

About anomalous superfluidity, superconductivity and ferromagnetism in nuclear systems

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Plan

- Superfluidity and superconductivity of condensate of complex scalar field (in potential well) & singlet $1S_0$ nn- and pp- Cooper pairings
- Condensates of vector boson fields (non-magnetic, diamagnetic, paramagnetic responses, ferromagnetic superfluidity and superconductivity)
- Triplet pairing in various systems, e.g. $3P_2$ nn –pairing, $3P_2$ pp and $3S_1$ np triplet pairings

Superfluidity and superconductivity of complex scalar field

placed in rectangular scalar pot. well U +e.m. field A^μ (for charged fields),

$$L = D_\mu \phi D^\mu \phi^* - m_{sc}^2 |\phi|^2 - \lambda |\phi|^4 / 2 - F_{\mu\nu} F^{\mu\nu} / (16\pi),$$

$$m_{sc}^2 = m^2 + U, \text{ where } m > 0$$

charge (electron)
↓
chemical potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu = \partial_\mu + ieA_\mu - i\mu\delta_{\mu 0},$$

Static mean-field (condensate) solution for $m_{ef}^2 = m_{sc}^2 - \mu^2 = m_{ef,0}^2 < 0$

A superfluid nonrelativistic motion of the system with the velocity \vec{v} is described with the help of the replacement

$$\vec{D} \rightarrow \vec{D} + im_{qp}\vec{v}, \text{ where } m_{qp} \text{ is a quasiparticle mass}$$

Uniform rotation $\mathbf{v}=[\boldsymbol{\omega} \times \mathbf{r}]$ acts as uniform magnetic field $\mathbf{A}=(1/2) [\mathbf{H} \times \mathbf{r}]$

Neutral complex scalar field+external mag. field $H=\text{const}$

Put $\mathbf{e}=0$, $\mu=0$, consider **semi-infinite matter** $x<0$, size $d_x \rightarrow \infty$

$$\phi = f_0 \text{th}[(x - x_0)/(\sqrt{2}l_\phi)], \quad x_0=0 \quad \text{For } H=\text{const}, \quad \vec{h} = \text{curl}\vec{A} = H$$

$$f_0 = \pm \sqrt{-m_0^2/\lambda} \theta(-m_0^2), \quad l_\phi = 1/|m_0| \quad m_{sc}^2 = m_0^2 < 0$$

$$\bar{G} = \frac{\int d^3x G}{\int d^3x} = -\frac{m_0^4}{2\lambda} \left(1 - \frac{4\sqrt{2}l_\phi}{3d_x} \right) \theta(-m_0^2), \quad \text{Magnetic and scalar fields decouple (nonmagnetic superfluid)}$$

No neutral complex scalar fields in nuclear systems, but model is realized as

Ginzburg-Landau model for $1S_0$ nn pairing in nuclei and NS, & Ginzburg-Pitaevskii description of superfluid ^4He

Free-energy density $F_L[\psi] = c_T |\hbar \nabla \psi|^2 / 2 - a_T |\psi|^2 + b_T |\psi|^4 / 2.$

$$m_{sc}^2 = m_0^2 < 0 \quad \leftrightarrow \quad \alpha_T(T) \text{ changes sign in critical point, e.g. at } T \text{ near } T_c$$

$$\alpha_T(T) = \alpha_0 (T_c - T); \quad \alpha_0, b(T), c(T) = \text{const} > 0 \quad \text{in mean-field treatment}$$

Fluctuations for T near T_c in a fluctuation region and change $a_T(T)$ and $b_T(T)$

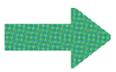
Fluctuations near critical point, scalar order parameter

Ginzburg criterion: $W \sim \exp(-\delta F(T)V_m/T)$,

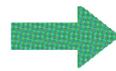
energy loss $\delta F \sim \alpha^2(T-T_c)^2/b$, minimal length $l_\varphi \sim 1/(T_c-T)^{1/2}$

when at $T=T_{fl}$ fluctuation of order parameter formed in a minimal volume $V_m \sim l_\varphi^3$ is probable ($W \sim 1$).

Fluctuations dominate for T near T_{cr} in a fluctuation region, estimated by **Ginzburg number** $Gi = |T_c - T_{fl}|/T_c$

For metallic superconductors fluctuation region is very narrow, $Gi \sim$ (typically) $10^{-8} \ll 1$, for superfluid ^4He , ^3He and for baryonic matter (strong interaction) $Gi \sim O(1/10-1)$ 

renormalization of mean-field coefficients

in ^4He C_v diverges only logarithmically  free energy density

$$F[\psi] = \frac{|\nabla\psi|^2}{2m\epsilon^*} - \alpha |T_c - T|^{4/3} |\psi|^2 + \frac{\lambda |T_c - T|^{2/3}}{2} |\psi|^4.$$

