Gravitational Wave Data Analysis with LIGO

EXPLORE Workshop, April 11 2023

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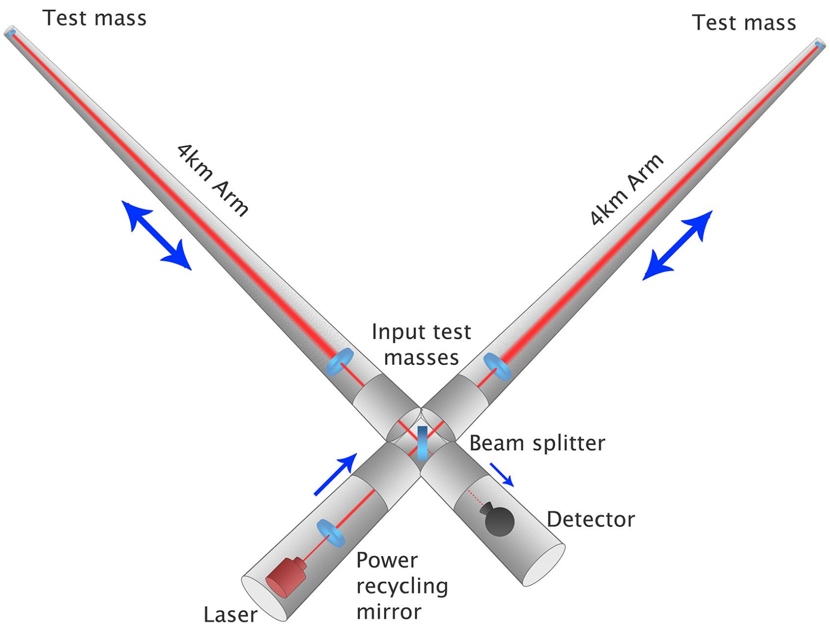
LIGO Observatory

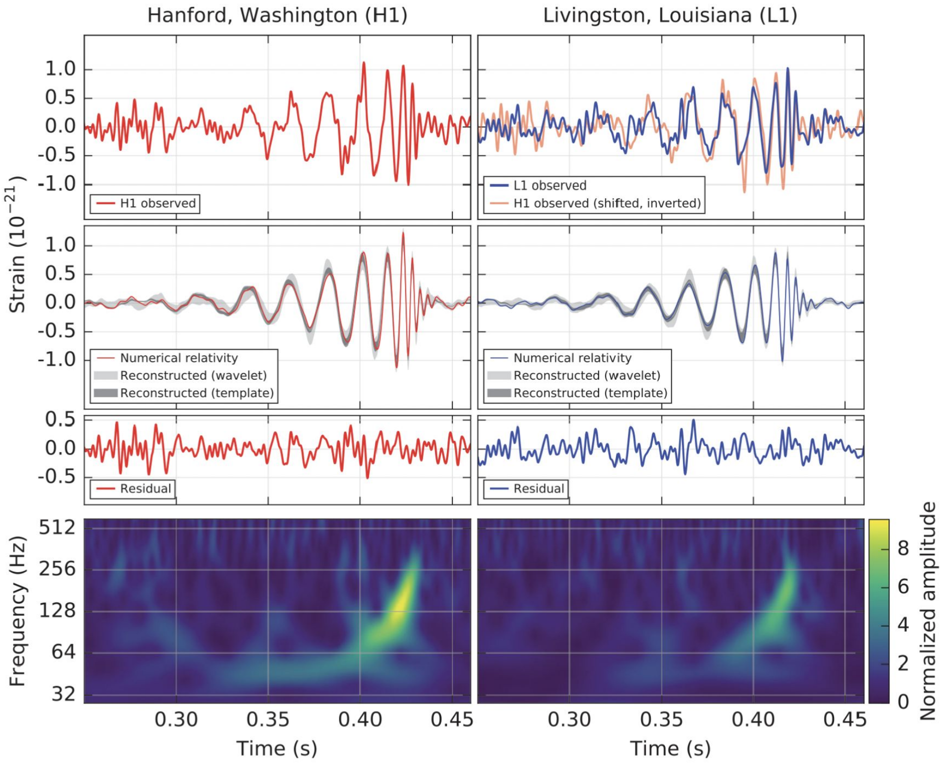
- Laser Interferometer Gravitational Wave (GW) Observatory
- The two detectors: Livingston and Hanford ~3000 km apart
- GW-induced phase shift of laser beams
- Frequency range: ~10 Hz - 10 kHz
- Astrophysical sources:
  Black Holes (BHs), Neutron Stars (NS), Exotic objects?



https://blogs.cardiff.ac.uk/chrisnorth/2016/02/14/

First Detection of Black Hole Binary: GW150914

- September 14th, 2015: Detection of binary black hole merger by LIGO/Virgo
- Constituent masses $\sim 30\, M_{sol}$
- Distance $\sim 1.3 \times 10^9\,\mathrm{ly}$



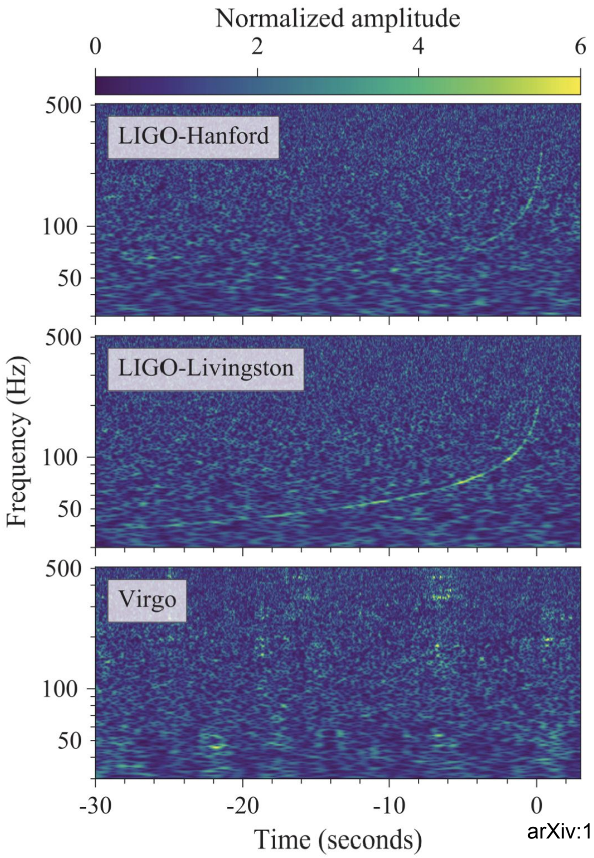
arXiv:1602.03837

GW170817: First Neutron Star Merger

How do we know it was a binary NS system?

