Effective Field Theory Approach to Binary Systems in Scalar-Tensor Theories

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with

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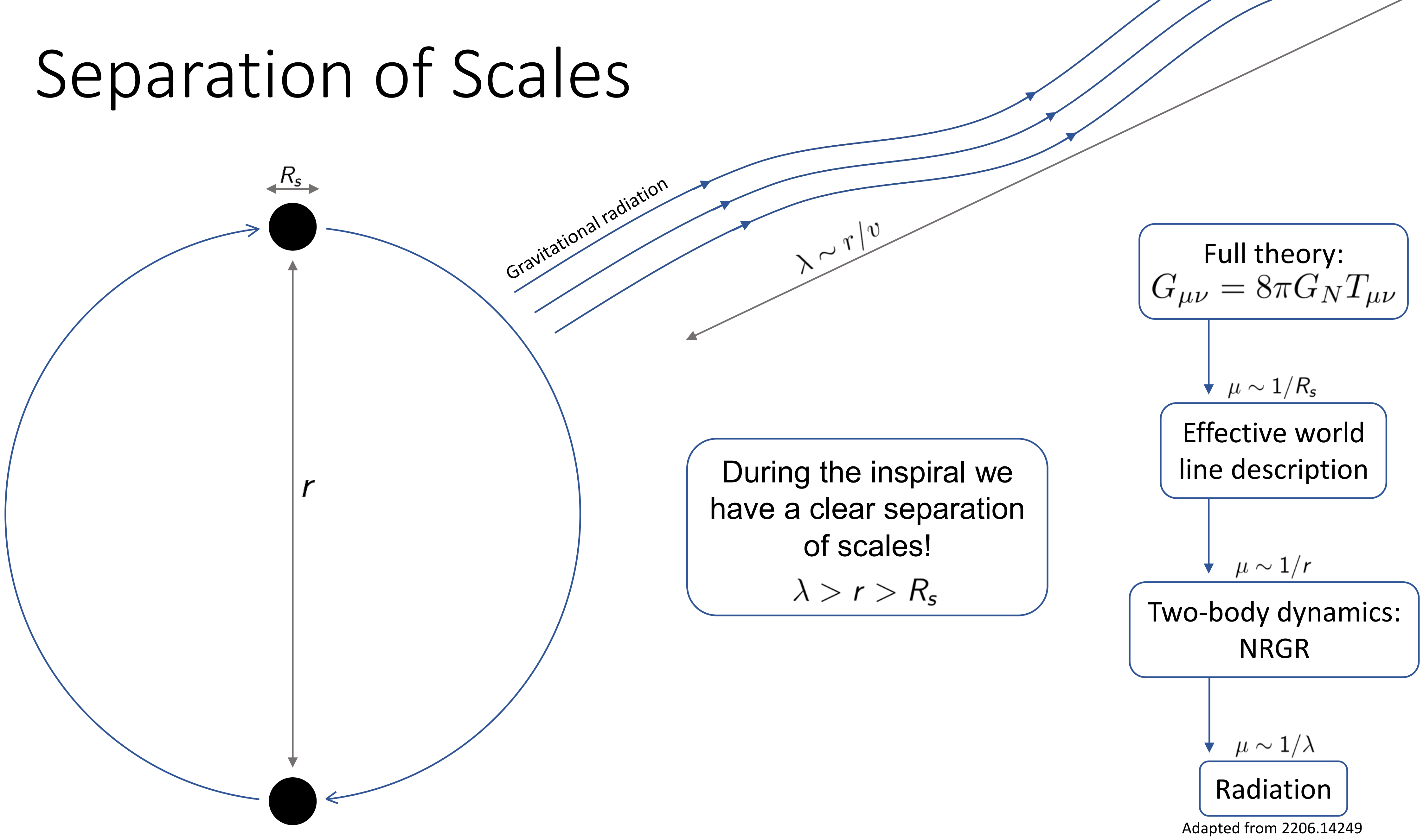
Based on: 2311.04274