Charged static complex scalar field $+H=\text{const}$

in semi-infinite matter $x < 0$,

Magnetic $\vec{h} = \text{curl}\vec{A}$ and scalar fields *couple*

Gibbs free energy density
in external mag. field H

$$G = |(\nabla - ie\vec{A})\phi|^2 + m_{\text{ef}}^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 + \frac{(\vec{h} - \vec{H})^2}{8\pi},$$

cf. Ginzburg-Landau
free energy

$$F = F_{n0} + \int \left\{ \frac{\mathbf{B}^2}{8\pi} + \frac{\hbar^2}{4m} \left| \left(\nabla - \frac{2ie}{\hbar c} \mathbf{A} \right) \psi \right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 \right\} dV$$

$$\alpha(T) = \alpha_0(T_c - T); \alpha_0 > 0$$

Typical length scales:

$$R_H = 1/\sqrt{|eH|}, \quad l_h = 1/\sqrt{8\pi e^2 f_0^2} \gg l_\phi, \quad l_\phi = 1/\sqrt{|m_0|},$$

Ginzburg-Landau parameter

$$\kappa = l_h/l_\phi = (\lambda/2e^2)^{1/2} \gg 1 \quad \rightarrow$$

2-order superconductor, Abrikosov mixed phase

For $H \ll H_{\text{cr}}$, $H_{\text{cr}} = \sqrt{4\pi}|m_{\text{ef}}^2|/\sqrt{\lambda}$ with $\vec{A} = (0, A_2(x), 0)$ for $\vec{H} \parallel z$

Meissner/Higgs effect

$$A_2(x \leq 0) = H l_h e^{x/l_h} \quad m_\gamma = 1/l_h$$

$$R_H = 1/\sqrt{|eH|} \sim l_h \quad H_{c1} \sim H_c/\kappa.$$

Vortices: surface energy is negative at $H > H_{cr1}$ (at $H_{cr1} < H_{cr}$) 

System loses uniformity: filamentary-vortex-structure for $H_{c1} < H < H_{c2}$,

Abrikosov triangular lattice:

$$R_H = 1/\sqrt{|eH|} \sim l_\phi, \quad H_{cr2} = |m_{ef}^2/e| = H_{cr}\sqrt{2\kappa}.$$

$$\phi = \phi_0 e^{-i\Phi} \quad \vec{J} = -2e\phi_0^2(\nabla\Phi + (e/\hbar c)\vec{A}).$$

In interior of the superfluid system at distances $d \gg l_h$ we have $\vec{J} = 0$ and $\text{curl}\vec{A} = 0$.

Aharonov-Bohm effect
$$\oint \vec{A}d\vec{l} = \int \text{curl}\vec{A}d\vec{S} = \int \vec{B}d\vec{S} = -\frac{\hbar c}{e} \oint \nabla\Phi d\vec{l} = \frac{\hbar c}{|e|} \delta\Phi.$$

$\delta\Phi = 2\pi n$, where n is integer number  **Filaments - quantum vortices**

Gap of singlet pairing \leftrightarrow complex scalar field ϕ

Consequences of pairing in NS

Glitches and m.b. star quakes in pulsars

NS cooling is sensitive to pairing (especially to $3P_2$ nn and $1S_0$ pp)

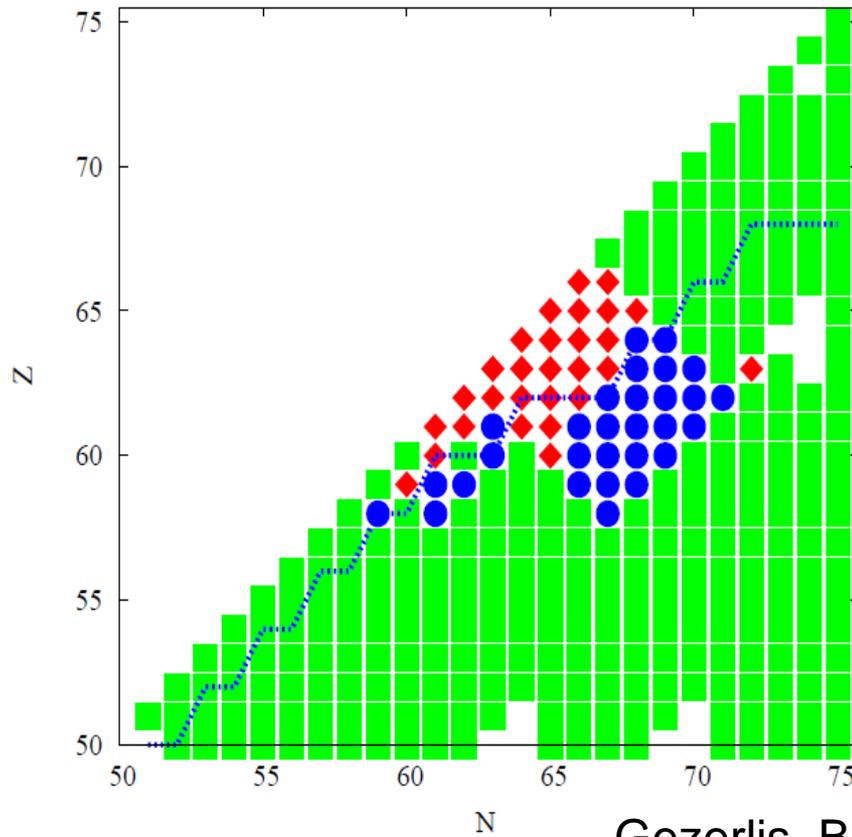
R-mode damping in millisecond pulsars (they have ~ 10 times smaller frequencies compared with that could be, shear and bulk viscosities depend on presence/absence of superfluidity)

