Late time cosmic acceleration in $f(Q, T)$ gravity

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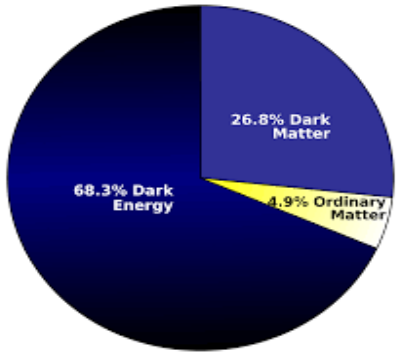
Outline of Presentation

- Introduction and Problem statement.
- Basic Formalism.
- The Models.
- Dynamical Parameters.
- Energy Conditions.
- Cosmographic Test.
- Results and Discussion.

Introduction

- Cosmological observations [1], confirm that the present Universe is expanding and accelerating.

- This is due to an unknown form of energy present in the Universe, known as the Dark Energy. Total cosmic energy budget is made up as:



- The dark energy is an exotic energy form that violates the strong energy condition and is associated with a fluid of negative pressure.
- Again there is some issue with GR in modern cosmology, as it has certain limitations in addressing the late time cosmic acceleration issue.

[1] A.G. Riess, et al., Astron. J. 116 (1998) 1009; S. Perlmutter, et al., Astrophys. J. 517 (1999) 565; P.A.R. Ade, et al., Astron. Astrophys. 594 (2016) A13; N. Aghanim, et al., Astron. Astrophys. 641 (2020) A6.

Problem Statement

- Post Supernovae, several extended gravity theories have been proposed by extending or modifying GR. The recent one is the $f(Q,T)$ gravity [2] , where Q be the non-metricity and T be the trace of energy momentum tensor.

- GR can be represented in two equivalent geometric representation: teleparallel representation and curvature representation.

- In teleparallel representation, the curvature and non-metricity vanishes, but torsion is non-zero whereas in curvature representation the torsion and non-metricity vanishes, but the curvature is non-zero.

- Another equivalent approach is the symmetric teleparallel gravity, where the basic geometry of the gravitational action is represented by the non-metricity Q of the metric [3]. The further development is the $f(Q)$ gravity [4].

[2] Y. Xu et al., Eur. Phys. J. C, **79**, 708 (2019).
[3] J.M. Nester, H-J Y, Chin. J. Phys., 37 (1999) 113.
[4] J.B. Jimenez et al., Phys. Rev. D, 98 (2018) 044048.

Introduction

- Standard cosmological model has many success stories in providing information on the evolution of the Universe.
- The occurrence of initial singularity makes the physical laws presuppose the space-time [5]. If singularity occurs, the description of space-time breaks down.
- The possible way to avoid the initial singularity is to find the non-singular approach without inflation, where the Universe before attaining singularity, would bounce back, known as the matter bounce scenario. However, in this scenario, the strong energy condition violates [6].
- So, in the context of GR, The cosmological bounce can not be described in a homogeneous and isotropic background.
- The investigation is on the role of $f(Q, T)$ gravity in getting a matter bounce scenario and addressing the late time cosmic speed up phenomena with the cosmological models.

[5] J. Earman et al., Singularities and Acausalities in Realtivistic space-time, Oxford University, Press, USA, 1995.

[6] M. Novello, J. Salim, Phys. Rev. D, 20, 377 (1979); V.N. Melnikov, S.V. Orlov, Phys. Lett. A, 70, 263 (1979).

The Mathematical Formalism

The action of $f(Q,T)$ gravity,

$$S=\int\left(\frac{1}{16\pi}f(Q,T)+\mathcal{L}_m\right)d^4x\sqrt{-g}, \tag{1}$$

The non-metricity Q can be defined as,

$$Q\equiv -g^{\mu\nu}(L^k{}_{l\mu}L^l{}_{\nu k}-L^k{}_{lk}L^l{}_{\mu\nu}), \tag{2}$$

where the disinformation tensor,

$$L^k{}_{l\gamma}\equiv -\frac{1}{2}g^{k\lambda}(\bigtriangledown_\gamma g_{l\lambda}+\bigtriangledown_l g_{\lambda\gamma}-\bigtriangledown_\lambda g_{l\gamma})$$

Varying the gravitational action (1), the field equation of $f(Q,T)$ gravity,

$$-\frac{2}{\sqrt{-g}}\bigtriangledown_k\left(f_Q\sqrt{-g}P^k_{\mu\nu}\right)-\frac{1}{2}fg_{\mu\nu}+f_T(T_{\mu\nu}+\Theta_{\mu\nu})-f_Q(P_{\mu kl}Q_\nu{}^{kl}-2Q^{kl}{}_\mu P_{kl\nu})=8\pi T_{\mu\nu}, \tag{3}$$

The super potential of the model can be expressed as,

$$P^k_{\mu\nu} = -\frac{1}{2}L^k_{\mu\nu} + \frac{1}{4}(Q^k - \tilde{Q}^k)g_{\mu\nu} - \frac{1}{4}\delta^k_{(\mu}Q_{\nu)}. \tag{4}$$

The trace of the energy momentum tensor and trace of the non-metricity tensor can be respectively denoted as,

$$\begin{aligned} T &= T_{\mu\nu}g^{\mu\nu} \\ Q_k &= Q_k{}^{\mu}{}_{\mu}, \tilde{Q}_k = Q^{\mu}{}_{k\mu} \end{aligned}$$

We consider the universe described by homogeneous, isotropic and spatially flat FLRW space-time as,

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{5}$$

The dilation rate $\tilde{T} = \frac{\dot{N}(t)}{N(t)}$ and in the standard case $N = 1$, the non-metricity reduces to, $Q = 6H^2$.

Now, the field equations of $f(Q,T)$ gravity [7] in the standard case for the FLRW metric can be expressed in the compact form [8] as,

$$p = -\frac{1}{16\pi}\left[f - 12FH^2 - 4\dot{\zeta}\right] \tag{6}$$

$$\rho = \frac{1}{16\pi}\left[f - 12FH^2 - 4\dot{\zeta}\kappa_1\right], \tag{7}$$

where $F = \frac{\partial f}{\partial Q}$ and $8\pi\kappa \equiv f_T = \frac{\partial f}{\partial T}$, $\kappa_1 = \frac{\kappa}{1+\kappa}$ and $\zeta = FH$. Adding eqns. (6) and (7), the evolution equation of Hubble's function can be obtained as,

$$\dot{\zeta} = 4\pi(p+\rho)(1+\kappa). \tag{8}$$

In comparison to the Friedman equations of Einstein's GR, the effective pressure (p_{eff}) and effective energy density (ρ_{eff}) can be characterized as,

$$2\dot{H} + 3H^2 = \frac{1}{F}\left[\frac{f}{4} - 2\dot{F}H + 4\pi[(1+\kappa)\rho + (2+\kappa)p]\right] = -8\pi p_{eff} \tag{9}$$

$$3H^2 = \frac{1}{F}\left[\frac{f}{4} - 4\pi[(1+\kappa)\rho + \kappa p]\right] = 8\pi\rho_{eff} \tag{10}$$

[7] Y. Xu et al., Eur. Phys. J. C, **79**, 708 (2019).

[8] L. Pati, B. Mishra and S. K. Tripathy, Phys. Scr., **96**, 105003 ((2021).

Three forms for $f(Q,T)$ have been suggested,

- $f(Q,T) = \lambda_1 Q + \lambda_2 T$,
- $f(Q,T) = \lambda_1 Q^m + \lambda_2 T$ (considered in this problem),
- $f(Q,T) = -\lambda_1 Q - \lambda_2 T^2$

$F = \lambda_1 m Q^{m-1}$, $\lambda_2 = 8\pi\kappa$, $\zeta = \lambda_1 m Q^{m-1} H$, $\dot{F} = 2(m-1)F\frac{\dot{H}}{H}$, $\dot{\zeta} = F\dot{H}(2m-1)$.

