Stiffening of matter
in **quark-hadron continuity**

--- peak in sound velocity ---

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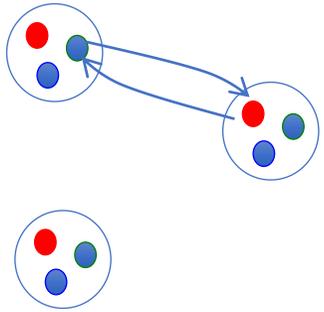
Ref)   TK,   2106.06687   [to be published in PRD]
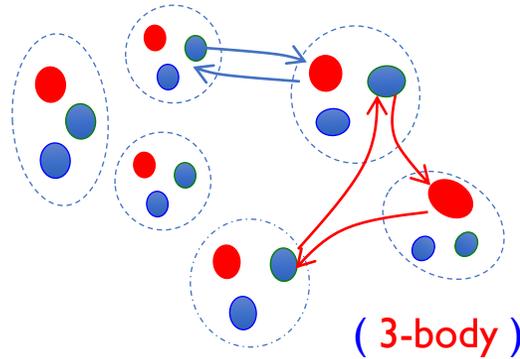
TK-Suenaga,   2110.xxxxx   [to appear]

A picture: **our proposal**

[Masuda-Hatsuda-Takatsuka '12; TK-Powell-Song-Baym '14]
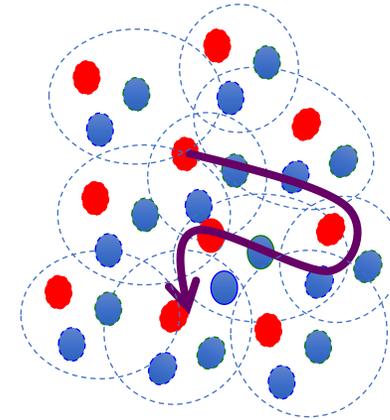
- **few** meson exchange
- nucleons **only**

- **many**-quark exchange
- structural change,...
- hyperons, Δ, ...

- Baryons overlap
- Quark Fermi sea

(3-body)

ab-initio nuclear cal.

laboratory experiments

steady progress

most difficult

(d.o.f ??)

strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

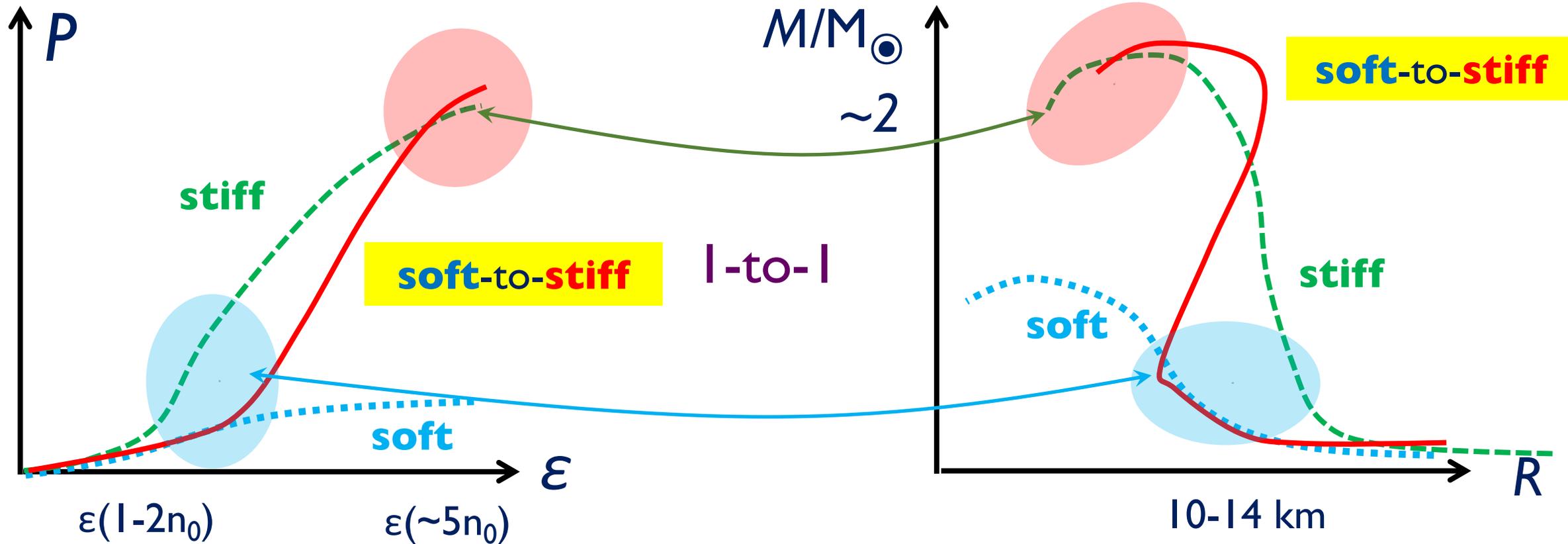$\sim 1.4\ M_\odot$

$\sim 2\ M_\odot$

n_B

Hints from NS

$\sim 2n_0$

$\sim 5n_0$

$\sim 40n_0$

EoS & Neutron Star M-R relation

gravity

(source: ε_{QCD})

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \dfrac{8\pi G}{c^4} T_{\mu\nu}$

QCD (+EW) EoS

QCD pressure

P

$M/M_\odot$

~2

stiff

soft-to-stiff

soft-to-stiff

1-to-1

stiff

soft

soft

ε

R

$\varepsilon(1\text{-}2n_0)$

$\varepsilon(\sim 5n_0)$

10-14 km

Observations: summary (2010-) [Miller+ '21]

quarks? (~ 3-5n_0)

M = 2.08 ±0.07 $M_\odot$

NICER obs.
J 0740+6620

$R_{2.08}$ = **12.35** ±0.75 km

1st order P.T.

GW170817

$R_{1.40}$ = **12.45** ±0.65 km

$R_{2.08} \sim R_{1.40}$ (!)

nuclear (n_B < 1.5-2n_0)