Separation of Scales



Originally presented by Goldberger and Rothstein in 2004 (hep-th/0409156)

Constructing the EFT

In pure GR the point particle action is

$$S_{\rm pp} = -M \int d\tau$$

Proper time

Total mass

Finite-size effects are encoded in higher order operators

$$S_{\rm pp} = -M \int d\tau \, (1 + c_R R + c_V R_{\mu\nu} u^\mu u^\nu + \dots)$$

Ricci scalar

Ricci tensor

four-velocity

Expanding the PP Action

Using

$$d\tau = \sqrt{v^\mu v^\nu g_{\mu\nu}}\, dt \qquad \text{and} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

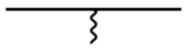
Metric perturbation

We can expand the pp action:

Velocity

$$S^{(0)}_{\rm pp} = -M \int dt \left(1 - \frac{1}{2}\mathbf{v}^2 - \frac{1}{8}\mathbf{v}^4\right) ,$$

$$S^{(1)}_{\rm pp} = -\frac{M}{2M_{\rm pl}} \int dt \left(h_{00} + 2h_{0i}\mathbf{v}^i + h_{ij}\mathbf{v}^i\mathbf{v}^j + \frac{h_{00}}{2}\mathbf{v}^2 + h_{0i}\mathbf{v}^i\mathbf{v}^2 + \frac{h_{ij}}{2}\mathbf{v}^i\mathbf{v}^j\mathbf{v}^2 + \frac{3}{8}h_{00}\mathbf{v}^4 \right) ,$$

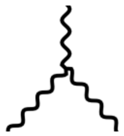


$$S^{(2)}_{\rm pp} = +\frac{M}{8M_{\rm pl}^2} \int dt \left(h_{00}^2 + 4h_{00}h_{0i}\mathbf{v}^i + 4(h_{0i}\mathbf{v}^i)^2 + 2h_{00}h_{ij}\mathbf{v}^i\mathbf{v}^j + \frac{3}{2}h_{00}^2\mathbf{v}^2 + 4h_{0i}h_{jk}\mathbf{v}^i\mathbf{v}^j\mathbf{v}^k + 6h_{00}h_{0i}\mathbf{v}^i\mathbf{v}^2 \right.$$
$$\left. + \left(h_{ij}\mathbf{v}^i\mathbf{v}^j\right)^2 + 6\left(h_{0i}\mathbf{v}^i\right)^2\mathbf{v}^2 + 3h_{00}h_{ij}\mathbf{v}^i\mathbf{v}^j\mathbf{v}^2 + \frac{15}{8}h_{00}^2\mathbf{v}^4 \right) .$$

Expanding the Einstein-Hilbert Action

Likewise we obtain

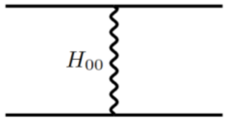
$$-2M_{\mathrm{Pl}}^2 \int dx^4 \sqrt{-g} R \approx \int dx^4 \left((\partial h)^2 + \frac{1}{M_{\mathrm{pl}}} h (\partial h)^2 + \mathcal{O}(h^4) \right)$$

$\approx$  $+$  $+ \cdots$

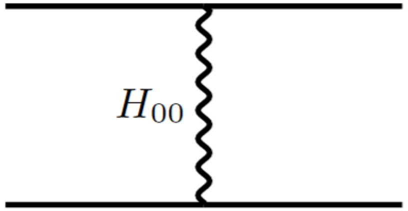
Propagator:

$$\left\langle T\left(h_{\mu\nu}(x) h_{\alpha\beta}(y) \right) \right\rangle = P_{\mu\nu;\alpha\beta} \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik(x-y)}}{k^2 + i\epsilon},$$

From Einstein to Newton

$S_{\mathrm{eff}} =$ 2x $+$ 

$+ \cdots$



$$= P_{00;00}\left(\frac{-i}{2M_{\mathrm{pl}}}\right)^2 M_1 M_2 \int dt_1 \int dt_2 \int \frac{dk^4}{(2\pi)^4}\frac{-ie^{-ik(x-y)}}{|\mathbf{k}|^2}$$

