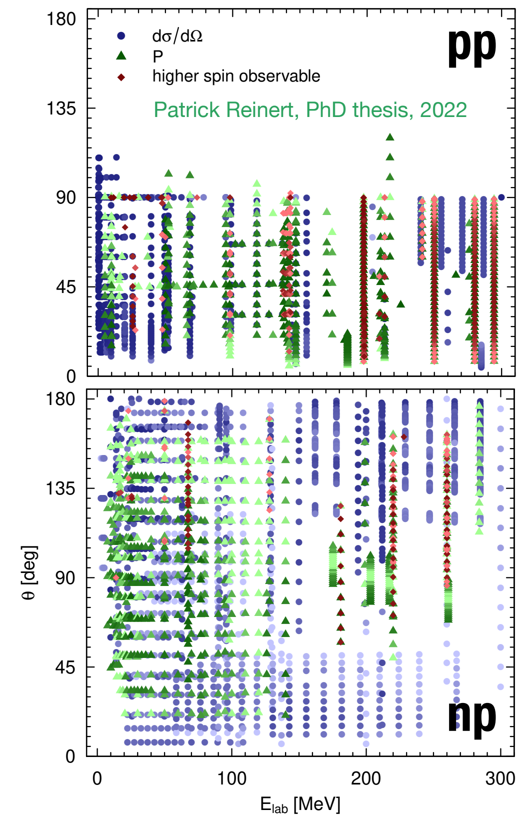
About 3000 proton-proton + 5000 neutron-proton data below 350 MeV.
Selection of mutually consistent data achieved via the $3\sigma$-criterion:
Nijmegen '94, Granada '13, Bochum '21 data bases.

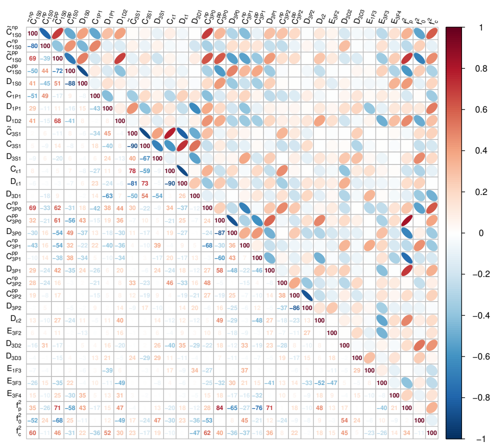
Longest-range interaction in all PWAs: $1\gamma + 2\gamma + 1\pi$
(complicated treatment of e.m. interactions beyond Coulomb…)

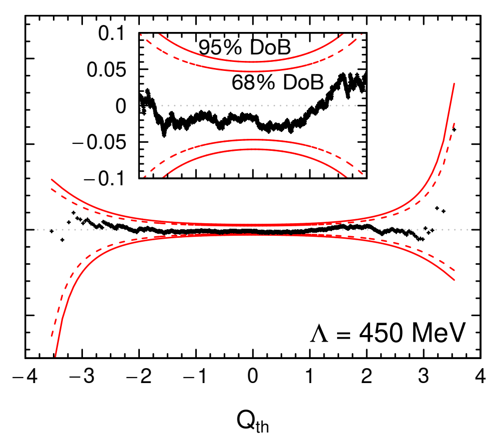
Shorter-range interactions:

- Nijmegen: Phenomenological boundary conditions
- Granada: Coarse graining ($\delta$-shells)
- Bochum: Chiral EFT Reinert, Krebs, EE, EPJA (18); PRL 126 (21)

| $E_{lab}$ bin | CD Bonn | Nijm I | Nijm II | Reid 93 | Bochum N$^4$LO$^+$ |
|---|---|---|---|---|---|
| 0-300 MeV | 1.042 | 1.061 | 1.070 | 1.078 | 1.013 |

