



BOUNDARY-INDUCED QUANTUM EFFECTS IN THE HYPERBOLIC VACUUM OF DS SPACETIME

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CONTENT

- ❑ Introduction
- ❑ Problem formulation
- ❑ Mode functions
- ❑ Hyperbolic vacuum
- ❑ Hadamard function
- ❑ Vacuum expectation values
- ❑ Numerical results
- ❑ Summary

DS SPACETIME

- ❑ **Scalar fields in dS spacetime** – QFT in curved spacetime, inflationary cosmology
- ❑ Described by **various coordinate systems**
- ❑ In open inflation two periods of inflation are separated by **nucleation of a bubble**
- ❑ **Interior region** of bubble includes an **open universe**, described by **curved space and hyperbolic coordinates**
- ❑ Any two separated regions eventually become **causally disconnected**
- ❑ Two open charts in dS space are described by open coordinates

THE CASIMIR EFFECT

- ❑ In a number of cosmological problems, additional **boundary conditions** are imposed on the operators of quantum fields
- ❑ Boundary conditions modify the spectrum of **quantum fluctuations** of fields
- ❑ Expectation values of physical observables are changed



Casimir effect

- ❑ Shift depends on the bulk and boundary geometries and on the boundary conditions

AIM OF THE WORK

- We consider the influence of the **cosmological expansion** on the local characteristics of the scalar vacuum
 - Background geometry is the **de Sitter spacetime with negative curvature spatial foliation**
 - Boundary geometry consists of a **spherical shell**
 - Scalar field operator obeys the **Robin** boundary condition on the spherical shell
- The two-point **Hadamard function** and the vacuum expectation values (VEVs) of the **field squared** and the **energy-momentum tensor** are evaluated

BACKGROUND SPACETIME

Background is the $(D + 1)$ -dimensional **de Sitter spacetime**, which is described by open coordinates and **foliated with time slices having constant negative curvature**

The line element is **conformally related** to the line element of **static spacetime with negative constant curvature space**

$$R = \frac{D(D + 1)}{\alpha^2} \quad \rightarrow \quad \text{Spacetime curvature}$$

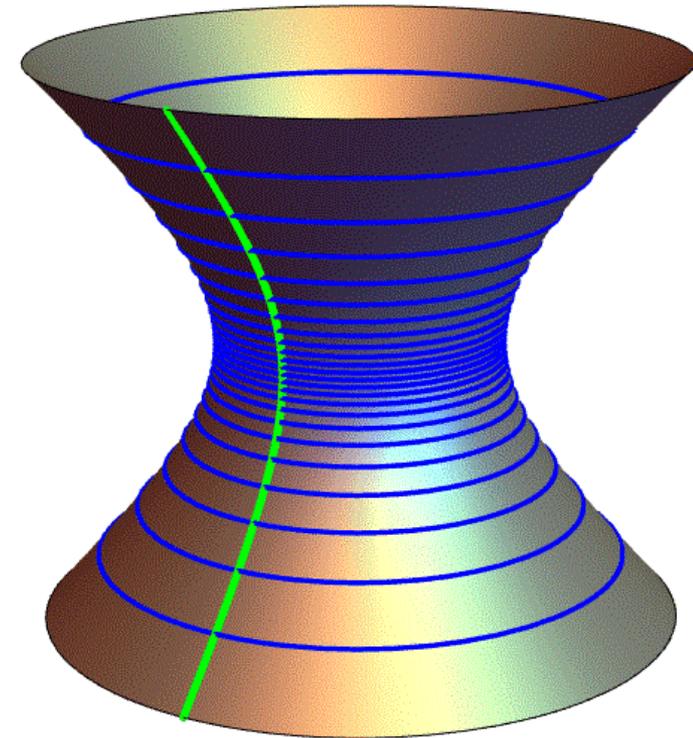
$$\alpha \quad \rightarrow \quad \text{Curvature radius}$$

$(D + 1)$ -dimensional dS spacetime as a **hyperboloid**

$$\eta_{MN} Z^M Z^N = -\alpha^2 \quad M, N = 0, 1, \dots, D + 1$$

in $(D + 2)$ -dimensional **Minkowski spacetime** with the line element

$$ds_{D+2}^2 = \eta_{MN} dZ^M dZ^N$$



BACKGROUND SPACETIME

- Coordinates with **negative curvature spatial foliation** (LI and LII regions)

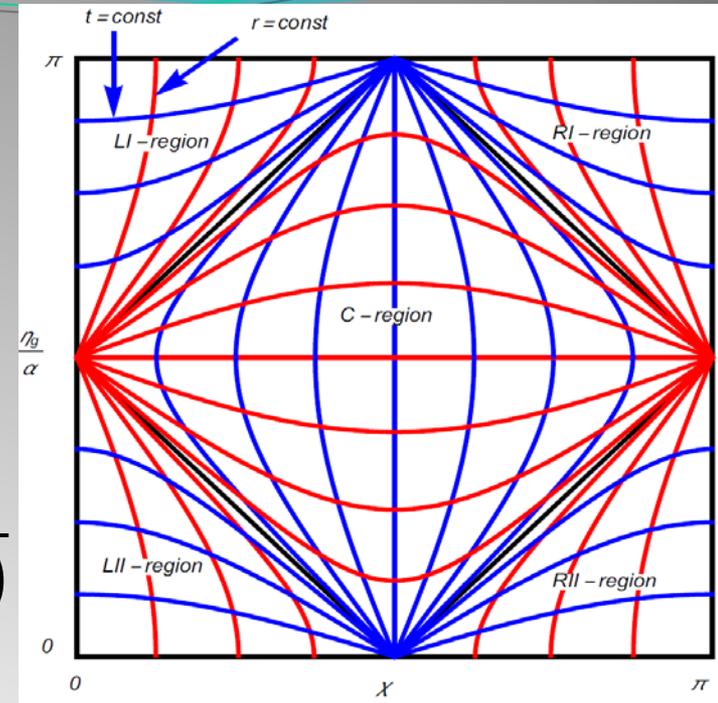
$$Z^0 = \alpha \sinh(t/\alpha) \cosh r$$

$$Z^1 = \alpha \cosh(t/\alpha)$$

$$Z^i = \alpha w^{i-1} \sinh(t/\alpha) \sinh r, i = 2, 3, \dots, D + 1$$

- Relations with **conformal global coordinates**

$$\cosh(t/\alpha) = \frac{\cos \chi}{\sin(\eta_g/\alpha)}, \quad \tanh r = -\frac{\sin \chi}{\cos(\eta_g/\alpha)}$$



The line element $\longrightarrow ds^2 = dt^2 - \alpha^2 \sinh^2(t/\alpha) (dr^2 + \sinh^2 r d\Omega_{D-1}^2)$

Conformal relation $\longrightarrow ds^2 = \frac{1}{\sinh^2(\eta/\alpha)} [d\eta^2 - \alpha^2 (dr^2 + \sinh^2 r d\Omega_{D-1}^2)]$