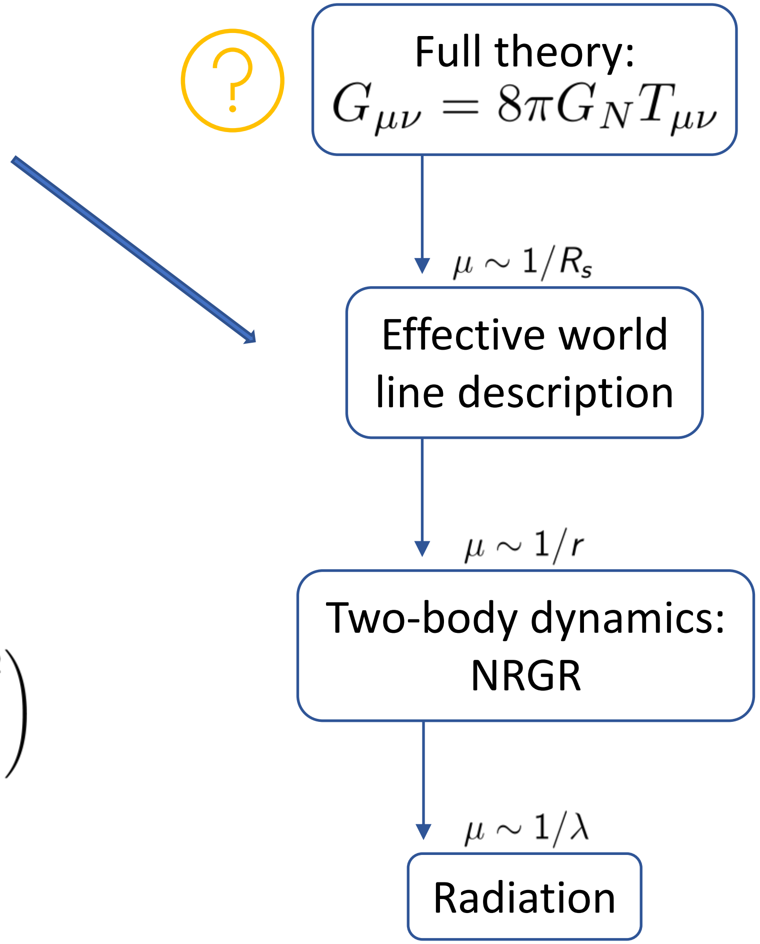
$$= P_{00;00}\left(\frac{-i}{2M_{\mathrm{pl}}}\right)^2 M_1 M_2 \int dt \frac{1}{4\pi r} = i\int dt \frac{G_N M_1 M_2}{r}$$

$$\Rightarrow S_{\mathrm{eff}} = \int dt\left(\frac{M_1}{2}\mathbf{v}_1^2 + \frac{M_1}{2}\mathbf{v}_1^2 + \frac{G_N M_1 M_1}{r} + \ldots\right)$$

Adding a Scalar

$$S_{\rm pp} = -\int d\tau \left(M + q\frac{\phi}{M_{\rm pl}} + p\left(\frac{\phi}{M_{\rm pl}}\right)^2 + \mathcal{O}\left(\phi^3\right) \right)$$

$$S = \int dx^4 \sqrt{-g}\left(-2M_{\rm Pl}^2 R + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_s^2\phi^2 - M_{\rm Pl}\frac{c_3}{3!}\phi^3 + \ldots \right)$$



Adapted from 2206.14249

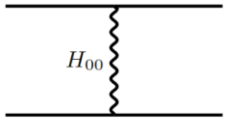
Example: R^2 Gravity

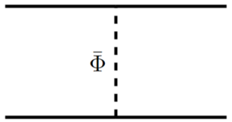
$$S = -2M_{\rm Pl}^2 \int dx^4 \sqrt{-g}\left(R + a_2 R^2\right)$$

$$\Rightarrow \tilde{S} = -2M_{\rm Pl}^2 \int dx^4 \sqrt{-\tilde{g}}\left(\tilde{R} + \frac{1}{2}(\partial\phi)^2 - \frac{3}{4}M_{\rm pl}^2 m_s^2 \left[1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\right]^2 \right)$$

