

# *The Modern Physics of Compact Stars and Relativistic Gravity*

## **Neutron stars in $f(R, T)$ gravity using realistic equations of state<sup>1</sup>**

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<sup>1</sup>R. Lobato, O. Lourenco, PHRS Moraes, C.H. Lenzi, M. de Avellar, W. de Paula, M. Dutra and M. Malheiro  
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## Compact stars

- **Compact stars are objects of highly compressed matter such that the geometry of space-time is changed considerably from flat space. Thus models of such stars are to be constructed in the framework of GR combined with theories of degenerated/superdense matter.**

**The generally covariant Lagrangian density is,**

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_G. \quad (1)$$

**The connection between the two branches is provided by Einstein's field equations:**

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi T^{\mu\nu}(\epsilon, P(\epsilon)). \quad (2)$$

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- The field equations and the many-body equations need to be solved simultaneously.
- The long range gravitational interaction can be separated from the short range interactions, the deviation from flat-space over the scale of the strong interaction  $\sim \text{fm}$ , is practically zero up to the highest densities reached in the stars' core ( $\sim 10^{15} \text{ g/cm}^3$ ). To have appreciable curvature on microscopic scale set by the strong interaction, densities greater than  $\sim 10^{40} \text{ g/cm}^3$  would be necessary.**

## Stellar structure

- The field equations of General Relativity and theories of extension can be derived from a variational principle
- The Einstein-Hilbert Lagrangian reads  $\sqrt{-g} \left( \frac{\mathcal{R}}{16\pi} + L_m \right)$ , and yields the field equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad (3)$$

- The metric is always written in its canonical form

$$ds^2 = e^\phi dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2, \quad (4)$$

- One can derive the standard TOV equilibrium equations<sup>23</sup>

$$\frac{dm}{dr} = 4\pi r^2 \rho. \quad (5)$$

$$\frac{dp}{dr} = -\frac{m\rho}{r^2} \left[ 1 + \frac{\rho}{\rho} \right] \left[ 1 + \frac{4\pi r^3 \rho}{m} \right] \left[ 1 - \frac{2m}{r} \right]^{-1} \quad (6)$$

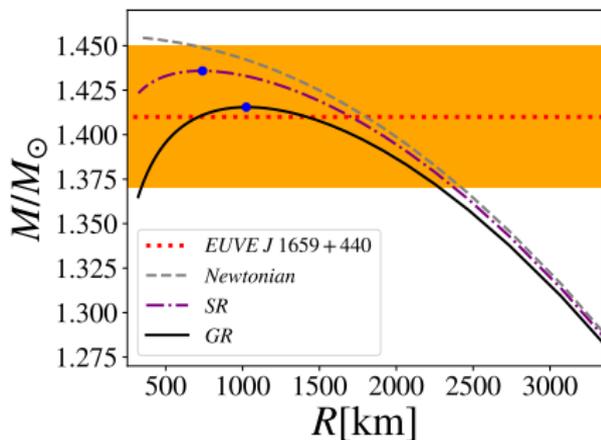
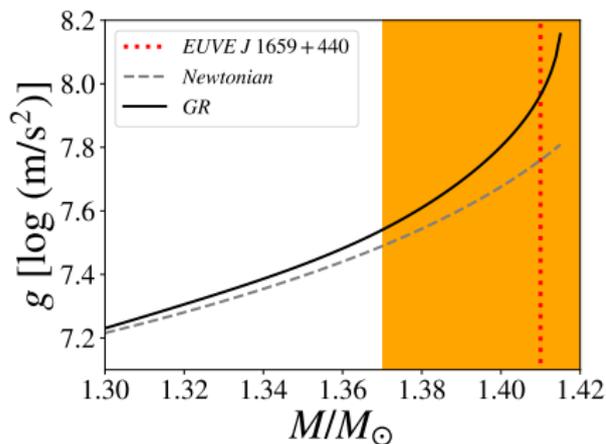
- Newtonian equilibrium equations

$$\frac{dp}{dr} = -\frac{Gm\rho_m}{r^2} \rightarrow \frac{dp}{dr} = -\frac{Gm\rho}{r^2 c^2} \quad (7)$$

<sup>2</sup>TOLMAN, R. C. Phys. Rev. **55**, 364-373 (1939).

<sup>3</sup>OPPENHEIMER, J. R.; VOLKOFF, G. M. Phys. Rev. **55**, 374-381 (1939).

- The general relativistic effects for WD with mass of  $M > 1.3M_{\odot}$ <sup>4</sup>



<sup>4</sup>G.A. Carvalho, R.M. Marinho and M. Malheiro, Gen. Relativ. Gravit. **50**, 38 (2018).

Table: Corresponding radii to fixed total star masses in Newtonian and general relativistic cases.  $R_{\text{Newton}}$  means the radius predicted by Newtonian case,  $R_{\text{SR}}$  is the radius given in the Special Relativistic case,  $R_{\text{GR}}$  is the radius in general relativistic case and the  $R_{\text{NR}}$  is the radius supplied by non-relativistic approximation, where the mass follows the relation  $M/M_{\odot} \propto 1/R^3$ .

Mass/ $M_{\odot}$	$R_{\text{Newton}}$ (km)	$R_{\text{SR}}$ (km)	$R_{\text{GR}}$ (km)	$R_{\text{NR}}$ (km)
1,300	3241	3222	3185	8140
1,312	3107	3081	3030	8114
1,325	2969	2937	2878	8087
1,338	2823	2788	2724	8062
1,351	2671	2633	2562	8036
1,364	2509	2467	2384	8011
1,376	2336	2286	2179	7986
1,389	2148	2085	1942	7961
1,402	1942	1859	1656	7937
1,415	1708	1595	1145	7913

- One of the most popular modified theory of gravity is the  $f(R)$ , where the Ricci scalar is replaced by an arbitrary function of it<sup>5</sup>.
- Cosmic acceleration with no need for dark energy<sup>6</sup>. Such extra terms can also elevate the maximum mass expected for neutron stars<sup>7</sup> and white dwarfs<sup>8</sup>.
- Its gravitational action is given by

$$S_G = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R). \quad (8)$$

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<sup>5</sup>A.A.Starobinsky, Phys. Lett. B **91**, 99 (1980).