1. Electromagnetic counterpart
   - Detection of Gamma ray burst after event
   - Not expected from BH mergers

2. Mass of the system
   - Relatively light: $M_{tot} \approx 2.7\, M_{sol}$
   - Not heavy enough to be black holes



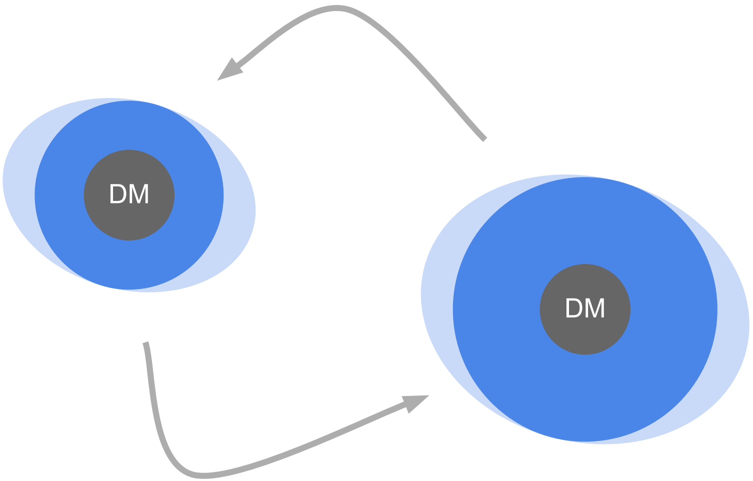
arXiv:1710.05832

Connection to exotic compact objects (ECOs)

- GW signal depends on inner structure of neutron stars
- Tidal deformability Λ: Deformation of a body responding to tidal forces
- Λ set by nuclear physics + (possibly) dark matter (DM)

↓

GW signature of new physics!



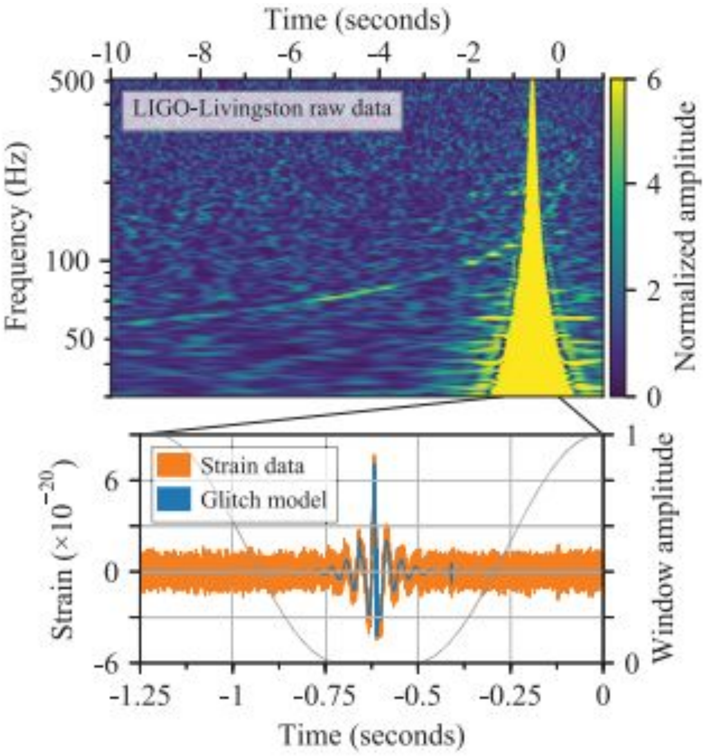
Theoretical background

Gravitational Waveform

- The Q-Transform of the GW strain data h(t) reveals a time dependency on the signal's frequency.

- The stationary phase approximation of the waveform $h(t) = \int_{-\infty}^{\infty} A f^{-7/6} e^{i(\Psi(f) - 2\pi f t)} df$ is given by

$$t = \frac{1}{2\pi} \frac{d\Psi(f)}{df}$$



10.1103/PhysRevLett.119.161101

PPN Equation

- The Post-Newtonian (PN) expansions are approximate solutions to Einstein field equations.

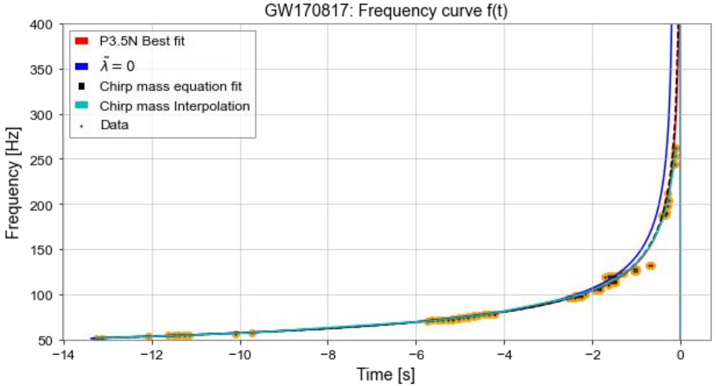
- $\Psi(f)$ consists of terms of up to P3.5N, with $v = (\pi M f)^{1/3}$

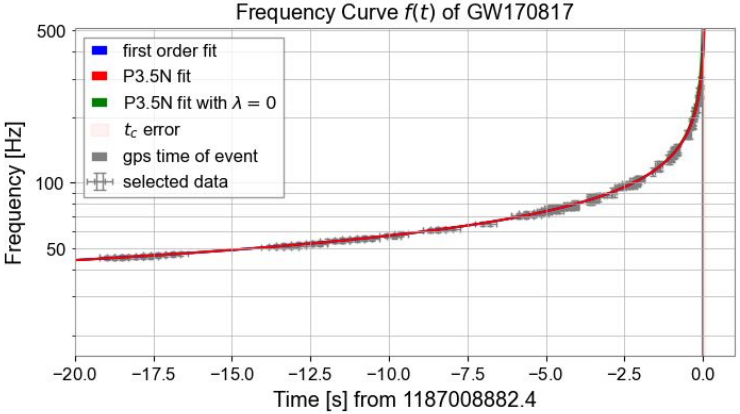
$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu} v^{-5} \left[1 + a_2 v^2 + a_3 v^3 + a_4 v^4 + a_5 \ln(v) v^5 + a_6 v^6 + a_7 v^7\right]$$

- Add P5N tidal correction term to the phase that includes $\tilde{\lambda}$: $\delta\psi = -\frac{9}{16} \frac{v^5}{\mu M^4} \tilde{\lambda}$

- The final model has parameters $\left\{t_c, \phi_c, M, \mu, \beta, \sigma, \tilde{\lambda}\right\}$.

Leading Order Chirp Mass Equation





- The frequency derivative of the steady phase approximation using the leading term of the phase yields

$$\frac{dt}{df} = \frac{1}{2\pi}\frac{d^2\Psi(f)}{df^2}$$

- By integrating the resulting equality we obtain the chirp mass M_c equation

$$\frac{3}{8}f^{-8/3} = -\frac{96}{5}M_c^{5/3}\pi^{8/3}t + b$$

- The chirp mass is the most defining constraint quantity of the frequency curve.

