Thermal Effects on Dark Boson Stars: Stellar Structure and Formation

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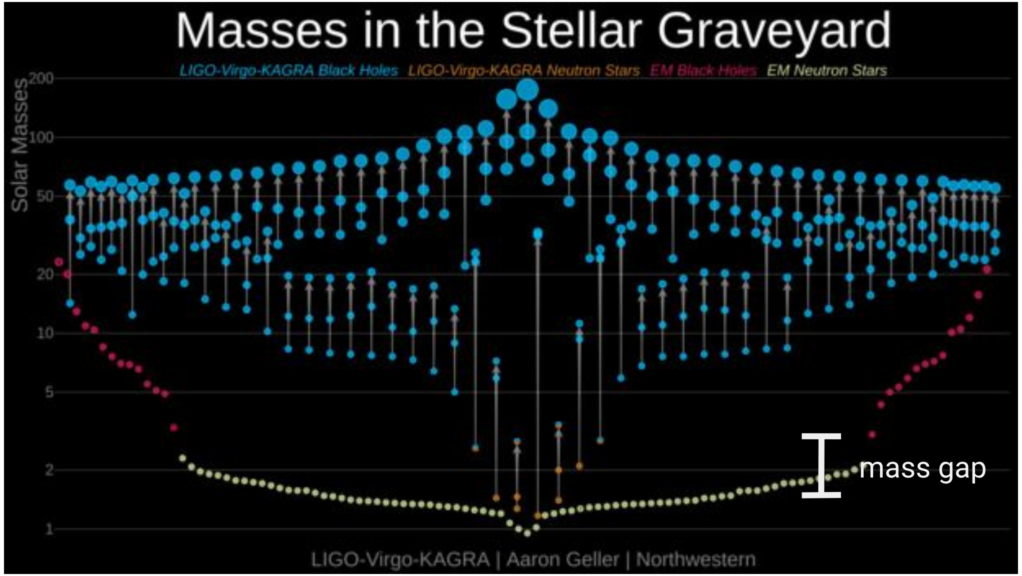
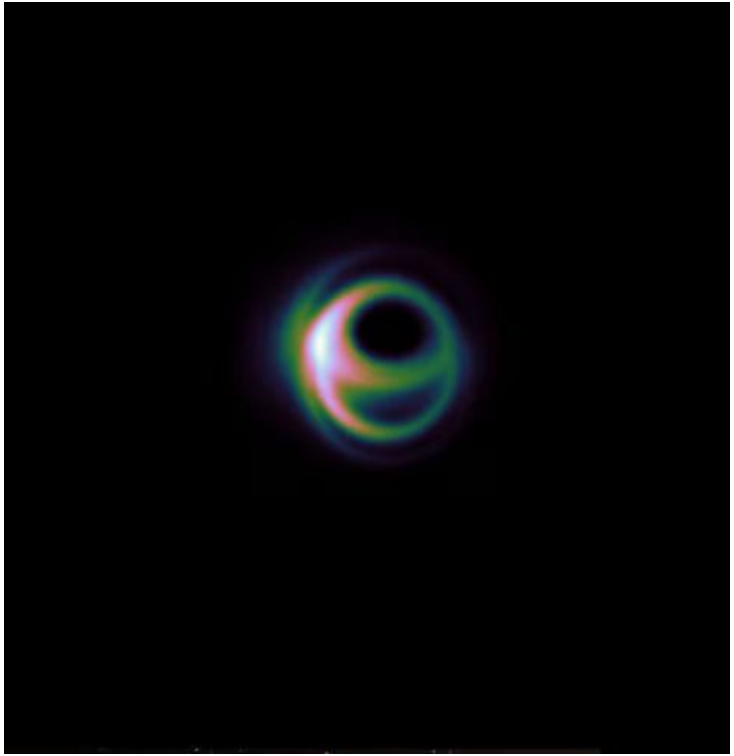
The Big Picture

What can we learn about the distribution of matter in our universe?