Now from eqns. (9) and (10), we obtain,

$$p = \frac{2\dot{\zeta}[2+\kappa-\kappa\kappa_1] - \lambda_1(6H^2)^m(1-2m)}{4\pi[(2+\kappa)(2+3\kappa)-3\kappa^2]}, \tag{11}$$

$$\rho = \frac{2\dot{\zeta}[3\kappa-(2+3\kappa)\kappa_1] + \lambda_1(6H^2)^m(1-2m)}{4\pi[(2+\kappa)(2+3\kappa)-3\kappa^2]}. \tag{12}$$

The equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ can be obtained as,

$$\omega = \frac{p}{\rho} = -1 + \frac{4\dot{\zeta}[(1+2\kappa)(1-\kappa_1)]}{2\dot{\zeta}[3\kappa-(2+3\kappa)\kappa_1] + \lambda_1(6H^2)^m(1-2m)} \tag{13}$$

Energy Conditions

If null energy condition is violated then all the other pointwise energy conditions would be violated. However, for dark energy dominated models with negative pressure cosmic fluid, the DEC is satisfied even if the NEC is violated. In some bouncing models or models dominated by phantom fields, the NEC has to be violated. Now, we can express the general form of energy conditions in the context of $f(Q,T) = \lambda_1 Q^m + \lambda_2 T$ gravity as,

$$\rho + p = \frac{1}{4\pi}\left[(1-\kappa_1)\dot{\zeta}\right], \text{NEC} \tag{14}$$

$$\rho + p = \frac{1}{4\pi}\left[(1-\kappa_1)\dot{\zeta}\right], \rho \geq 0 \text{WEC} \tag{15}$$

$$\begin{aligned} \rho + 3p &= \frac{1}{16\pi}\left[-2f + 24FH^2 + 4\dot{\zeta}(3-\kappa_1)\right] \text{SEC} \\ &= \frac{1}{16\pi(1+2\kappa)}\left[-2(1-2m)\lambda_1 Q^m + 2\dot{\zeta}(6+6\kappa-2\kappa_1-6\kappa\kappa_1)\right], \end{aligned} \tag{16}$$

$$\begin{aligned} \rho - p &= \frac{1}{8\pi}\left[f - 12FH^2 - 2\dot{\zeta}(1+\kappa_1)\right] \text{DEC} \\ &= \frac{1}{8\pi(1+2\kappa)}\left[(1-2m)\lambda_1 Q^m + 2\dot{\zeta}(-1+\kappa-\kappa_1-\kappa\kappa_1)\right]. \end{aligned} \tag{17}$$

Some Constraints

Since we are interested to discuss some bouncing models, therefore we expect that the NEC should be violated through out the cosmic evolution. The conditions necessary for NEC violation within the framework of $f(Q,T)$ gravity.

$\rho + p = \frac{1}{4\pi}\left[(1-\kappa_1)\dot{\zeta}\right]$,

NEC needs to be violated, i.e., $\rho + p < 0$ leads to the condition that either $\kappa_1 < 1$ or we have a negative value for $\dot{\zeta}$.

If the first condition holds, then $\kappa = \frac{1}{8\pi}\frac{\partial f}{\partial T} < -1$.

If the second condition holds, we have $\dot{\zeta} < 0$ implying $\dot{H}\frac{\partial f}{\partial Q} + \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial Q}\right)H < 0$.

The symmetric bounce models satisfy, $H = 0$ and $\dot{H} > 0$ at the bounce. So, at least at the bouncing epoch, violation of NEC with $\kappa_1 > 0$ requires $\frac{\partial f}{\partial Q} < 0$.

In an earlier work [9] , we have shown that, $f(Q,T)$ is a negative quantity. Also, it is a decreasing function of the redshift. In view of this, the second condition is mostly satisfied.

[9] L. Pati, B. Mishra and S. K. Tripathy, Phys. Scr., **96**, 105003 ((2021).

- $a(t) = e^{\alpha t}.t^{\beta}$, α , β are constants. Subsequently, $H = \alpha + \frac{\beta}{t}$ and $q = -1 + \frac{\beta}{(\alpha t+\beta)^2}$
- $a(t) = \left(\frac{\alpha}{\chi} + t^2\right)^{\frac{1}{2\chi}}$, α, χ both are positive constants. Subsequently, $H = \frac{t}{\alpha+\chi t^2}$ and $q = -1 - \frac{\alpha}{t^2} + \chi$.
- $a(t) = \sqrt[3]{\frac{3\rho_c t^2}{4} + 1}$, where ρ_c is a constant parameter. The model bounces at the epoch $t_b = 0$ and $a(t_b) = 1$, $H = \frac{\rho_c t}{2}\left(\frac{3\rho_c t^2}{4} + 1\right)^{-1}$.

Model I

Table: Hybrid scale factor parameters value

Models	α	β	z_{da}
HSF11	0.3	0.585	0.8
HSF21	0.2	0.65	0.8
HSF12	0.3	0.47	0.5
HSF22	0.2	0.51	0.5

Model I

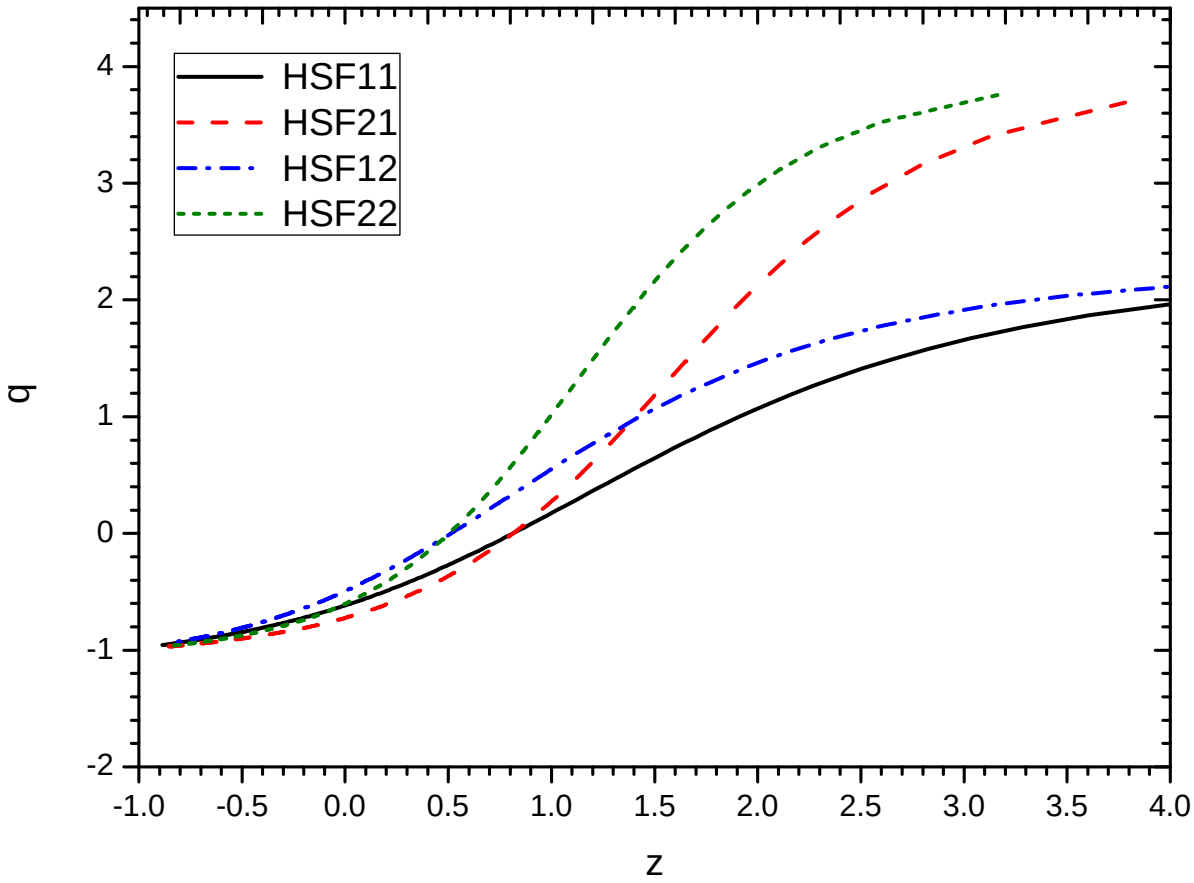


Figure: Plot for the variation of the deceleration parameter in redshift with $a = -4.4$, $b = 0.01$, $m = 0.6$.