Tidal Deformability

- A quadrupole moment Q_{ij} is induced by a quadrupolar tidal field ε_{ij}

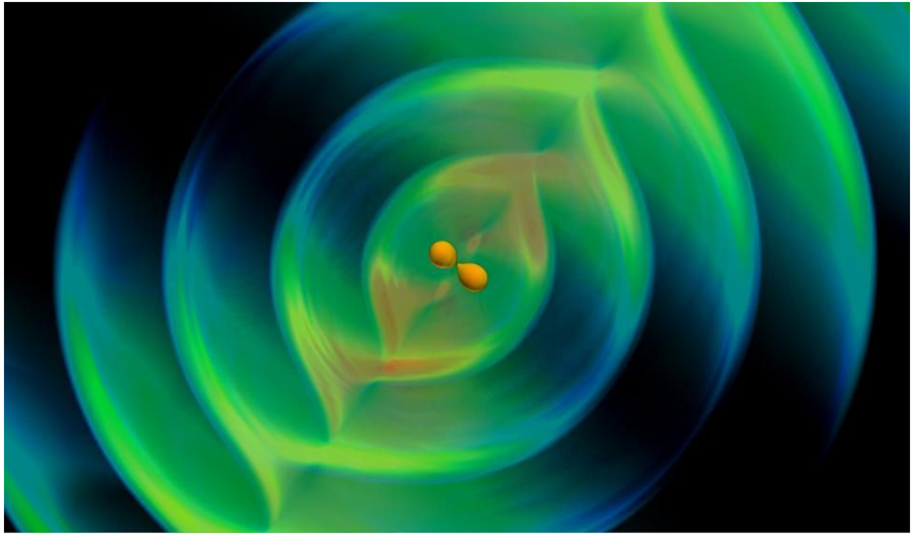
$$Q_{ij} = -\lambda\varepsilon_{ij}$$

The tidal deformability λ describes how much a body is deformed.

- The weighted average tidal deformability of a BNS is $\tilde{\lambda}$.

- The love number is a dimensionless unit of tidal deformation

$$k_2 = \frac{3}{2}\lambda R^{-5}$$



https://www.forbes.com/sites/startswithabang/2017/10/16/astronomys-rosetta-stone-merging-neutron-stars-seen-with-both-gravitational-waves-and-light/?sh=27a504331725

Methodology

Resources: GWOSC & GWpy

GWOSC (**G**ravitational **W**ave **O**pen **S**cience **C**enter)

- Open access to LIGO and Virgo data
- Event Catalogs
- Workshops & Tutorials

R. Abbott et al. (LIGO Scientific Collaboration, Virgo Collaboration and KAGRA Collaboration), "Open data from the third observing run of LIGO, Virgo, KAGRA and GEO", arXiv:2302.03676 (2023)

GWpy (**G**ravitational **W**ave **py**thon)

- Python library for GW physics
- Direct access to GWOSC data
- easy-to-use yet powerful

*D. M. Macleod et al, SoftwareX, **13**, 100657 (2021)*

Version: 3.0.1 (DOI: 10.5281/zenodo.7305083)

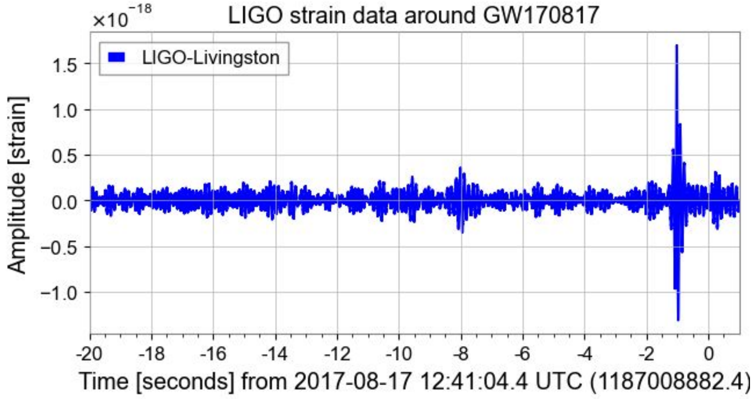
Code Example: Download and Plot Time Series

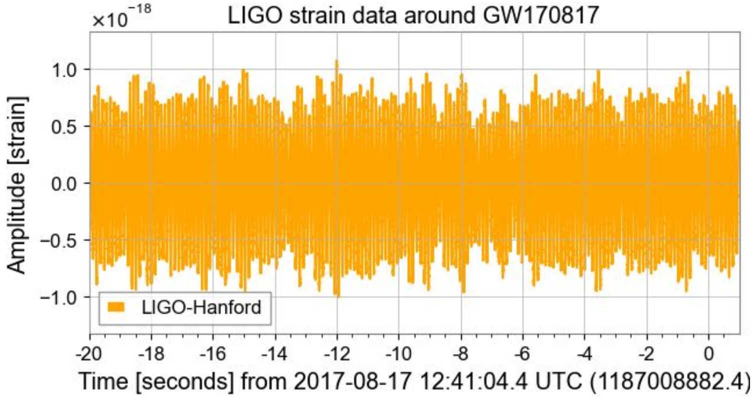
[1]:

```python
# get gps time of event
from gwosc import datasets
gps = datasets.event_gps("GW170817")
# download TimeSeries
from gwpy.timeseries import TimeSeries
TS_L1 = TimeSeries.fetch_open_data("L1", gps-30,gps+30) # Livingston
TS_H1 = TimeSeries.fetch_open_data("H1", gps-30,gps+30) # Hanford
```

[2]:

```python
# plot TimeSeries (Hanford strain only)
from gwpy.plot import Plot
plot = Plot(figsize=[8, 4])
ax = plot.gca()
ax.plot(TS_H1, label='LIGO-Hanford', color='orange')
ax.set_title('LIGO strain data around GW170817')
ax.set_xlim(gps-20, gps+1)
ax.set_xscale('seconds', epoch=gps)
ax.set_ylabel('Amplitude [strain]')
ax.legend();
```





Code Example: Q-Transform

[4]:

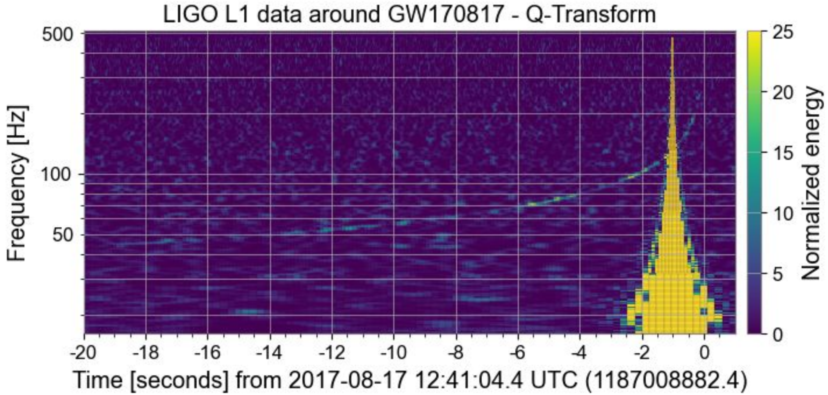
```
# calculate q-transform
from gwpy.segments import Segment
seg = Segment(gps-0.5, gps+0.5)
QG_L1 = TS_L1.q_gram(qrange=(2,256),
                     frange=(16,512),
                     mismatch=0.3,
                     snrthresh=0,
                     search=seg)
```

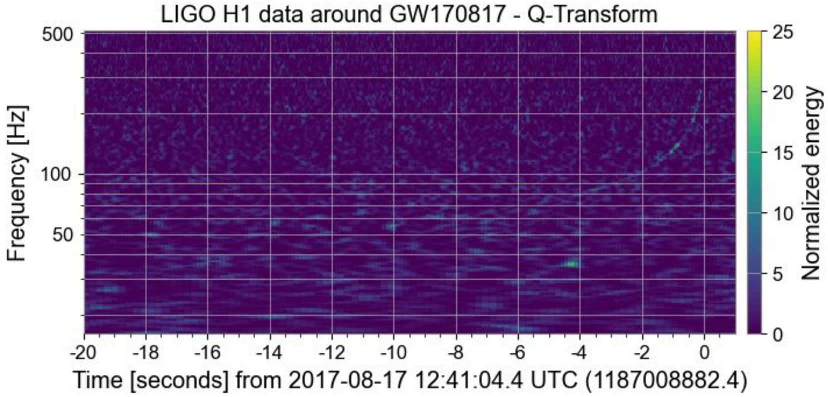
variable-Q transform

tile mismatch

[5]:

```
# plot q-transform (Livingston strain only)
from matplotlib.cm import get_cmap
cmap = get_cmap('viridis')
plot_L1 = QG_L1.tile('time', 'frequency', 'duration', 'bandwidth',
                     color='energy', figsize=[8, 4], linewidth=0.1,
                     edgecolor=cmap(0), antialiased=True)
ax = plot_L1.gca()
ax.set_title('LIGO L1 data around GW170817 - Q-Transform')
ax.set_xscale('seconds')
ax.set_xlim(gps-20, gps+1)
ax.set_epoch(gps)
ax.set_yscale('log')
ax.set_ylim(16,512)
ax.set_ylabel('Frequency [Hz]')
ax.grid(True, axis='y', which='both')
ax.colorbar(cmap='viridis', label='Normalized energy', clim=[0, 25])
ax.set_facecolor(cmap(0))
```

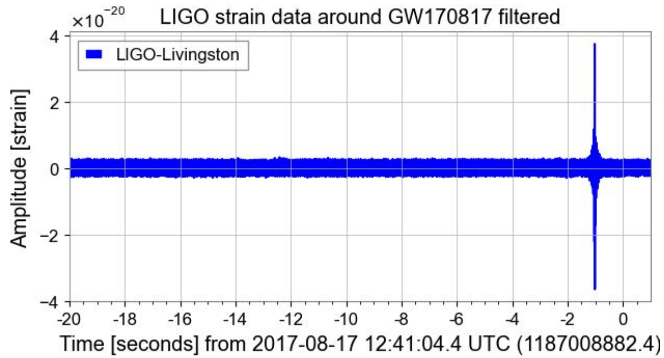




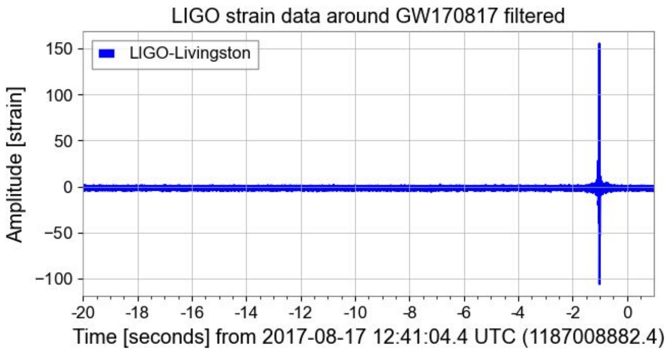
Frequency Curve Extraction

1) Downloading
2) Filtering (Bandpass + AC Notches)
3) **Whitening**
4) Q-transform (fine-tune parameters)
5) Thresholds
6) Errors (dimensions of q-tiles)
7) Sectioning
8) Cleanup (by hand)
9) Final Data

before:



now:



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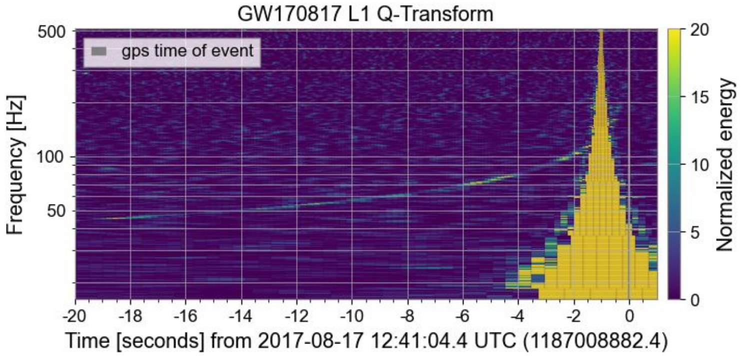

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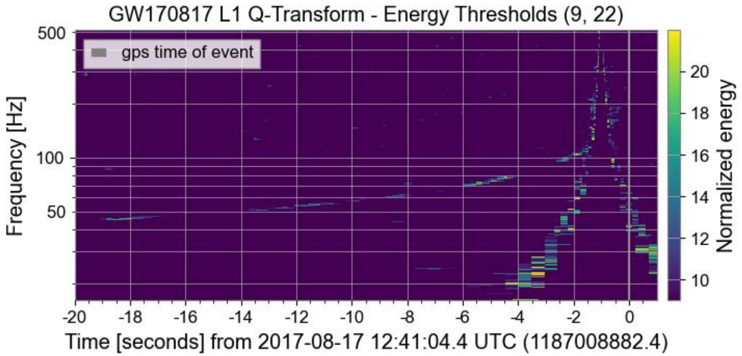

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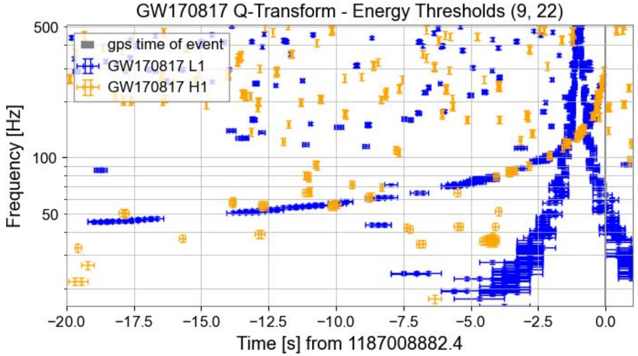