- ➔ What is the nature of **dark matter**?
- ➔ New observational technology: **gravitational waves.**
- ➔ Gravitational wave **observation reveals a “mass gap”.**

(LIGO-Virgo / Aaron Geller / Northwestern University)




(H. Olivares et al. 2020)

Contents

Theoretical Background: Dark Matter + Boson Stars

Dark Matter Models

- ΛCDM (cold, collisionless dark matter)
- SIDM (self-interacting dark matter)

Boson stars

- Bosons: Have integer spin, can occupy the same quantum state.
- Small + very dense compared to other objects → **compact stars** (Schaffner - Bielich 2020)

Theoretical Background: Our Model

Our dark matter is made up of:

- Dark bosons→ building blocks of our dark stars, self-interacting
- Dark neutrinos → cooling mediators

The model → clumps of dark boson gas arise from primordial overdensities, which then cool and collapse to make dark matter boson stars!

Theoretical Background: Thermodynamics

To describe our stars -> need an **equation of state (EOS)**. The goal is to:

- Describe the energy density and pressure of the object in terms of number density and temperature
- EOS must be **thermodynamically consistent,** so we must derive the EOS from a thermodynamic quantity, eg. the free energy (F) of the system

$$\mu = \left(\frac{\partial F}{\partial N}\right)\bigg|_{V,T} \qquad P = -\left(\frac{\partial F}{\partial V}\right)\bigg|_{N,T} \qquad S = -\left(\frac{\partial F}{\partial T}\right)\bigg|_{N,V}$$

μ=chemical potential

P = pressure

S = entropy

(Pathria 1994)

Research Goal

EXPLORE IV project: Dark matter boson stars at T= 0.
GREP Project: Introduce temperature to formulate a description for **hot** dark boson stars!

Equation of State

Tolman - Oppenheimer - Volkoff Equation → **Structure**

Jeans Criterion + Hierarchical Fragmentation Arguments → **Formation**

Equation of State (EOS) Derivation: Free Energy

Free Energy: Total Energy to assemble a system - heat from environment (Pathria 1994)

Hamiltonian: → Partition Function: → Free Energy:

$$H = \sum_{n=1}^{N} \frac{p_i^2}{2m} + m + \varphi(n)$$

$$Q_N(V,T) = \frac{1}{N!h^{3N}} \int e^{-\frac{H}{kT}} d\omega$$

$$F(N,V,T) = -kTln(Q_N)$$

$\varphi(n)$ = interaction potential

Free Energy:

$$F = mN + \varphi(n)N + NkT[ln(\frac{N}{V}(\frac{h^2}{2\pi mkT})^{\frac{3}{2}}) - 1]$$

Equation of State (EOS) Derivation - for $\varphi(n) = cn$

Free Energy

Internal Energy

$$E = F + TS$$

÷volume

Energy Density

$$\varepsilon(n) = mn + cn^2 + \frac{3}{2}nkT$$

Pressure

$$P = cn^2 + nkT$$

Entropy

$$S = Nk[ln(\frac{V}{N}(\frac{2\pi mkT}{h^2})^{\frac{3}{2}}) + \frac{5}{2}]$$

ε, P in terms of n, T

Structure: Tolman - Oppenheimer - Volkoff Equation

Hydrostatic equilibrium in static spherical spacetime:

$$\frac{dp'}{dr'} = -\frac{m'_r \varepsilon'}{r'^2}(1 + \frac{p'}{\varepsilon'})(1 + \frac{4\pi r'^3 p'}{m'_r})(1 - \frac{2m'_r}{r'})^{-1}$$