11 – 13 km

P

quark

nuclear

1st order

ε

Soft to *stiff* is challenging: **a new quality**

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*) ➡ *nuclear* & *quark* physics constrain each other
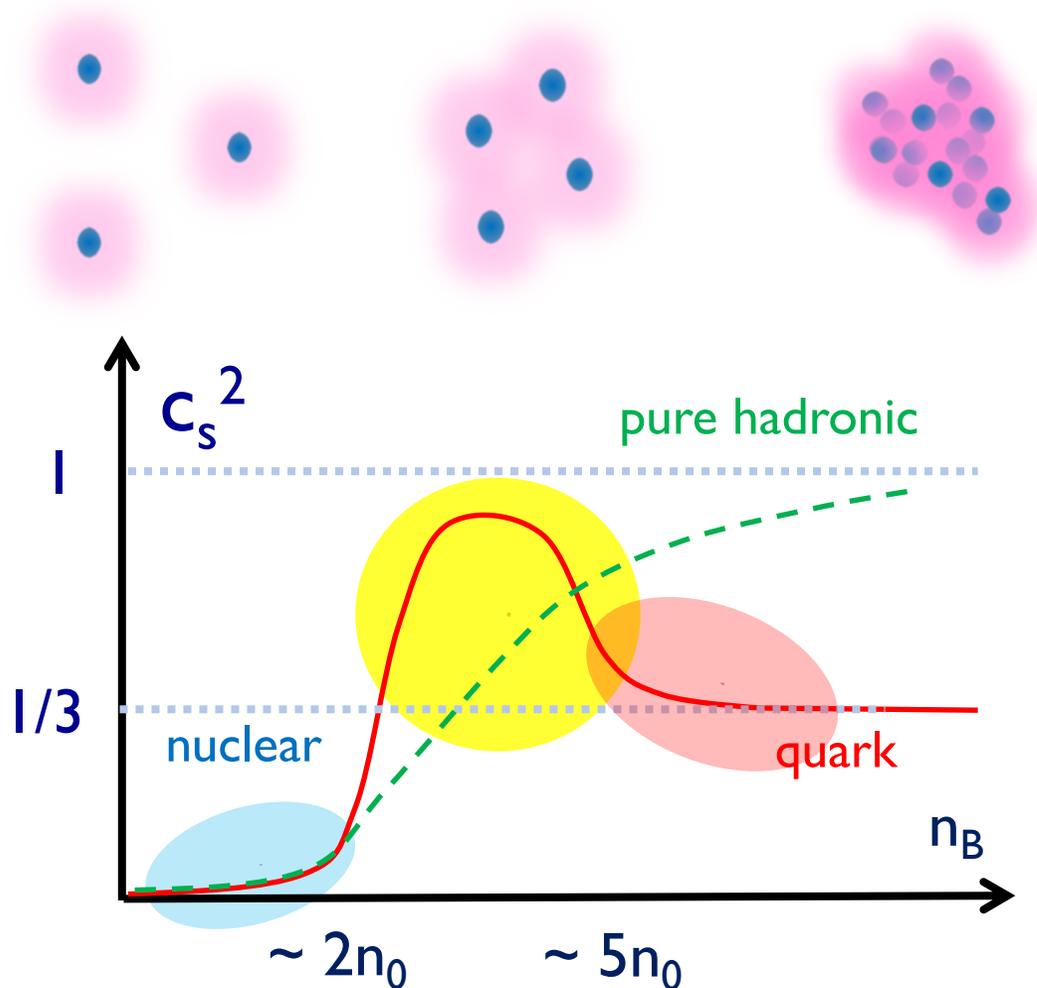

P

$M > 2M_\odot$

stiff

$R_{1.4} \sim 12$ km

soft

$c_s^2 > 1$

ε

$\varepsilon(1\text{-}2n_0)$
nuclear

$\varepsilon(\sim 5n_0)$
quarks?

new quality

c_s^2

microphysical insights ?

forbidden

1

$1/3$

n_B

$\sim 2n_0$

$\sim 5n_0$

baseline: *quark-hadron continuity*

Goals in this work

Direct descriptions of the "c_s-peak"

[cf) McLerran-Reddy (MR), PRL '19,...]

the statements include:

1) *built in a quark-hadron continuity model*

2) *appears before baryon cores overlap*

3) *nuclear repulsive forces are NOT major driving forces*
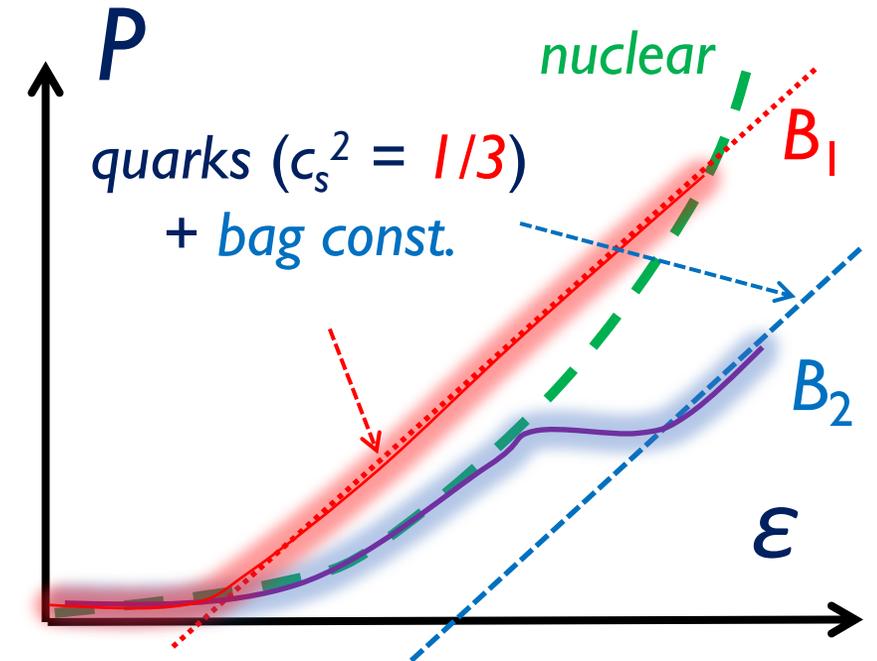
[rather nuclear int. smear the peak]

c_s^2

pure hadronic

1

1/3

nuclear

quark

n_B

$\sim 2n_0$ $\sim 5n_0$

Contents

Problem

- *Switching from baryonic to quark bases*

  → *a source of confusions in hybrid models*

  (e.g. normalization of energy)

Strategy

Follow quark states from nuclear to quark matter
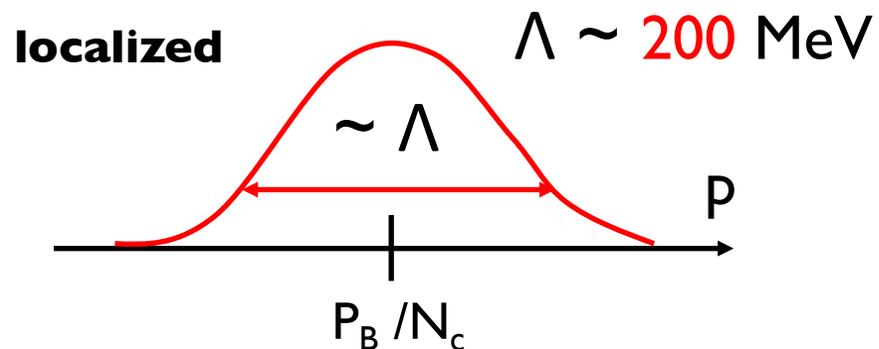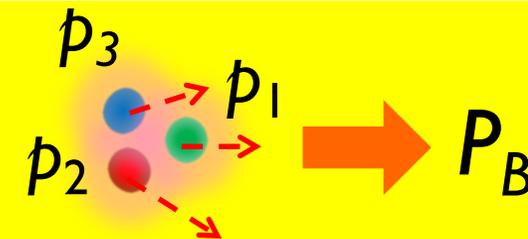
(within a *single* model)

Quarks in a baryon

N_c (=3): number of colors

probability density:

$$Q_{\text{in}}(\boldsymbol{p}, \boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2}$$

p_3
p_1
p_2
P_B

localized $\Lambda \sim$ 200 MeV

$\sim \Lambda$

P

P_B / N_c

mean: $\quad \langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\text{in}}(\boldsymbol{p}, \boldsymbol{P}_B)$

variance: $\quad \left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$ *energetic !*

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B=0} \left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