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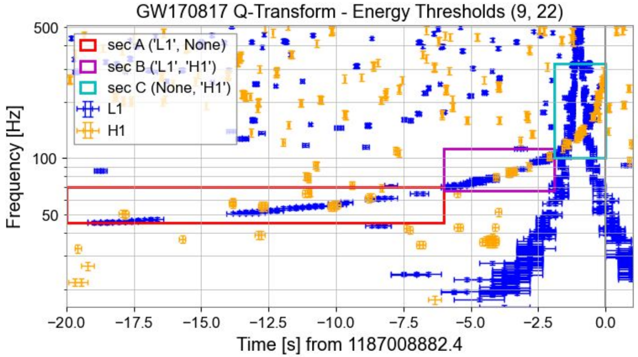

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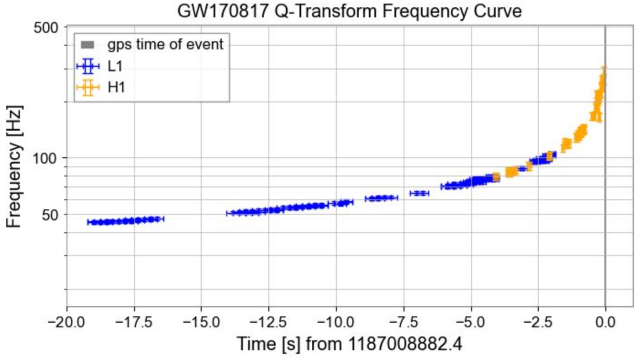
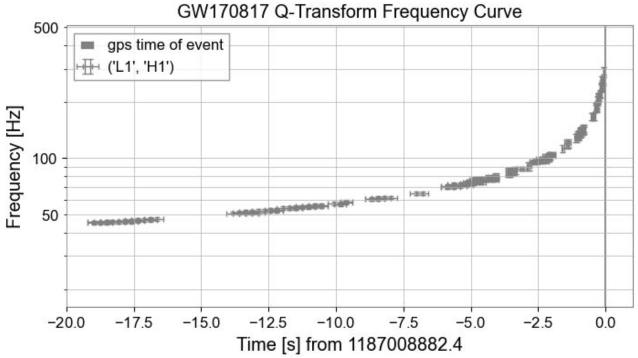
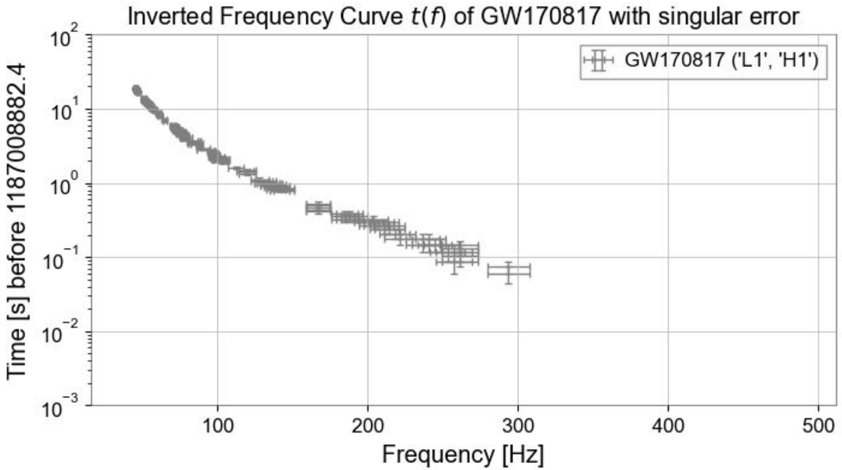


now:



Data to fit: t(f)

Invert frequency curve f(t) -> t(f)



Fit function:

$$t(f) = t_c + t(f, M, \mu, \beta, \sigma) + \partial_f \delta\Psi(f, M, \mu, \tilde{\lambda})$$

Parameters:

- $\mu = M_c \left((1+q)^2 / q \right)^{-2/5}$
- $M = M_c \left((q+1)^2 / q \right)^{3/5}$
- fix: $q = m_1/m_2 = 1,\ \beta = 0,\ \sigma = 0$
- free: $\left\{ M_c,\ t_c,\ \tilde{\lambda} \right\}$

absolut error:

$$\sigma_t = \sqrt{(\text{“duration”})^2 + (\tfrac{dt}{df}\, \textit{“bandwidth”})^2}$$

Results

Leading Order Chirp Mass Fit

- The weighted average of the chirp mass calculations is

$$M_c = 1.187 \pm 0.002\, M_\odot$$

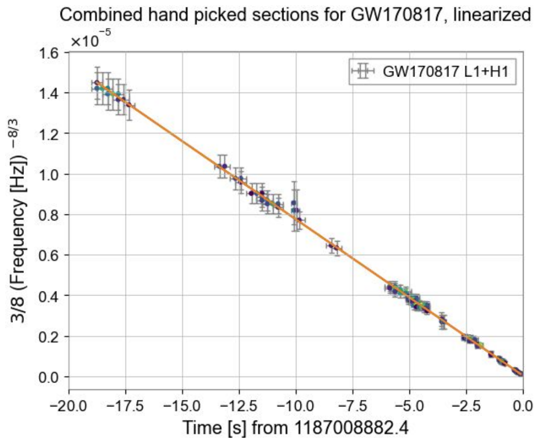
- Comparing with the result obtained by *Abbott et al.**

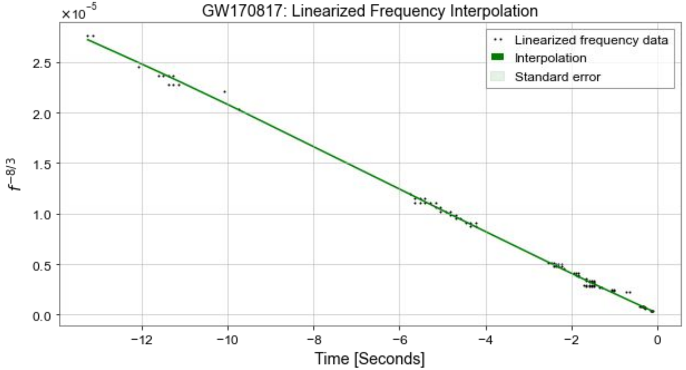
$$M_{c_{Abbott}} = 1.188\,^{+0.004}_{-0.002} M_\odot$$