Conservation of mass in general relativity:

$$\frac{dm'_r}{dr'} = 4\pi r'^2 \varepsilon'$$

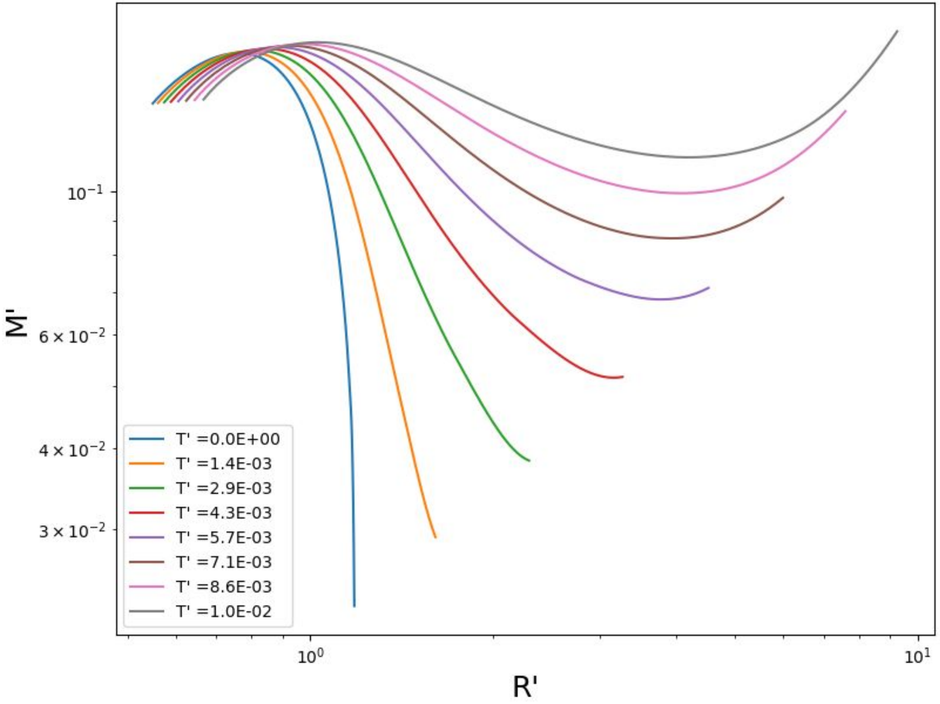
By solving the TOV equation we can obtain:

➔ R’ and M’, R’ is defined as the radius at which p → 0, M’(R’) = M

➔ Maximum mass and corresponding radius

(J. Schaffner-Bielich 2020)

Results: M’ vs R’ Plot for constant temperature



- Blue curve → same as T=0 case done in EXPLORE
- As T increases a minimum mass appears!
- Maximum mass scales ~ linearly with temperature
- Radius gets very large for nonzero temperatures

Formation: Conditions for collapse

Jeans criterion: When the gravitational force sufficiently counteracts pressure force.

- ➔ Jeans mass = m_J= minimum mass at which criterion is fulfilled
- ➔ Jeans length = λ_J = critical radius at which criterion is fulfilled
- ➔ If $m > m_J$, the collapsing gas cloud can form a star

Derivation: Arises from studying small linear perturbations of a uniform fluid.

- ➔ Find condition for perturbations large enough to cause gravitational instability.
- ➔ Instability -> collapse -> star formation

c_s = speed of sound

$$\lambda_J = c_s \left(\frac{\pi}{G\rho}\right)^{\frac{1}{2}} \qquad m_J = \frac{\pi}{6} c_s^3 \left(\frac{\pi}{G}\right)^{\frac{3}{2}} \frac{1}{\sqrt{\rho}}$$

