Twin star solutions considering exotic matter in compact stars
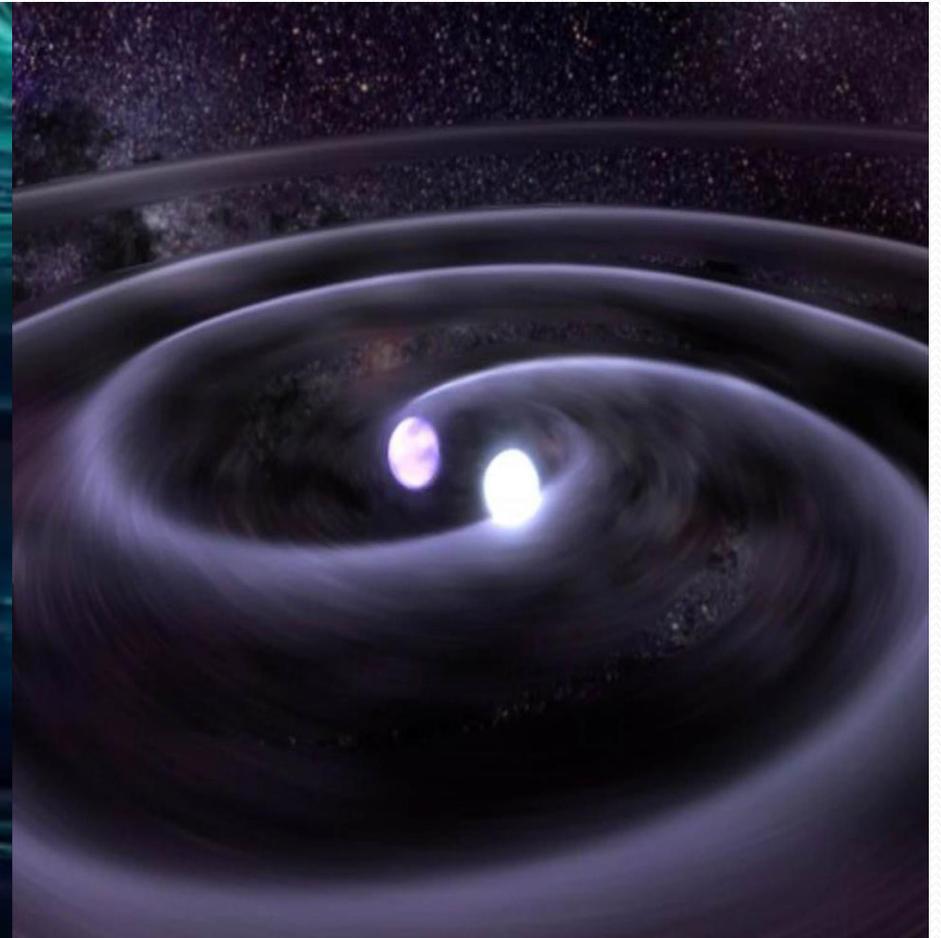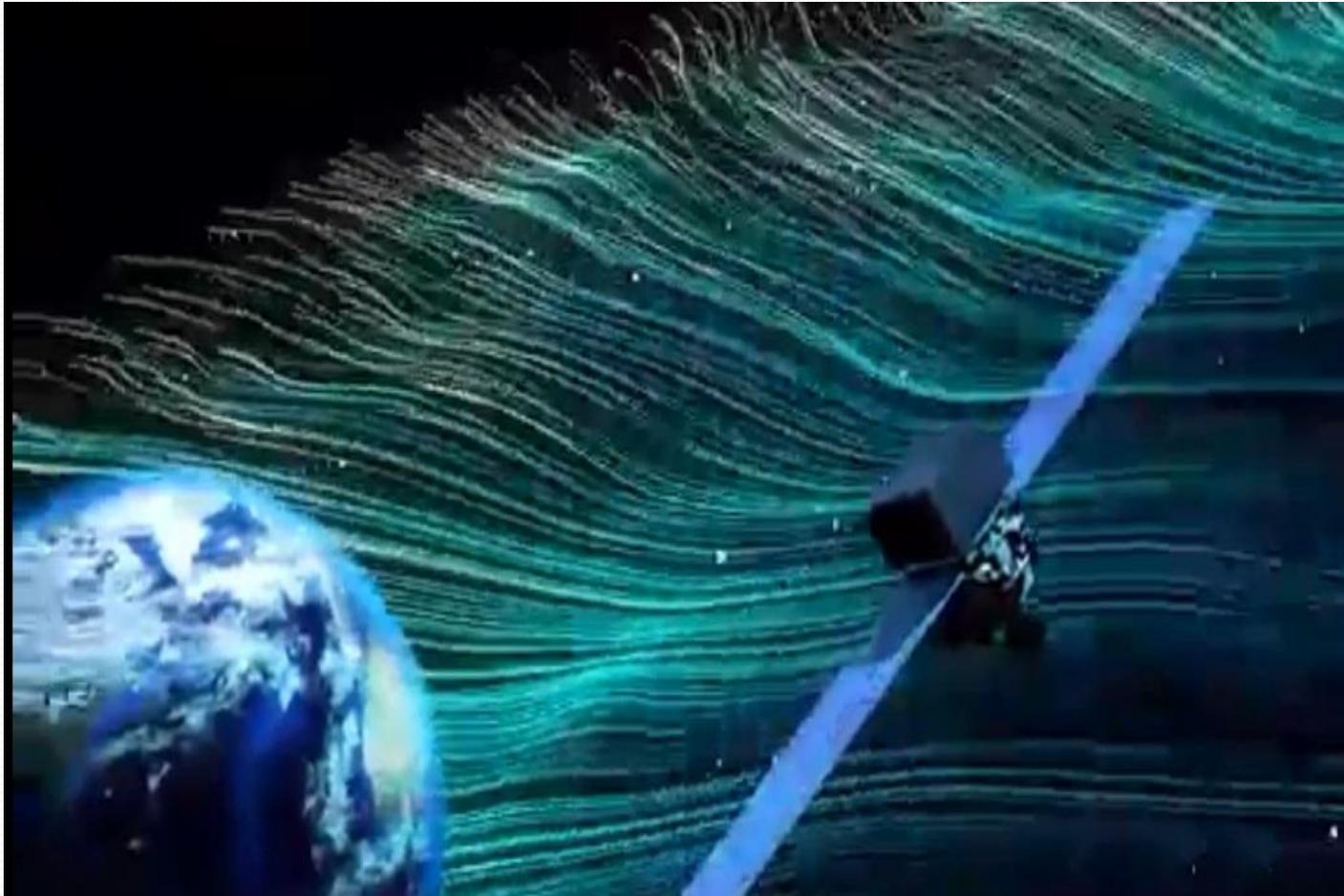
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- GW170817 provided valuable constraints on the equations of state of binary neutron star mergers.