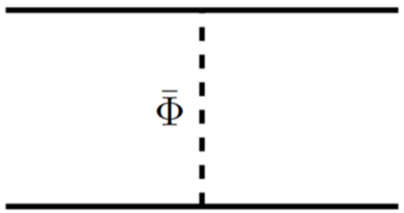
$$\Rightarrow m_s = \sqrt{\frac{1}{6a_2}}, \quad c_3 = -\sqrt{6}\frac{m_s^2}{M_{\rm Pl}^2}$$

Scalar Corrections to the Newtonian Potential

$S_{\text{eff}} =$ 2x $+$ 

$+$ 

$+ \cdots$



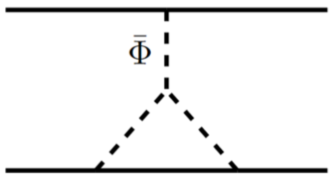
$$= \left(\frac{-i}{M_{\text{pl}}}\right)^2 q_1 q_2 \int dt_1 \int dt_2 \int \frac{dk^4}{(2\pi)^4} \frac{-ie^{-ik(x-y)}}{|\mathbf{k}|^2 + m_s^2}$$

$$= \left(\frac{-i}{M_{\text{pl}}}\right)^2 q_1 q_2 \int dt \frac{1}{4\pi r} e^{-m_s r} = 8i \int dt \frac{G_N q_1 q_2}{r} e^{-m_s r}$$

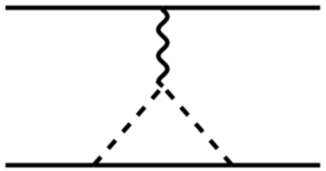
$$\Rightarrow S_{\text{eff}} = \int dt \left(\frac{M_1}{2}\mathbf{v}_1^2 + \frac{M_1}{2}\mathbf{v}_1^2 + \frac{G_N M_1 M_2}{r} + 8\frac{G_N q_1 q_2}{r} e^{-m_s r} + \ldots \right)$$

$$\Rightarrow \omega^2 = \frac{GM}{r^3}\left[1 + 8\frac{q_1 q_2}{M_1 M_2}(1 + m_s r)e^{-m_s r} + \ldots\right]$$

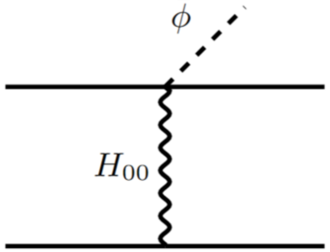
Higher-order diagrams



$$= i\frac{G_N}{2\pi}\frac{q_1 q_2(q_1+q_2)c_3}{m_s}\int dt\frac{1}{r}\left[-\mathrm{Ei}(-3m_s r)e^{m_s r}+\mathrm{Ei}(-m_s r)e^{-m_s r}+\log(3)e^{-m_s r}\right]$$