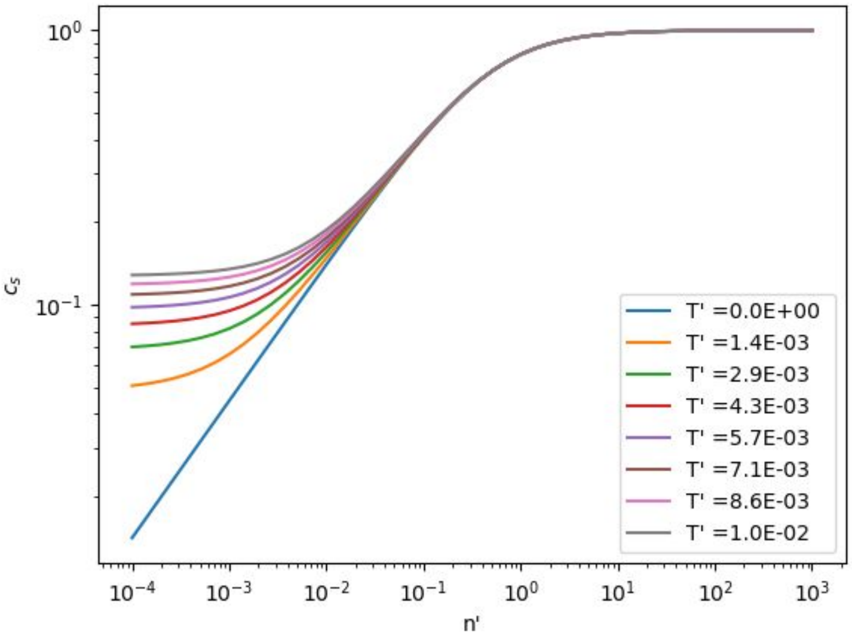
(Ryden 2007)

Results: Speed of Sound

Speed of sound: The speed of pressure waves moving through a medium.

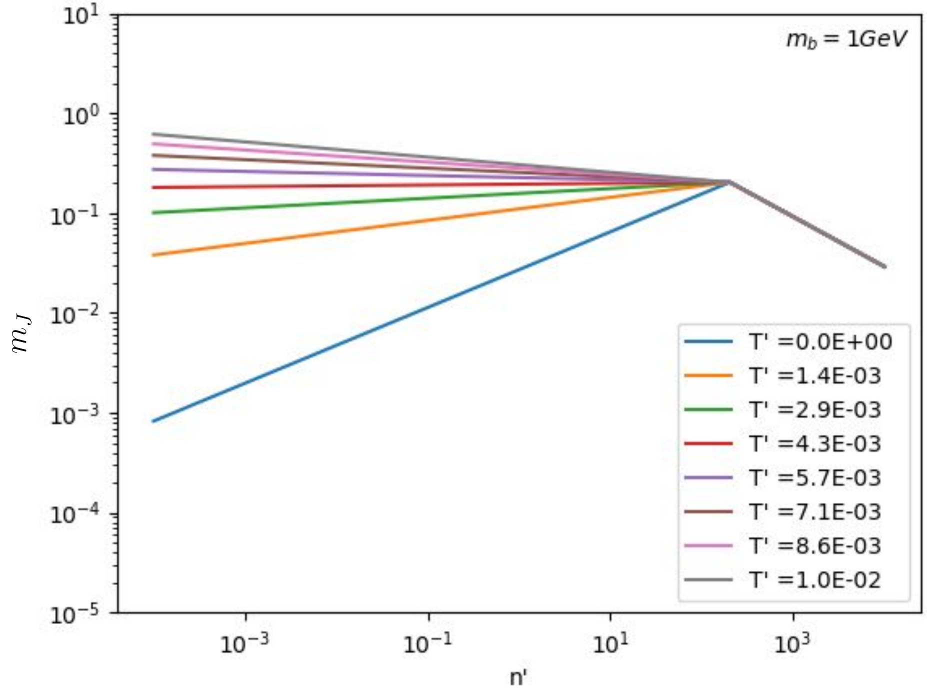
Temperature Effects ➡

$$c_s^2 = \left(\frac{dP}{d\varepsilon}\right)\bigg|_S = \frac{2n_b' + \frac{10}{3}\pi n_b'^{\frac{2}{3}}(e^{s_n' - \frac{5}{2}})^{\frac{2}{3}}}{1 + 2n_b' + 5\pi n_b'^{\frac{2}{3}}(e^{s_n' - \frac{5}{2}})^{\frac{2}{3}}}$$



(Ryden 2007)