Static spacetime with negative
constant curvature space \curvearrowright

where

$$e^{\eta/\alpha} = \tanh(t/2\alpha), \quad -\infty < \eta \leq 0,$$

BACKGROUND SPACETIME

- The line element in C-region is

$$ds^2 = -dt_C^2 + \alpha^2 \cos^2(t_C/\alpha) (dr_C^2 - \cosh^2 r_C d\Omega_{D-1}^2)$$

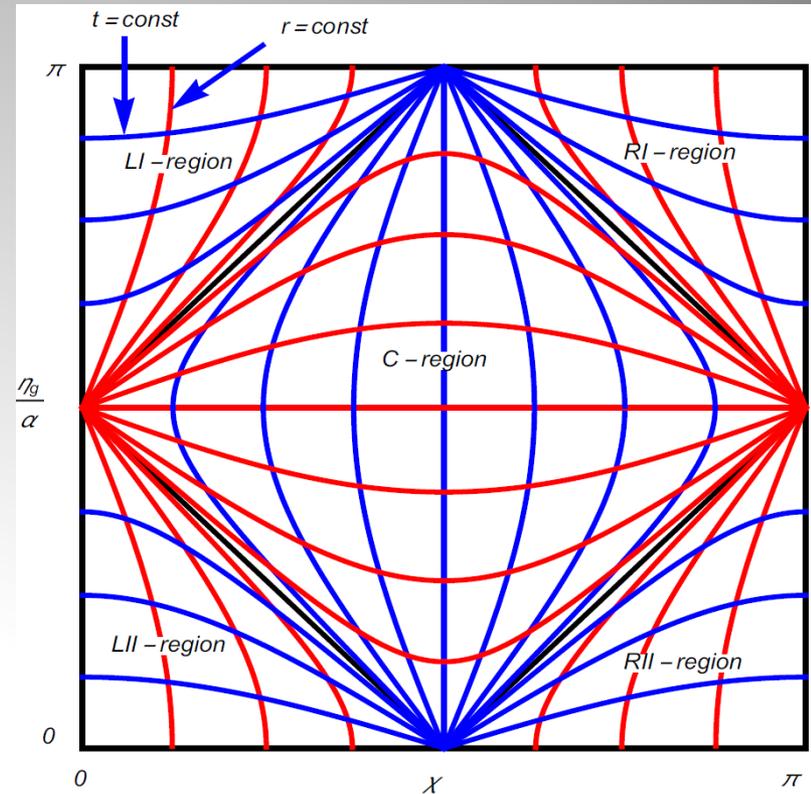
- Inflationary** coordinates cover only the half of dS spacetime

$$\frac{t_I}{\alpha} = \ln[\cosh(t/\alpha) + \sinh(t/\alpha) \cosh r]$$

$$\frac{r_I}{\alpha} = e^{-t_I/\alpha} \sinh(t/\alpha) \sinh r$$

- The line element is

$$ds^2 = dt_I^2 - e^{2t_I/\alpha} (dr_I^2 + r_I^2 d\Omega_{D-1}^2)$$



PROBLEM FORMULATION

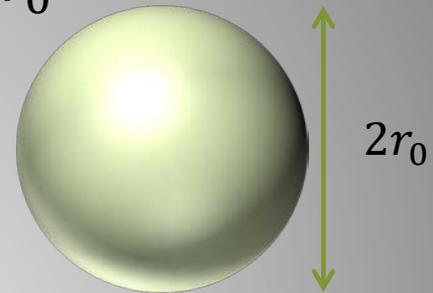
□ **Scalar field** φ

mass - m

curvature coupling parameter - ξ

□ Boundary geometry consists of a sphere of radius r_0 with **Robin** boundary condition (BC)

$$(A - \delta_{(j)} B \partial_r) \varphi(x) \Big|_{r=r_0} = 0,$$
$$j = i, e, \quad \delta_{(i)} = 1, \delta_{(e)} = -1$$



□ Special case - **Dirichlet** BC

$$\downarrow$$
$$A = 1, B = 0$$

□ Field equation

$$(\nabla_\mu \nabla^\mu + m^2 + \xi R) \varphi(x) = 0$$

$$\text{Minimal coupling} \rightarrow \xi = 0$$

$$\text{Conformal coupling} \rightarrow \xi = \frac{D-1}{4D}$$

RELATED PAPERS

1. Minkowski spacetime

A.A. Saharian, Phys. Rev. D **63**, 125007 (2001)

2. dS spacetime, Bunch-Davies vacuum

K.A. Milton, A.A. Saharian, Phys. Rev. D **85**, 064005 (2012)

3. Static spacetime with negative constant curvature space

S. Bellucci, A.A. Saharian and N.A. Saharyan, Eur. Phys. J. C **74**, 3047 (2014)

4. Milne universe, conformal and adiabatic vacua

A.A. Saharian and T.A. Petrosyan, Symmetry **12**, 619 (2020)

5. dS spacetime, hyperbolic vacuum

A.A. Saharian and T.A. Petrosyan, Phys. Rev. D **104**, 065017 (2021)

MODE FUNCTIONS

$$\varphi_{\sigma}(t, r, \vartheta, \phi) = \frac{X_{\nu}^{iz}(\cosh(t/\alpha))}{\sinh^{(D-1)/2}(t/\alpha)} \frac{Z_{iz-1/2}^{-\mu}(\cosh r)}{\sinh^{D/2-1} r} Y(m_p; \vartheta, \phi)$$

where

σ - the set of quantum numbers specifying the modes

$X_{\nu}^{iz}(\cosh(t/\alpha))$ \rightarrow are expressed in terms of associated Legendre
 $Z_{iz-1/2}^{-\mu}(\cosh r)$ \rightarrow functions of the first and second kinds

$$X_{\nu}^{iz}(y) = d_1 P_{\nu-1/2}^{iz}(y) + d_2 Q_{\nu-1/2}^{iz}(y) = b_1 P_{\nu-1/2}^{iz}(y) + b_2 P_{\nu-1/2}^{-iz}(y)$$

$$Z_{iz-1/2}^{-\mu}(u) = c_1 P_{iz-1/2}^{-\mu}(u) + c_2 Q_{iz-1/2}^{-\mu}(u)$$

where

$$y = \cosh(t/\alpha),$$

$$u = \cosh r$$

$$\nu = \sqrt{D^2/4 - \xi D(D+1) - m^2 \alpha^2},$$

$$\mu = l + D/2 - 1$$

$Y(m_p; \vartheta, \phi)$ - hyperspherical harmonics of degree l

MODE FUNCTIONS

The relations between coefficients are found from **normalization conditions** and **BC**

$$(A - \delta_{(j)} B \partial_r) \varphi(x) \Big|_{r=r_0} = 0 \quad \longrightarrow \quad \frac{c_2}{c_1}$$

$$-i \int d^D x \sqrt{|g|} \varphi_\sigma(x) \overleftrightarrow{\partial}_t \varphi_{\sigma'}^*(x) = \delta_{\sigma\sigma'} \quad \longrightarrow \quad c_1$$

$$f^*(x)f'(x) - f(x)f^{*'}(x) = -\frac{i}{\sinh^D x}$$



$$\begin{cases} |b_1|^2 - |b_2|^2 = \frac{\pi}{2 \sinh(\pi z)} \\ \frac{2}{\pi} |d_1|^2 \sinh(\pi z) + i (d_1 d_2^* - d_2 d_1^*) e^{-\pi z} = 1 \end{cases}$$