<sup>6</sup>L. Amendola et al., Phys. Rev. D **75** (2007) 083504.

<sup>7</sup>S. Capozziello et al., Phys. Rev. D **93** (2016) 023501.

<sup>8</sup>U. Das and B. Mukhopadhyay, J. Cosm. Astrop. Phys. **05** (2015) 045.

- The gravitational action of  $f(R, T)$  theory is<sup>9</sup>

$$\mathcal{S}_G = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R, T). \quad (9)$$

- The field equations of  $f(R, T) = R + f(T)$  gravity are obtained from the variation of (9) in respect to the metric:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} + \frac{1}{2} f(T) g_{\mu\nu} + f_T(R, T) (T_{\mu\nu} + p g_{\mu\nu}), \quad (10)$$

with the covariant derivative of the energy-momentum tensor being

$$\begin{aligned} \nabla^\mu T_{\mu\nu} &= \frac{f_T(R, T)}{8\pi - f_T(R, T)} [(3T_{\mu\nu} + p g_{\mu\nu}) \nabla^\mu \ln f_T(R, T) \\ &+ \nabla^\mu (2T_{\mu\nu} + p g_{\mu\nu}) - (1/2) g_{\mu\nu} \nabla^\mu T]. \end{aligned} \quad (11)$$

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<sup>9</sup>Harko et al.,  $f(R, T)$  gravity, Phys. Rev. D **84** (2011) 024020

## Hydrostatic equilibrium equations in $f(R, T)$ gravity

- Proposed by Harko<sup>10</sup>, the theory assumes the gravitational part of the action depends on a generic function of  $R$  and  $T$ . The total action reads

$$S = \frac{1}{16\pi} \int d^4x f(R, T) \sqrt{-g} + \int d^4x \mathcal{L}_m \sqrt{-g}. \quad (12)$$

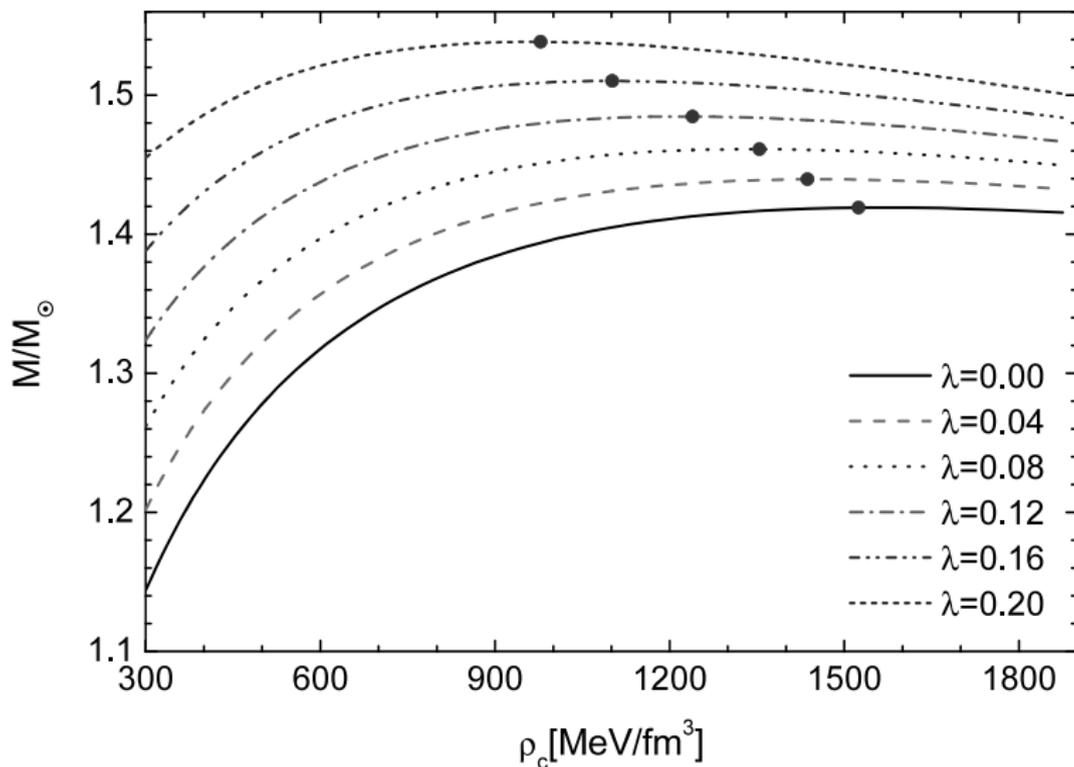
- The hydrostatic equilibrium equations for a spherical object assuming in the  $f(R, T) = R + 2\lambda T$  are<sup>11</sup>

$$\frac{dm}{dr} = 4\pi\rho r^2 + \frac{\lambda}{2}(3\rho - p)r^2, \quad (13)$$

$$\frac{dp}{dr} = -(\rho + p) \frac{4\pi pr + \frac{m}{r^2} - \frac{\lambda(\rho - 3p)r}{2}}{\left(1 - \frac{2m}{r}\right) \left[1 + \frac{\lambda}{8\pi + 2\lambda} \left(1 - \frac{dp}{dp}\right)\right]}, \quad (14)$$

<sup>10</sup>Tiberiu Harko et al. en. In: *Physical Review D* 84.2 (July 2011).

<sup>11</sup>P. H. R. S. Moraes et al. en. In: *Journal of Cosmology and Astroparticle Physics* 2016.06 (2016), p. 005, G. A. Carvalho et al. en. In: *The European Physical Journal C* 77.12 (Dec. 2017).

Figure:  $M \times \rho_c$  for neutron stars.