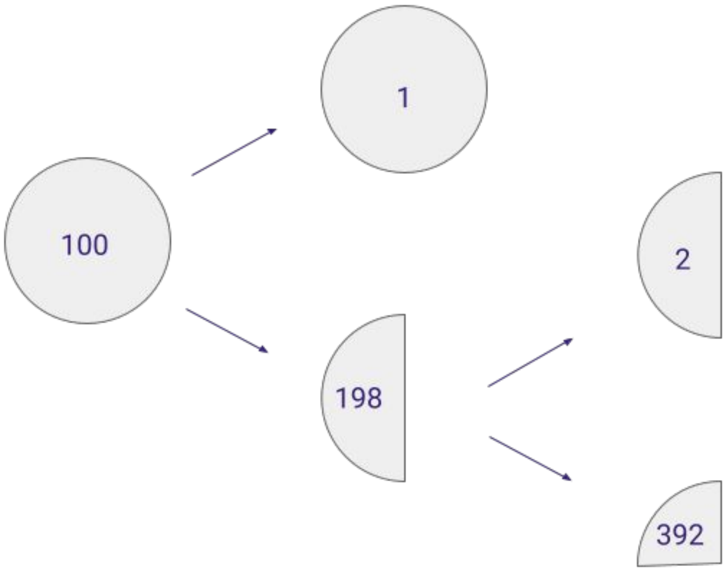
Results: Jeans Mass



$$M_J = \frac{1}{G^{\frac{3}{2}}} \frac{\pi^{\frac{5}{2}}}{6} \left(\frac{2n_b' + \frac{10}{3}\pi n_b'^{\frac{2}{3}} (e^{s_n' - \frac{5}{2}})^{\frac{2}{3}}}{1 + 2n_b' + 5\pi n_b'^{\frac{2}{3}} (e^{s_n' - \frac{5}{2}})^{\frac{2}{3}}} \right)^{\frac{3}{2}} \frac{1}{\sqrt{n_b' m_b^3}}$$

- ➔ Temperature effect prominent at low number densities
- ➔ All behave more similar at higher number densities (interaction effects more dominant)

Formation: Hierarchical Fragmentation



- ➔ Assume failure rate of ~1%.
- ➔ If you start with 100 cores, only 99 will fragment in half, and 1 will stay the same size.
- ➔ 2^n mass distribution.

(Ryden 2007)

. . . and continues until cooling is no longer sufficient.

Formation: Cooling

Time of fragmentation: When the cooling timescale and the collapse time scale are $\sim$ equal (J. H. Chang et. al., 2016).

- ➔ Free fall/collapse time scale: $t_{\rm ff} = \left(\frac{1}{16\pi G n_b' m_b^3}\right)^{1/2}$
- ➔ Cooling timescale dependant on cooling rate, Λ $t_{cooling} \equiv \frac{3T}{m_b \Lambda}$
- ➔ Our cooling rate, used for neutrino cooling (Mendes et al., 2024):

$$\Lambda \propto 10^{-10} n'^{\frac{1}{3}} T'^{6} \left(\frac{4\pi}{3}\lambda_J^3\right)$$

Results: Fragments and Abundance

Fragmentation stops when gas becomes optically thick -> cooling is suppressed

- This happens when the mean free path of the dark neutrinos is similar to the jeans length of the cloud, dark neutrinos don't escape cloud $\lambda_J \sim \ell_{mfp}$

$$\ell_{mfp} = \frac{1}{n\sigma} = \frac{1}{nG_F^2 T^2}$$

σ = cross section = $G_F^2 E^2$

E~T

- The jeans mass associated with this point would be the mass of the smallest fragments (J. H. Chang et. al., 2016)