$f(x) \equiv f(t/\alpha)$ - temporal part of $\varphi_\sigma(x)$

VACUUM STATE – CONFORMAL VACUUM

- **Massless conformally coupled field**

Temporal part of mode functions
in **dS** spacetime

$$X_{1/2}^{iz}(y) = \frac{b_1 e^{-iz\eta/\alpha}}{\Gamma(1-iz)} + \frac{b_2 e^{iz\eta/\alpha}}{\Gamma(1+iz)}$$

Mode functions in **static** spacetime

$$\varphi_\sigma^{(s)}(x) = \frac{e^{-iz\eta/\alpha} Z_{iz-1/2}^{-\mu}(\cosh r)}{\sinh^{D/2-1} r} Y(m_k; \vartheta, \phi)$$

$$b_2 = 0, \quad |b_1|^2 = \frac{\pi}{2 \sinh(\pi Z)}$$

- Mode functions in dS spacetime for the **conformal vacuum**

$$\varphi_\sigma(x) = \sinh^{\frac{D-1}{2}}(|\eta|/\alpha) e^{-iz\eta/\alpha} \frac{Z_{iz-1/2}^{-\mu}(\cosh r)}{\sinh^{D/2-1} r} Y(m_k; \vartheta, \phi)$$

where

$$\varphi_\sigma(x) = \Omega^{(1-D)/2} \varphi_\sigma^{(s)}(x)$$

$$\Omega^2 = \sinh^{-2}(\eta/\alpha) \longrightarrow \text{Conformal factor}$$

- **Massive field with general curvature coupling**

$$X_{iz}(mt) = b_1 P_{\nu-1/2}^{iz}(y)$$

VACUUM STATE – ADIABATIC VACUUM

- Temporal part of equation of motion

$$\partial_{\eta}^2 h(\eta) + \omega^2(z, \eta) h(\eta) = 0$$

where

$$h(\eta) = \sinh^{(D-1)/2}(t/\alpha) f(t/\alpha) \quad \omega(z, \eta) = \frac{1}{\alpha} \left(z^2 - \frac{v^2 - 1/4}{\sinh^2(\eta/\alpha)} \right)^{1/2}$$

effective frequency 

- Time dependence of the effective frequency is weak for $|\eta| \gg \alpha$
- Asymptotically static region (**static in-region**) $\rightarrow \eta \rightarrow -\infty \leftrightarrow t/\alpha \ll 1$

Temporal part of mode functions

$$X_v^{iz}(\cosh(t/\alpha)) \approx \frac{b_1 e^{-iz\eta/\alpha}}{\Gamma(1-iz)} + \frac{b_2 e^{iz\eta/\alpha}}{\Gamma(1+iz)}$$

Zeroth adiabatic order for the modes realizing in-vacuum

$$h^{(0)}(\eta) \sim e^{-iz\eta/\alpha}$$

$$b_2 = 0, \quad |b_1|^2 = \frac{\pi}{2 \sinh(\pi z)}$$

- **Coincides with conformal vacuum**

HADAMARD FUNCTION

$$G(x, x') = \sum_{\sigma} [\varphi_{\sigma}(x) \varphi_{\sigma}^*(x') + \varphi_{\sigma}(x') \varphi_{\sigma}^*(x)] = G_0(x, x') + G_S(x, x')$$

- Interior region of the sphere, hyperbolic vacuum

$$G_S(x, x') = -\frac{2\alpha^{1-D}}{nS_D} \sum_{l=0}^{\infty} \mu C_l^{n/2}(\cos \theta) \int_0^{\infty} dz \frac{ze^{-i\mu\pi} \bar{Q}_{z-1/2}^{\mu}(u_0)}{\sin(\pi z) \bar{P}_{z-1/2}^{-\mu}(u_0)} \\ \times \frac{P_{z-1/2}^{-\mu}(u) P_{z-1/2}^{-\mu}(u')}{(\sinh r \sinh r')^{D/2-1}} \frac{\sum_{j=+,-} P_{\nu-1/2}^{jz}(y) P_{\nu-1/2}^{-jz}(y')}{[\sinh(t/\alpha) \sinh(t'/\alpha)]^{(D-1)/2}}$$

- Exterior region of the sphere, hyperbolic vacuum

$$G_S(x, x') = -\frac{2\alpha^{1-D}}{nS_D} \sum_{l=0}^{\infty} \mu C_l^{n/2}(\cos \theta) \int_0^{\infty} dz \frac{ze^{-i\mu\pi} \bar{P}_{z-1/2}^{-\mu}(u_0)}{\sin(\pi z) \bar{Q}_{z-1/2}^{\mu}(u_0)} \\ \times \frac{Q_{z-1/2}^{\mu}(u) Q_{z-1/2}^{\mu}(u')}{(\sinh r \sinh r')^{D/2-1}} \frac{\sum_{j=+,-} P_{\nu-1/2}^{-jz}(y) P_{\nu-1/2}^{jz}(y')}{[\sinh(t/\alpha) \sinh(t'/\alpha)]^{(D-1)/2}}$$

HADAMARD FUNCTION

- Interior region \longrightarrow Summation over z_k

$$\bar{P}_{iz_k-1/2}^{-\mu}(u_0) = 0, u_0 = \cosh r_0$$

Summation formula

$$\sum_{k=1}^{\infty} T_{\mu}(z_k, u_0) h(z_k) = \frac{e^{-i\mu\pi}}{2} \int_0^{\infty} dx \sinh(\pi x) h(x) - \frac{1}{2\pi} \int_0^{\infty} dx \frac{\bar{Q}_{x-1/2}^{-\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} \cos[\pi(x - \mu)] \sum_{j=\pm} h(xe^{j\pi i/2})$$

where

$$h(z) = z \frac{|\Gamma(\mu + iz + 1/2)|^2}{\sinh(\pi z)} P_{iz-1/2}^{-\mu}(u) P_{iz-1/2}^{-\mu}(u') \sum_{j=+,-} P_{v-1/2}^{jiz}(y) P_{v-1/2}^{-jiz}(y')$$

$$T_{\mu}(z, u) = \frac{\bar{Q}_{iz-1/2}^{-\mu}(u)}{\partial_z \bar{P}_{iz-1/2}^{-\mu}(u)} \cos[\pi(\mu - iz)]$$