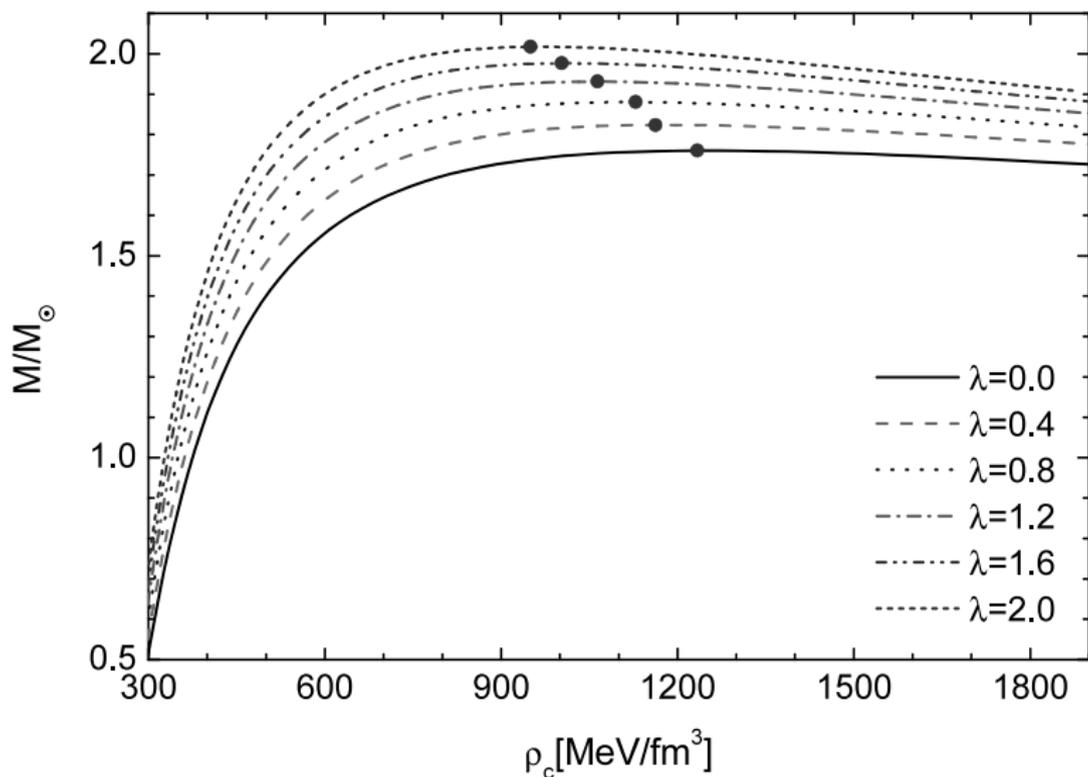
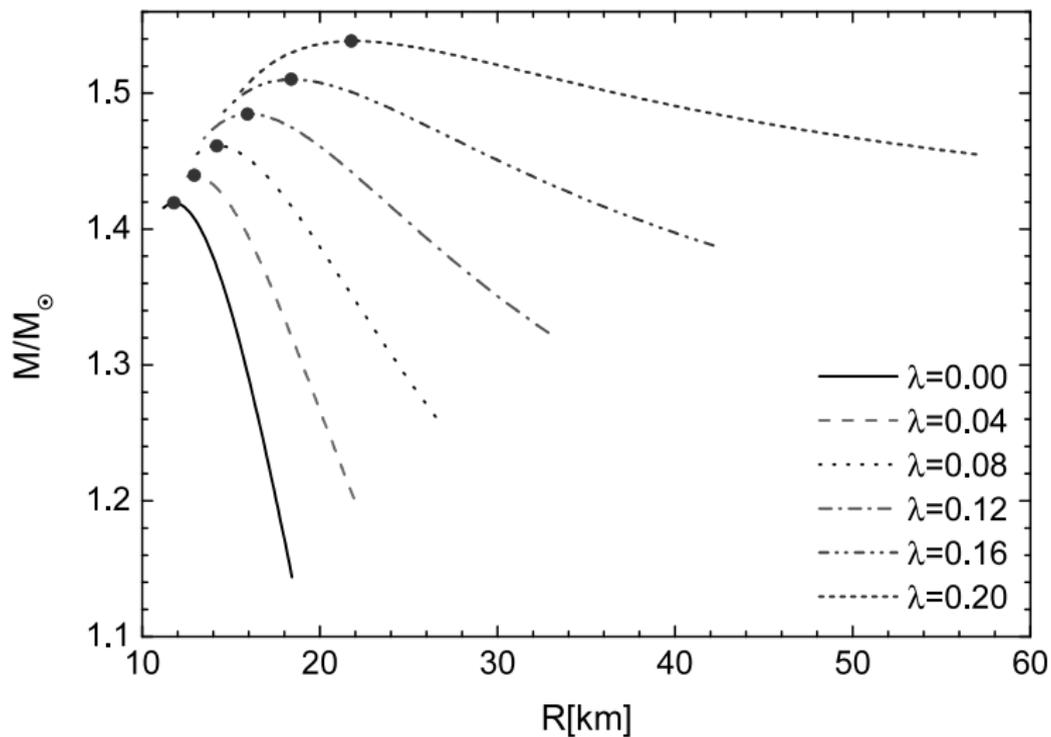
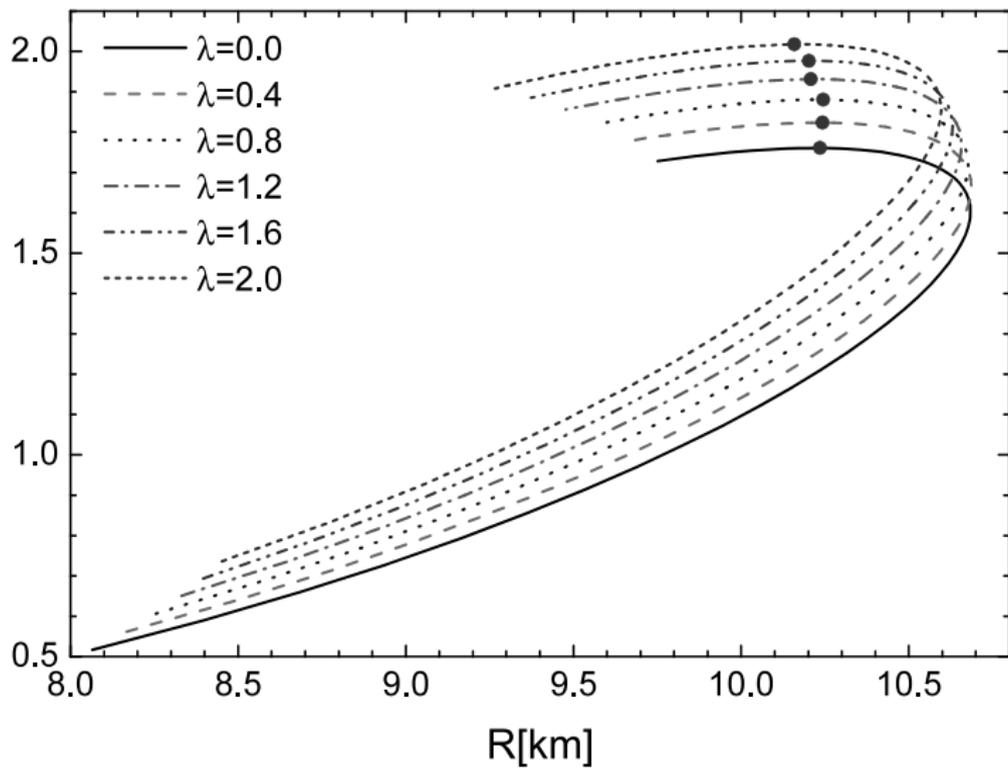