$\times N_c$

$\times N_c$

$\sim N_c (M_q + \Lambda) \quad \gg \quad \sim P_B{}^2 / (N_c E_q)$

baryon mass

baryon kin. energy

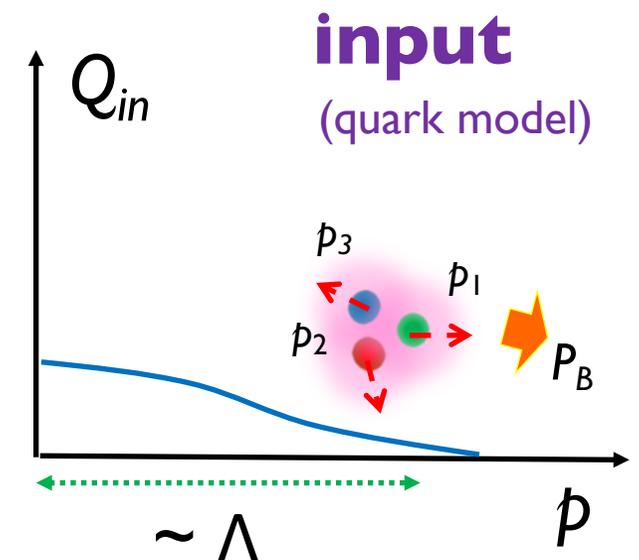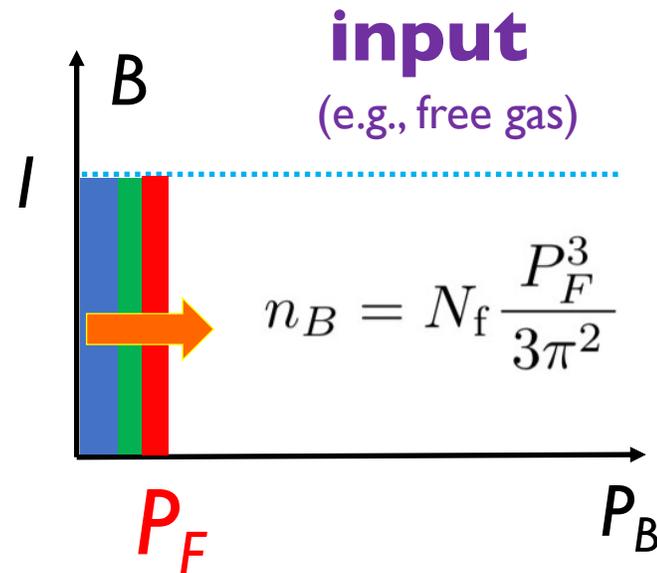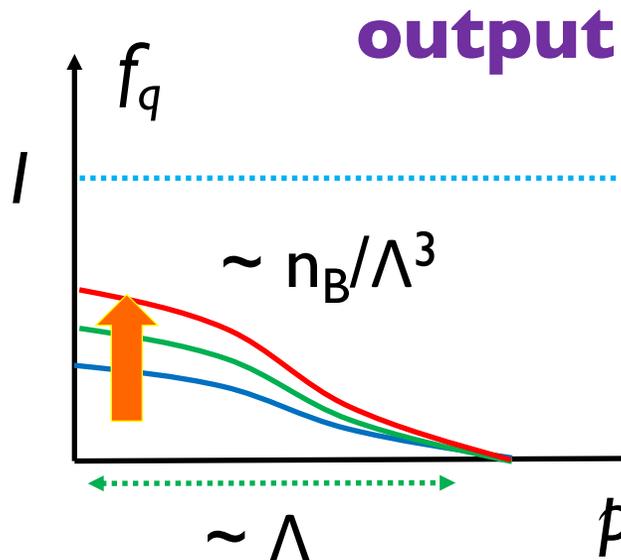# Occupation probability of quark states

occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_q(\underline{p}; n_B) = \int_{\underline{P_B}} \mathcal{B}(\underline{P_B}; n_B) Q_{\text{in}}(\underline{p}, \underline{P_B})$$

input

input

e.g.) in **dilute** baryonic matter

output

f_q

I

$\sim n_B/\Lambda^3$

$\sim \Lambda$

p

input
(e.g., free gas)

B

I

$n_B = N_{\text{f}} \dfrac{P_F^3}{3\pi^2}$

$\otimes$

P_F

P_B

input
(quark model)

Q_{in}

p_3

p_1

p_2

P_B

$\sim \Lambda$

p

Quarks in **ideal baryon gas**

$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\text{in}}(\boldsymbol{p}, \boldsymbol{P}_B)$$
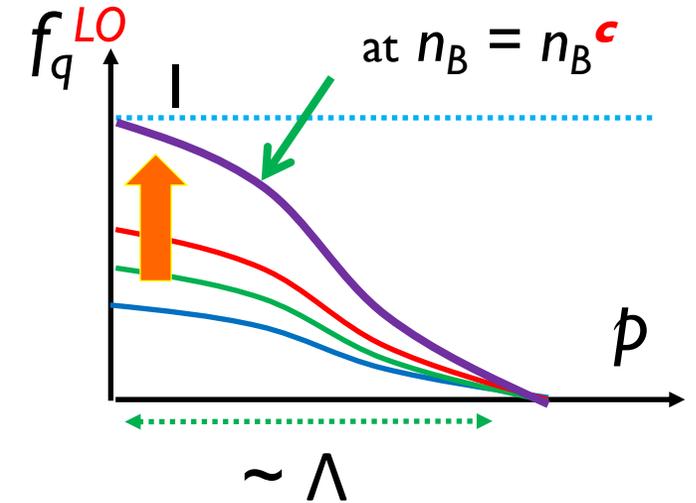
ideal baryon gas:

$$f_q(p; n_B) = \frac{n_B}{n_B^c} e^{-p^2/\Lambda^2} + O(1/N_c^2)$$

LO:
- the **shape** does not change
- the **height** grows linearly in n_B

f_q^{LO} at $n_B = n_B^c$

$\sim \Lambda$

p

energy density

kin. energy

$$\varepsilon(n_B) = N_c \int_{\boldsymbol{p}} E_q(p) \left(f_q^{\text{LO}}(p; n_B) + O(1/N_c^2) \right) = n_B M_B + O(1/N_c)$$

pressure $\quad \mathcal{P} = n_B^2 \partial(\varepsilon/n_B)/\partial n_B \sim 0 + O(1/N_c)$

$\sim$ const. \qquad very soft

Saturation of quark states

at $n_B = n_B^c$

f_q

for ideal baryon gas:

p

$\sim \Lambda$

$\Lambda \sim 0.2\text{-}0.3$ GeV $\rightarrow$ $n_B^c \sim 0.5\text{-}2\ n_0$

should happen **before** baryon cores overlap

cf) *Soft Deconfinement* [Fukushima-TK-Weise, '20]
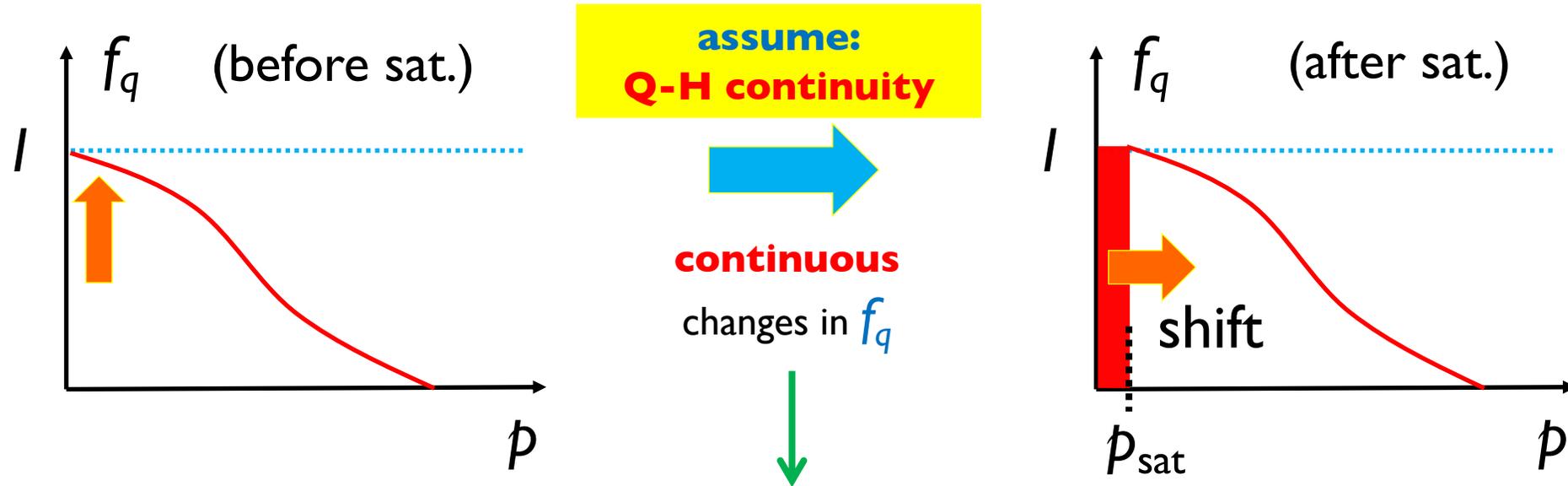
Contents