- Exterior region \longrightarrow Integration by z

VEV OF THE FIELD SQUARED

$$\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_0 + \langle \varphi^2 \rangle_s$$

- Interior region

$$\langle \varphi^2 \rangle_s = -\frac{\alpha^{1-D} \sinh^{2-D} r}{S_D \sinh^{D-1}(t/\alpha)} \sum_{l=0}^{\infty} e^{-i\mu\pi D_l} \int_0^{\infty} dx \frac{x}{\sin(\pi x)} \frac{\bar{Q}_{x-1/2}^{\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} \\ \times P_{\nu-1/2}^x(y) P_{\nu-1/2}^{-x}(y) [P_{x-1/2}^{-\mu}(u)]^2$$

$$\langle \varphi^2 \rangle_s = -\frac{2\alpha^{1-D} \sinh^{2-D} r}{\pi S_D \sinh^{D-1}(t/\alpha)} \sum_{l=0}^{\infty} e^{-i\mu\pi D_l} \sum_{k=0}^{\infty} ' (-1)^k \int_0^{\infty} dx \frac{x F(x) - k F(k)}{x^2 - k^2}$$

where

$$F(x) = x \frac{\bar{Q}_{x-1/2}^{\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} P_{\nu-1/2}^x(y) P_{\nu-1/2}^{-x}(y) [P_{x-1/2}^{-\mu}(u)]^2$$

- Exterior region

$$\bar{P}_{x-1/2}^{-\mu}(u_0) \Leftrightarrow \bar{Q}_{x-1/2}^{\mu}(u_0)$$

$$P_{x-1/2}^{-\mu}(u) \Leftrightarrow Q_{x-1/2}^{\mu}(u)$$

ENERGY-MOMENTUM TENSOR (EMT)

VEV of EMT



$$\langle T_{ik} \rangle = \lim_{x' \rightarrow x} \partial_{i'} \partial_k W(x, x') + [(\xi - 1/4)g_{ik} \nabla_p \nabla^p - \xi \nabla_i \nabla_k - \xi R_{ik}] \langle \varphi^2 \rangle$$

$$\langle T_{ik} \rangle = \langle T_{ik} \rangle_0 + \langle T_{ik} \rangle_s$$

$$\langle T_k^i \rangle_s = \begin{pmatrix} \varepsilon & \langle T_1^0 \rangle_s & 0 & 0 & 0 \\ \langle T_0^1 \rangle_s & -p & 0 & 0 & 0 \\ 0 & 0 & -p_\perp & 0 & 0 \\ 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -p_\perp \end{pmatrix}$$

- Non-diagonal component corresponds to the **energy flux** along radial direction

ENERGY FLUX AND ENERGY DENSITY

- Interior region

$$\langle T_0^1 \rangle_s = \frac{\sinh^{-3}(t/\alpha)}{\alpha^{D+2} S_D} \sum_{l=0}^{\infty} e^{-i\mu\pi D_l} \int_0^{\infty} dx \frac{x}{\sin(\pi x)} \frac{\bar{Q}_{x-1/2}^{\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} \times \left[\left(\frac{1}{4} - \xi \right) (y^2 - 1) \partial_y + \xi y \right] \partial_r F^{(i)}(x, y, u)$$

Energy flux

Energy density

$$\langle T_0^0 \rangle_s = - \frac{\sinh^{-2}(t/\alpha)}{\alpha^{D+1} S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} dx \frac{x e^{-i\mu\pi}}{\sin(\pi x)} \frac{\bar{Q}_{x-1/2}^{\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} [\hat{F}_0^{(0)}(y) - \hat{F}_0^{(1)}(u)] F^{(i)}(x, y, u)$$

where

$$\hat{F}_0^{(0)}(y) = (y^2 - 1) \left\{ \frac{1}{4} (y^2 - 1) \partial_y^2 + \left[D(\xi + \xi_D) + \frac{1}{2} \right] y \partial_y + m^2 \alpha^2 + \xi D^2 + \frac{(D-1)^2}{4} - x^2 \right\}$$

$$\hat{F}_0^{(1)}(u) = \left(\xi - \frac{1}{4} \right) [(u^2 - 1) \partial_u^2 + Du \partial_u]$$

$$F^{(i)}(x, y, u) = \frac{P_{\nu-1/2}^x(y) P_{\nu-1/2}^{-x}(y) [P_{x-1/2}^{-\mu}(u)]^2}{\sinh^{D-1}(t/\alpha) \sinh^{D-2} r}$$

VACUUM STRESSES

- Interior region

$$\langle T_1^1 \rangle_s = -\frac{\sinh^{-2}(t/\alpha)}{\alpha^{D+1} S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} dx \frac{x e^{-i\mu\pi} \bar{Q}_{x-1/2}^{\mu}(u_0)}{\sin(\pi x) \bar{P}_{x-1/2}^{-\mu}(u_0)} [\hat{F}_1^{(0)}(y) - \hat{F}_1^{(1)}(u)] F^{(i)}(x, y, u)$$

Normal stress

Azimuthal stress

$$\langle T_2^2 \rangle_s = -\frac{\sinh^{-2}(t/\alpha)}{\alpha^{D+1} S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} dx \frac{x e^{-i\mu\pi} \bar{Q}_{x-1/2}^{\mu}(u_0)}{\sin(\pi x) \bar{P}_{x-1/2}^{-\mu}(u_0)} [\hat{F}_2^{(0)}(y) - \hat{F}_2^{(1)}(u)] F^{(i)}(x, y, u)$$

where

$$\hat{F}_k^{(0)}(y) = (y^2 - 1) \left\{ \left(\xi - \frac{1}{4} \right) (y^2 - 1) \partial_y^2 + \left[D(\xi - \xi_D) - \frac{1}{2} \right] y \partial_y - \xi D \right\} + \delta_{1k} \left[x^2 - \frac{(D-1)^2}{4} \right]$$

$$\hat{F}_1^{(1)}(u) = \frac{1}{4} (u^2 - 1) \partial_u^2 + \left[\xi(D-1) + \frac{D}{4} \right] u \partial_u - \frac{l(l+n)}{u^2 - 1}$$

$$\hat{F}_2^{(1)}(u) = \hat{F}_0^{(1)}(u) - \xi u \partial_u + \frac{1}{D-1} \frac{l(l+n)}{u^2 - 1}$$

CONSERVATION EQUATION

- Interior region

$$\langle T_2^2 \rangle_s = \langle T_3^3 \rangle_s = \dots = \langle T_D^D \rangle_s$$