Dynamical Parameter

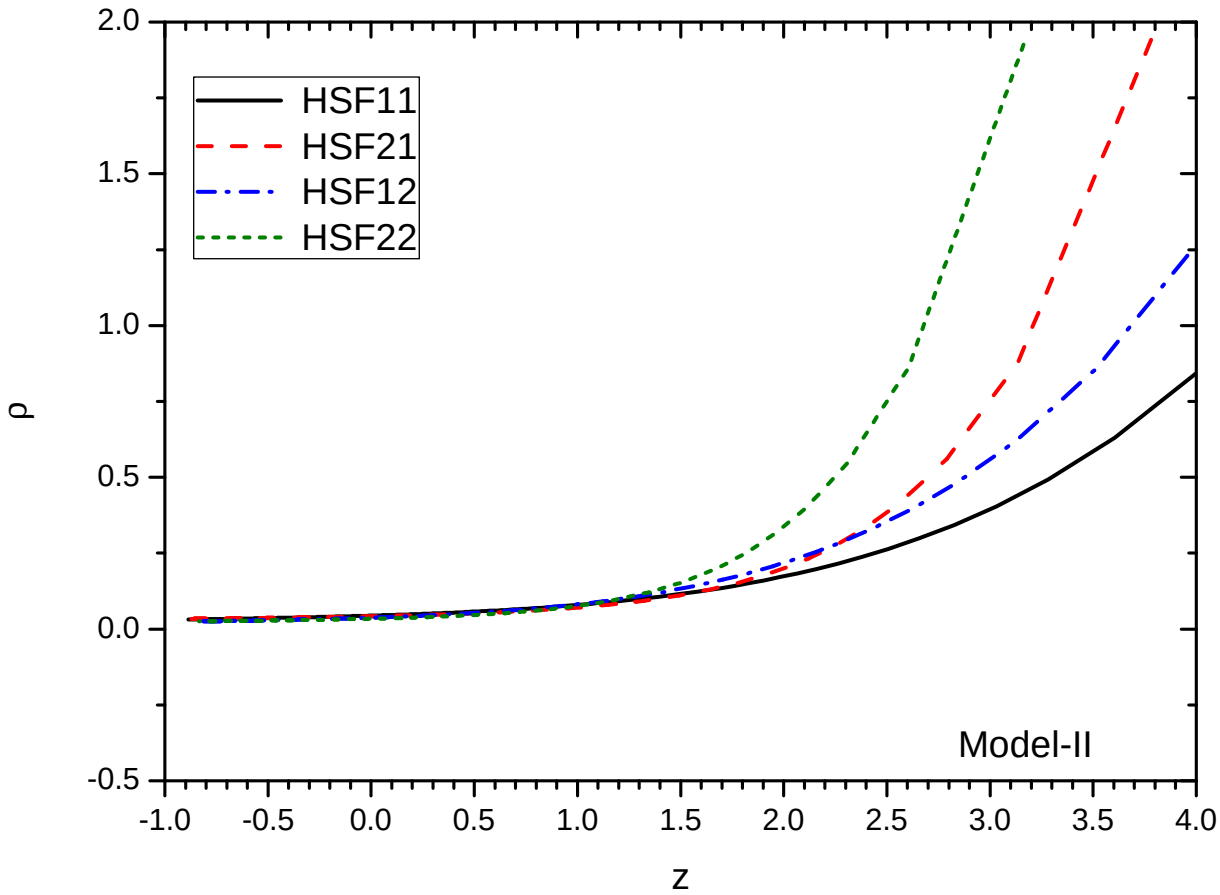


Figure: Plot for the variation of the energy density in redshift with $a = -4.4$, $b = 0.01$, $m = 0.6$.

EoS Parameter

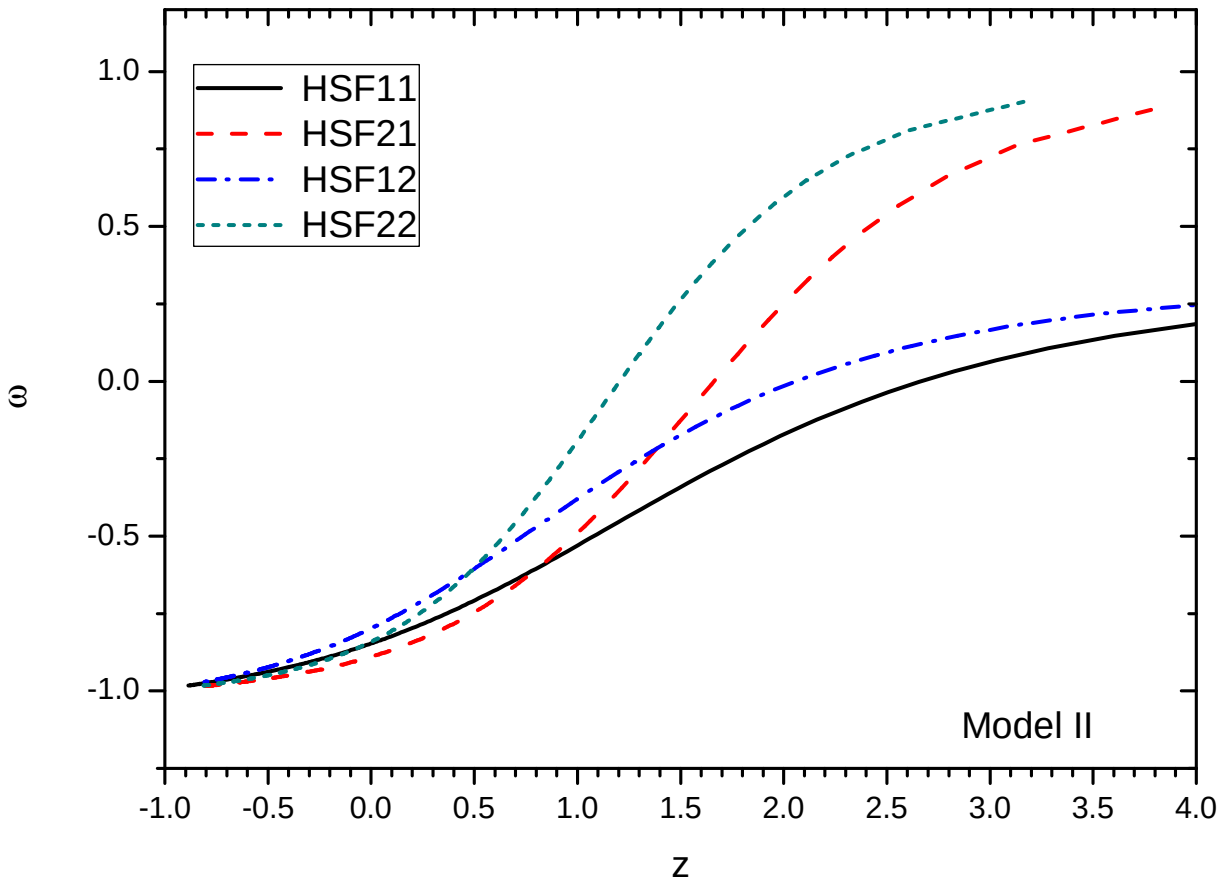


Figure: Plot for the variation of the EoS parameter in redshift with $a = -4.4$, $b = 0.01$, $m = 0.6$.

The HSF

Table: EoS parameter at the present epoch as predicted by the HSF models within $f(Q, T)$ gravity theory.

Models	HSF11	HSF21	HSF12	HSF22
m	ω_0	ω_0	ω_0	ω_0
0.6	-0.847	-0.889	-0.798	-0.841
0.8	-0.796	-0.852	-0.730	-0.788
1.0	-0.745	-0.815	-0.663	-0.736
1.1	-0.719	-0.797	-0.629	-0.709
1.5	-0.617	-0.723	-0.494	-0.603
2	-0.489	-0.631	-0.325	-0.471