Results: Investigating behaviour at fragmentation

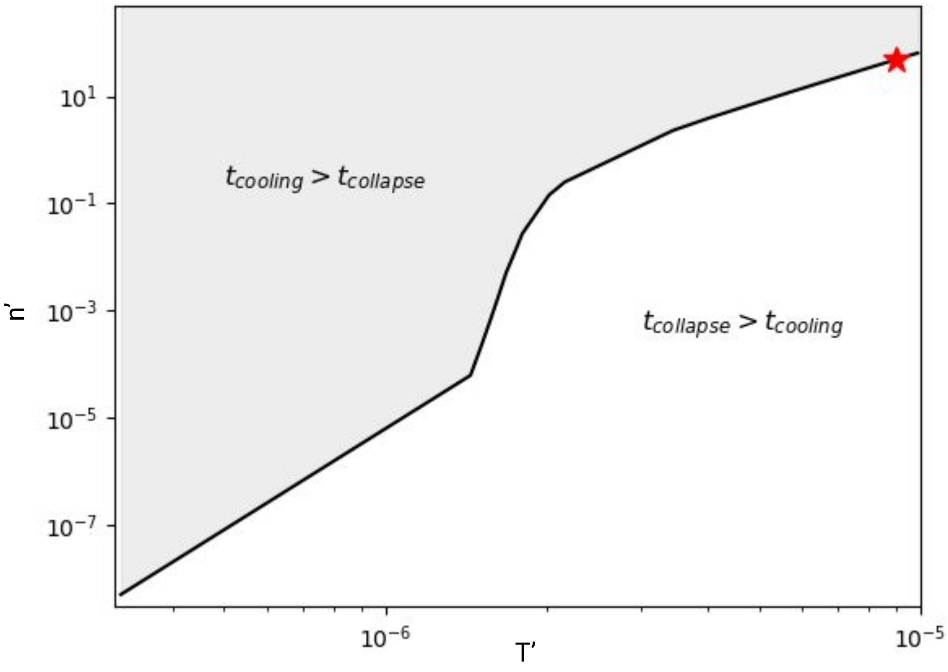
Black Line: n’ and T’ for when timescales are ~ equal

Cooling is efficient at very high temperature, and up to very high number densities

Very dense boson stars can be formed!

Star: End of fragmentation

➔ $M_J = 0.42 M_\odot$ for m_b = 1GeV



Summary

- ➔ Derived a simple EOS of a boson star including temperature.
- ➔ TOV results -> M’ vs. R’ plots reveal minimum mass for increasing T’ and large radius for increasing T’.
- ➔ Neutrino cooling model is very efficient, produces extremely compact boson stars.

Open Questions

- How can we refine our model?
  - More complicated EOS?
  - Using different cooling rates/processes?
- Observation of boson stars/ boson star mergers via gravitational wave detectors?

Dimensionless scaling relations

Mass: $m_r' = m_r (G^3 \varepsilon_0)^{\frac{1}{2}}$

Radius: $r' = r(G\varepsilon_0)$

Number Density: $n_b' = \frac{n_b}{m_b^3}$

Temperature: $T' = \frac{T}{m_b}$

ε_0 = constant with dimensions of energy density

Free Energy Derivation

$$Q_N = \frac{1}{N!h^{3N}} \int e^{-\beta\left(\sum_{n=1}^{N} \frac{p_i^2}{2m} + m + \varphi(n)\right)} \prod_{i=1}^{N} \left(d^3 q_i d^3 p_i\right) = \frac{V^N}{N!h^{3N}} \left(e^{-\frac{m+\varphi(n)}{kT}} \int_0^\infty e^{-\frac{p^2}{2mkT}} 4\pi p^2 dp\right)^N$$

$$Q_N = \frac{e^{-N\frac{m+\varphi(n)}{kT}}}{N!} \left(\frac{V}{h^3}(2\pi mkT)^{\frac{3}{2}}\right)^N$$

$$A = -kT \ln\left(Q_N\right) = -kT\left(\ln\left(\frac{e^{-N\frac{m+\varphi(n)}{kT}}}{N!}\left(\frac{V}{h^3}(2\pi mkT)^{\frac{3}{2}}\right)^N\right)\right)$$

$$A = mN + \varphi(n)N + NkT\left[\ln\left(\frac{N}{V}\left(\frac{h^2}{2\pi mkT}\right)^{\frac{3}{2}}\right) - 1\right]$$