Covariant conservation equation $\longrightarrow \nabla_k \langle T_i^k \rangle_s = 0$

$$\sum_{k=0,1} \partial_k \langle T_0^k \rangle_s + (D-1) \coth r \langle T_0^1 \rangle_s + \frac{1}{\alpha} \coth(t/\alpha) \left[(D+1) \langle T_0^0 \rangle_s - \langle T_k^k \rangle_s \right] = 0$$

$$\sum_{k=0,1} \partial_k \langle T_1^k \rangle_s + \frac{D}{\alpha} \coth(t/\alpha) \langle T_1^0 \rangle_s + (D-1) \coth r \left(\langle T_1^1 \rangle_s - \langle T_2^2 \rangle_s \right) = 0.$$

The vacuum energy induced by the sphere in the spherical shell $r_1 \leq r \leq r_2$

$$E_{(s)} = \alpha \sinh(t/\alpha) \int_{r_1}^{r_2} dr S_p(r) \langle T_0^0 \rangle_s$$

$S_p(r) = S_D [\alpha \sinh(t/\alpha) \sinh r]^{D-1} \longrightarrow$ proper surface area of the sphere with radius r

$$\partial_0 E_{(s)} = -\alpha \sinh(t/\alpha) S_p(r) \langle T_0^1 \rangle_s \Big|_{r=r_1}^{r=r_2} + \cosh(t/\alpha) \int_{r_1}^{r_2} dr S_p(r) \sum_{k=1}^D \langle T_k^k \rangle_s$$

Energy flux density per unit proper surface area \longrightarrow

$$\langle \tilde{T}_0^1 \rangle_s = \alpha \sinh(t/\alpha) \langle T_0^1 \rangle_s$$

- Exterior region

$$\begin{aligned} \bar{P}_{x-1/2}^{-\mu}(u_0) &\rightleftharpoons \bar{Q}_{x-1/2}^{\mu}(u_0) \\ P_{x-1/2}^{-\mu}(u) &\rightleftharpoons Q_{x-1/2}^{\mu}(u) \end{aligned}$$

ASYMPTOTIC BEHAVIOR

- Early stages of expansion. $t/\alpha \ll 1$

$$\langle \varphi^2 \rangle_s \approx -\frac{\sinh^{2-D} r}{\pi S_D t^{D-1}} \sum_{l=0}^{\infty} e^{-i\mu\pi D_l} \int_0^{\infty} dx \frac{\bar{Q}_{x-1/2}^{-\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} [P_{x-1/2}^{-\mu}(u)]^2$$

- Late stages of expansion. $t/\alpha \gg 1 \longrightarrow (\nu > 0; \quad \nu = 0; \quad \nu = i|\nu|)$

$$\nu = 0 \longrightarrow \langle \varphi^2 \rangle_s \approx -\frac{2^{D+1} t^2 e^{-Dt/\alpha}}{\pi^2 S_D \alpha^{D+1} \sinh^{D-2} r} \sum_{l=0}^{\infty} e^{-i\mu\pi D_l} \int_0^{\infty} dx x \cot(\pi x) \frac{\bar{Q}_{x-1/2}^{-\mu}(u_0)}{\bar{P}_{x-1/2}^{-\mu}(u_0)} [P_{x-1/2}^{-\mu}(u)]^2$$

- Near the sphere center. $r \ll 1$

$$\langle \varphi^2 \rangle_s \approx -\frac{e^{-i\pi(D/2-1)} (2\alpha)^{1-D}}{\pi^{D/2} \Gamma(D/2) \sinh^{D-1}(t/\alpha)} \int_0^{\infty} dx \frac{x}{\sin(\pi x)} \frac{\bar{Q}_{x-1/2}^{-D/2-1}(u_0)}{\bar{P}_{x-1/2}^{1-D/2}(u_0)} P_{\nu-1/2}^x(y) P_{\nu-1/2}^{-x}(y)$$

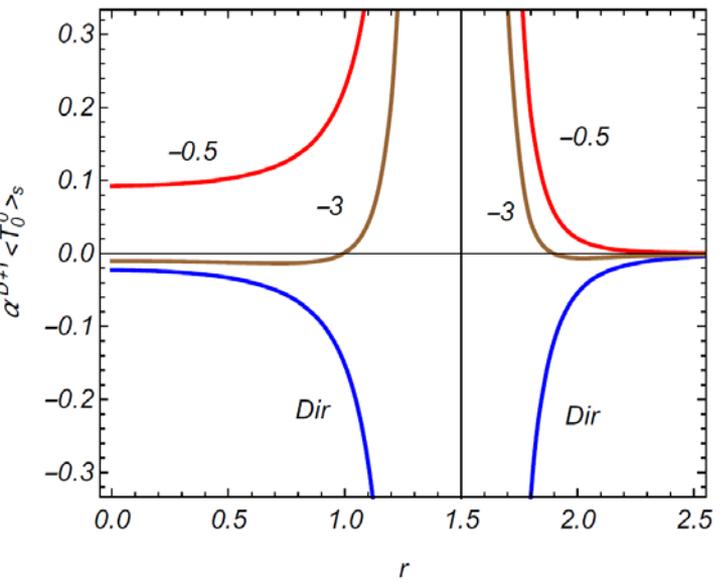
- Near the sphere. $r \approx r_0(-)$

$$\langle \varphi^2 \rangle_s \approx \frac{(1 - 2\delta_{0B}) \Gamma((D-1)/2)}{(4\pi)^{(D+1)/2} [\alpha \sinh(t/\alpha) (r_0 - r)]^{D-1}} \longleftrightarrow \langle \varphi^2 \rangle_s \approx \frac{\langle \varphi^2 \rangle_s^{(st)}}{\sinh^{D-1}(t/\alpha)}$$

- Far from the sphere. $r \gg 1$ ↪ Proper distance from the sphere

$$\langle \varphi^2 \rangle_s \approx -\frac{2^{D-3} \alpha^{1-D} [P_{\nu-1/2}^0(y)]^2}{S_D \sinh^{D-1}(t/\alpha) r e^{(D-1)r}} \sum_{l=0}^{\infty} \frac{D_l \bar{P}_{-1/2}^{-\mu}(u_0)}{e^{-i\mu\pi} \bar{Q}_{-1/2}^{-\mu}(u_0)} \Gamma^2(\mu + 1/2)$$

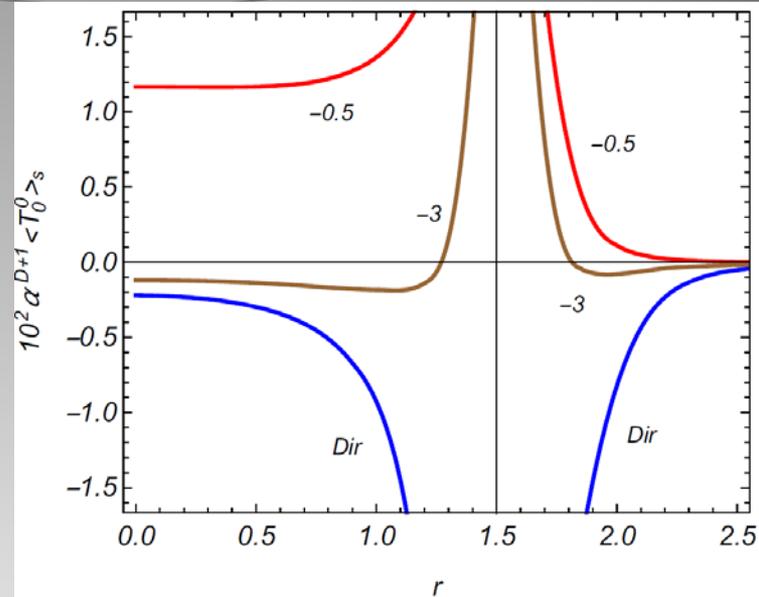
ENERGY DENSITY, ENERGY FLUX



Energy density

Minimal
($\xi = 0$)