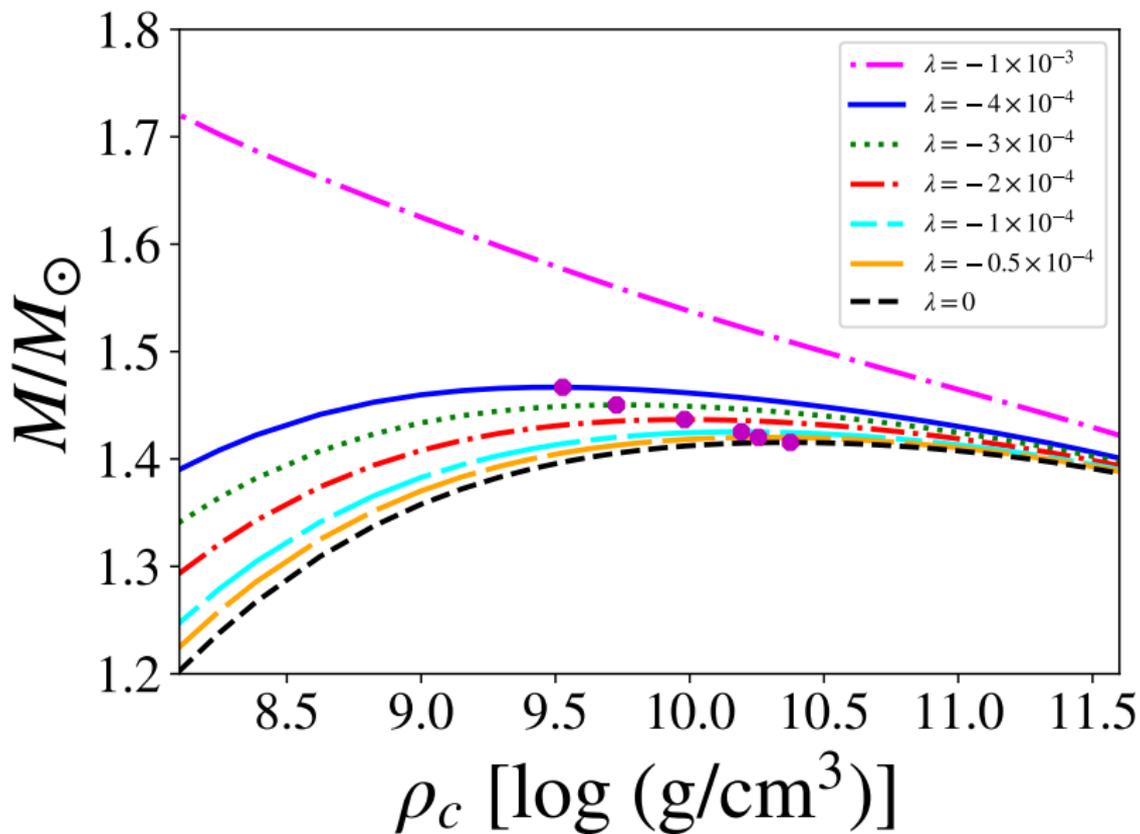
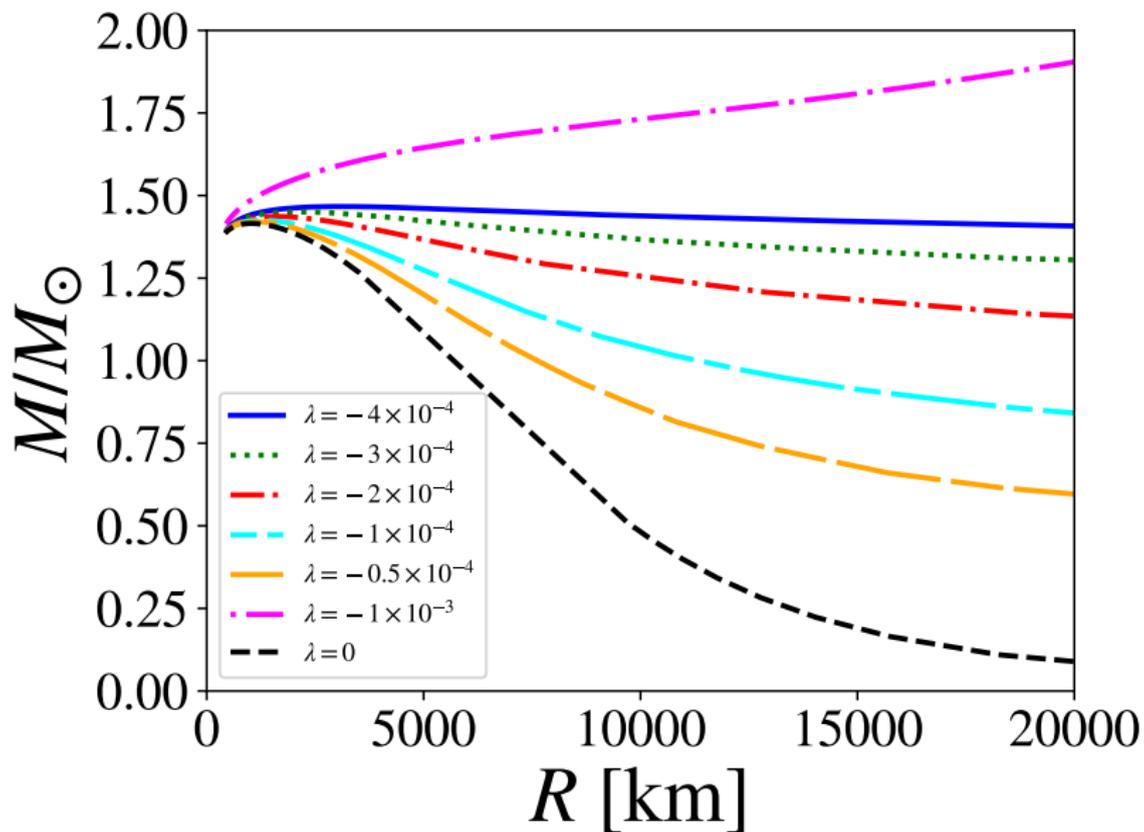