Population of magnetic field in NS & superfluidity and then time-decay at $>10^4$ yr

Rotation & superfluidity

Pairing in nuclei

Pairing below the $N=Z$ line



Gezerlis, Bertsch, Luo, PR 106, 252502, 2011

FIG. 1: (color online) Chart of nuclides with $Z \leq N$ for neutron numbers from 50 to 75. Blank squares denote nuclei that exhibit practically no pairing ($E_{corr} < 0.5$ MeV), green squares signify the case where the pairing condensate is mostly spin-singlet, red diamonds are used for the nuclei that exhibit spin-triplet pairing, while blue circles denote nuclei for which the pairing is a mixture of spin-singlet and spin-triplet. The blue dashed line is the proton-drip line from Ref. [16].

- I. S. Shapiro, V. A. Khangulyan, and R. A. Barankov, Triplet cooper pairing in nuclear matter and rotational states of superdeformed nuclei, *Phys. At. Nucl.* **63**, 1533 (2000).
- G. F. Bertsch and Y. Luo, Spin-triplet pairing in large nuclei, *Phys. Rev. C* **81**, 064320 (2010).
- W. Guo, U. Lombardo, and P. Schuck, Medium-polarization effects in 3SD_1 spin-triplet pairing, *Phys. Rev. C* **99**, 014310 (2019).
- B. Cederwall *et al.*, Evidence for a spin-aligned neutron-proton paired phase from the level structure of ^{92}Pd , *Nature (London)* **469**, 68 (2011).

Influence on moments of inertia

Neutron gas with Zeeman coupling. Ferromagnetism.

$$L = \bar{\psi}(i\gamma^\nu \partial_\nu + i\gamma^0 \mu - m_F)\psi - U + \boxed{\eta \vec{S} \vec{h}} - \vec{h}^2/8\pi, \quad \text{Spin} \quad \vec{S} = \frac{1}{2} \bar{\psi} \vec{\gamma} \gamma_5 \psi,$$

Zeeman term

m_F is the bare fermion mass, $\vec{M} = \eta \vec{S}$ is the magnetic moment of the fermion, U is a fermion interaction term not depending on h .

$$\vec{h} = \vec{H} + \vec{n}_3 \cdot 2\pi\eta \langle (\bar{\psi} \gamma^3 \gamma_5 \psi) \rangle,$$

where $\vec{n}_3 = (0, 0, 1)$, and

spin-spin coupling

$$G_h = -\pi\eta^2 \langle \bar{\psi} \gamma^3 \gamma_5 \psi \rangle^2 / 2 + \eta \langle (\bar{\psi} \gamma^3 \gamma_5 \psi) \rangle H_3 / 2.$$

Even for $H=0$:

$$E - E(h=0) = \frac{3^{5/3} \pi^{4/3} (2^{2/3} - 1) n^{5/3}}{10m_F^*} - \frac{\pi\eta^2 n^2}{2}, \quad n > n_{\text{cr}} = \frac{3^5 \pi (2^{2/3} - 1)^3}{125m_F^* \boxed{\eta^6}},$$

In this model ferromagnetic tr. is not realized at densities reachable in NS

In presence of π^0 condensate, $\eta \rightarrow \eta_{\text{ef}}$ due to axial anomaly term, then m.b. averaged nucleon spin $\neq 0$

K. Hashimoto, Possibility of ferromagnetic neutron matter, Phys. Rev. D **91**, 085013 (2015).

Relation between ferromagnetism and superfluidity (?)

Gap of triplet pairing \leftrightarrow complex vector field φ

Condensates of complex vector fields

Let $\phi_\nu^{(j)} = (\phi_\nu^{(1)}, \phi_\nu^{(2)}, \phi_\nu^{(3)})$ be the field of a massive vector-isospin-vector boson, such as ρ meson, with $\phi_\nu^{(1)}, \phi_\nu^{(2)}, \phi_\nu^{(3)}$ as real quantities. Latin superscripts (1),

(2), and (3) describe isospin $\phi_\nu = (\phi_\nu^{(1)} - i\phi_\nu^{(2)})/\sqrt{2}$, $\phi_\nu^* = (\phi_\nu^{(1)} + i\phi_\nu^{(2)})/\sqrt{2}$.

$$\text{Spin } S_k \propto i\epsilon_{jik}\phi_j^*\phi_i$$

Examples:

W^\pm condensation in strong magnetic field $H \sim 10^{24}$ G, cf. below B phase

J. Ambjørn and P. Olesen, Phys. Lett. B 214 (1988) 565.

ρ^\pm meson condensation in dense isospin-asymmetric matter, if effective ρ mass decreases with density n ,
cf. below A phase of vector cond.

D. N. Voskresensky, On the possibility of the condensation of the charged rho meson field in dense isospin asymmetric baryon matter, Phys. Lett. B **392**, 262 (1997).

ρ^\pm meson condensation in strong magnetic field $H \sim 10^{20}$ G in vacuum,
cf. B phase of vector cond.