Quark matter **formation:** $f_q\left(p\,;n_B\right)$

a **model** after the saturation:

same shape

$$f_q^{\text{after}} = \theta\big(p_{\text{sat}} - p\big) + \theta\big(p - p_{\text{sat}}\big)\,\underline{f_q(p - p_{\text{sat}}; n_B^c)}$$
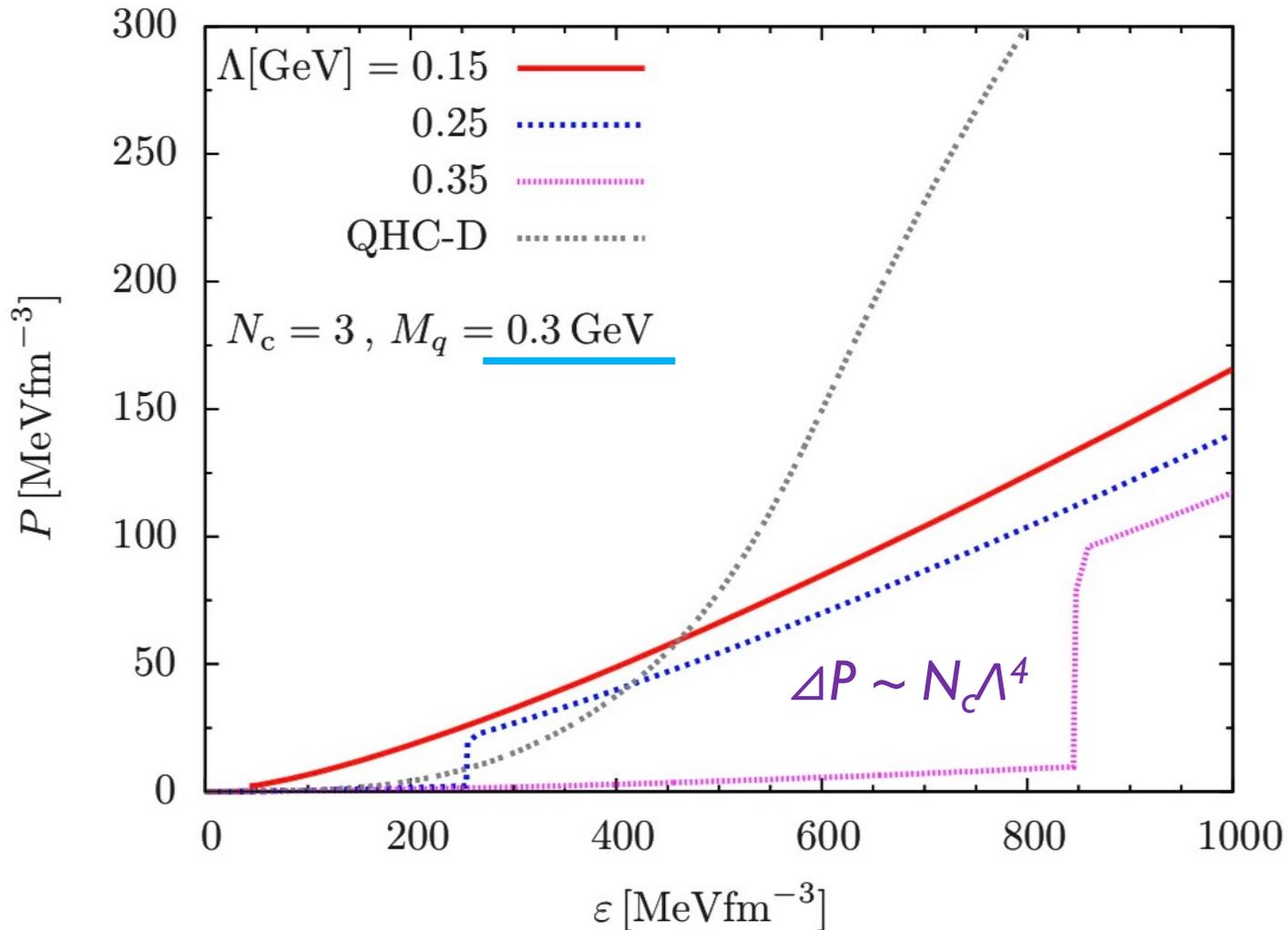

f_q (before sat.)

assume:
Q-H continuity

continuous

changes in f_q

f_q (after sat.)

shift

p_{sat}

ε, n_B are **continuous** before and after the saturation.

any nontrivial consequences ?

jumps in μ_{B}, P

(Nc = 3, M$_q$ = 0.3 GeV)

a model of f_q looks innocent,

but its consequence is unphysical:

ε, n_B are **continuous,**

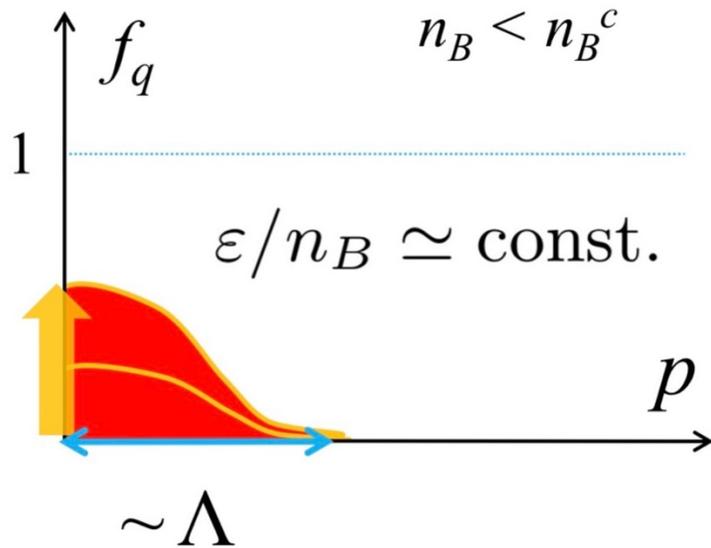**jumps** in μ_B & P

($\Delta\mu_B \sim N_c \Lambda$)
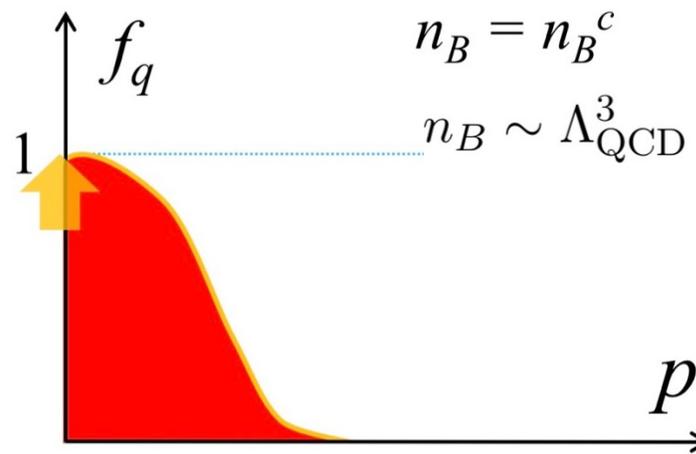
opposite to usual Ist order P.T. (!)