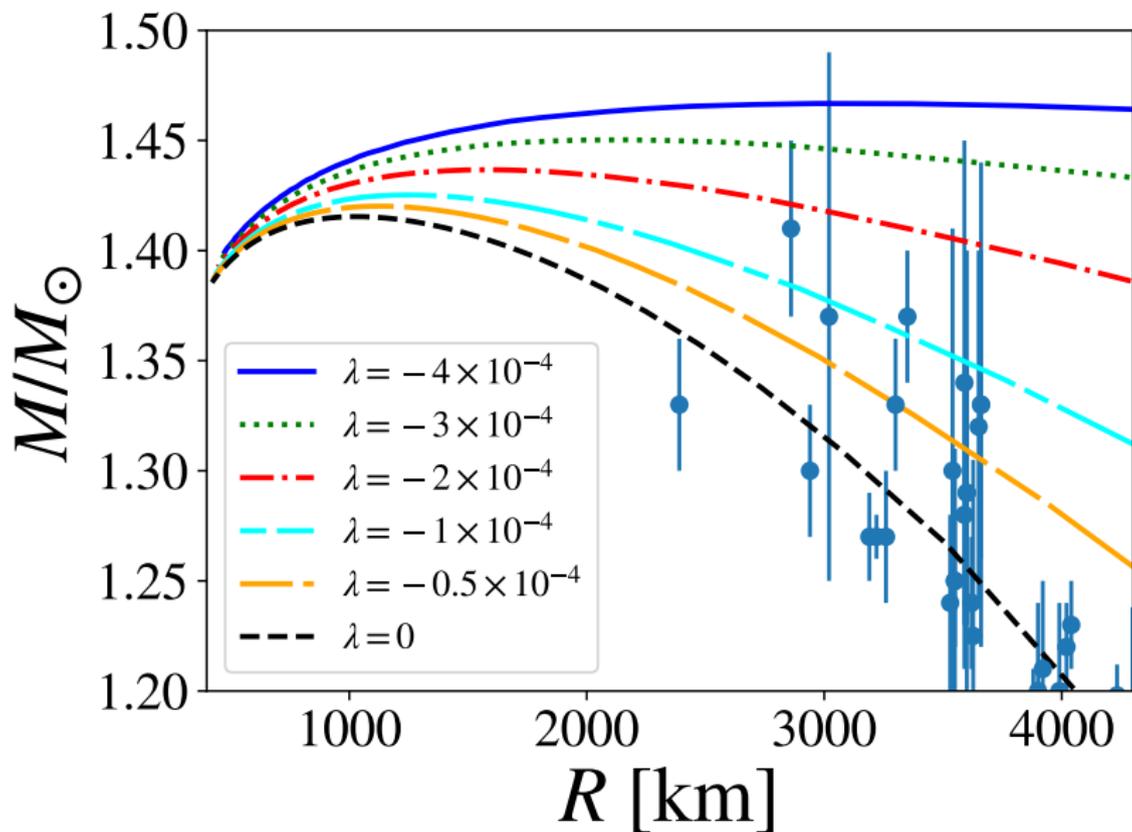
Figure:  $M \times \rho_c$  for strange stars.