- The chirp mass in the detectors frame is redshifted by $m^{\text{det}} = m^s(1+z)$

$$M^{\text{det}}_{c_{Abbott}} = 1.1977^{+0.0008}_{-0.0003} M_\odot$$

10.1103/PhysRevLett.119.161101





Tidal Deformability Fit

- 90% upper limit of tidal deformability:

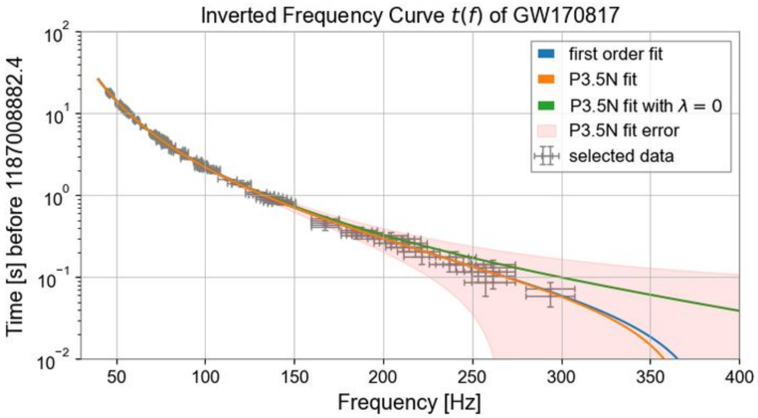
$$\lambda_F \leq 71\left(10^{36} g\, cm^2 s^2\right) \qquad \lambda_E \leq 53\left(10^{36}\, g\, cm^2\, s^2\right)$$

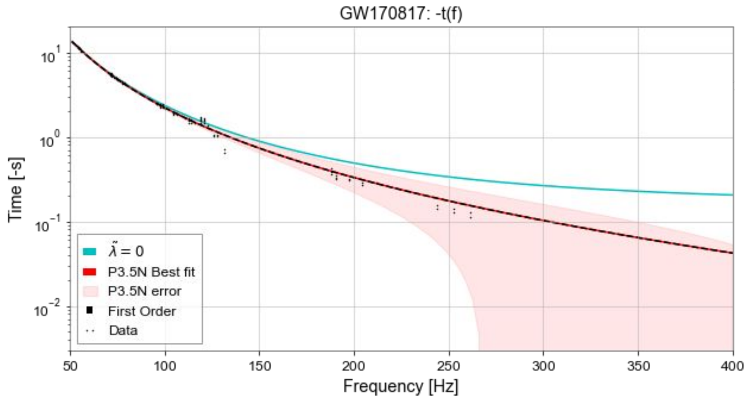
- Upper limit of dimensionless tidal deformability:

$$\Lambda \leq 13599 \qquad \Lambda_{\text{Abbott}} \leq 720$$

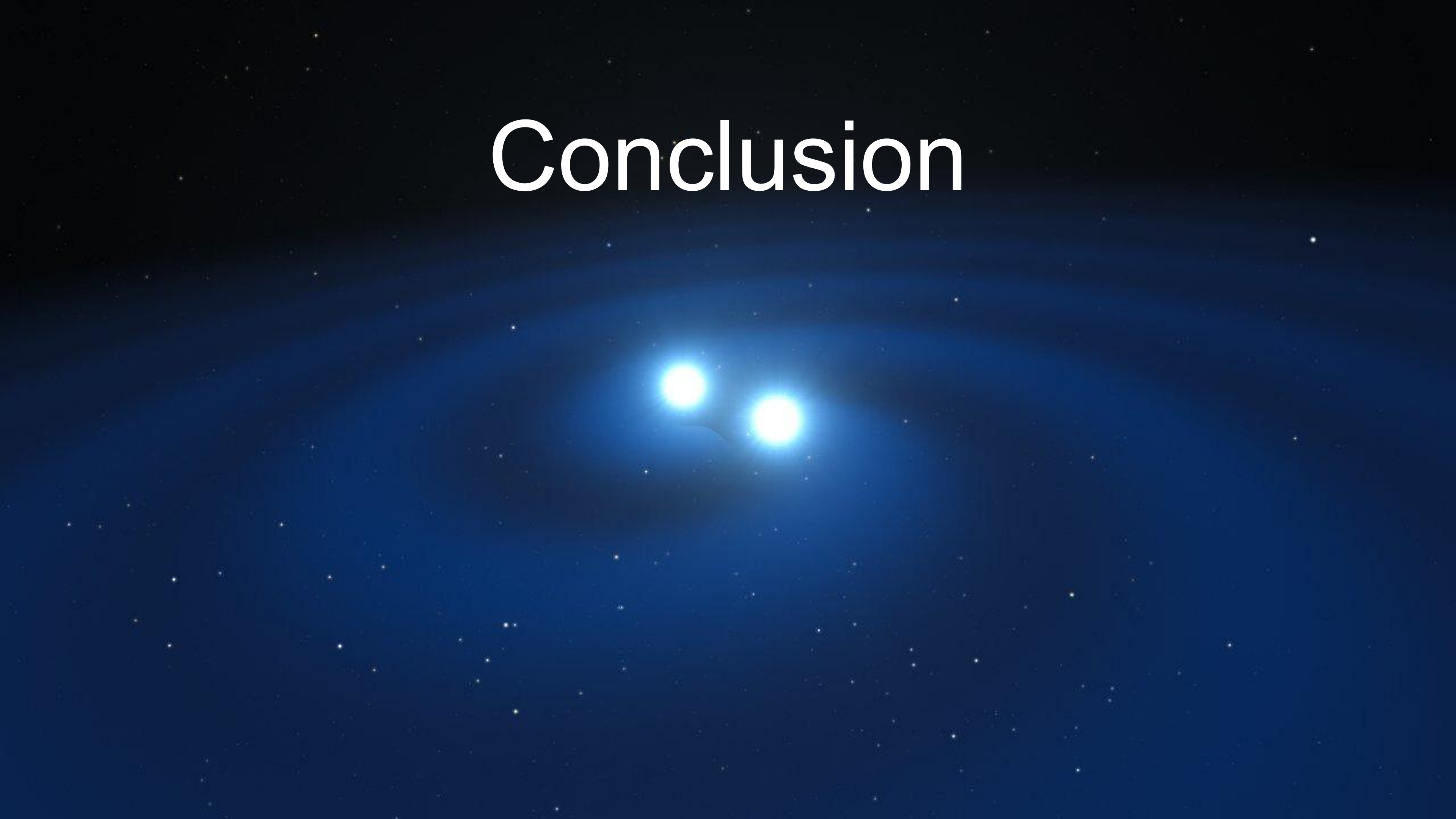
- Upper limit for Love Number:

$$k_2 \leq 7 \quad \text{with} \quad (R = 10\ \text{km})$$





Conclusion



Conclusion

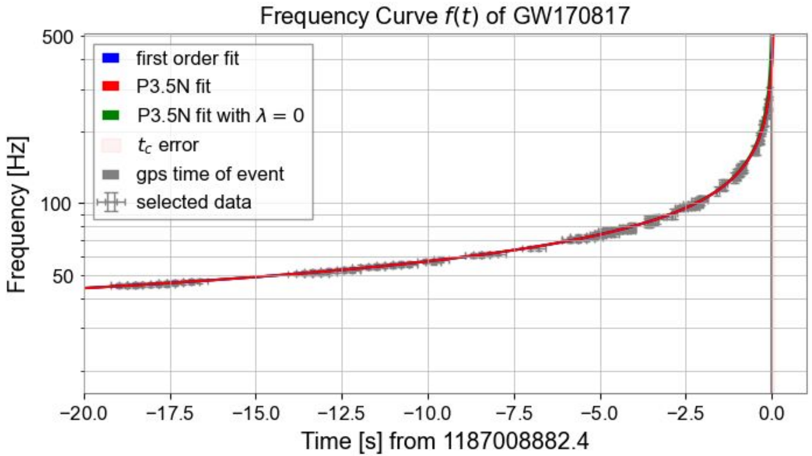
Love Number k_2: closely linked to internal structure of NS

- $k_2 = \frac{3}{2}\frac{\lambda}{R^5}$
- $k_2 \leq 7 \quad \text{with} \quad (R = 10 \text{ km})$

Theory:

- NS models: $k_2 = 0.05 - 0.15$
- black hole: $k_2 = 0$
- Impossible: $k_2 < 0$

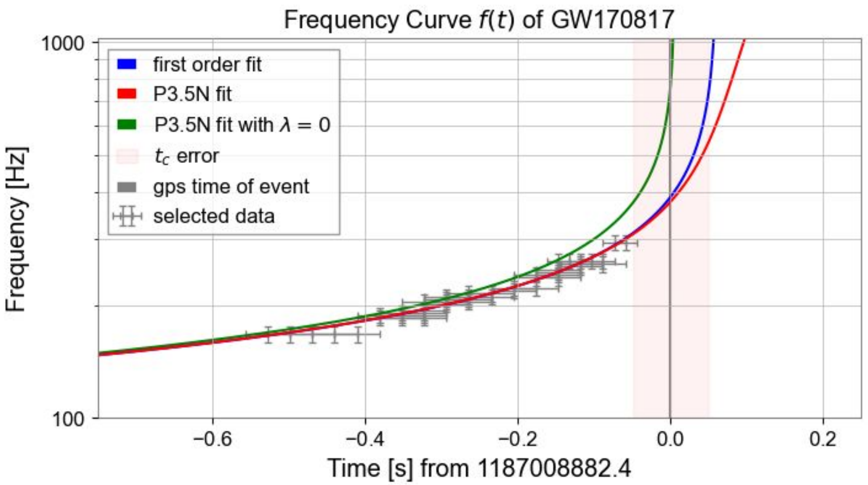
- Upper limits in realm of possibility
- Uncertainties very high at >100%
- Very subtle tidal effects at low frequencies
- Only chirp mass M_c well constrained



Limitations, Improvements and Outlook

Full fit results:

- $M_c = (1.200 \pm 0.005)\,\mathrm{M}_\odot$
- $t_c = (1 \pm 50)\,\mathrm{ms}$
- $\tilde{\lambda} = (-4989 \pm 6837)10^{36}\ \mathrm{g\ cm^2\ s^2}$



Potential Improvements

- Higher frequency data (new events, q-Transform tweaks)
- Fixing t_c (bounds)
- P5.5N Phase term $\Psi(f)$

Outlook:

- GW190425: BNS candidate
- Future events in closer proximity
- Einstein Telescope

Thank You!

Questions?

Backup

GW170817 Abbott et al.

- Distance of event: (40 ± 11) Mpc
-

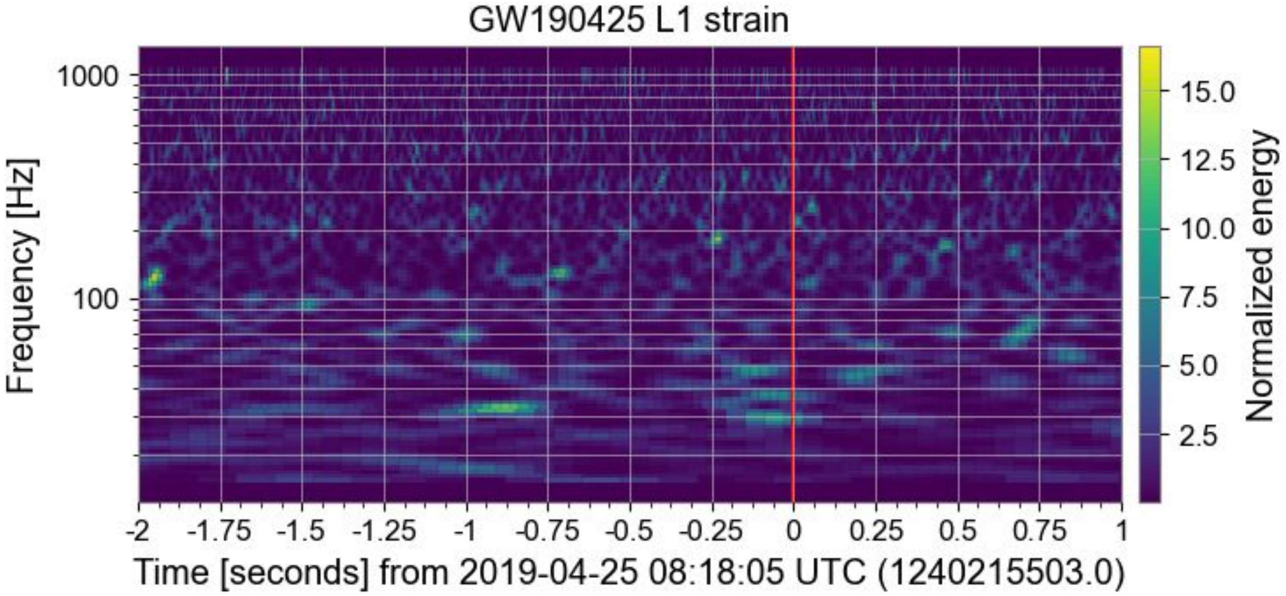
PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass m_{tot}	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy E_{rad}	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance D_{L}	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