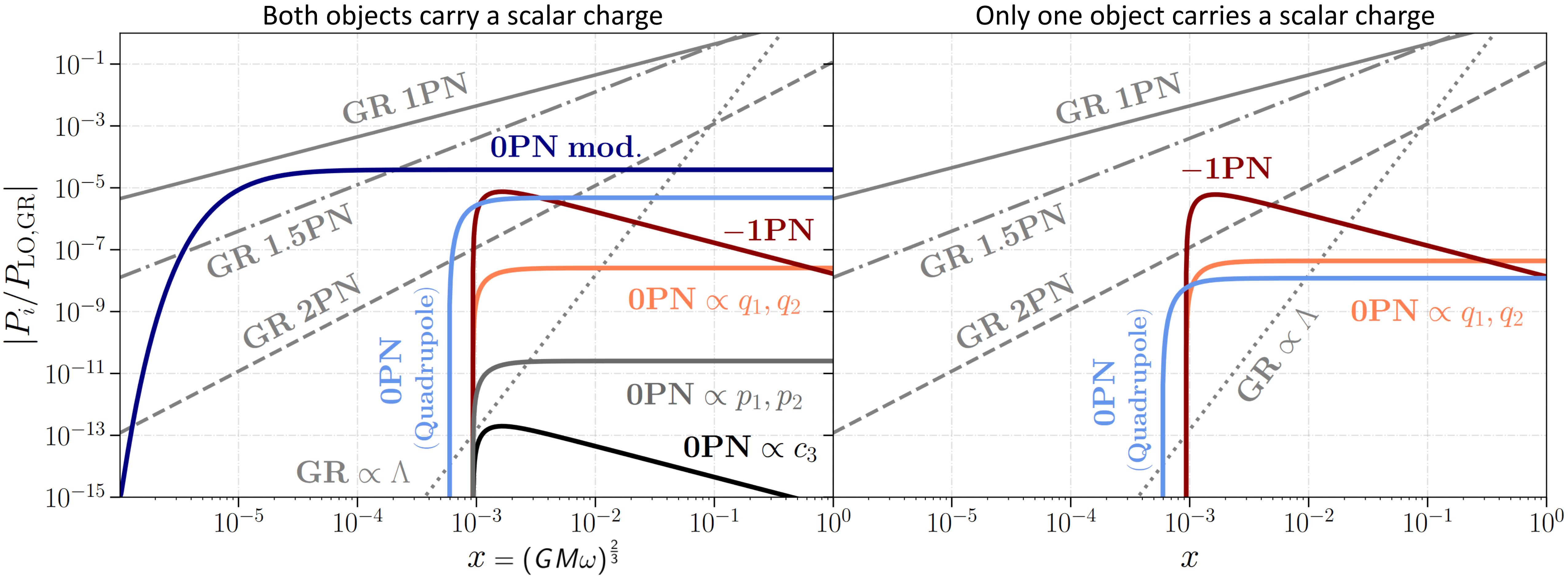


$$= -i4G^2\left(q_2^2 M_1+q_1^2 M_2\right)\int dt\left[-2m_s^2\mathrm{Ei}(-2m_s r)+\frac{m_s}{r}-\frac{m_s}{r}e^{-2m_s r}\right]$$



$$= i\frac{q_1 M_2+q_2 M_1}{32\pi M_{\mathrm{Pl}}^2}\int dt\frac{1}{r}\frac{\bar{\phi}}{M_{\mathrm{Pl}}}$$