- Binary neutron stars, natural laboratories of extreme temperature and density, may lead to the estimation of some exotic matter like deconfined quark matter in their cores.

Content

- **Neutron star matter equation of state:**

  VLOCV method

- **Hybrid equation of state with first-order phase transition:**

  Constant Speed of Sound (CSS)

  Categories of twin stars

- **Tidal deformability**

- **Conclusion**

Neutron star matter equation of state

We consider neutron matter as an infinite system of strongly interacting A neutrons with finite size.

The proper volume: $b = \dfrac{16}{3}\pi r^3$ **The number density of the system:** $\rho = A/V$

Energy per particle and pressure $\longrightarrow$ **VLOCV method**

$$E(\rho) = E^*_{\text{nuc}}(\rho^*),$$
$$p_H(\rho) = \rho^2 \frac{\partial E(\rho)}{\partial \rho} = \rho^{*2}\frac{\partial E^*(\rho^*)}{\partial \rho^*} = p^*_H(\rho^*)$$

Energy per nucleon which is calculated by LOCV model

$$\rho* = \rho/(1 - b\rho)$$

LOCV approach

Ψ a trial many-body wave function: $\psi = \mathcal{F}\phi$

F   Jastrow form of A-body correlation operator: $\mathcal{F} = \mathcal{S}\prod_{i>j} f(ij)$

$\mathcal{S}$ Symmetrizing operator

ϕ uncorrelated ground state wave function of A-independent nucleons

The cluster expansion of the energy functional up to the two-body term:

$$E^*_{\text{nuc}}([f]) = \frac{1}{A}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = E_1 + E_2$$

LOCV approach

One-body term:
$$E_1 = \sum_{\tau=n,p} \sum_{k \le k_\tau^F} \frac{\hbar^2 k^2}{2m_\tau}$$

Fermi momentum of nucleons
$$k_n^F = (3\pi^2 \rho^*)^{1/3}$$

two-body energy:
$$E_2 = \frac{1}{2A} \sum_{i,j} \langle i,j | -\hbar^2/2m[f(12),[\nabla_{12}^2, f(12)]] + f(12)V(12)f(12) | ij - ji \rangle$$

Two-body correlation operator

$V(12)$
two-body potential:

AV18

Three Nucleon Interaction (TNI)

$$E = E(\text{AV}i + \text{TNR}) + \text{TNA}, \quad i \equiv 18, 8', 6'$$

Three nucleon attractive part of TNI contribution

$$\text{TNA} = \gamma_2 \rho^2 \exp(-\gamma_3 \rho)(3 - 2\beta^2)$$

Three nucleon repulsive part of TNI contribution

$$\propto \quad \exp(-\gamma_1 \rho)$$

I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. A**359**, 349 (1981).

TNI parameters are adjusted to give the correct saturation properties of SNM.

The functional minimization of the two-body cluster energy with respect to the variations in the correlation functions but subject to the normalization constraint,

$$\frac{1}{A}\sum_{ij}\langle ij|\left[1 - \frac{9}{2}\left(\frac{J_j^2(k_\tau^F r)}{k_\tau^F r}\right)^2\right]^{-1} - f^2(12)|ij\rangle_a = 0$$

- It is worth mentioning that the radius of neutrons may be variable inside the compressed nuclear medium.

- One of the simplest models that can be applied to have such an assumption is Bag model.

Bag model for nucleon properties

In this model, nucleons have the spherical volume of Ω_N as the systems with internal components of three quarks in the lowest state, which have **the energy** as,

$$E_{\text{Bag}}^0 = \frac{3\omega_0 - Z_0}{r_0} + \frac{4\pi}{3} B(\rho_0) r_0^3$$

The pressure inside the bag:

$$p_B = -\left(\frac{\partial E_{\text{Bag}}^0}{\partial \Omega_N}\right)_{\text{surface}} = 0$$

$$r_0 = \left[\frac{3\omega_0 - Z_0}{4\pi B(\rho_0)}\right]^{1/4} \leftrightarrow B(\rho_0) = \frac{3\omega_0 - Z_0}{4\pi r_0^4}$$
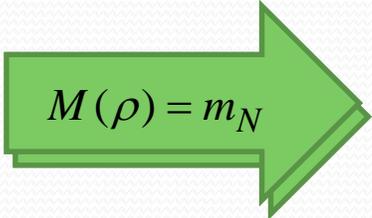
K. Johnson, Acta Phys. Pol. B **6**, 865 (1975).