$f(Q,T)$ Gravity

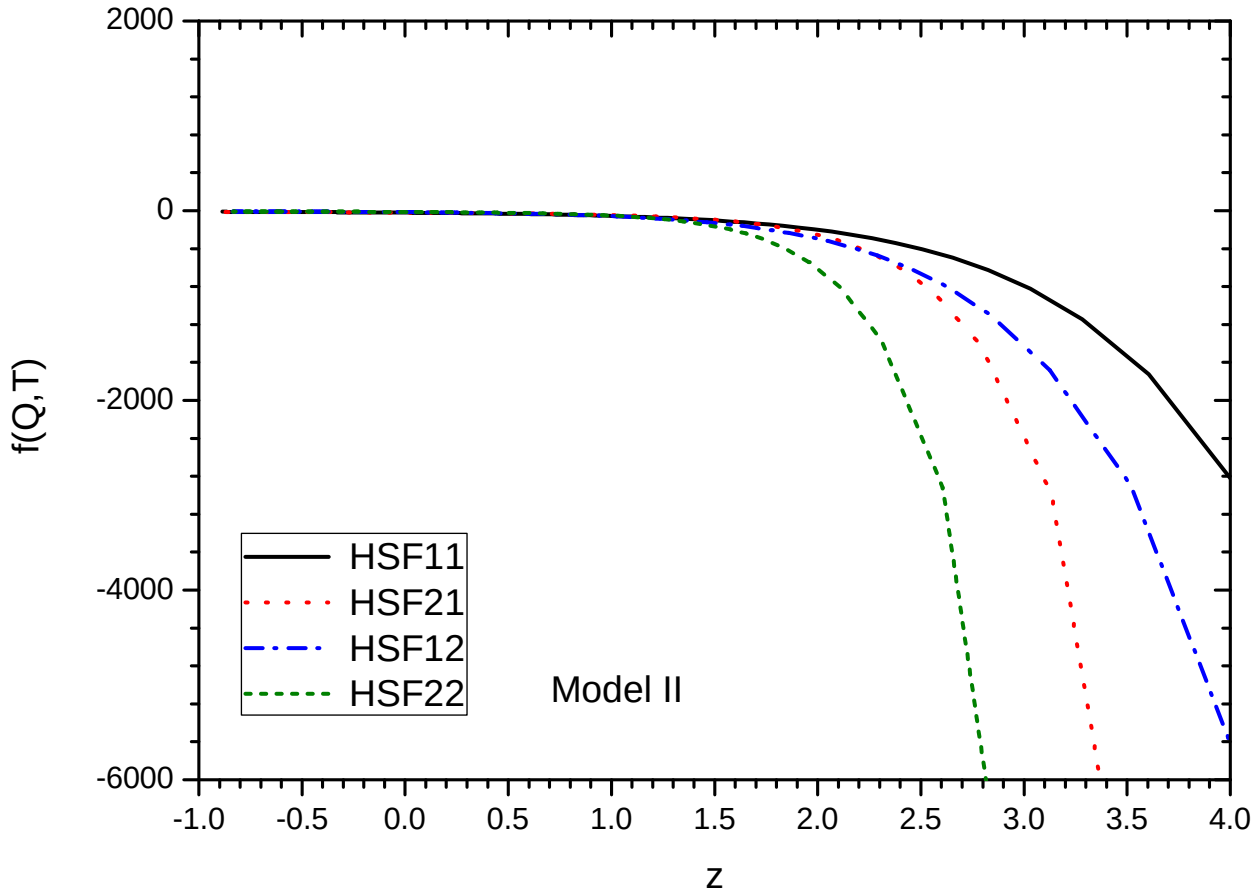
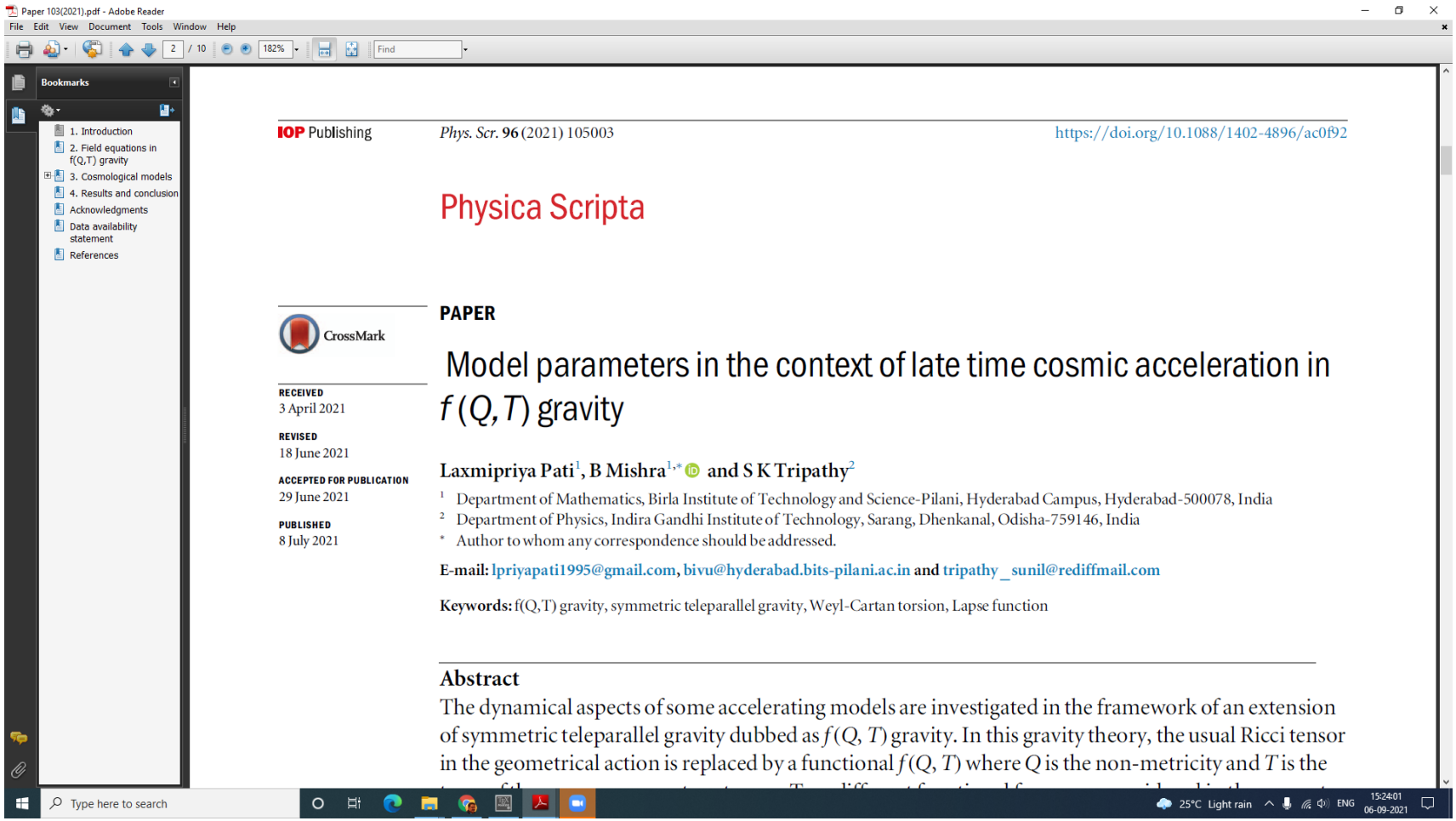


Figure: Plot for the variation of $f(Q,T)$ in redshift with $a = -4.4$, $b = 0.01$, $m = 0.6$.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1. Introduction
2. Field equations in f(Q,T) gravity
3. Cosmological models
4. Results and conclusion
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Model parameters in the context of late time cosmic acceleration in f (Q, T) gravity
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Abstract
The dynamical aspects of some accelerating models are investigated in the framework of an extension of symmetric teleparallel gravity dubbed as f (Q, T) gravity. In this gravity theory, the usual Ricci tensor in the geometrical action is replaced by a functional f (Q, T) where Q is the non-metricity and T is the
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Matter Bounce Scenario

- At the bouncing epoch, the scale factor a contracts to non-zero finite size, the Hubble parameter $H = 0$, $q = -1 + \frac{\dot{H}}{H^2}$ becomes singular.
- The NEC is violated, since the Hubble parameter changes sign from the bouncing point, therefore this phenomena is ruled out in GR.
- The slope of the scale factor increases after the bounce. During the matter contraction phase the Hubble parameter remains negative. It becomes positive during the matter expansion process.

Model II

$$H(z) = \frac{\left(\chi - \alpha(1+z)^{2\chi}\right)^{\frac{1}{2}}(1+z)^{\chi}}{\chi^{\frac{3}{2}}} \tag{18}$$

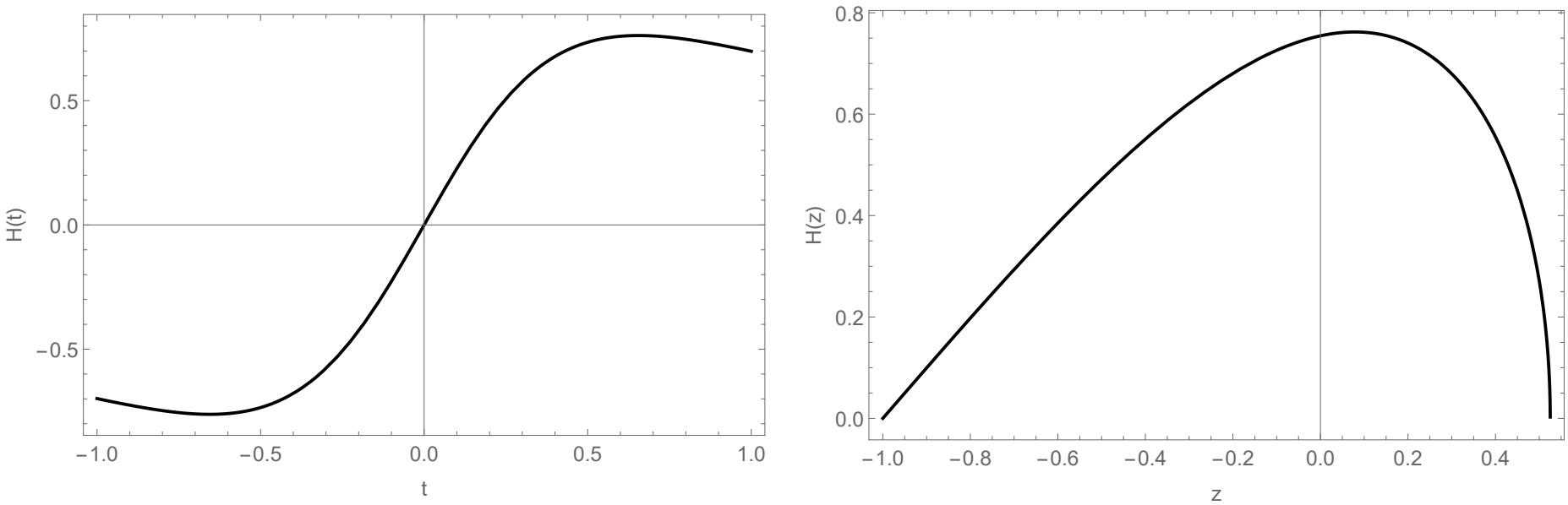


Figure: Plot for the variation of the Hubble parameter vs. cosmic time (Left panel) and Hubble parameter vs. redshift (Right panel) with $\alpha = 0.43$ and $\chi = 1.001$

Dynamical Parameters

The pressure, energy density and EoS parameter for the $f(Q, T)$ model.