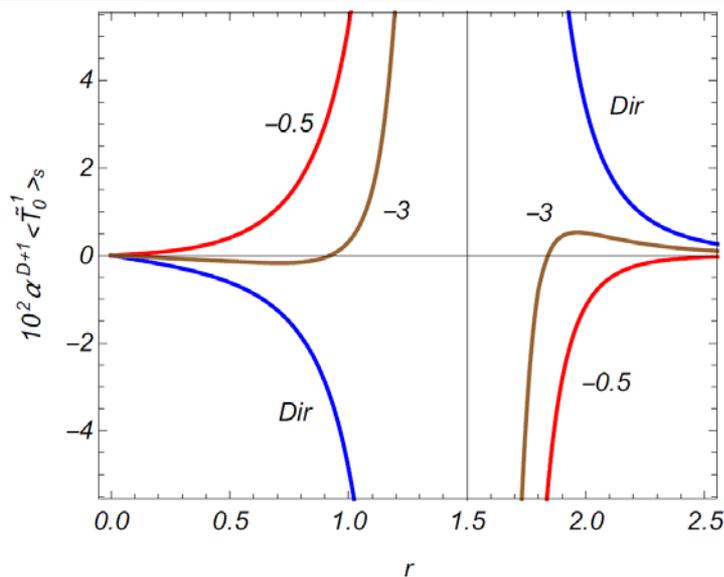
Conformal
($\xi = \xi_D$)



$D = 3, \quad r_0 = 1.5,$

$m\alpha = t/\alpha = 1,$

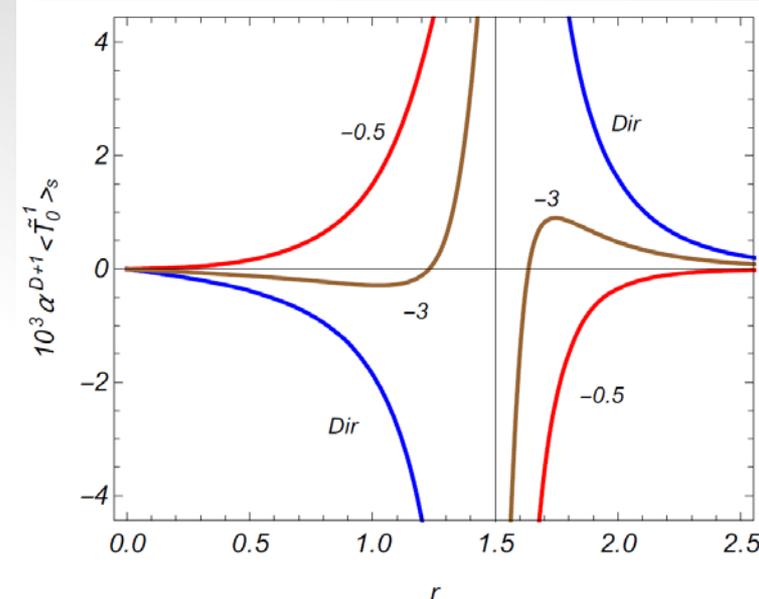
$\beta \equiv A/B = -3, -0.5, -\infty$



Minimal
($\xi = 0$)

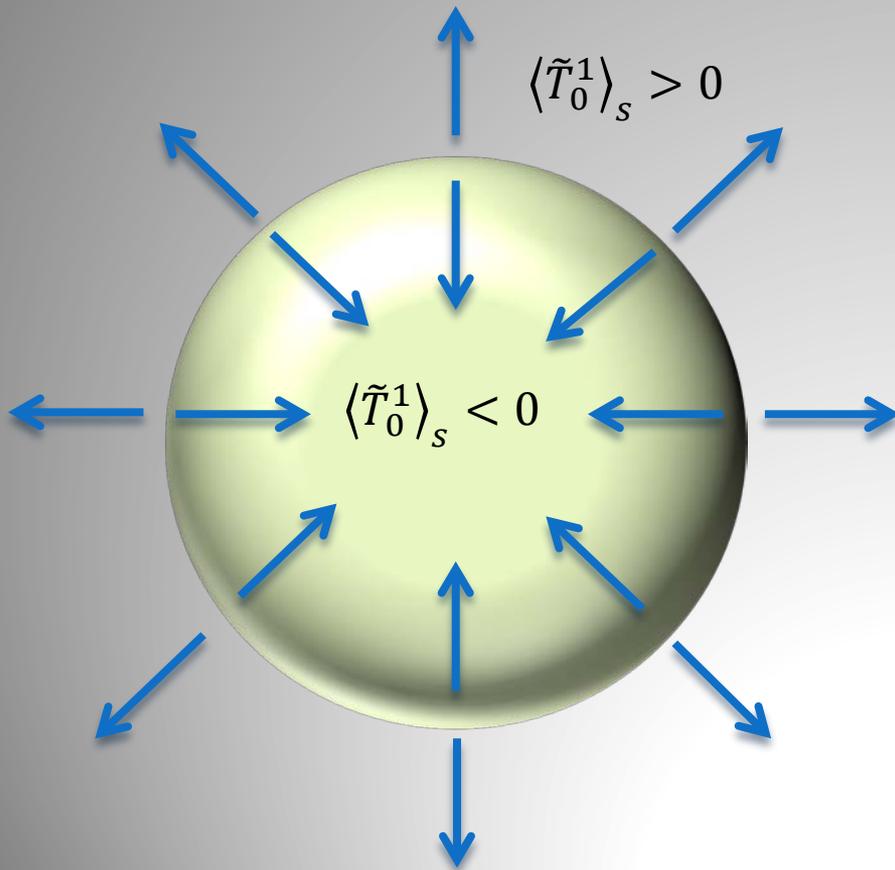
Conformal
($\xi = \xi_D$)