Quark matter formation: *EoS*

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$
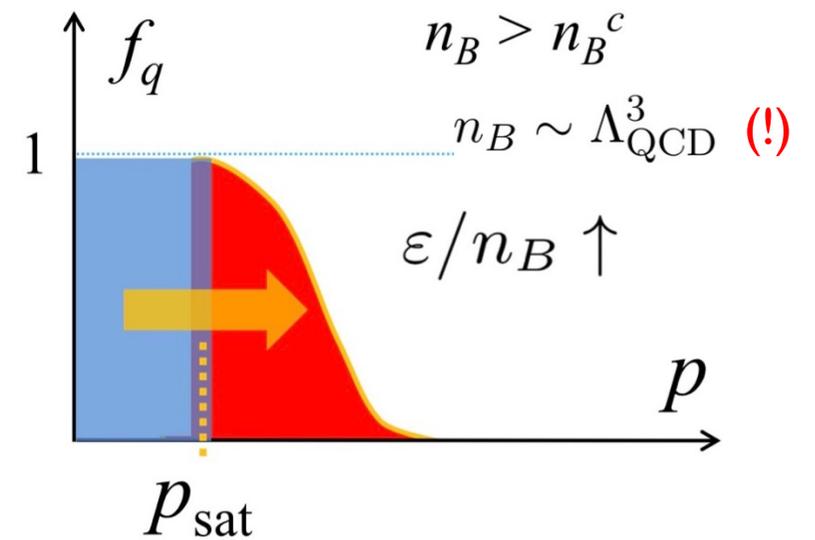
[cf) McLerran-Reddy (MR), PRL '19,...]

energy per particle

$n_B < n_B^c$

f_q

1

$\varepsilon/n_B \simeq \text{const.}$

p

$\sim \Lambda$

$\varepsilon \sim n_B \times N_c \Lambda_{\mathrm{QCD}}$

$\mathcal{P} \sim n_B^{5/3}/\underline{N_c}\Lambda_{\mathrm{QCD}}$

$n_B = n_B^c$

f_q

1

$n_B \sim \Lambda_{\mathrm{QCD}}^3$

p

$n_B > n_B^c$

f_q

1

$n_B \sim \Lambda_{\mathrm{QCD}}^3$ (!)

$\varepsilon/n_B \uparrow$

p

p_{sat}

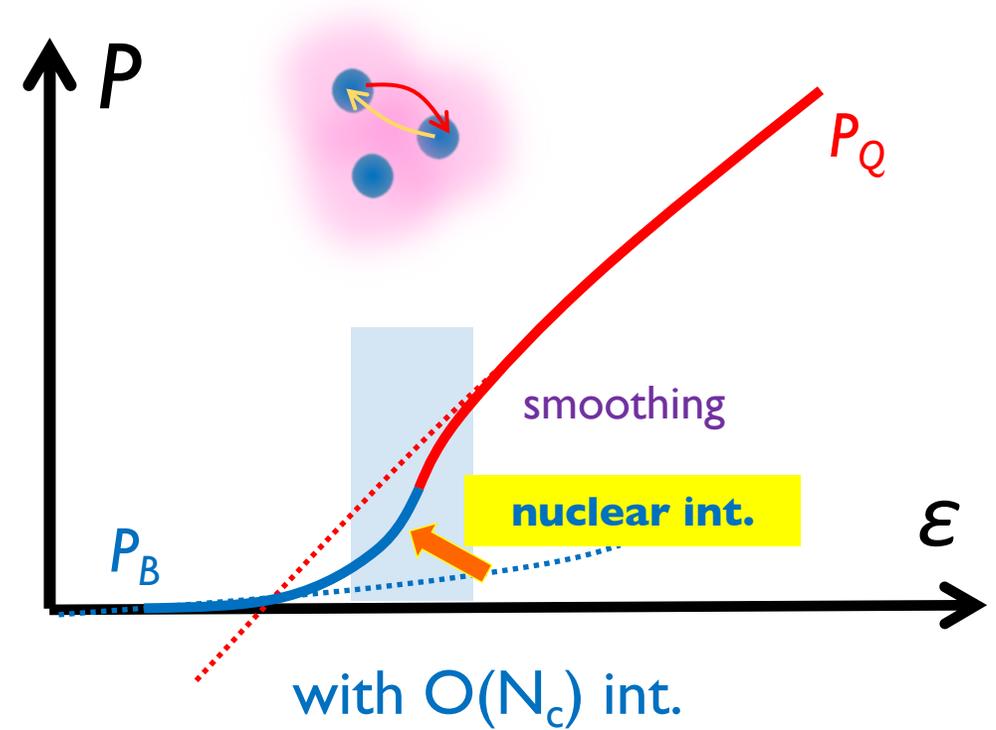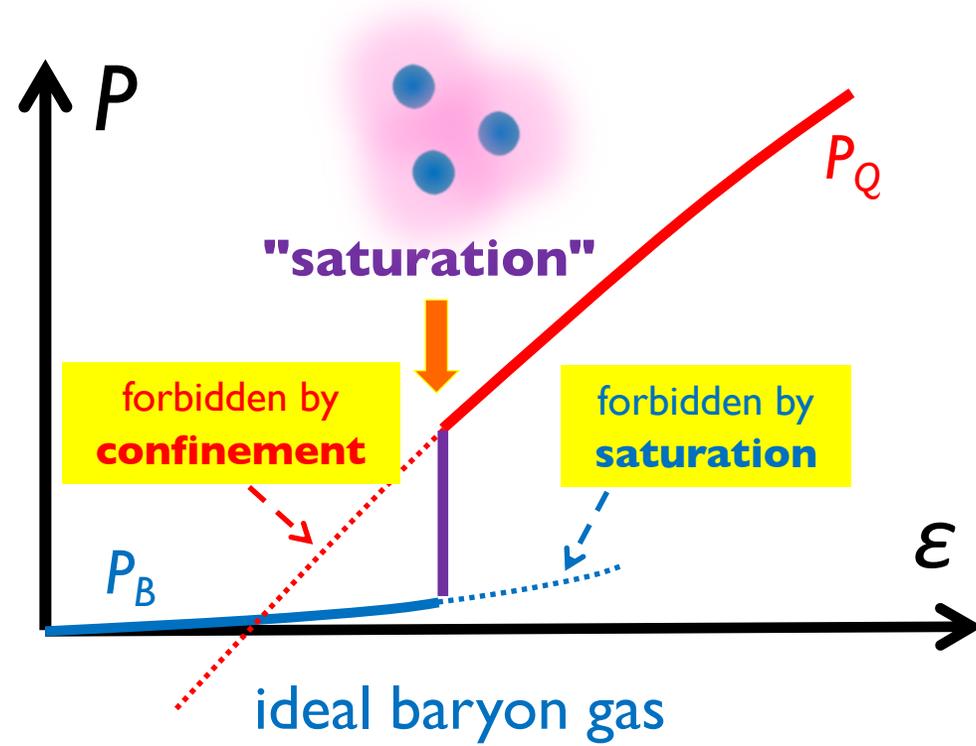$$\varepsilon \sim N_c \Lambda_{\mathrm{QCD}}^4$$