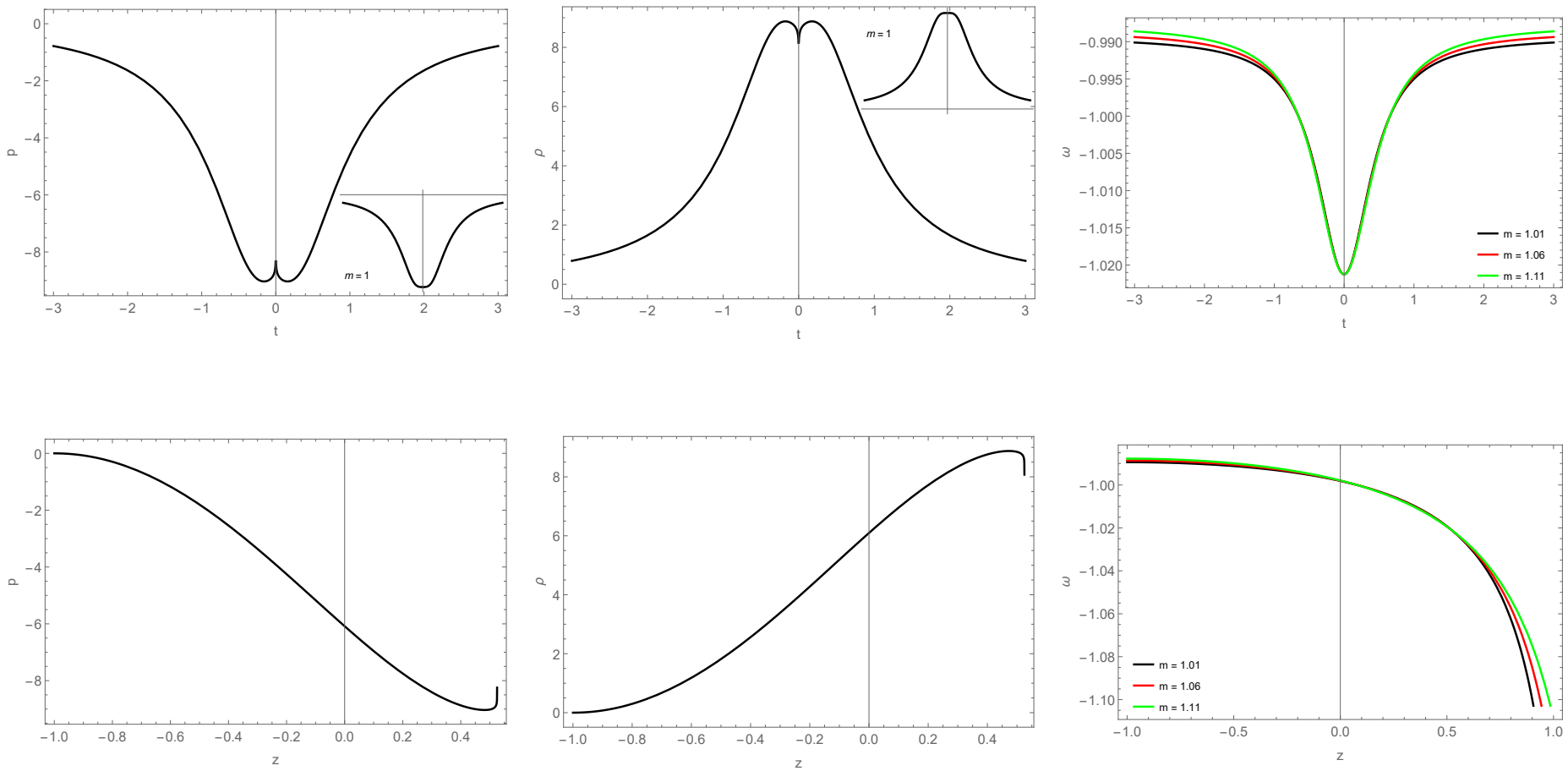


Figure: Plot for the variation of the pressure (Left panel), energy density (Middle panel) and equation of state parameter (Right panel) with the $\alpha = 0.43$, $\chi = 1.001$, $\lambda_1 = -0.5$, $\lambda_2 = -12.5$ and $m = 1.01$.

Dynamical Parameters

- The energy density increases in the pre bounce epoch as we move closer to the bouncing point. It forms a ditch near the bounce and then in the post bounce region, it decreases. For a choice of the teleparallel gravity parameter $m = 1$ (inset in the left panel figure), the ditch in the energy density near the bounce disappears. With respect to the redshift, the energy density decreases from a higher positive value to a smaller positive value.
- The pressure looks like a mirror image of the curve for energy density. The pressure is a negative quantity through out the cosmic evolution. In the pre bounce region, it decreases from a small negative value to large negative value at the bounce. In the post bounce region, it increases from large negative values to small negative values. A small bump is formed in the curve of the pressure at the bouncing epoch which disappears for $m = 1$. In view of this, we believe that, the choice of the model parameter m has a substantial effect on the dynamical aspects of the model. The pressure as a function of redshift, increases almost linearly after showing a sharp decrease in an initial epoch around $z = 0.525$.
- The $p < 0$ and $\rho > 0$ behaviour in the post bounce period, illustrates an accelerating Universe.

Dynamical Parameters

- As we move from the pre bounce period to post bounce period through the bouncing epoch, the EoS parameter is observed to decrease initially, passes the phantom divide and then increases again after forming a well near the bounce. The depth of the well depends on the choice of the parameter m.
- In the post bounce period, it increases from a phantom field dominated phase to a quintessence like phase.
- The role of the modified gravity parameter is to split the tail region of the EoS parameter because the rate of increment in ω is higher for large values of m. For all the values of m chosen in the present work, ω tends to -0.99 as z tends to -1, and at $z = 0$ it has a value close to -0.998.
- This prediction shows that, the present model provides viable bouncing scenario alongside providing a suitable explanation for the late time cosmic speed up issue.

Model III

$$H(z) = \frac{\sqrt{\rho_c}}{\sqrt{3}} \left(1 - (1+z)^3\right)^{\frac{1}{2}} (1+z)^{\frac{3}{2}} \tag{19}$$

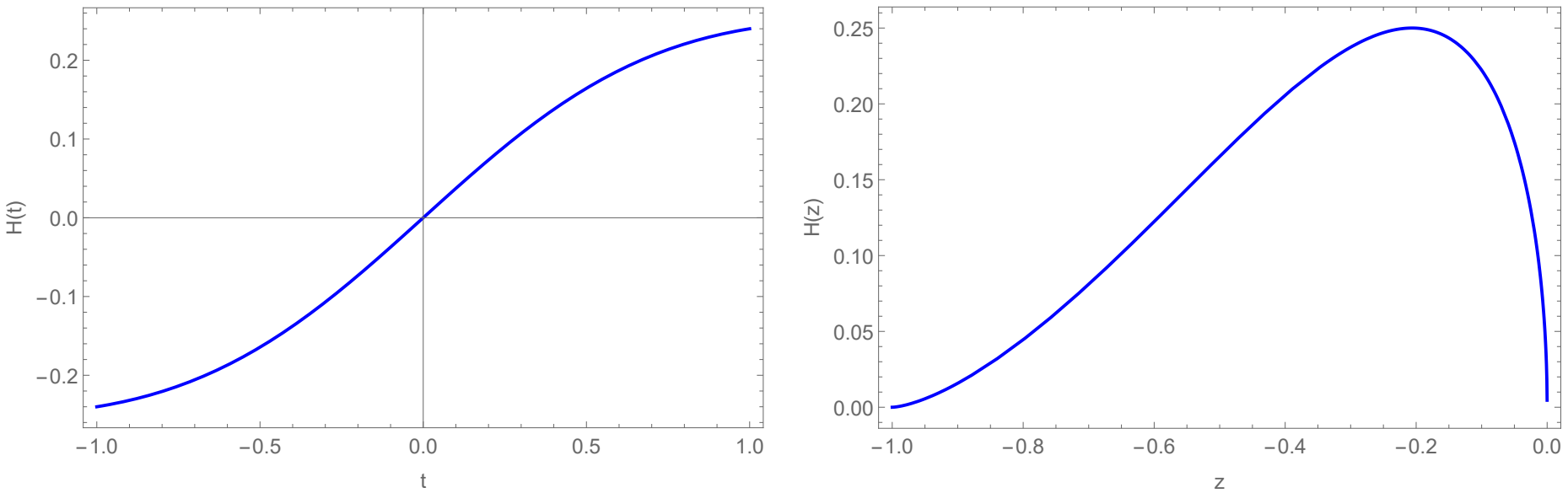


Figure: Plot for the variation of the Hubble parameter vs. cosmic time (Left panel) and Hubble parameter vs. redshift (Right panel) with $\rho_c = 0.75$.