Energy flux

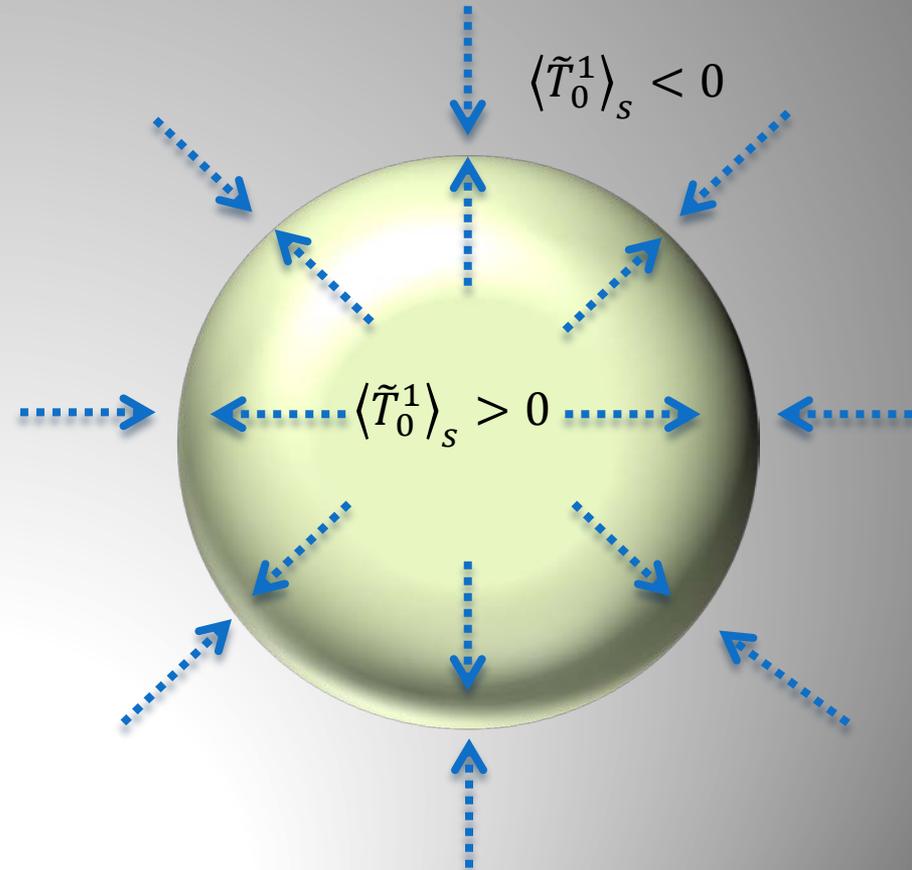


NUMERICAL RESULTS - ENERGY FLUX

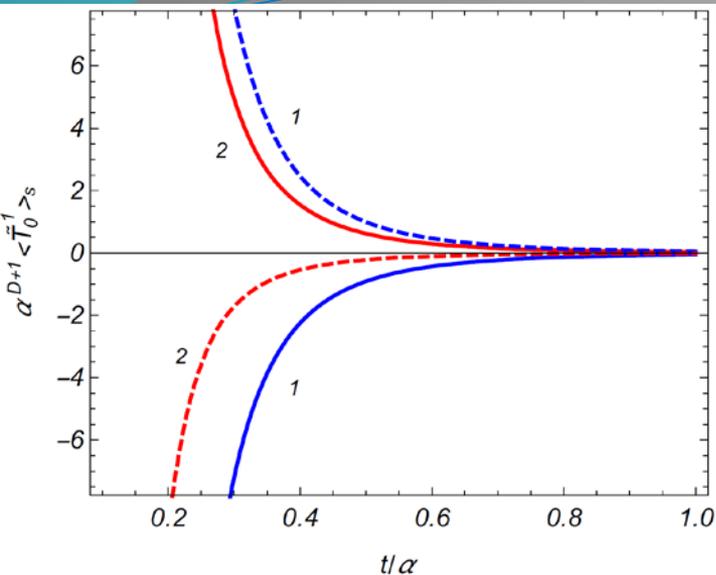
Dirichlet BC



non-Dirichlet BC



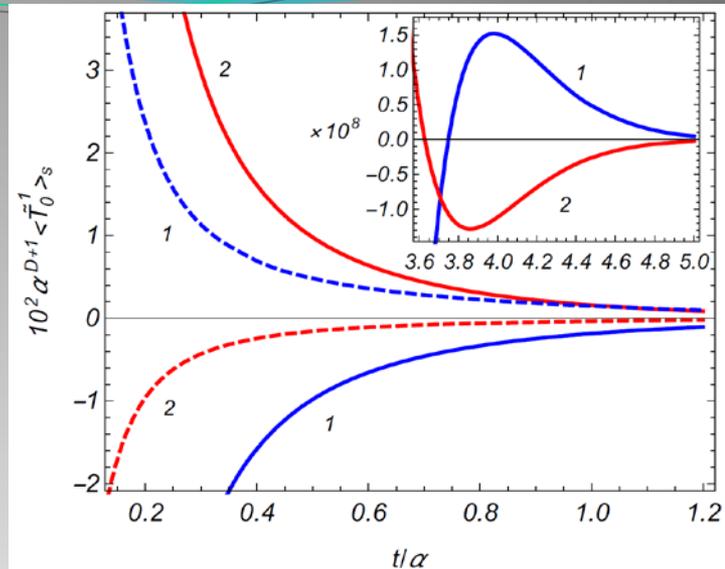
NUMERICAL RESULTS - ENERGY FLUX



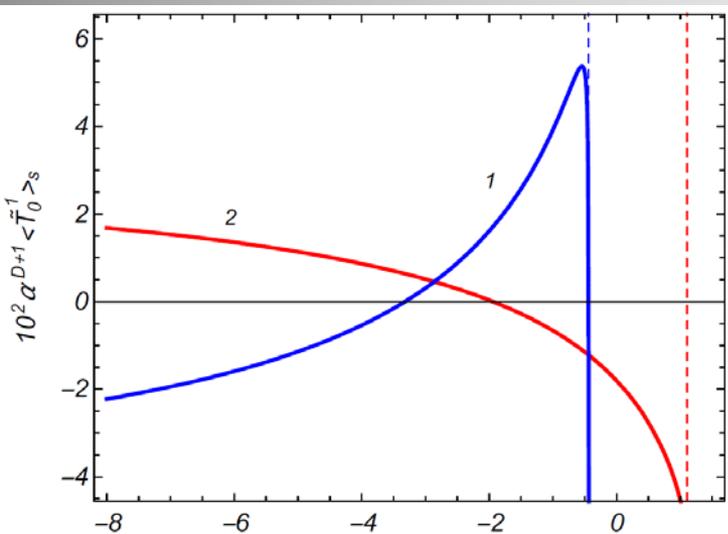
— Dirichlet BC
 - - - Robin BC
 ($\beta = -0.5$)

Minimal
 ($\xi = 0$)
 Conformal
 ($\xi = \xi_D$)