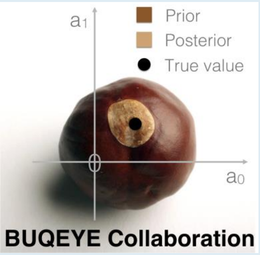





← Tail-sensitive normality test
Aldor-Noiman '13

# Truncation uncertainty




Bayesian approach to truncation error estimation Furnstahl, Phillips, Klco, Melendez, EE, …

$$\underbrace{X(p)}_{\text{observable}} = \underbrace{X^{\mathrm{LO}} + \Delta X^{\mathrm{NLO}} + \Delta X^{\mathrm{N^2LO}} + \ldots + \Delta X^{\mathrm{N^kLO}}}_{\text{known from explicit calculations}} \underbrace{+ \Delta X^{\mathrm{N^{k+1}LO}} + \ldots}_{\delta X^{(k)} \text{ to be estimated}}$$

Idea: Bayesian inference of the size of higher-order contributions from the known low-order ones

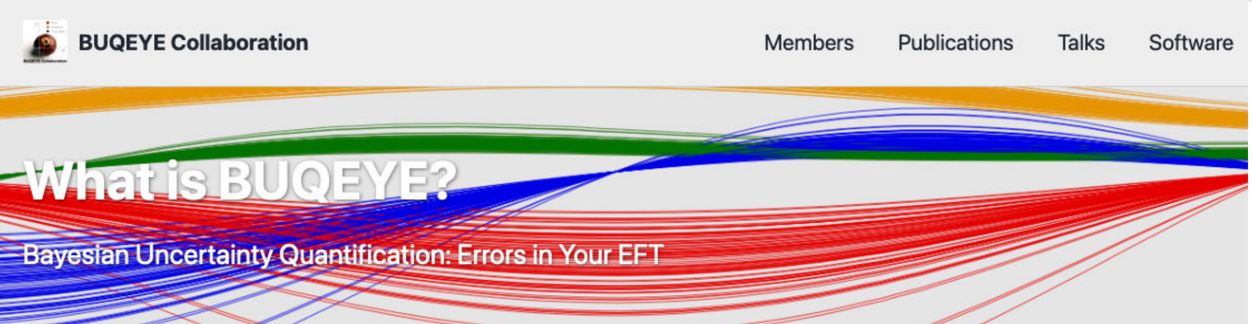
$$= X^{\mathrm{LO}} \left(1 + c_2 Q^2 + c_3 Q^3 + \ldots + c_k Q^k + c_{k+1} Q^{k+1} + \ldots\right)$$

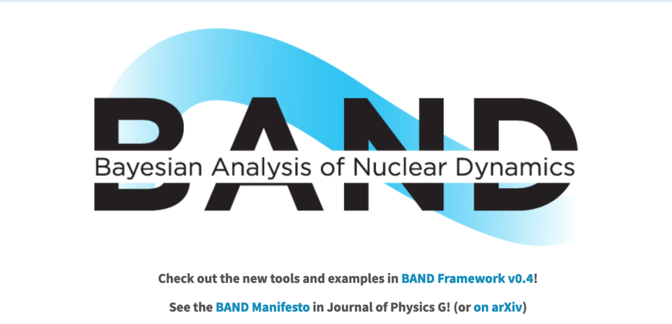
$\Rightarrow$ compute $\mathrm{p}\left(\delta X^{(k)} \,|\, \{c_2, \ldots, c_k\}, \mathrm{model}\right)$ $\Rightarrow$ truncation error (DoB intervals)

Assumptions (model): Same expansion for $X$ as for the Hamiltonian $H$; expansion parameter $Q$; $\forall c_i$ obey the same probability distribution