M. N. Chernodub, Superconductivity of QCD vacuum in strong magnetic field, Phys. Rev. D **82**, 085011 (2010).

General consideration of triplet NN pairing and pairing in various materials, cf.

D.V. Phys.Rev. D101 (2020)

Simplest Lorentz invariants

$$L_{\phi,A} = \frac{F_{\mu\nu}F^{\mu\nu}}{16\pi} - \frac{\phi_{\mu\nu}\phi^{*\mu\nu}}{2} + m_{\text{sc}}^2\phi_\nu\phi^{*\nu} \\ + L_{\phi\phi} + i\eta F_{\mu\nu}\phi^{*\mu}\phi^\nu,$$

$\phi_{\mu\nu} = D_\mu\phi_\nu - D_\nu\phi_\mu$, as above m_{sc}^2 is the squared bare mass shifted by an attractive scalar potential.

Zeeman coupling term, $L_{\text{Zeeman}} = i\eta F_{\mu\nu}\phi^{*\mu}\phi^\nu$,

$$L_{\phi\phi} = -\Lambda[(\phi_\nu\phi^{*\nu})^2 + \xi_1(\phi_\nu\phi^\nu)(\phi_\mu^*\phi^{*\mu})],$$

where Λ is a positive coupling constant.

A non-Abelian form of the self-interaction $\xi_1 = -1$.

Let us for a while for simplicity employ $\xi_1 = 0$,

$$D^\mu D_\mu\phi^\nu - D^\nu D_\mu\phi^\mu - i(e + \eta)F^{\mu\nu}\phi_\mu$$

$$+ m_{\text{sc}}^2\phi^\nu - 2\Lambda(\phi_\mu^*\phi^\mu)\phi^\nu = 0,$$

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu, \quad \text{with}$$

$$J^\nu = ieD^\nu\phi_\mu \cdot \phi^{*\mu} - ie\phi^{*\mu}D_\mu\phi^\nu + \text{c.c.} \\ - i(e + \eta)\partial_\mu(\phi^{*\mu}\phi^\nu - \phi^{*\nu}\phi^\mu).$$

Neutral complex vector field. A-phase in infinite matter

Phase A, 3 degenerate sub-phases $\langle \mathbf{S} \rangle = 0$ since $S_k \propto i\epsilon_{jik}\phi_j^*\phi_i$

$$\mathbf{A1} \quad \boldsymbol{\varphi} = (\varphi_1, 0, 0), \quad \mathbf{A2} \quad \boldsymbol{\varphi} = (0, \varphi_1, 0), \quad \mathbf{A3} \quad \boldsymbol{\varphi} = (0, 0, \varphi_1),$$

nonmagnetic superfluid phase (as He-II)

$$\vec{h} = \vec{H}, \quad |\phi|^2 = -\frac{m_{sc}^2}{2\Lambda} \theta(-m_{sc}^2), \quad G_A = -\frac{m_{sc}^4}{4\Lambda} \theta(-m_{sc}^2).$$

A-sub-phases in semi-infinite matter $x < 0$

dependence of $\boldsymbol{\varphi}$ and \mathbf{A} only on x 

A1 $\boldsymbol{\varphi} = (\varphi_1, 0, 0)$ is forbidden since kinetic term $-D_i^2\phi_j + D_jD_i\phi_i = 0$

A2 $\boldsymbol{\varphi} = (0, \varphi_1, 0)$, A3 $\boldsymbol{\varphi} = (0, 0, \varphi_1)$, nonmagnetic superfluid phase

Same solution as was for scalar complex field $m_{sc}^2 = m_0^2 < 0$,

$$\bar{G}_{A_2} = \frac{\int d^3x G}{\int d^3x} = -\frac{m_0^4}{4\Lambda} \left(1 - \frac{4\sqrt{2} l_\phi}{3 d_x} \right) \theta(-m_0^2), \quad l_\phi = 1/\sqrt{|m_0|},$$

$$\bar{G}_{A_2} = \bar{G}_{A_3}.$$

Neutral complex vector field B-phase *in infinite matter*

B1 $\varphi = (0, \varphi_1, \varphi_2)$, **B2** $\varphi = (\varphi_1, 0, \varphi_2)$, **B3** $\varphi = (\varphi_1, \varphi_2, 0)$, **profitable** $\varphi_1 = \pm i\varphi_2$

Ferromagnetic superfluidity: two possibilities ($\$ \parallel \mathbf{H}$, $\$ \uparrow \downarrow \mathbf{H}$), and $\$ \perp \mathbf{H}$

B1 $\varphi = (0, \varphi_1, i\varphi_1)$, for $\vec{A} = (0, 0, Hy \mp 4\pi\eta|\tilde{\psi}|^2 y)$, $\tilde{\psi} = \phi_1(x)\sqrt{2}$

$$h_1 = H \mp 4\pi\eta|\tilde{\psi}|^2 = \text{const}, \quad h_2 = h_3 = 0, \quad |\tilde{\psi}|^2 = \frac{-m_{sc}^2 \mp \eta H}{2\tilde{\Lambda}} \theta(-m_{sc}^2 \mp \eta H),$$