Dynamical Parameters

The pressure, energy density and EoS parameter for the II model.

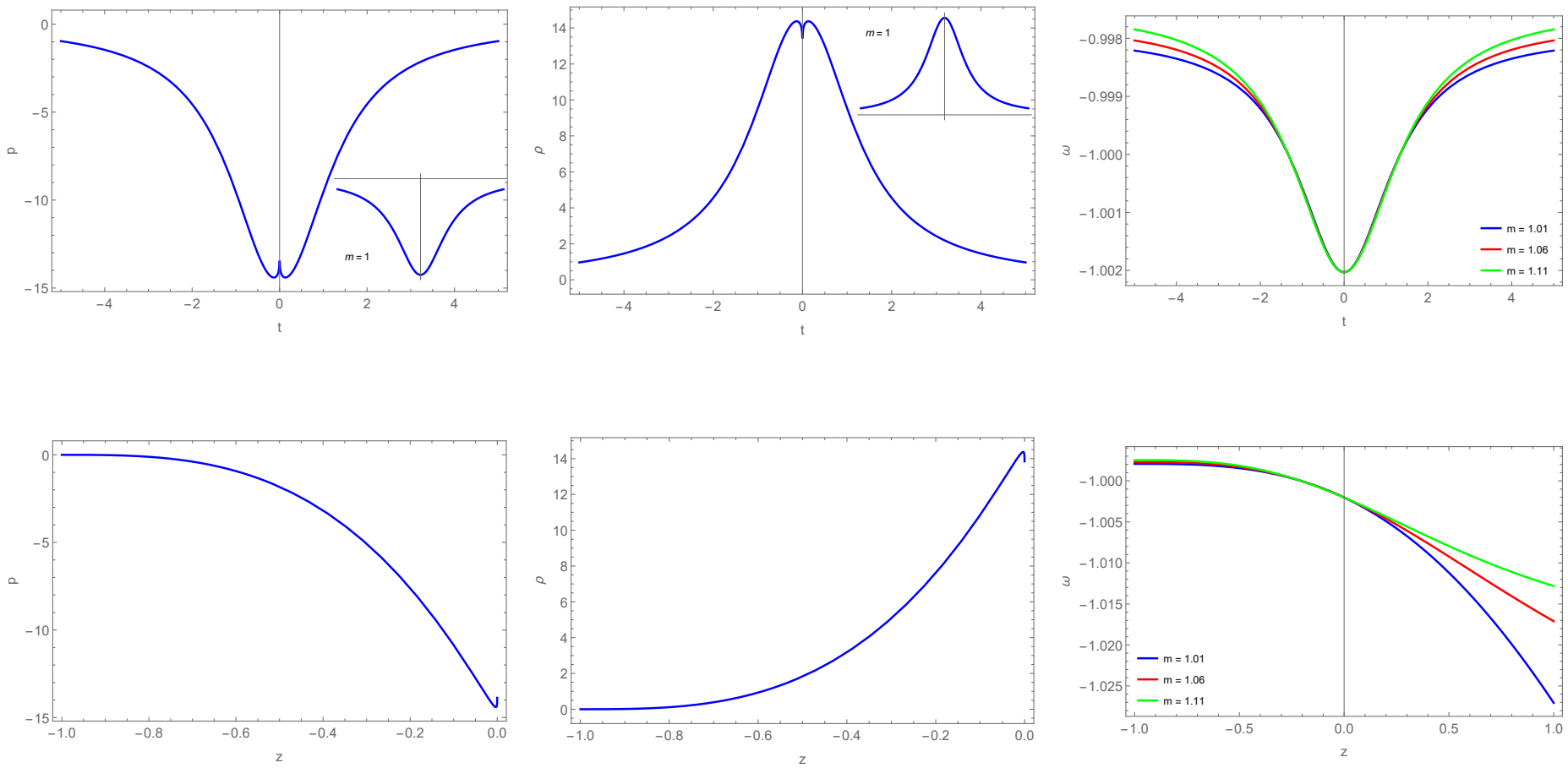
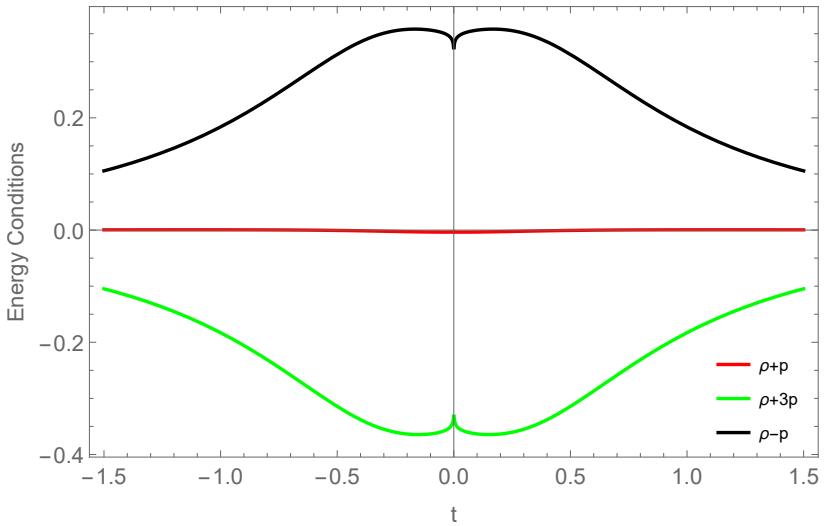


Figure: Plot for the variation of the pressure (Left panel), energy density (Middle panel) and equation of state parameter (Right panel) with the $\rho_c = 0.75$, $\lambda_1 = -0.5$, $\lambda_2 = -12.56$ and $m = 1.01$.

Dynamical Parameters

- The energy density demonstrates the bounce at $t = 0$ with a well-shaped curve. The parameters have been limited to ensure that the energy density remains positive during the bounce and in both the negative and positive time zones.
- It evolves from a small positive value to form a ditch at bounce and then it decreases in the positive time zone. The peak value of the energy density and the formation of the ditch depends on the choice of the parameter m. For $m = 1$, the ditch disappear and the energy density curve almost follow a Gaussian pattern.
- The pressure evolves with negative values through out. In the negative time zone, as the cosmic time approaches the bouncing epoch, the value of the pressure drops more rapidly. In the post bounce period, the pressure increases rapidly after forming a bump near the bounce. The formation of the bump depends on the choice of m.
- Throughout the cosmic evolution, the EoS parameter remains near the concordant ΛCDM value $\omega = -1$. It evolves from a lower negative value, crosses the phantom divide and attains a minimum in the phantom like region at the bouncing epoch. In the post bounce period, it again rises from a phantom like region to a quintessence region. At the present epoch, it assumes a value of $\omega_0 = -1.002$ which is closer to the recent estimates.