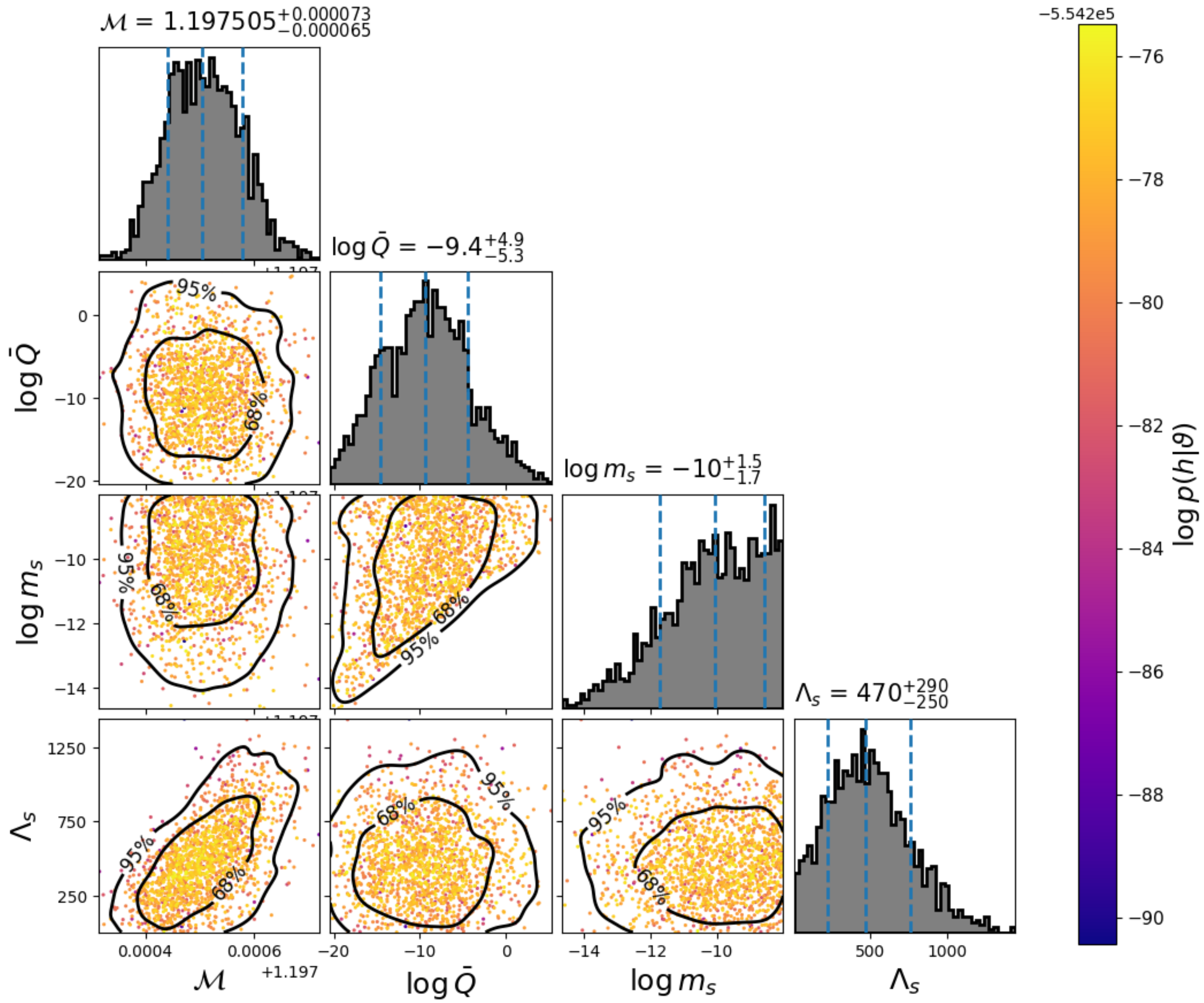
Powerloss



	m_s [eV]	q_1 [M_1]	q_2 [M_2]	p_1 [q_1]	p_2 [q_2]	c_3
Left	10^{-15}	10^{-3}	10^{-3}	5×10^{-2}	5×10^{-2}	m_s^2/M_{Pl}^2
Right	10^{-15}	10^{-4}	0	5×10^{-3}	0	m_s^2/M_{Pl}^2

$$P_{\bar{\phi}}^{l=1,\mathrm{LO}} = \frac{8}{3} G \frac{(M_1 q_2 - M_2 q_1)^2}{M^2} r^2 \omega^4 \left(1 - \frac{m_s^2}{\omega^2}\right)^{3/2}$$

Constraints from GW170817

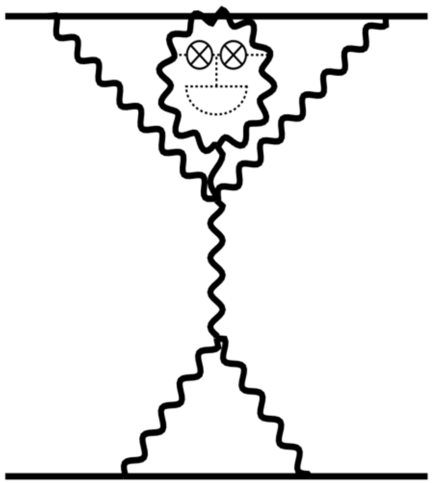


Conclusion

- Effective field theory provides a systematic and elegant way to calculate PN corrections!
- Easy to extent to beyond GR scenarios!
  - Can Investigate generic scalar-tensor theories such as
    - f(R) gravity
    - Axion models

Outlook

- Get constraints on the deviations to GR from LIGO/Virgo/KAGRA data
- Push to higher PN orders



Thank you!