Own magnetic field h_1 even at $H=0$ \rightarrow **Ferromagnetic superfluidity**

$$G_{B_1}(\vec{H} \parallel x) = -\frac{(-m_{sc}^2 \mp \eta H)^2}{4\tilde{\Lambda}} \theta(-m_{sc}^2 \mp \eta H). \quad \tilde{\Lambda} = \Lambda - 2\pi\eta^2 > 0$$

$$G_{B_1}(\vec{H} \parallel z) = -\frac{m_{sc}^4}{4\tilde{\Lambda}} \theta\left(\frac{-m_{sc}^2}{4\tilde{\Lambda}}\right). \quad h_1 = \mp 4\pi\eta|\tilde{\psi}|^2 = \text{const}, \quad h_2 = 0, \quad h_3 = H,$$

profitable is $\$ \parallel \mathbf{H}$ for $\eta < 0$, or $\$ \uparrow \downarrow \mathbf{H}$ for $\eta > 0$

for $m_{sc}^2 < 0$ increasing H yields gain in energy !

Condensate arises even for $m_{sc}^2 > 0$ for $H > m_{sc}^2/|\eta|$!

1-order phase transition to C-phase if were $\tilde{\Lambda} = \Lambda - 2\pi\eta^2 < 0$

In hadronic case e.g. for p condensate $\Lambda \sim 1$, $\eta \sim e$ and C-phase is not realized

B-sub-phases in semi-infinite matter $x < 0$

B1 $\boldsymbol{\varphi} = (0, \varphi_1, \varphi_2)$ has larger kinetic (surface energy) term than B2

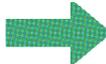
B2 $\boldsymbol{\varphi} = (\varphi_1, 0, \varphi_2)$, $\varphi_1 = i\varphi_2$ profitable \$ $\|\mathbf{H}\|_y$ for $\eta < 0$, or \$ $\uparrow\downarrow\mathbf{H}$ for $\eta > 0$

$$\vec{A} = (0, 0, -Hx \pm 4\pi\eta \int^x |\tilde{\psi}|^2 dx), \quad h_1 = 0, \quad h_2 = H \mp 4\pi\eta |\tilde{\psi}(x)|^2, \quad h_3 = 0$$

$$\tilde{\psi}(x) = \pm \sqrt{\frac{-m_0^2 \mp \eta H}{2\tilde{\Lambda}}} \theta(-m_0^2 \mp \eta H) \text{th} \frac{x}{\sqrt{2}l_\phi^{\text{B}_2}}, \quad \text{own magnetic field even for } H=0$$

$$l_\phi^{\text{B}_2} = l_\phi / \sqrt{2},$$

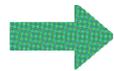
$$\bar{G}_{\text{B}_2}(\vec{H} \parallel y) = -\frac{(-m_0^2 \mp \eta H)^2}{4\tilde{\Lambda}} \left(1 - \frac{4\sqrt{2}l_\phi^{\text{B}_2}}{3d_x} \right) \times \theta(-m_0^2 \mp \eta H).$$

 **Ferromagnetic superfluidity accompanied by self-rotation**
for $m_{sc}^2 < 0$ increasing H is profitable !

Condensate also arises for $m_{sc}^2 > 0$ for $H > m_{sc}^2 / |\eta|$

$$\tilde{\Lambda} = \Lambda - 2\pi\eta^2 < \Lambda$$

+smaller surface energy



even for $H=0$ one of B phases is more profitable than A phases

Domains, white noise with re-orientation of domains

Charged complex vector field

A-phase in infinite matter

$$\mathbf{A1} \quad \boldsymbol{\varphi} = (\varphi_1, 0, 0), \quad \mathbf{A2} \quad \boldsymbol{\varphi} = (0, \varphi_1, 0), \quad \mathbf{A3} \quad \boldsymbol{\varphi} = (0, 0, \varphi_1), \quad \langle \mathbf{S} \rangle = 0$$

$$\text{Nonmagnetic superfluid phase} \quad \vec{h} = \vec{H}, \quad |\phi|^2 = -\frac{m_{sc}^2}{2\Lambda} \theta(-m_{sc}^2), \quad G_A = -\frac{m_{sc}^4}{4\Lambda} \theta(-m_{sc}^2).$$

A-sub-phases in semi-infinite matter $x < 0$

$$\mathbf{A1} \quad \boldsymbol{\varphi} = (\varphi_1, 0, 0) \parallel \mathbf{x} \text{ is forbidden since kinetic term} \quad -D_i^2 \phi_j + D_j D_i \phi_i = 0$$

$$\mathbf{A2} \quad \boldsymbol{\varphi} = (0, \varphi_1, 0), \quad \text{for } \mathbf{H} \parallel \mathbf{z}$$

$$\bar{G}_{A_2}(\vec{H} \parallel z) = -\frac{m_{ef,0}^4}{4\Lambda} \left(1 - \frac{4\sqrt{2}l\phi}{3d_x} \right) \theta(-m_{ef,0}^2), \quad \vec{h} = \vec{H}.$$

Nonmagnetic phase! Condensate and \mathbf{h} decouple.