$D = 3, \quad r_0 = 1.5, \quad r = 1, 2, \quad m\alpha = 1$

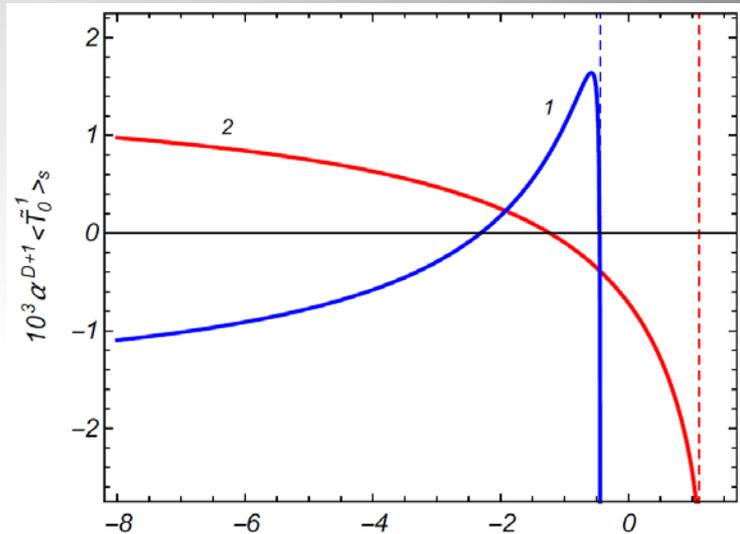


$m\alpha = 1$



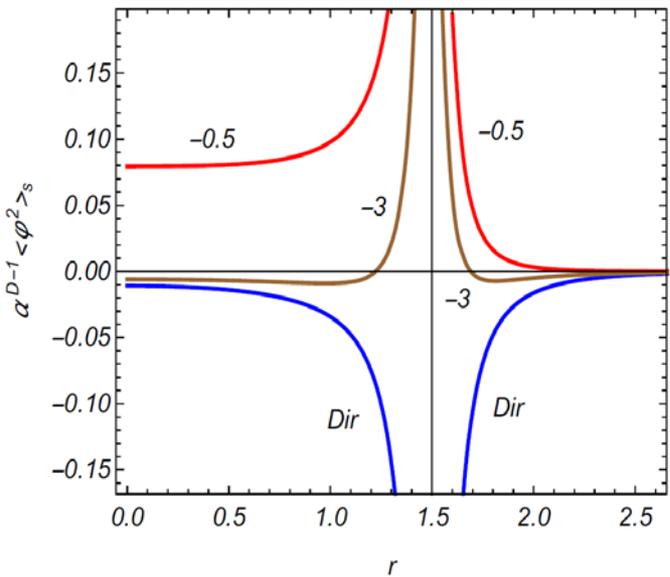
Minimal
 ($\xi = 0$)
 Conformal
 ($\xi = \xi_D$)

$\beta \quad D = 3, \quad r_0 = 1.5, \quad r = 1, 2,$



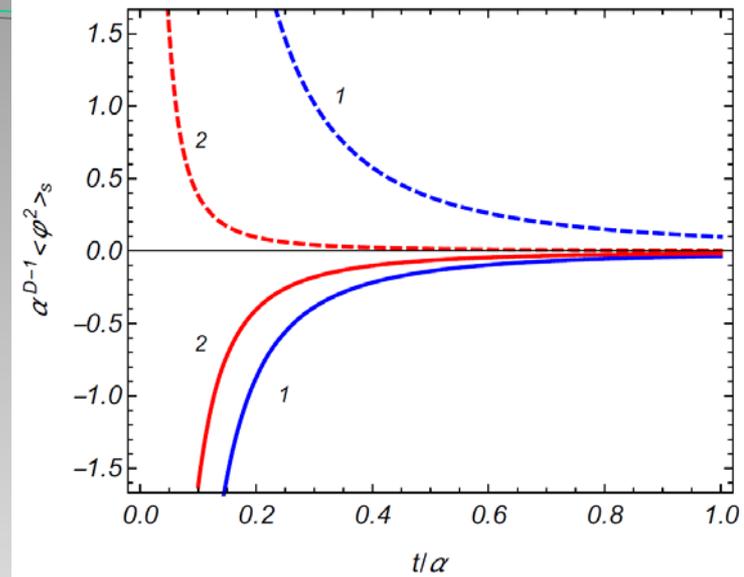
$m\alpha = t/\alpha = 1 \quad \beta$

NUMERICAL RESULTS - FIELD SQUARED



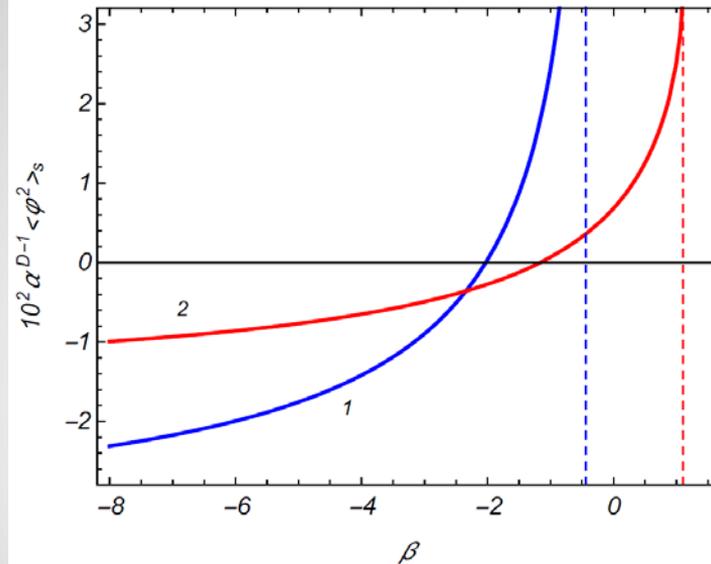
Minimal coupling
($\xi = 0$)

— Dirichlet BC
- - Robin BC
($\beta = -0.5$)



$D = 3,$ $r_0 = 1.5,$ $m\alpha = t/\alpha = 1,$
 $\beta = -3, -0.5, -\infty$

$D = 3,$ $r_0 = 1.5,$
 $r = 1, 2,$ $m\alpha = 1$



$D = 3,$ $r_0 = 1.5,$
 $r = 1, 2,$ $m\alpha = t/\alpha = 1$

SUMMARY

- ❑ The Hadamard function and the vacuum expectation values (VEVs) of the field squared and of the energy-momentum tensor (EMT) are investigated in the geometry of a spherical shell on background of the dS spacetime. The field obeys the Robin boundary condition on the sphere.
- ❑ The boundary-induced contribution is explicitly extracted and the renormalization is done only for the boundary-free contribution
- ❑ Rapidly convergent integral representations are provided for the boundary-induced parts
- ❑ Adiabatic and conformal vacuum states are realized in the same conditions (hyperbolic vacuum)
- ❑ There is a nonzero energy flux along the radial direction (the off-diagonal component of the VEV of the EMT)
- ❑ Depending on the coefficients in the Robin BC, sphere-induced part in the energy density can be either positive or negative



THANK YOU