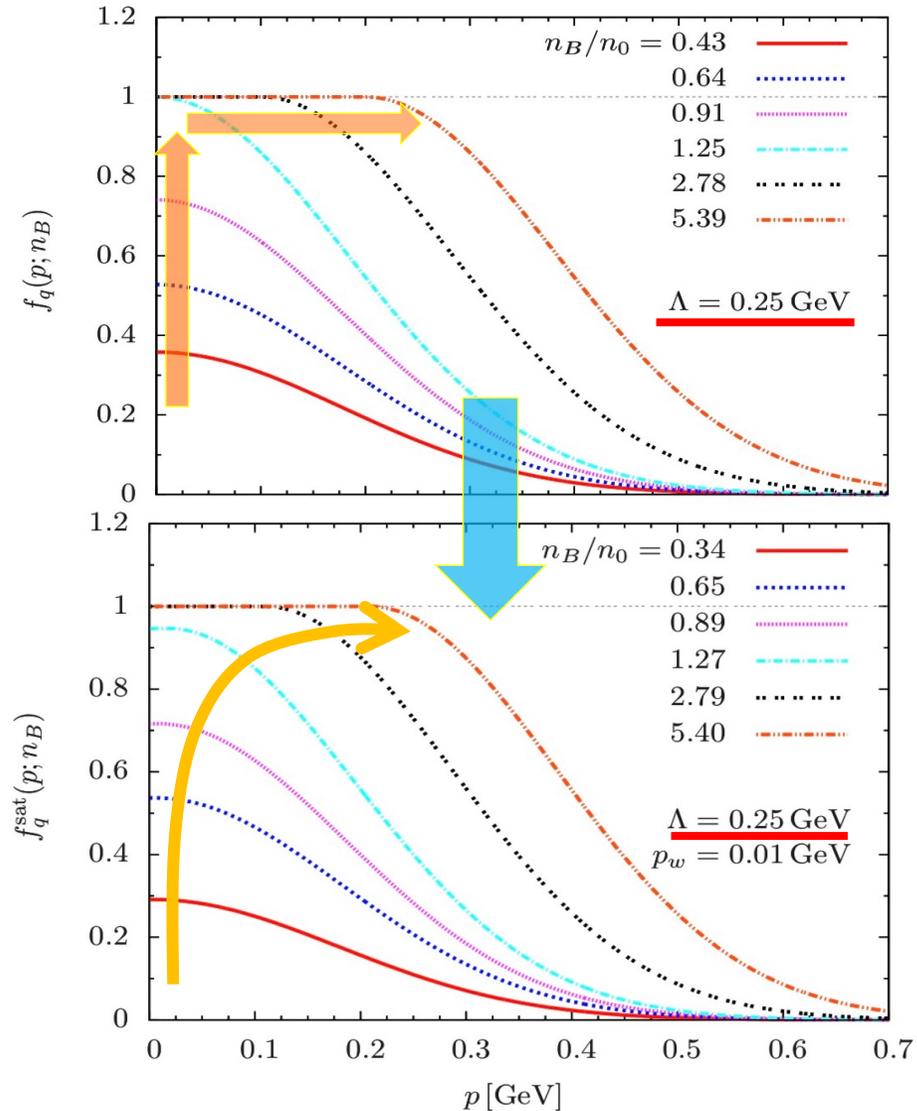$$\mathcal{P} \sim \Lambda_{\mathrm{QCD}}^4/\underline{N_c}$$

$$\mathcal{P} \sim \underline{N_c}\Lambda_{\mathrm{QCD}}^4 \ (!)$$
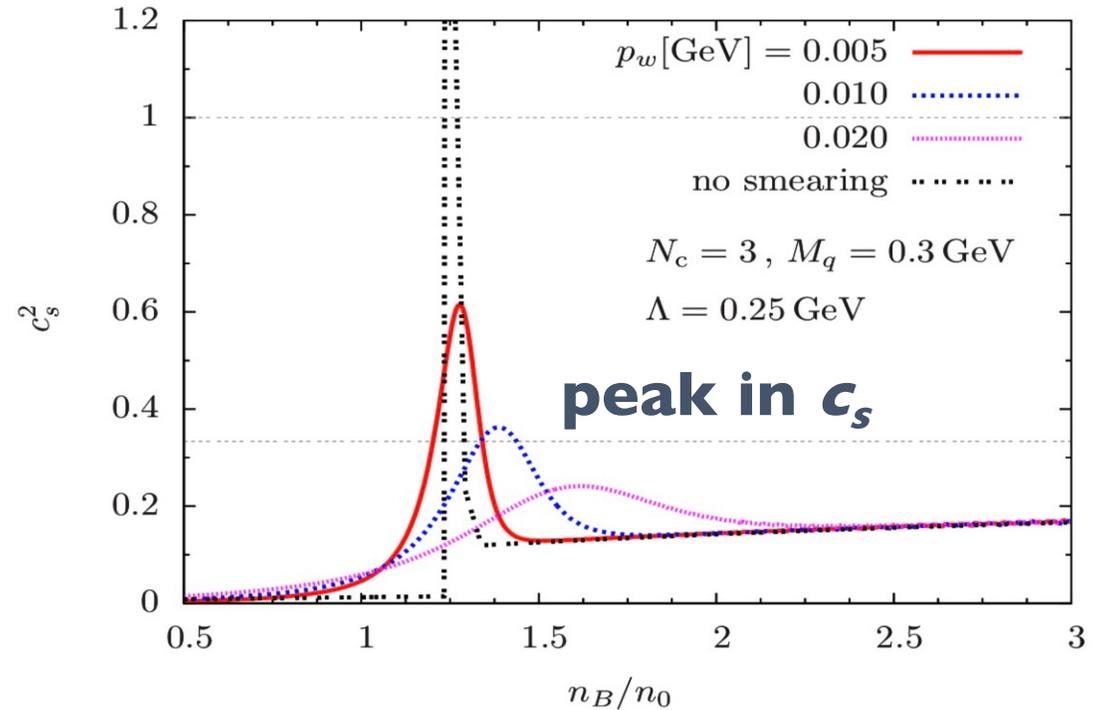
More realistic picture

[P_B & P_Q from a **single** model; f_q continuous]

"saturation"

forbidden by **confinement**

forbidden by **saturation**

P_Q

P_B

ε

ideal baryon gas

smoothing

nuclear int.

P_Q

P_B

ε

with O(N$_c$) int.

B

I

P_F

P_B

B

I

P_B

Smooth version

$$f_q^{\mathrm{sat}}(p;p_{\mathrm{sat}}) = \tanh(p_{\mathrm{sat}}/p_w) \times \left[\theta(p_{\mathrm{sat}} - p) + \theta(p - p_{\mathrm{sat}})\,\mathrm{e}^{-(\tilde{p}-\tilde{p}_{\mathrm{sat}})^2} \right]$$

peak in c_s

- *location* primarily determined by Λ (or baryon size)
- *width* primarily determined by p_w (or interactions)
- interactions: *NOT driving forces,* just *temper* the peak

Contents

For stiff quark EoS : *a guide*

cf) [TK-Powell-Song-Baym, '14]

kin. energy   interactions

$$\varepsilon(n) = an^{4/3} + bn^{\alpha}$$

(*n*: quark density)

ideal gas   interactions

$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right)n^{\alpha}$$

For **stiff** EoS:

(for large P)

for **α > 4/3**:   b > 0   (e.g. bulk repulsion, ~ $+ n_B^2/\Lambda^2$)

for **α < 4/3**:   b < 0   (e.g. surface pairings, ~ $- \Lambda^2 n_B^{2/3}$)

attraction ← − − − 2- or 3-quark correlations

quark
Fermi sea
(ideal combo)

repulsion

"Exotic" Fermi surface stiffens EoS !
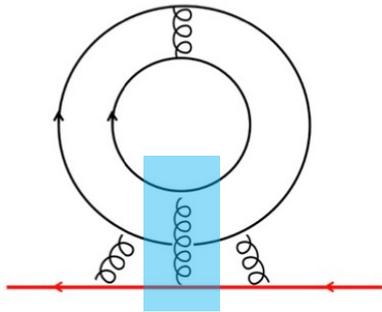
Reminder: QCD int. are very **channel dependent**!