There was no such phase for case of charged scalar field

Superconducting A3 subphase for $H \parallel z$

$$G_{A_3}(\vec{H} \parallel z) = |\partial_1 \phi_3|^2 + e^2 A_2^2 |\phi_3|^2 + m_{\text{ef}}^2 |\phi_3|^2 + \Lambda |\phi_3|^4 + \frac{(\vec{h} - \vec{H})^2}{8\pi}.$$

for $H < H_{\text{cr1}}$ Meissner ef. $\bar{G}_{A_3}(\vec{H} \parallel z) \simeq \frac{H^2}{8\pi} - \frac{m_{\text{ef},0}^4}{4\Lambda} \left(1 - \frac{4\sqrt{2}\tilde{l}_\phi}{3d_x}\right) \theta(-m_{\text{ef},0}^2).$

$H_{\text{cr1}} < H < H_{\text{cr2}}$ Abrikosov lattice

For $H \neq 0$ preferable is nonmagnetic subphase

Condensate differently responses on direction of external field (**compass**), with energy gain or loss taken from external field.

Exo-endo-thermic reactions by changing direction of external magnetic field  possible applications

Charged complex vector field

B-sub-phases in semi-infinite matter

Subphase B₃.—Let $\vec{H}\parallel z$ and employ $\vec{A} = (A_1(x, y), A_2(x, y), 0)$, i.e., $\vec{h}\parallel z$. for $m_{\text{ef},0}^2 < 0$

$$\text{for } H < H_{\text{cr1}} \quad \bar{G}_{B_3}(\vec{H}\parallel z) \simeq \frac{H^2}{8\pi} - \frac{m_{\text{ef},0}^4}{4\Lambda} \left(1 - \frac{4\sqrt{2}\tilde{l}_\phi^B}{3d_x} \right) \theta(-m_{\text{ef},0}^2).$$

$H_{\text{cr1}} < H < H_{\text{cr2}}$ Abrikosov lattice for $\eta > 0$, $e < 0$, $H_{\text{cr2}} = -m_{\text{ef}}^2/\eta > 0$

No H_{c2} for $\eta < 0$, $e < 0$

also solution exists for $m_{\text{ef}}^2 > 0$, for $\eta < 0$, $e < 0$ at

$$H > H_{\text{cr2}} = -m_{\text{ef}}^2/\eta > 0. \quad \partial_1^2 A_2 + 8\pi\eta e A_2 |\tilde{\psi}|^2 = 0,$$

anti-screening effect

For $H \rightarrow 0$ $\bar{G}_{B_3}(\vec{H}\parallel z) < \bar{G}_{A_2}(\vec{H}\parallel z)$ due to a smaller surface energy

Then increasing H (for $H < H_{c2}$) $\bar{G}_{A_2}(\vec{H}\parallel z) < \bar{G}_{A_3}(\vec{H}\parallel z) < \bar{G}_{B_3}(\vec{H}\parallel z)$,

For $H > H_{c2}$ at $m_{\text{ef}}^2 > 0$ only B phase is possible

Spin-triplet pairing in condensed matter

(i) $^3\text{He-A1}$ phase in external magnetic field behaves as magnetic superfluid

$$|\psi\rangle = \psi_{A_1}(\mathbf{R}) Y_{11} |\uparrow\uparrow\rangle$$

V. Ambegaokar and N. D. Mermin, Thermal Anomalies of He^3 : Pairing in a Magnetic Field, *Phys. Rev. Lett.* **30**, 81 (1973).

(ii) Paramagnetic Meissner effect in Nb-Ho-Au, superconductivity enhances magnetic signal

A. Di Bernardo *et al.*, Intrinsic paramagnetic Meissner effect due to S-wave odd-frequency superconductivity, *Phys. Rev. X* **5**, 041021 (2015).

(iii) Triplet pairing in time-reversal Dirac semimetals in presence of magnetic impurities or if exchange interaction is strong

B. Rosenstein, B. Y. Shapiro, and I. Shapiro, Ginzburg-Landau theory of the triplet superconductivity in 3D Dirac semi-metal, [arXiv:1501.07910](https://arxiv.org/abs/1501.07910).

(iv) Spin-ordered pairing in heavy-fermion systems

J. A. Sauls, The order parameter for the superconducting phases of UPt_3 , *Adv. Phys.* **43**, 113 (1994).

D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Flouquet, J.-P. Brison, E. Lhotel, and C. Paulsen, Coexistence of superconductivity and ferromagnetism in URhGe, *Nature (London)* **413**, 613 (2001).