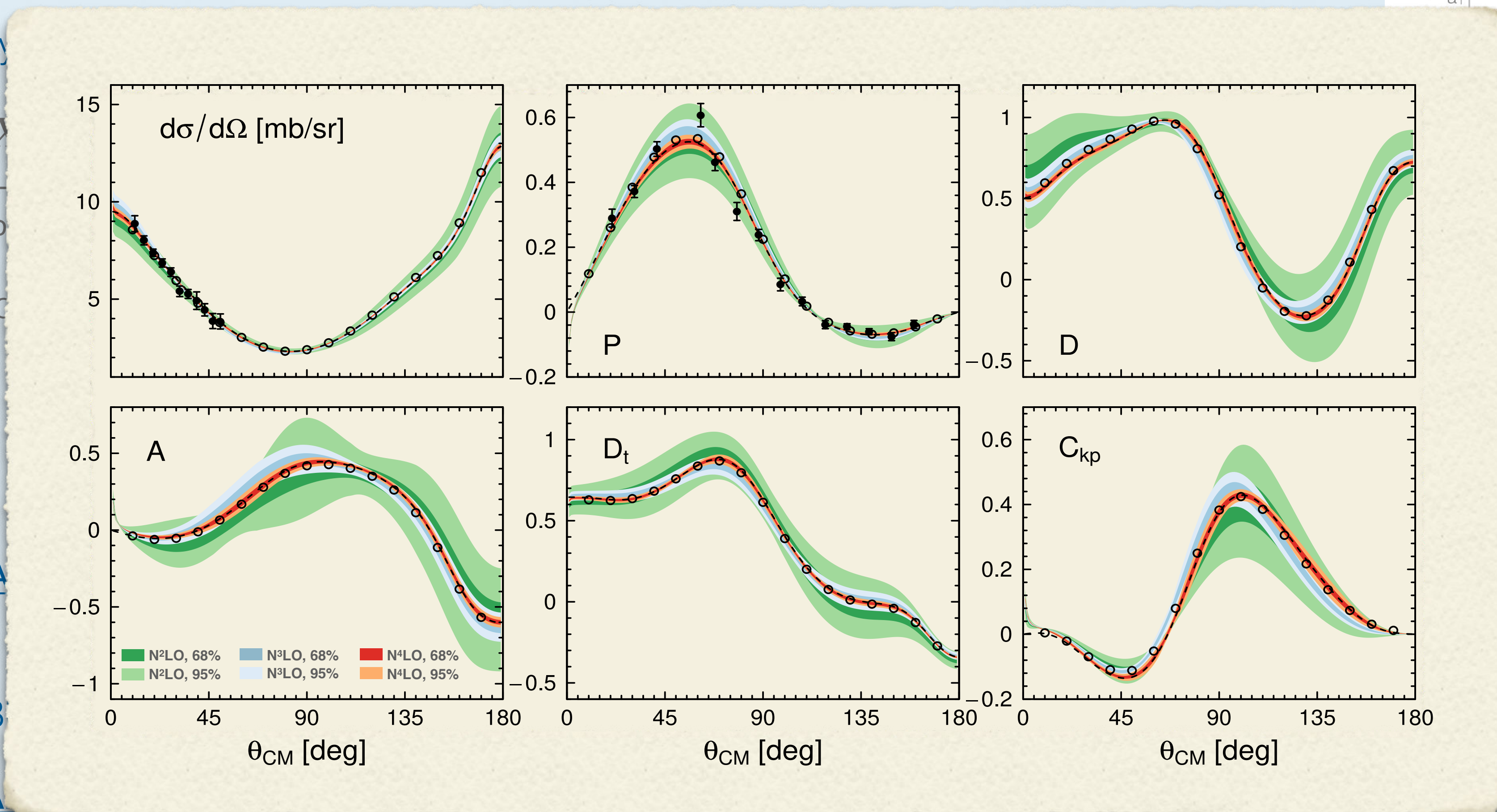
BUQEYE Software (Python, Jupiter): correlated observables, diagnostics of EFT models, emulators

Alternative: Explicit marginalization over higher orders (samples of potentials) S. Heihoff, EE, in progress