Figure:  $M \times R$  for neutron stars.

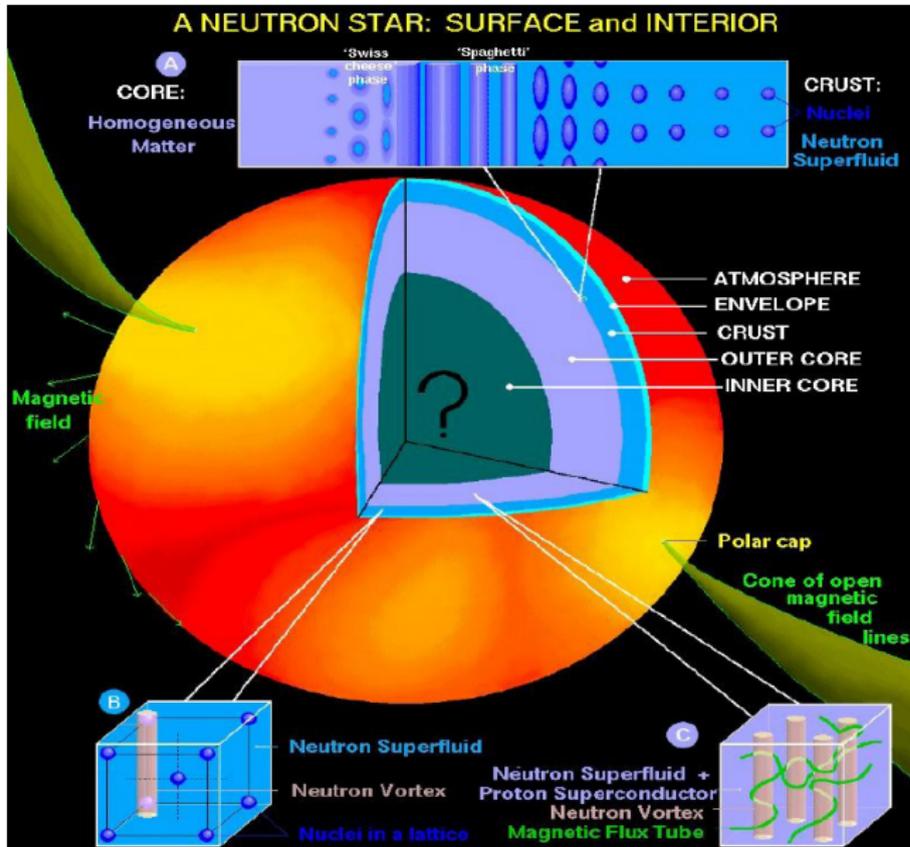
Figure:  $M \times R$  for strange stars.



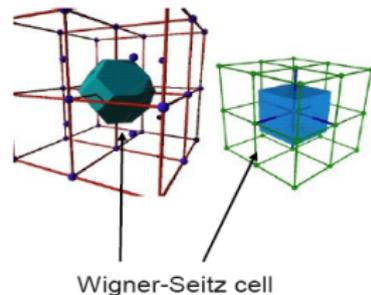




# Neutron star crust



The crust is made of a lattice of nuclei



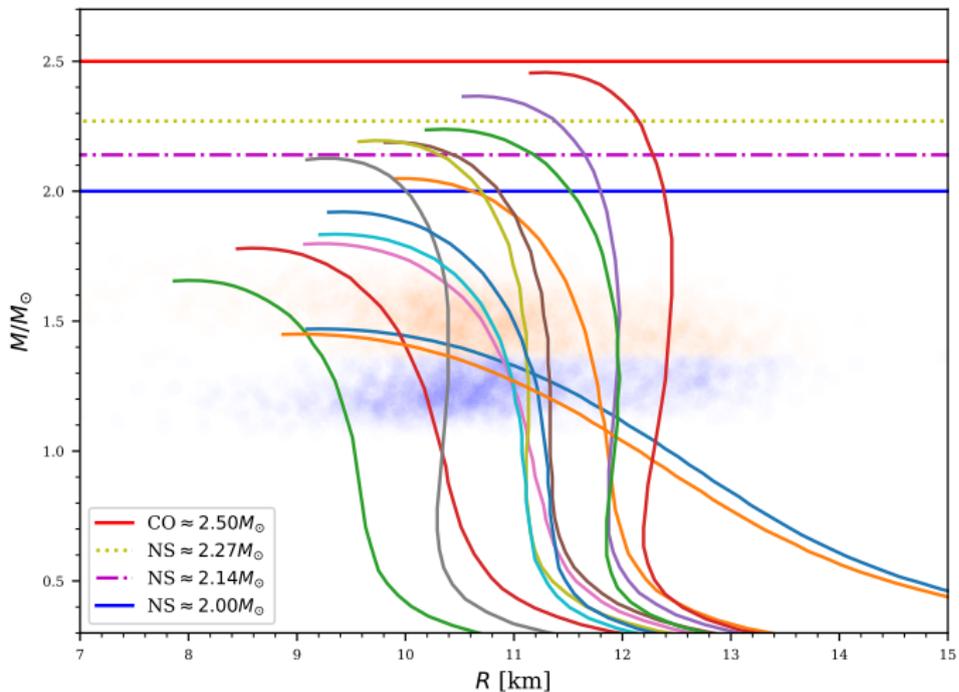
Mass-radius with constrains of GW170817<sup>12</sup> and massive pulsars<sup>13</sup> (GR)

Figure: Mass vs radius. Hadronic EOS.