(v) Ferromagnetic behavior of P-wave pairing in Ru-based superconductors

A. Knigavko, B. Rosenstein, *Phys. Rev.* **B58**, 9354 (1998).

Spin-triplet pairing in neutral fermion system

$$G = G_{\text{grad}}^{\text{neut}} + G_{\text{hom}}, \text{ Cooper gap } \hat{\Delta}(\vec{k}) = \psi_i \Phi_i(\vec{k}),$$

In difference with QFT $G_{\text{grad}}^{\text{neut}} = c_1 |\partial_i \psi_j|^2 + c_2 |\partial_i \psi_i|^2 + c_3 (\partial_i \psi_j)^* \partial_j \psi_i,$

$$G_{\text{hom}} = -a |\psi_i|^2 + b_1 (\psi_i \psi_i^*)^2 + b_2 (\psi_i \psi_i) (\psi_j^* \psi_j^*) \\ + \mathcal{M} h_i i C \epsilon_{ijk} \psi_j^* \psi_k + (h_i - H_i)^2 / (8\pi) \\ + b_3 \sum |\psi_j|^4 + \{\gamma_k \psi_i\}^6, \vec{\mathcal{M}}_{\text{pair}} = \mathcal{M}_{\text{pair}} \vec{s}_{\text{pair}},$$

Spin density $S_i = -i C \epsilon_{iik} \psi_i^* \psi_k, a = \alpha_0 t \quad t = (T_{\text{cr}} - T) / T_{\text{cr}}$

at negligible spin-orbital interactions $\mathcal{M}_{nn} \simeq g_{nn} \mathcal{M}_N, g_{nn} = -2 \times 1.91$
 where $C > 0$ is a normalization constant.

Relations between c_i depend on system under consideration

Dirac semimetals $c_3 = [u_L - u_T] / 4, c_1 = u_T / 4, c_2 = 0,$ BCS: $c_1 \simeq c_2 \simeq c_3 > 0.$

$u_L = u_T / 32, u_T = \frac{7\zeta(3)N(0)v_F^2}{15\pi^2 T_{\text{cr}}^2},$ i.e., $c_1 \simeq -c_3, c_2 = 0.$

QFT: $c_1 = -c_3 > 0, c_2 = 0.$

Reference [21] for the description of a new class of Ru-based superconductors uses the simplest choice $c_2 = c_3 = 0,$ Ref. [2] employs also the choice $c_2 = c_3 \ll c_1 \sim N(0)v_F^2 / (\pi^2 T_{\text{cr}}^2)$ (E_2 model), v_F is the Fermi velocity.

Spin-triplet pairing. Charged channel

$$G_{\text{grad}}^{\text{ch}} = c_1 |D_i \psi_j|^2 + c_2 |D_i \psi_i|^2 + c_3 (D_i \psi_j)^* D_j \psi_i, \quad i[D_i, D_j]_- = e \epsilon_{ijk} h_k,$$

Many new choices, **two GL parameters in A phase, no non-magnetic phase**

except case we have considered $c_1 = -c_3 > 0, c_2 = 0.$

in B phase there arises extra term in effective magnetic moment

Zeeman

$$\vec{M} = C\vec{\mathcal{M}} - \vec{n}_3 e_* (c_3 - c_2)/2.$$

intrinsic magnetism accompanied by self-rotation

$$H_{c2}^{\text{B}} = -a/M_{\pm}. \quad \text{Here } M_+ = C\mathcal{M}_3 - e_*(c_1 + c_3) \text{ corresponds to } e_* > 0,$$

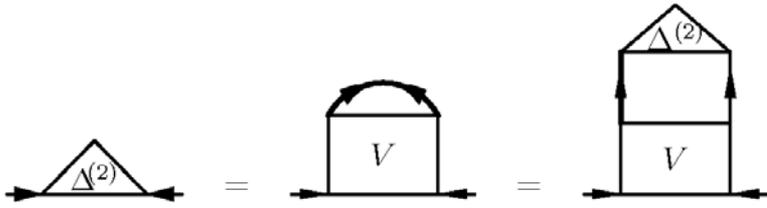
$$M_- = C\mathcal{M}_3 - |e_*|(c_1 + c_2) \text{ relates to } e_* < 0.$$

For $M_{\pm} > 0$, solution with $\psi \neq 0$ exists

for $H > H_{c2}^{\text{B}}$ at $a < 0$ (i.e., for $T_{\text{cr}}^{\text{BH,CH}} > T > T_{\text{cr}}$),

any H at $a > 0$ (i.e., for $T < T_{\text{cr}}$).

3P₂ nn pairing in nucleon matter



$$[\hat{\Delta}(\mathbf{k})]_{ab} = \sum_{JSL} \sum_{M_J M_L} \Delta_{LS}^{JM_J}(p) [G_{LJ}^{M_J}(\mathbf{n}_k)]_{ab}$$

$$\hat{\Delta}(\mathbf{k}) = \sigma_i (i\sigma_2) A_{ij} \mathbf{n}_{k,j} \quad \mathbf{n}_k = \mathbf{k}/|\mathbf{k}| \quad \text{cf. } \sim \mathbf{S}_k \boldsymbol{\tau} \boldsymbol{\varphi} \text{ NN}\pi\text{-cond. interaction}$$

$$A = \begin{bmatrix} \frac{a_{-2}}{2} - \frac{a_0}{\sqrt{6}} + \frac{a_2}{2} & \frac{i}{2}(a_2 - a_{-2}) & \frac{1}{2}(a_{-1} - a_1) \\ \frac{i}{2}(a_2 - a_{-2}) & -\frac{a_{-2}}{2} - \frac{a_0}{\sqrt{6}} - \frac{a_2}{2} & -\frac{i}{2}(a_{-1} + a_1) \\ \frac{1}{2}(a_{-1} - a_1) & -\frac{i}{2}(a_{-1} + a_1) & \sqrt{\frac{2}{3}}a_0 \end{bmatrix}.$$