quark energy; *parameterization of MF*

$$\mathcal{V}_{\mathrm{CE}}[f_q] = -C_E^{\mathcal{A}} \times \left(1 - f_q^{\beta}\right) + C_E^{\mathcal{S}} f_q^{\beta}$$

for $f_q(p) \ll 1$

$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq -C_E^{\mathcal{A}}$$

dilute in momentum space

for $f_q(p) \sim 1$

$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq C_E^{\mathcal{S}}$$

for *saturated levels*

repulsive   *attractive*
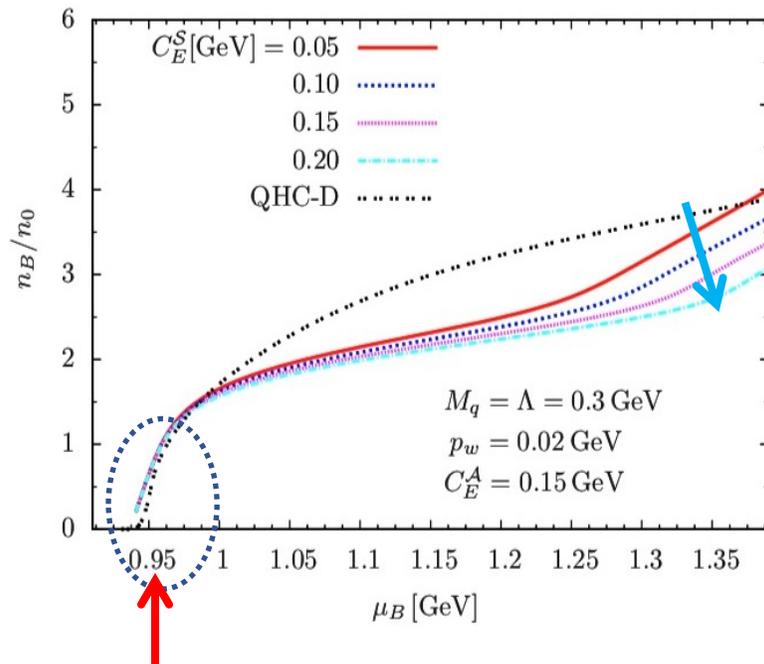
color-*antisym.* channels dominate
→ the quark feels *attractive* correlations

color-*sym.*
channels also enter

→ the quark feels
repulsive correlations also

EoS with interactions

f_q

I

C_E^S C_E^A

p

repulsive   *attractive*

stiffening

larger C_E^S

high density stiffening

adjust C_E^A (fit M_B = 939 MeV)

peak in c_s

Contents

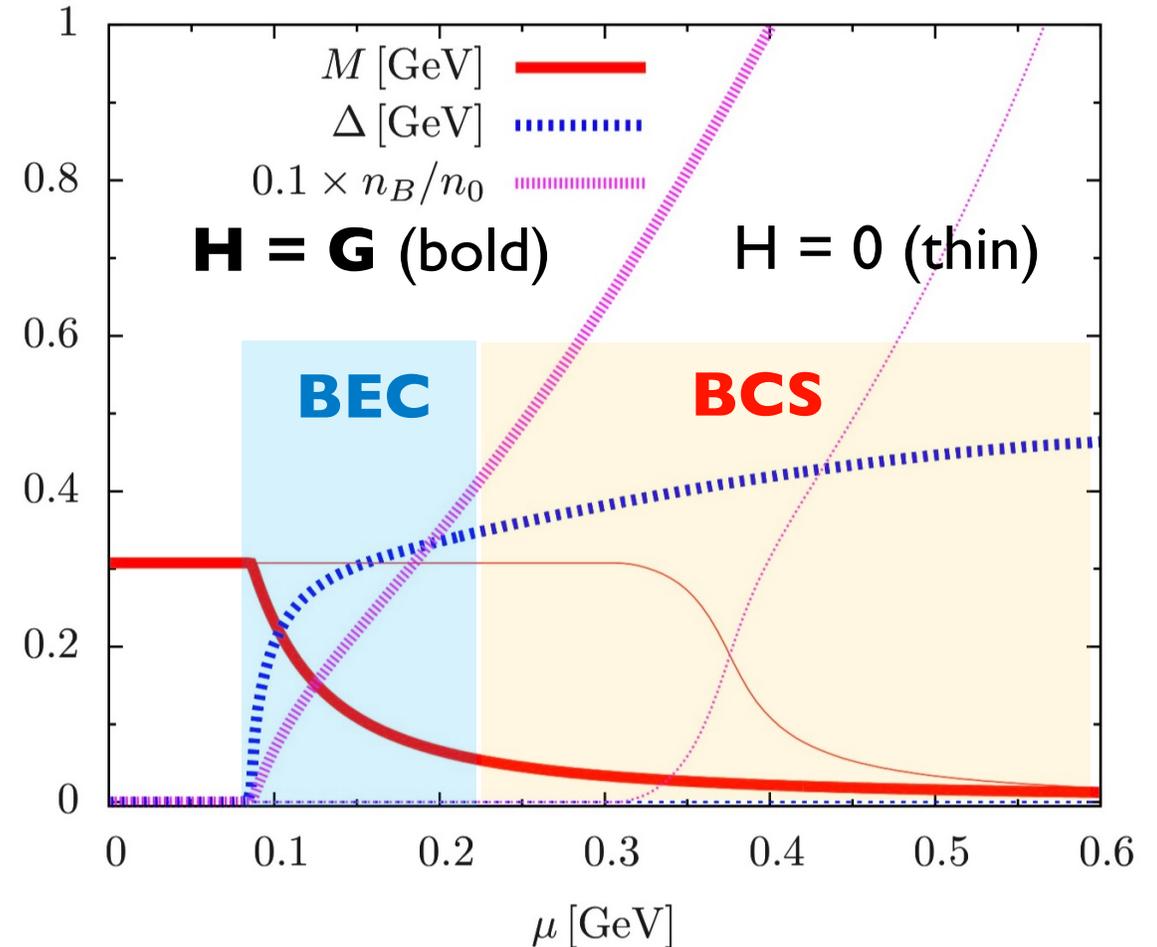# Example) 2-color NJL model   [TK-Suenaga, to appear]