Powerloss

$$P_{\bar{\phi}}^{l=1,\mathrm{LO}} = \frac{8}{3} G M^2 \delta q^2 r^2 \omega^4,$$

$$P_{\bar{\phi}}^{l=1,p_1,p_2} = \frac{256}{3} G^2 M^3 \Xi_p \, r \omega^4,$$

$$P_{\bar{\phi}}^{l=1,c_3} = \frac{G M_1 M_2 M}{3\pi m_s} \Xi_c (m_s r - 1) r^2 \omega^4,$$

$$P_{\bar{\phi}}^{l=1,\mathrm{NLO}} = \frac{8GM^2}{15} \delta q \Big[- 10 g_1 G M + g_2 r^3 (m_s^2 - 6\omega^2) \Big] r \omega^4,$$

Lagrangian

$$
\begin{aligned}
L_\phi =& 8Gq_1q_2\frac{e^{-m_s r}}{r}\left[1 - G\frac{M_1+M_2}{r} - \frac{\boldsymbol{v}_1^2+\boldsymbol{v}_2^2}{2} - \frac{(\boldsymbol{v}_1\cdot\boldsymbol{r}_1)(\boldsymbol{v}_2\cdot\boldsymbol{r}_2)}{2\,r^2}(1+m_s r) + \frac{\boldsymbol{v}_1\cdot\boldsymbol{v}_2}{2}\right] \\
& - 64G^2(p_1q_2^2+p_2q_1^2)\frac{e^{-2m_s r}}{r^2} + 4m_sG^2\frac{M_1q_2^2+M_2q_1^2}{r}\left[e^{-2m_s r} + 2m_s r\mathrm{Ei}(-2m_s r)\right] \\
& - 8m_sG^2q_1q_2\,\frac{M}{r}\left[\log(2m_s r)e^{-m_s r} - \mathrm{Ei}(-2m_s r)e^{m_s r}\right] \\
& + Gc_3q_1q_2\frac{q_1+q_2}{2\pi m_s r}\left[\mathrm{Ei}(-m_s r)e^{-m_s r} - \mathrm{Ei}(-3m_s r)e^{m_s r}\right],
\end{aligned}
$$

Phase

$$\Psi_E^{0\mathrm{PN}} = \frac{5\bar{Q}}{6\nu \bar{m}_s^{5/2}} \left[\Gamma\left(\frac{5}{2}, \frac{\bar{m}_s}{v^2}\right) + \Gamma\left(\frac{7}{2}, \frac{\bar{m}_s}{v^2}\right) + 2\,\Gamma\left(\frac{9}{2}, \frac{\bar{m}_s}{v^2}\right)\right] - v^3 \frac{5\bar{Q}}{6\nu \bar{m}_s^4} \left[\Gamma\left(4, \frac{\bar{m}_s}{v^2}\right) + \Gamma\left(5, \frac{\bar{m}_s}{v^2}\right) + 2\,\Gamma\left(6, \frac{\bar{m}_s}{v^2}\right)\right],$$

$$\Psi_{l=1}^{-1\mathrm{PN}} = \delta q^2 \frac{5}{896\nu^3} \left[-\frac{1}{v^7}\,{}_3F_2\left(-\frac{3}{2}, \frac{5}{3}, \frac{7}{6}; \frac{8}{3}, \frac{13}{6}; \frac{\bar{m}_s^2}{v^6}\right) - \frac{90 \cdot 2^{1/3} 3^{1/2} v^3 \Gamma\left(\frac{2}{3}\right)^3}{247\pi \bar{m}_s^{10/3}} + \frac{63\,\Gamma\left(\frac{1}{3}\right)^3}{256 \cdot 2^{1/3} \pi \bar{m}_s^{7/3}}\right]$$