Order parameters

J.A. Sauls and J.W. Serene, Phys. Rev. D 17, 1524 (1978).

$$a_{M_J} = \sqrt{\frac{3}{8\pi}} \Delta_{11}^{2M_J} \quad \text{Now sub-phases with } M_J = \pm 2, \pm 1, 0$$

$$G = F_{\text{gr}}[A] + F[A] + \mathbf{hS} + \frac{1}{8\pi}(\mathbf{h} - \mathbf{H})^2$$

$$F_{\text{grad}} = c_1 D_i A_{\nu k} D_i^* A_{\nu k}^* + c_2 D_i A_{\nu i} D_j^* A_{\nu j}^* + c_3 D_i A_{\nu j} D_j^* A_{\nu i}^*$$

$$F[A] = -\bar{\alpha} \text{Tr}(A A^*)$$

$$+ \bar{\beta}_1 \text{Tr}(A A) \text{Tr}(A^* A^*) + \bar{\beta}_2 \text{Tr}(A A^*) \text{Tr}(A A^*) + \bar{\beta}_3 \text{Tr}(A A A^* A^*)$$

$$+ \{\bar{\gamma} A^6\}$$

$$\mathbf{S}_i = -i \eta \epsilon_{ijk} A_{lj} A_{lk}^*$$

Phases at one fixed spin projection: $m_J = 0, -1, \text{ or } -2$

$$G_0 = -\bar{\alpha} |a_0|^2 + (\bar{\beta}_1 + \bar{\beta}_2 + \frac{1}{2}\bar{\beta}_3) |a_0|^4 + \frac{1}{8\pi}(\mathbf{h} - \mathbf{H})^2$$

$$G_{-1} = -\bar{\alpha} |a_{-1}|^2 + (\bar{\beta}_2 + \frac{1}{4}\bar{\beta}_3) |a_{-1}|^4 + \frac{1}{2} \eta h |a_{-1}|^2 + \frac{1}{8\pi}(\mathbf{h} - \mathbf{H})^2,$$

$$G_{-2} = -\bar{\alpha} |a_{-2}|^2 + \bar{\beta}_2 |a_{-2}|^4 + \eta h |a_{-2}|^2 + \frac{1}{8\pi}(\mathbf{h} - \mathbf{H})^2$$

sub-phases similar to A,B,C phases we have considered
but now characterized by various projections m

Some estimates in BCS approx. (weak coupling):

nn 3P2 pairing in NS:

In BCS approx. for $T < T_c$ B-magnetic $3P_2$ nn phase in NS matter is less profitable than A ($m=0$) for $T < T_c$ *No ferromagnetic superfluidity*
but for $T_c < T < T_c^{BH}$ $3P_2(-1)$ -B₃ phase is realized

Beyond BCS, e.g with UPt3 data $\bar{\beta}_3/\bar{\beta}_2 > 0$, $G_{-2,B_3}^{BCS} \simeq \frac{5}{4} G_0 < 0$ 

$3P_2(-2)$ -B is profitable for all $0 < T < T_{cr}^{B_3H}$ *Ferromagnetic superfluidity*

pp 3P2 pairing if existed in NS (?): owing to intrinsic magnetism

$\tilde{\eta}_{\pm 1} \sim \tilde{\eta}_{\pm 2} \sim -(1 - 10^4)$  $3P_2(2)$ -B even within BCS approx. is profitable,

Typical $h \sim 10^{16}$ G, magnetars (?), growth of critical temperature T_c^{BH}

np 3S₁ pairing in nuclei (?): np pairing gaps \sim (several – 10) MeV.

B phase is profitable, typical $h \sim 10^{16}$ G

Peripheral low-energy HIC $H \sim 10^{18}$ G  growth of T_c^{BH} above 10 MeV

+Superfluid self-rotation in B (and C) -phases

Conclusion

Condensate of complex scalar field with $m_{\text{ef}}^2 < 0$: neutral superfluid - nonmagnetic phase. 1S0 pp pairing-superconductivity: Abrikosov lattice.

Zeeman coupling and ferromagnetism. Condensates of complex neutral fields. Nonmagnetic A superfluid subphases, and B **ferromagnetic superfluid subphases (more profitable)**. Domains. For $m_{\text{ef}}^2 < 0$ with increasing H gain in energy. For $m_{\text{ef}}^2 > 0$ ferromagnetic superfluidity for $H > H_{c2}$.

Charged nonmagnetic and superdiamagnetic A-sub-phases. Ferromagnetic superconducting B sub-phases (more profitable at some H). **Anti-screening.**

Considered triplet pairing in absence of spin-orbit interaction.

Considered 3P2 nn and pp, and 3S1 np pairings. BCS estimates and beyond.

Other relevant issues:

phases in CSC systems;

superfluidity/superconductivity of systems with roton-like (moat spectrum)

example of p-wave charged pion condensate;

analogy between ferromagnetic properties of neutral pion cond. ($\Gamma \sim \mathbf{S} \mathbf{k} \tau \phi_{\mathbf{k}}$) and dipolar cold Fermi gases;

condensation of Bose excitations at $k \neq 0$ in moving, e.g. in rotating systems