Energy Conditions of the Models



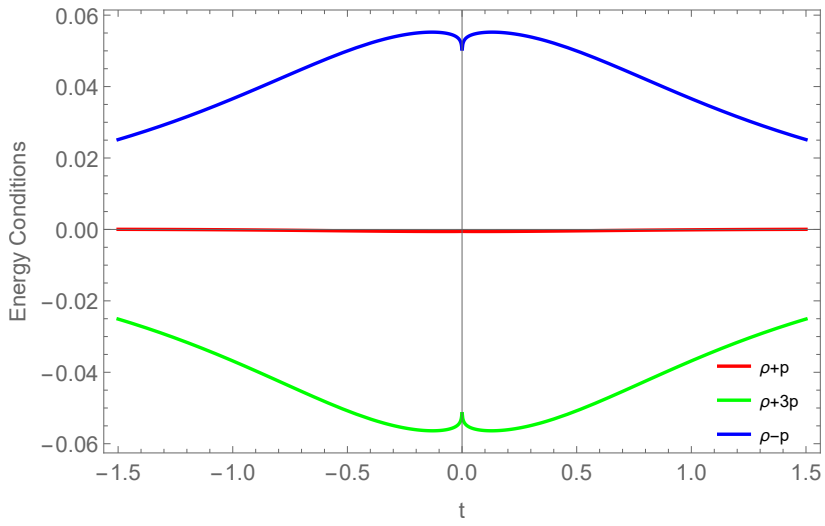
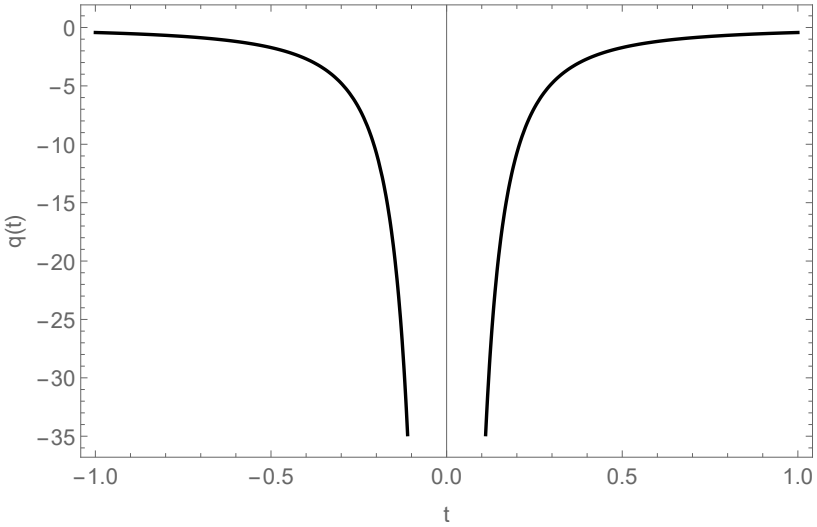
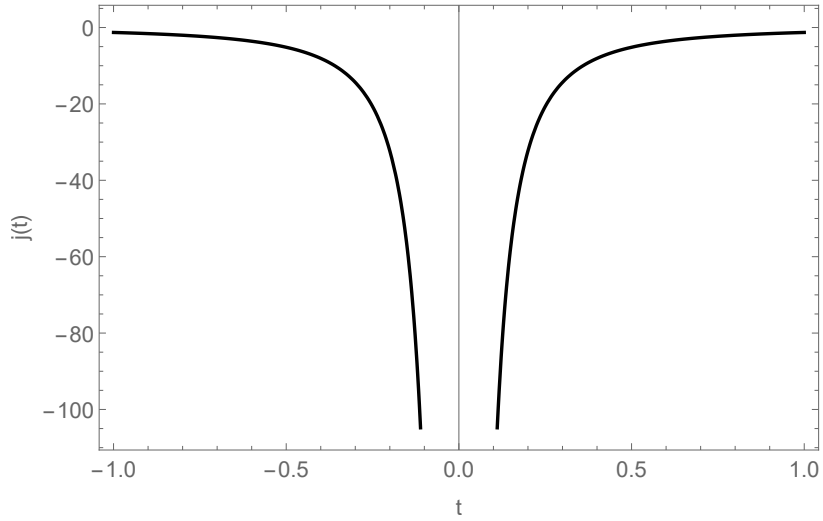
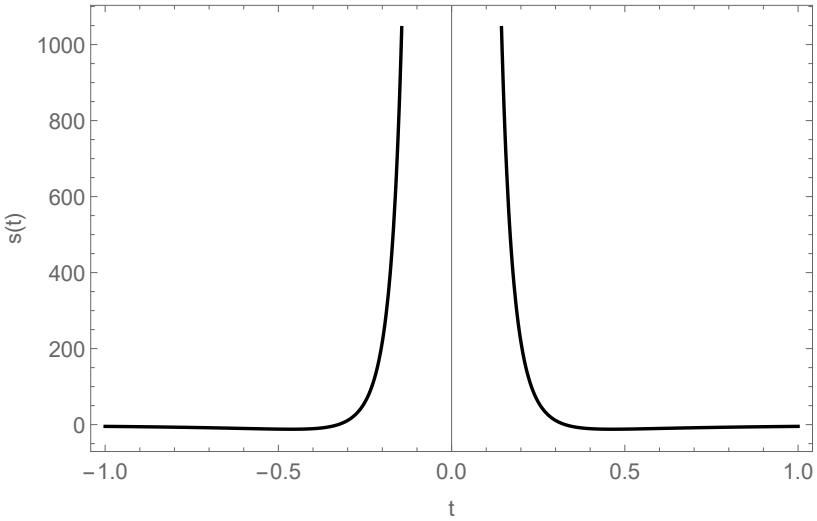


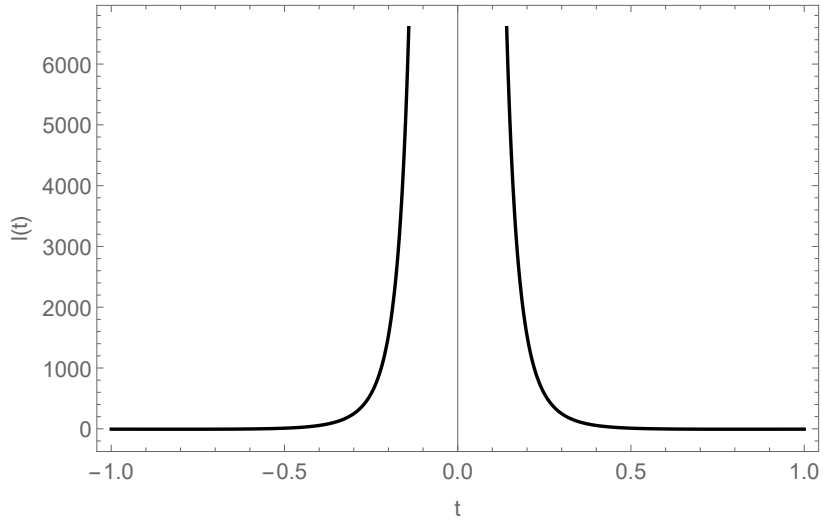
Figure: Plot for the variation of the energy conditions vs. cosmic time.