- **lattice QCD doable for 2-color**

  - baryons = diquarks

  - diquark mass = m_π << M_q
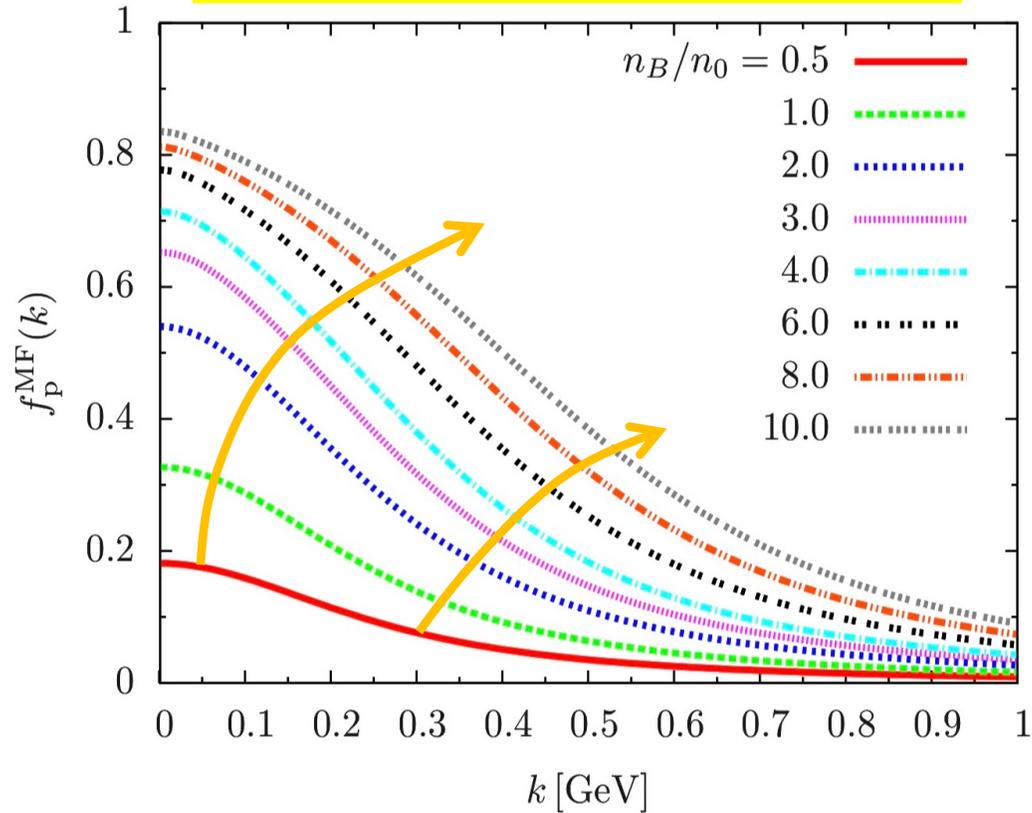
  - BEC-BCS crossover
    (diquark condensate)

$$\mathcal{L}_4 = G\big[\,(\bar{q}\tau_a q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau_a q)^2\,\big]$$
$$+ H\big[\,|\bar{q}\mathrm{i}\gamma_5\tau_2\sigma_2 q_C|^2 + |\bar{q}\tau_2\sigma_2 q_C|^2\,\big]$$

Legend:
M [GeV] (red solid)
Δ [GeV] (blue dotted)
$0.1 \times n_B/n_0$ (magenta dotted)

H = G (bold)   H = 0 (thin)

BEC   BCS

μ [GeV]

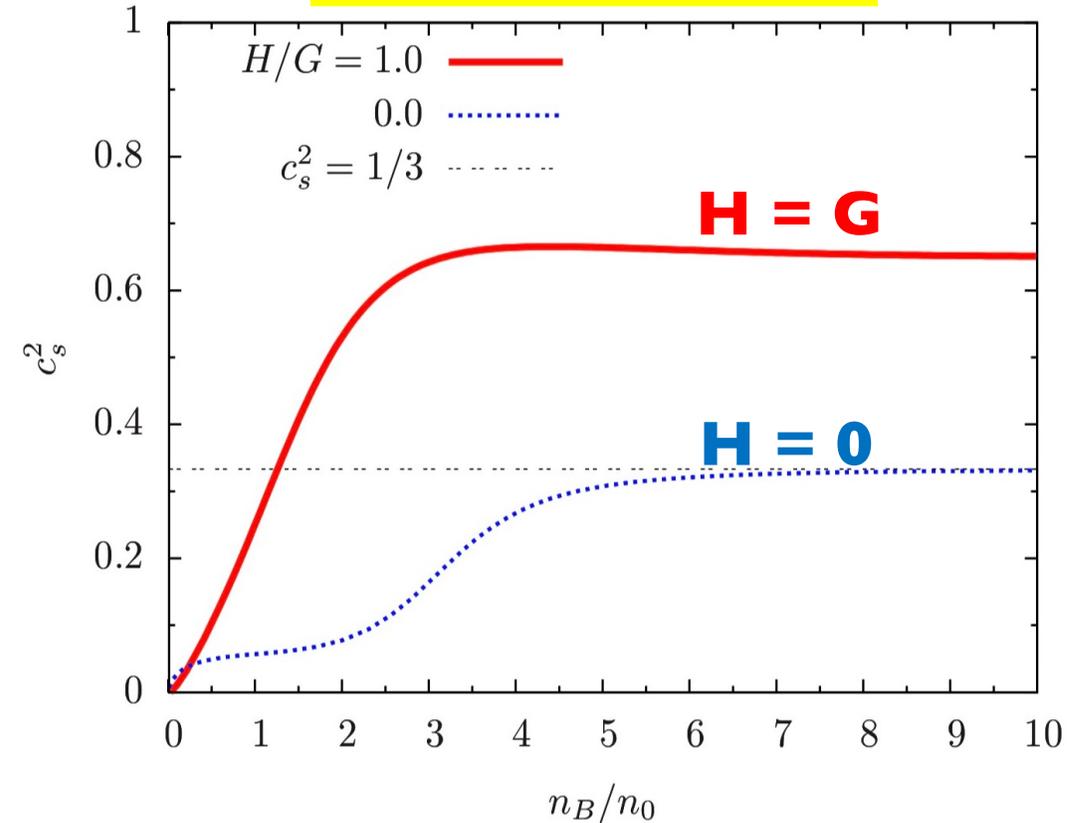# Example) 2-color NJL model [TK-Suenaga, to appear]

occupation probability

sound velocity

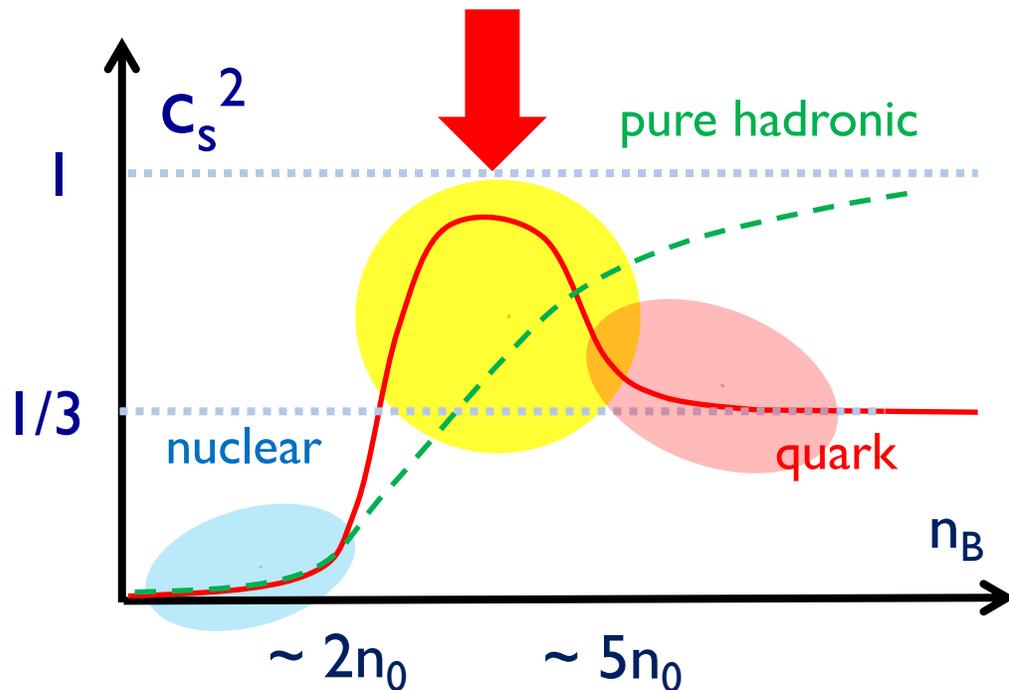
BEC → BCS & $c_s^2\uparrow$ occur at 0.5-1n_0 (early stiffening)

Summary

Direct descriptions of the "c_s-peak"

[cf) McLerran-Reddy (MR), PRL '19,...]

new quality

c_s^2

1

pure hadronic

1/3

nuclear

quark

n_B

$\sim 2n_0$ $\sim 5n_0$

the statements include:

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[rather nuclear int. smear the peak]