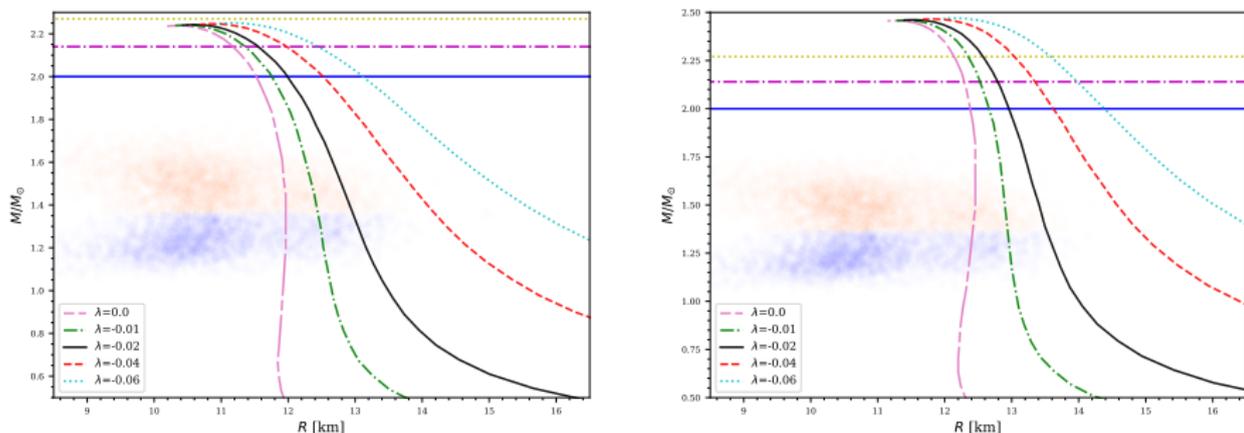
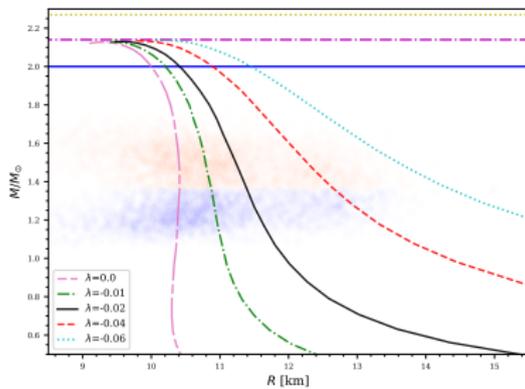
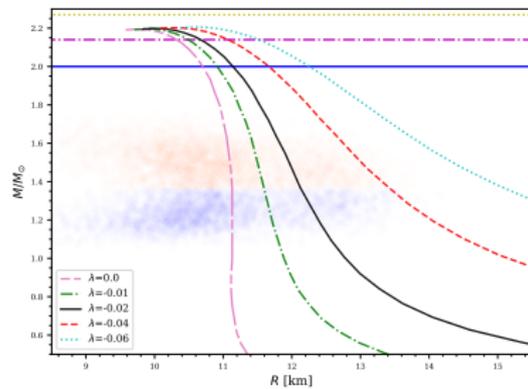


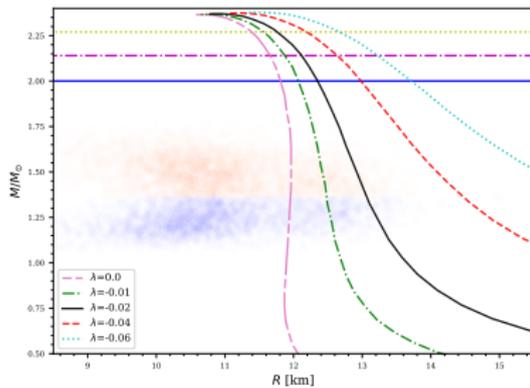
Figure: On the left side, the mass-radius relation for the ENG equation of state. On the right side, the mass-radius relation for the MPA1 equation of state. It was considered five values of  $\lambda$  in the mass-radius for each EoS, going from  $\lambda = -0.06$  to  $0.0$ , for  $\lambda = 0$ , the theory retrieves general relativity. The blue and orange cloud region is the constraints for mass-radius from the GW170817 event, which was a merger of two neutron stars with an observation in the electromagnetic and gravitational spectrum. The blue continuous line at  $2.0 M_{\odot}$ , the magenta dot-dashed line at  $2.14 M_{\odot}$  and the yellow dot line at  $2.27 M_{\odot}$  represent the most massive pulsars observed up to now. The pulsar with  $2.14 M_{\odot}$  has a 95.4% credibility level.



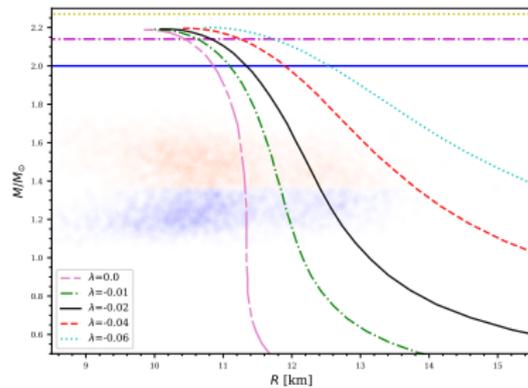
(a) WFF1 equation of state.



(b) WFF2 equation of state.



(c) APR3 equation of state.



(d) APR4 equation of state.

- We also study neutron stars in  $f(\mathcal{R}, \mathcal{T})$  gravity through RFMs models<sup>14</sup>,

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + g_\sigma \sigma \bar{\psi}\psi - g_\omega \bar{\psi}\gamma^\mu \omega_\mu \psi - \frac{g_\rho}{2} \bar{\psi}\gamma^\mu \vec{\rho}_\mu \vec{\tau}\psi + \frac{1}{2}(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) \\ &\quad - \frac{A}{3} \sigma^3 - \frac{B}{4} \sigma^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{C}{4} (g_\omega^2 \omega_\mu \omega^\mu)^2 - \frac{1}{4} \vec{B}^{\mu\nu} \vec{B}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu \\ &\quad + g_\sigma g_\omega^2 \sigma \omega_\mu \omega^\mu (\alpha_1 + \frac{1}{2} \alpha'_1 g_\sigma \sigma) + g_\sigma g_\rho^2 \sigma \vec{\rho}_\mu \vec{\rho}^\mu (\alpha_2 + \frac{1}{2} \alpha'_2 g_\sigma \sigma) + \frac{1}{2} \alpha'_3 g_\omega^2 g_\rho^2 \omega_\mu \omega^\mu \vec{\rho}_\mu \vec{\rho}^\mu. \end{aligned}$$

<sup>14</sup>Bao-An Li et al. en. In: *Physics Reports* 464.4 (Aug. 2008), pp. 113–281, M. Dutra et al. In: *Physical Review C* 90.5 (Nov. 2014), p. 055203.

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- We choose **BKA20**, **BSR8**, **IU-FSU**, and **Z271s4** as representative parametrizations of the “families” **BKA**, **BSR**, **FSU**, and **Z271**.