Cosmographic Parameters for Model I

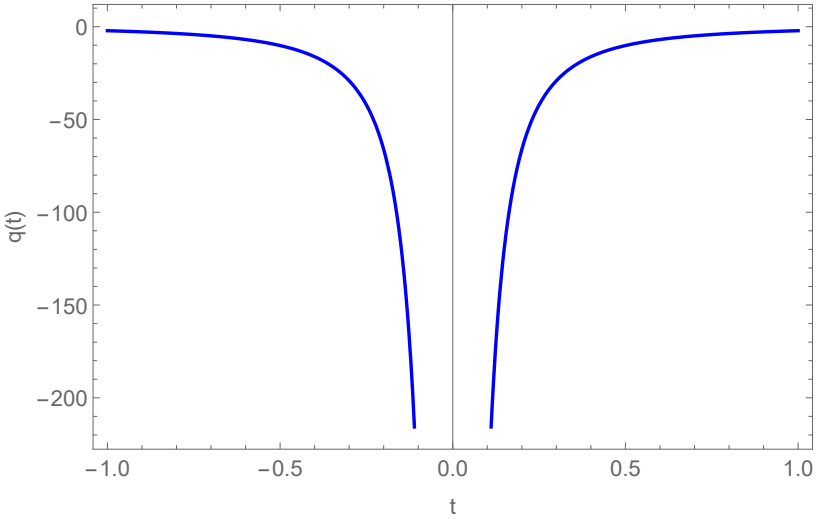


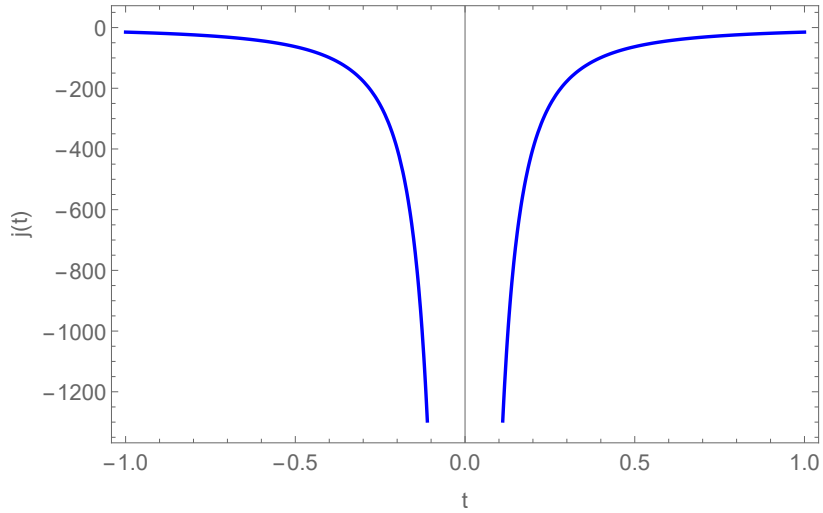


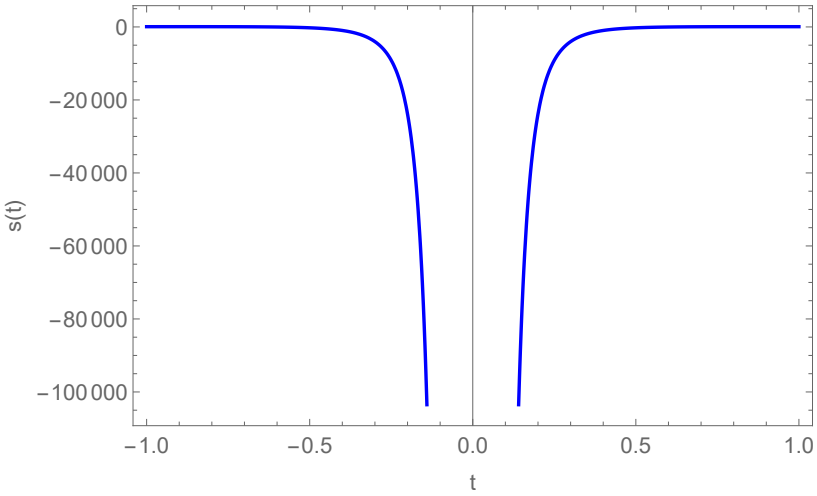


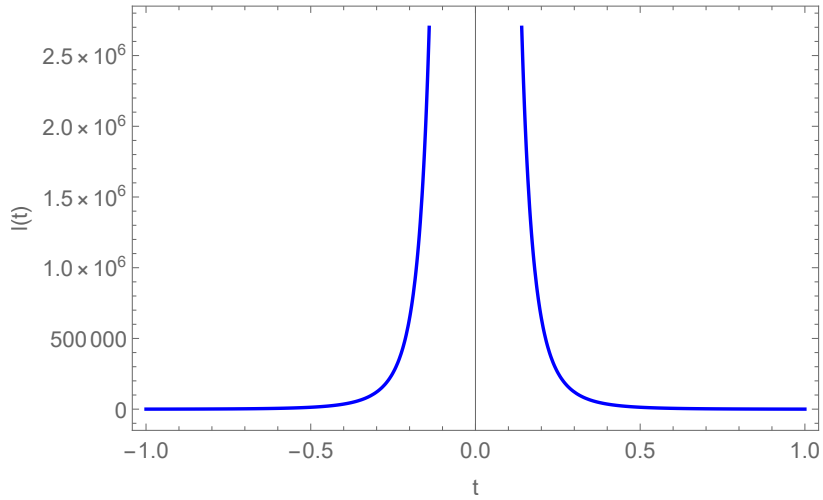


Cosmographic Parameters for Model II









Results and Discussion

- The late time cosmic acceleration issue can be addressed in the context of $f(Q, T)$ gravity.
- The matter bounce scenario are possible within the formalism of $f(Q, T)$ gravity theory besides providing a suitable behaviour for the EoS parameter.
- The evolutionary aspect of the dynamical parameters such as the pressure, energy density and the EoS parameter show a kind of ditch/bump behaviour near the bounce.
- In order to justify the models, we have validated the models through the calculation of the cosmographic parameters and the energy conditions.
- As it should be for any dark energy models, the strong energy condition is observed to be violated through the cosmic evolution both in the positive and negative time frame.

Results and Discussion

- Also, for bouncing scenario to be materialised, the null energy condition be violated at the bounce. Although, we obtain marginal violation of the NEC for our models, we fix this responsibility to the choice of the model parameters.
- As such in bouncing models, the violation of NEC leads to thermodynamical and mechanical instability in the models. The instability of the models are also shown.
- We hope this new version of symmetric teleparallel gravity may provide suitable geometrical alternatives to dark energy models.
- However, further investigations in this direction is required to understand the stability and the issue of non-conservation of the energy-momentum in this geometric theory.

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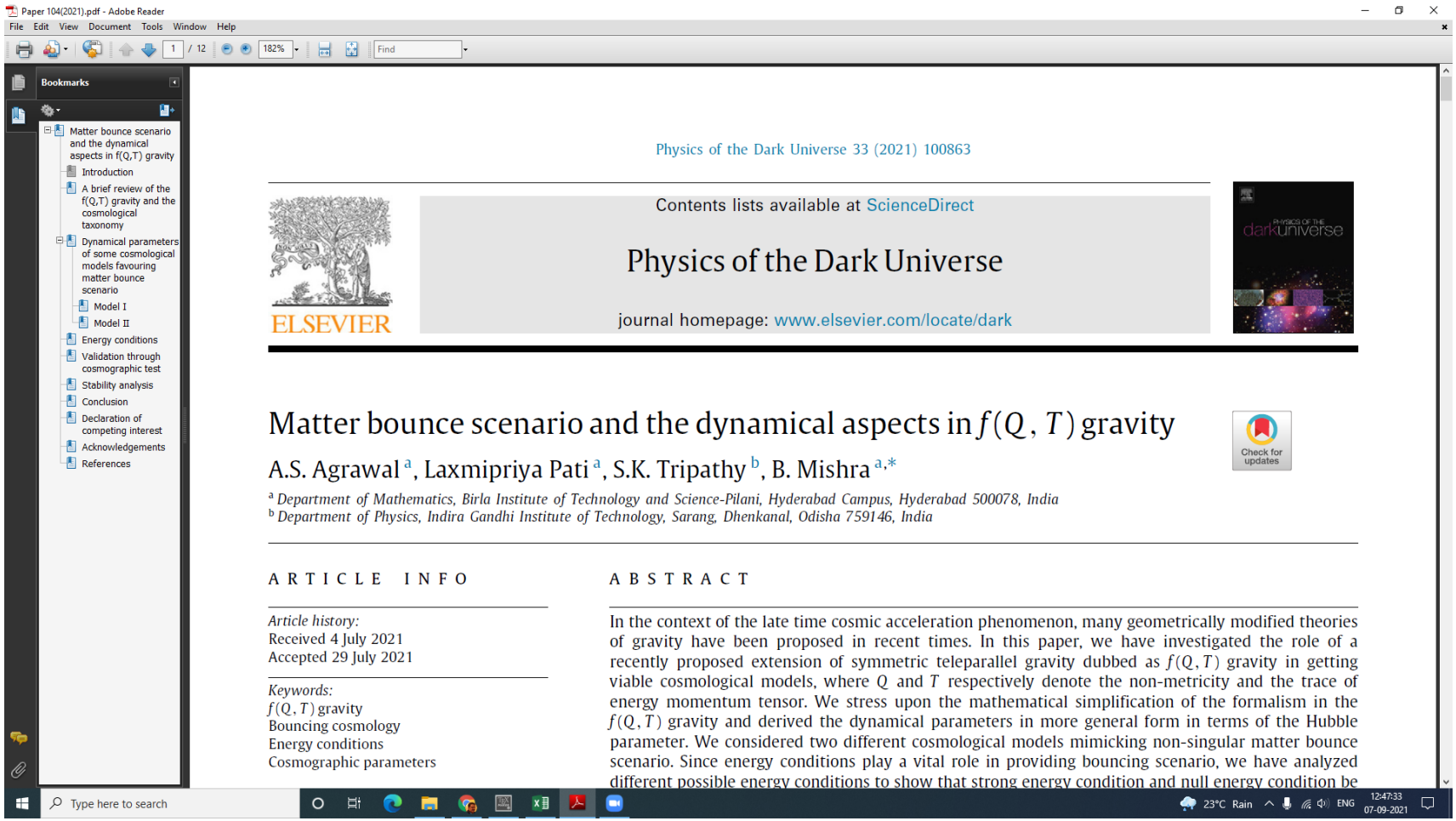

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Bookmarks

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Matter bounce scenario and the dynamical aspects in $f(Q, T)$ gravity

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ABSTRACT

In the context of the late time cosmic acceleration phenomenon, many geometrically modified theories of gravity have been proposed in recent times. In this paper, we have investigated the role of a recently proposed extension of symmetric teleparallel gravity dubbed as $f(Q, T)$ gravity in getting viable cosmological models, where Q and T respectively denote the non-metricity and the trace of energy momentum tensor. We stress upon the mathematical simplification of the formalism in the $f(Q, T)$ gravity and derived the dynamical parameters in more general form in terms of the Hubble parameter. We considered two different cosmological models mimicking non-singular matter bounce scenario. Since energy conditions play a vital role in providing bouncing scenario, we have analyzed different possible energy conditions to show that strong energy condition and null energy condition be

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