Explore Neutron Stars!

For the following questions, make the following assumptions about all stars:

- Newtonian gravity
- Spherical symmetry
- Constant density

Feel free to look up any constants or extra equations that you might need! The subscript $\odot$ means "The Sun". For most questions, it will be useful to express your energies in both Joules and MeV so that you can easily compare orders of magnitude.

1. **Constant Density Star in Newtonian Gravity**
Notation: R is the outer radius of the star, and M is the total mass contained in the star.

(a) The equation for the mass inside a sphere of radius r is

$$m(r) = 4\pi \int_0^r dr r^2 \rho(r).$$

Solve for $m(r)$ for all locations inside the star in terms of M and R.
(b) Use your answer from part (a) to solve for the gravitational binding energy of a constant density star. Express your answer in terms of M and R.
(c) Use your Newtonian estimate from part (b) to calculate the gravitational binding energy of: (i) The Sun; (ii) The white dwarf star Sirius B with $M = 1.0M_\odot$, $R = 0.007R_\odot$; and a neutron star with $M = 1.4M_\odot$ and $R = 12km$. Express your answer in Joules and MeV.

2. **Baryon Number of the Sun**
Protons and neutrons are the simplest baryons. For this question, you can assume that neutrons have the same mass as the proton. The baryon number is simply the number of baryons in a nucleus. Calculate the baryon number of the Sun! (All you need to know is the mass of the Sun and the mass of a proton.) Explain what approximations you are making when you do this.

3. **Energy Lost from the Sun**
The Sun radiates energy today at the rate of 3.8×10^{26} Watts. Assume that the Sun's luminosity doesn't change with time, and that it lives for 10 billion years. How much energy does it lose over its lifetime? Express your answer in Joules.

4. **Energy from Core Collapse**
In a core collapse supernova, the iron core collapse inwards. You should express answers in both Joules and MeV to make it easier to compare the orders of magnitudes of the various values.
(a) If all of the gravitational potential energy is converted to heat, how much heat energy is created when an Iron core with $M = 1.4M_\odot$ collapses from the size of the Sun to the size of a typical neutron star with $R = 12km$?
(b) Estimate the average temperature and the energy per photon ($2.7kT$) resulting from the collapse in part (a).

5. **Photodissociation**
Photodissociation is a process that takes place in a core-collapse supernova.
(a) Take a star with the same mass as the Sun, but make it out of iron-56. The photo-dissociation reaction

$$Q + \gamma +^{56} Fe \rightarrow 13^4 He + 4n,$$

where $Q = 124.4$ MeV uses up photons and destroys iron. How much energy will be used up if all of the iron in this star is photodissociated?
(b) Similarly, Helium nuclei can be photodissociated by photons into 2 protons and 2 neutrons if the photon has an energy of at least 28.3 MeV. How much energy will be used up if all of the Helium in part (a) is photodissociated?
(c) Where does the energy Q come from?

6. **Fermi Energy and Momentum**
A fermion in a degenerate gas has a momentum called the Fermi momentum given by

$$p_F = \frac{\hbar}{\Delta x}$$

where Δx is the average distance between particles in the gas.
(a) Assume a constant density, spherical star, with mass M, radius R, that is a charge neutral mixture of protons and electrons. What is the average distance between electrons in the star?
(b) Assume that the electrons are non-relativistic. Use the usual relation between energy and momentum for non-relativistic particles to calculate the Fermi energy of the electrons (in terms of M, R, m_e). Do the same for the protons. Which particle type will have a larger Fermi energy?
(c) What changes in part (b) if the particles are relativistic? (Ie. $v\ c$)
(d) Calculate the (non-relativistic) Fermi energy in MeV of an electron in a Carbon white dwarf star at densities of $10^3, 10^6, and 10^9 kg/m^3$.
(e) Calculate the (non-relativistic) Fermi energy in MeV of a neutron in a neutron star at densities of 10^8 and $10^{11} kg/m^3$.
(f) Give the equivalent temperature in Kelvin (ie using E=kT) for the Fermi energies calculated in parts (d) and (e)

7. **Neutronization**
Neutronization is one of the processes involved in creating a neutron star!
(a) The neutronization reaction

$$Q + e +^{12} C \rightarrow^{12} B$$

can take place for electrons with an energy of $Q = 13.4$ MeV. Assume that the white dwarf star Sirius B is composed of pure Carbon-12. How much energy will be lost if all of the Carbon is neutronized into Boron?
(b) Calculate the density of white dwarf material required for electrons to have a Fermi energy of 13.4 MeV.