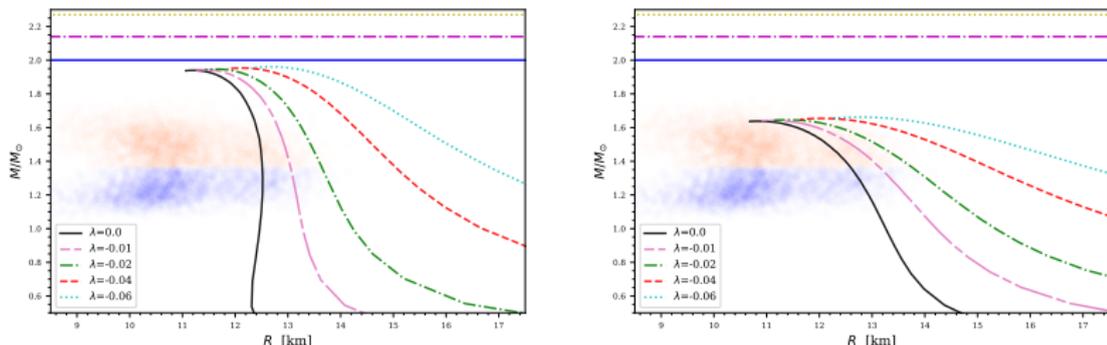


Figure: On the left side, the mass-radius relation for the IU-FSU equation of state. On the right side, the mass-radius relation for the Z271s4 equation of state.

<sup>14</sup>Bao-An Li et al. en. In: *Physics Reports* 464.4 (Aug. 2008), pp. 113–281, M. Dutra et al. In: *Physical Review C* 90.5 (Nov. 2014), p. 055203.

## Hydrostatic equilibrium equations in $f(R, T)$ gravity

- The hydrostatic equilibrium equations for a spherical object assuming in the  $f(R, T) = R + 2\lambda T$  are<sup>15</sup>

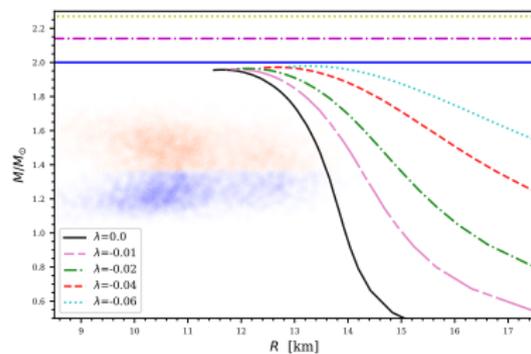
$$\frac{dm}{dr} = 4\pi\rho r^2 + \frac{\lambda}{2}(3\rho - p)r^2, \quad (16)$$

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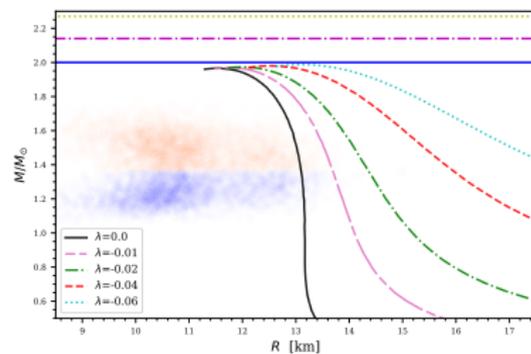
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<sup>15</sup>P. H. R. S. Moraes et al. en. In: *Journal of Cosmology and Astroparticle Physics* 2016.06 (2016), p. 005, G. A. Carvalho et al. en. In: *The European Physical Journal C* 77.12 (Dec. 2017).

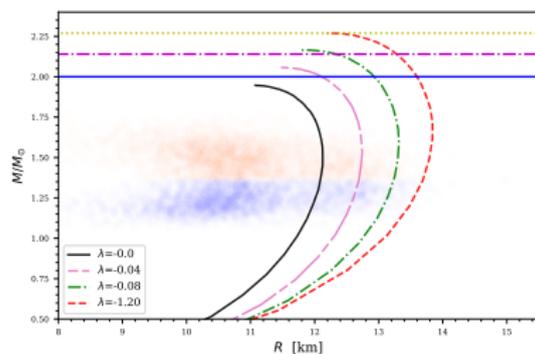
## Effect of the crust



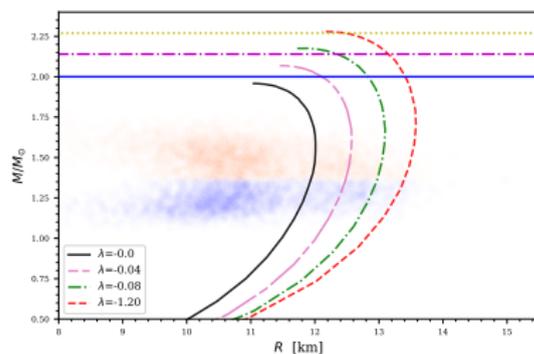
(a) BKA20 equation of state.



(b) BSR8 equation of state.



(c) BKA20 equation of state without crust.



(d) BSR8 equation of state without crust.

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- ⑤ **Stellar mass and radii changes depend only on crust, where the EoS is essentially the same for all the models. The NS crust effect implying very small values of  $|\lambda|$  does not depend on the theory's function chosen, since for any other one the hydrostatic equilibrium equation in  $f(R, T)$  would always have the dependence  $1/v_s$ .**

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- ⑥ **Finally, we highlight that our results indicate that conclusions obtained from NS studies done in modified theories of gravity without using realistic EoS that describe correctly the NS interior can be unreliable.**

## Acknowledgments

- U.S. Department of Energy (DOE) under grant DE-FG02-08ER41533.
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- Coordenação de aperfeiçoamento de Pessoal de Nível Superior (CAPES).
- Project INCT-FNA Proc. No. 464898/2014-5.
- FAPESP Thematic Project 2013/26258-4
- Committee of The modern physics of compact stars and relativistic gravity

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