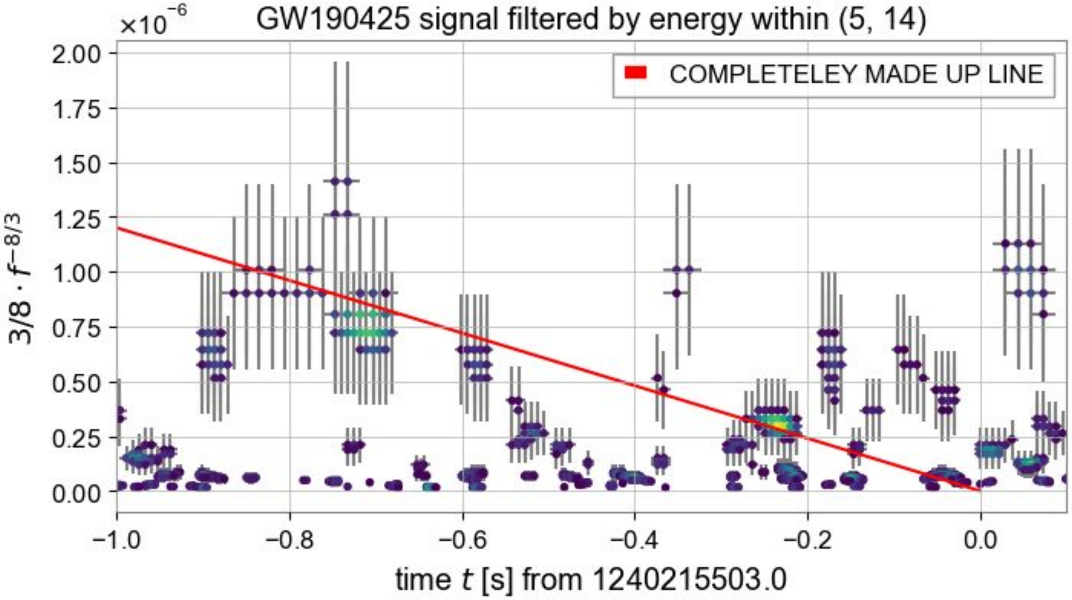
GW190425 (1/2)

Q-Transform has barely visible curve => SNR to low for frequency curve extraction?



GW190425 (2/2)

Linearized to first order (chirp mass)



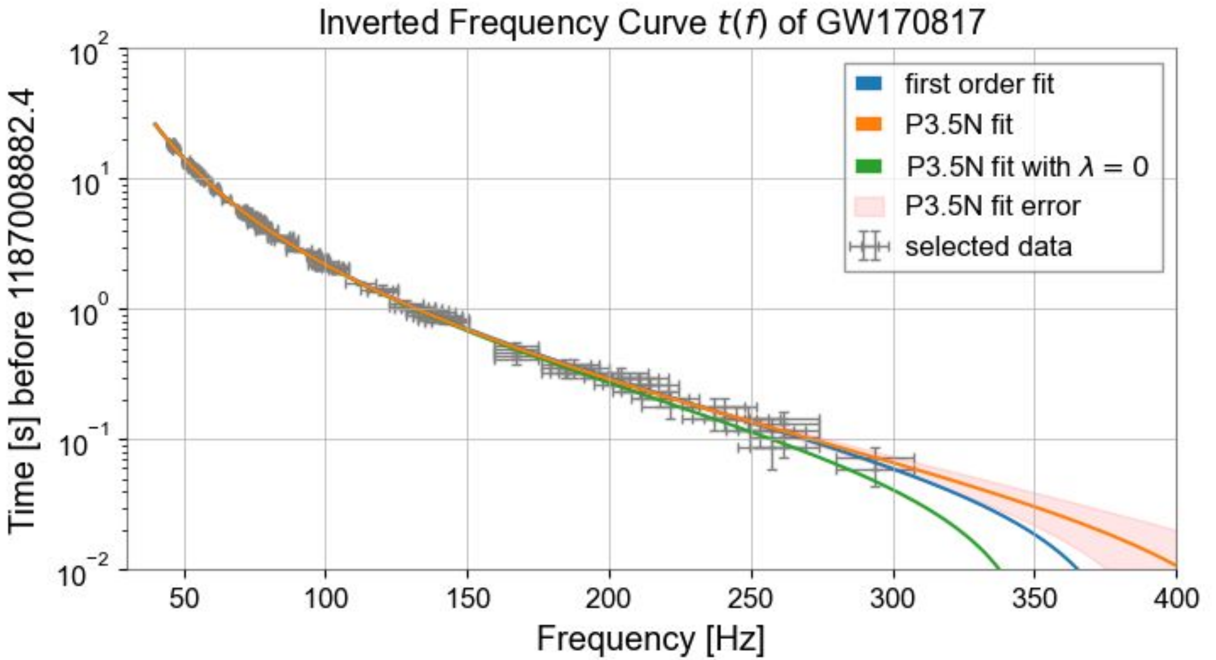
Fixed time of coalescence $t_c \sim 50\text{ms}$

Time of coalescence from first order!

$$\Lambda \leq 29000$$

$$\lambda \leq 150$$

$$k_2 \leq 15$$




```
M_chrip = (1.195 ± 0.003) M_sun
lambda_tilde = (2995 ± 858) 10^36 g cm^2 s^2 -> upper limit: 3853 10^36 g cm^2 s^2 ??
Lambda = (22470 ± 6446) -> upper limit: 28916 ??
k_2 = (12 ± 3) -> upper limit: 15 ??
```

Full P3.5N $\Psi(f)$

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3M}{128\mu}(\pi M f)^{-5/3}\Bigg[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)v^2 - 4(4\pi - \beta)v^3 + 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\frac{\mu}{M} + \frac{617}{144}\frac{\mu^2}{M^2} - \sigma\right)v^4 + \left(\frac{38645\pi}{252} - \frac{65}{3}\frac{\mu}{M}\right)\ln v + \left(\frac{11583231236531}{4694215680} - \frac{640\pi^2}{3} - \frac{6848\gamma}{21}\right)v^6 + \frac{\mu}{M}\left(\frac{15335597827}{3048192} + \frac{2255\pi^2}{12} + \frac{47324}{63} - \frac{7948}{9}\right)v^6 + \left(\frac{76055}{1728}\frac{\mu^2}{M^2} - \frac{127825}{1296}\frac{\mu^3}{M^3} - \frac{6848}{21}\ln(4v)\right)v^6 + \pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\frac{\mu}{M} - \frac{74045}{756}\frac{\mu^2}{M^2}\right)v^7\Bigg], \quad (12)$$

$$\delta\Psi = -\frac{9}{16}\frac{v^5}{\mu M^4}\left[\left(11\frac{m_2}{m_1} + \frac{M}{m_1}\right)\lambda_1 + 1 \leftrightarrow 2\right], \quad (10)$$

where $v = (\pi M f)^{1/3}$, β and σ are spin parameters, and γ is Euler's constant [25]. The tidal term (10) adds linearly to this, yielding a phase model with 7 parameters $(t_c, \phi_c, M, \mu, \beta, \sigma, \tilde{\lambda})$, where $\tilde{\lambda} = [(11m_2 + M)\lambda_1/m_1 + (11m_1 + M)\lambda_2/m_2]/26$ is a weighted average of λ_1 and λ_2. We incorporate the maximum spin constraint for the

Template