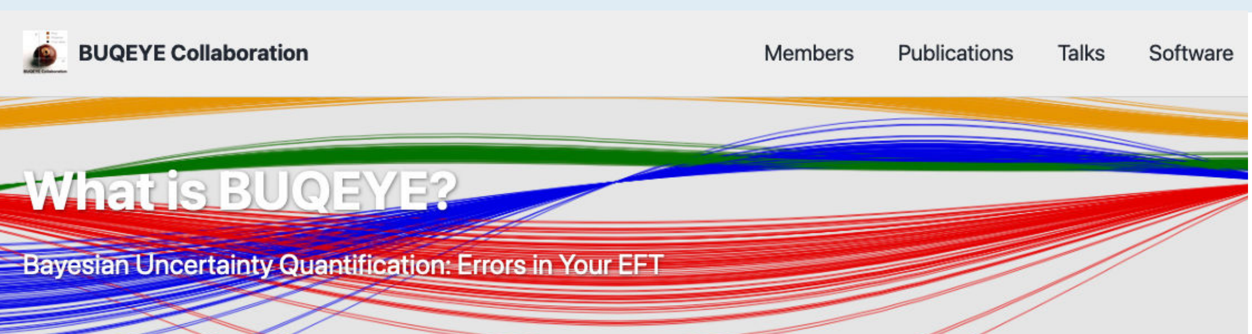


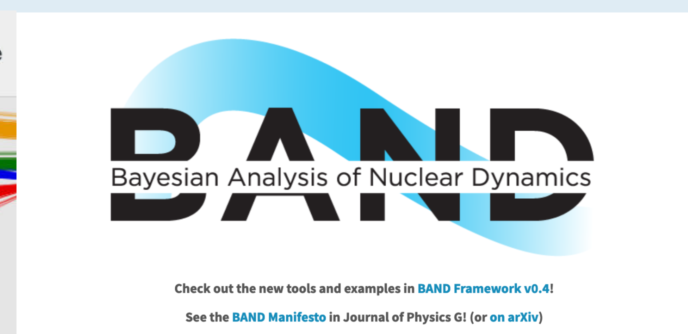



# Truncation uncertainty







# Matching nuclear $\chi$EFT to lattice QCD

**Lattice-QCD**

input for few-B systems
(crucial for $S \neq 0$, BSM MEs and variable $m_q$)

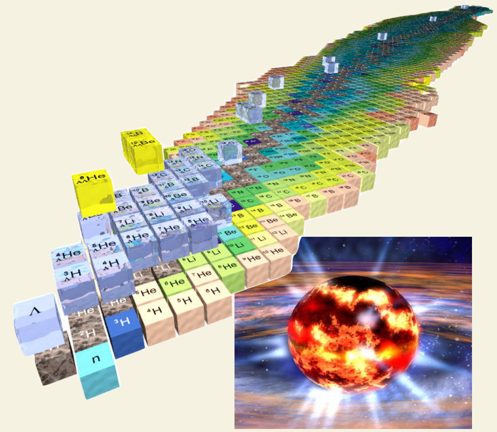
**Experiment**

Data for 2B & 3B observables at physical $m_q$

**Chiral EFT**

(Hyper-) nuclear interactions
& current operators derived from the effective chiral Lagrangian

**(Hyper-) nuclear physics, neutron stars, …**



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**Chiral EFT**

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FAIR Jefferson Lab J-PARC

**(Hyper-) nuclear physics, neutron stars, …**



Finite volume energy spectra as an efficient interface between lattice-QCD and chiral EFT

Lu Meng, EE, JHEP 10 (21); Lu Meng, Baru, EE, Filin, Gasparyan, PRD 109 (24)

— infinite-V extrapolations without Lüscher

— solves the t-channel cut problem

— partial wave mixing included

*known function of FV energies*

$$\det \left[ M^{(\Gamma, \boldsymbol{P})}_{ln,l'n'} - \delta_{ll'}\delta_{nn'} \cot \delta_l \right] = 0$$

*Lüscher's quantization condition is not valid below the left-hand cut*

# Democratizing chiral EFT: A long-term vision