Bag model for nucleon properties

The density dependent radius of the nucleon:

$$p_H = p_B \rightarrow r(\rho)$$

$$= \left[\frac{3\omega_0 - Z_0}{4\pi(B(\rho) + p_H(\rho))} \right]^{1/4}$$

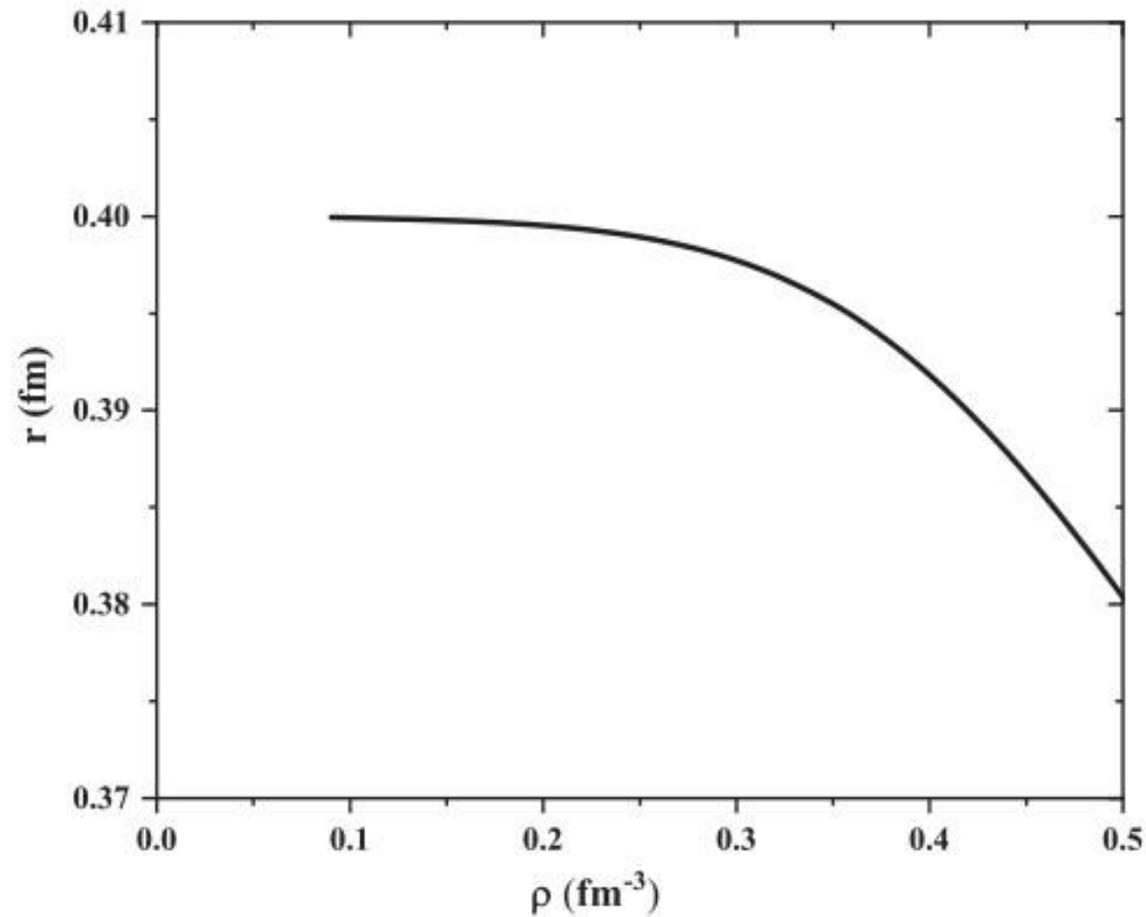$$\leftrightarrow B(\rho) = \frac{3\omega_0 - Z_0}{4\pi r^4(\rho)} - p_H(\rho)$$

Mass in nuclear medium:

$$m(\rho) = m_N \frac{r_0}{r(\rho)} - p_H \Omega_N$$

$$M(\rho) = m_N$$

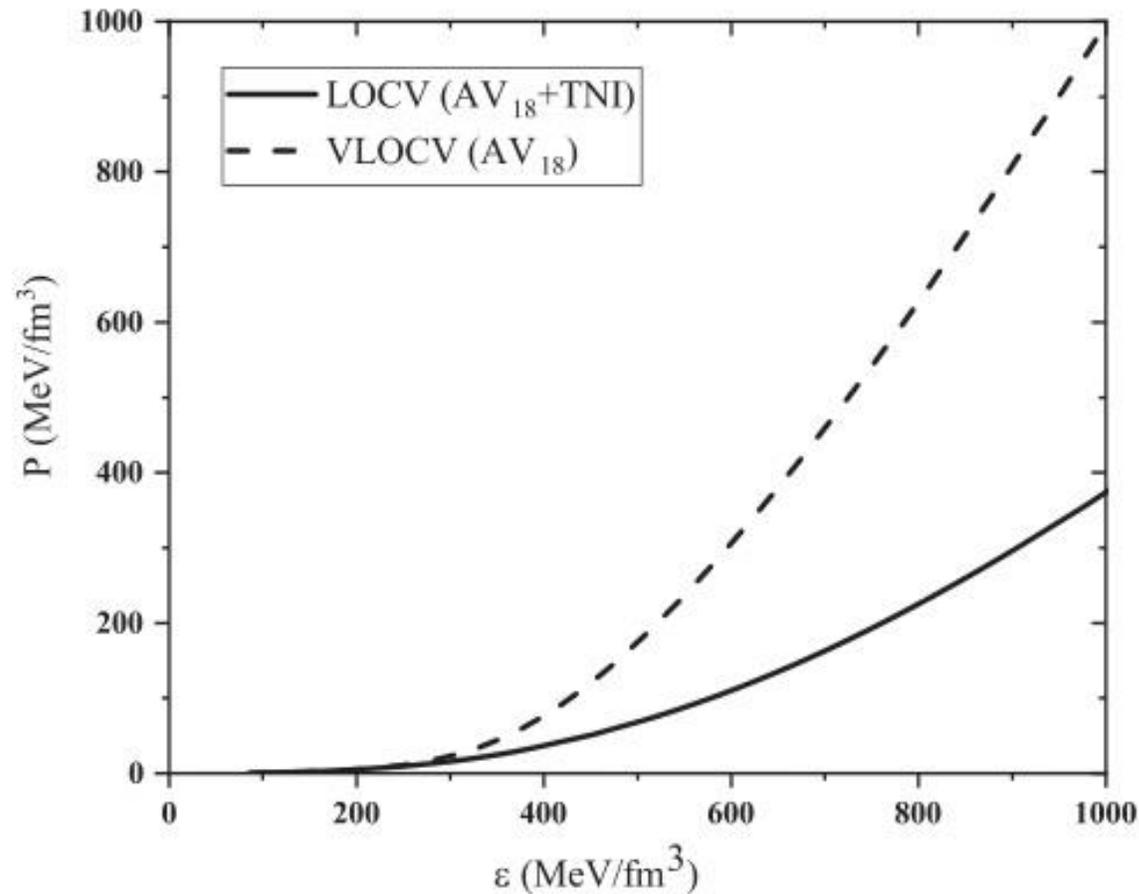$$m_N r(\rho) + p_H \frac{4\pi}{3} r^4(\rho) = m_N r_0$$

J. Rożynek, J. Phys. G **42**, 045109 (2015).

Bag model for nucleon properties

$$r_0 = 0.4 \ fm$$

Radius remains constant for densities of $\simeq (0.1 - 0.25) \ fm^{-3}$.

Equation of state

The maximum mass of neutron stars:

EoS	LOCV (AV_{18})	LOCV $(AV_{18} + TNI)$	VLOCV (AV_{18})
M_{max}	1.623	2.11	2.19

Z.A. Aghbolaghi and M. Bigdeli, Eur. Phys. J. Plus 134, 430 (2019).
M. Bigdeli and S. Elyasi, Eur. Phys. J. A **51**, 38 (2015).

Hybrid equation of state with first-order phase transition

Constant speed of sound equation of sate (CSS):

$$\varepsilon = \begin{cases} \varepsilon(p) & p < p_{\text{trans}} \\ \varepsilon(p_{\text{trans}}) + \Delta\varepsilon + c_{QM}^{-2}(p - p_{\text{trans}}) & p > p_{\text{trans}} \end{cases}$$
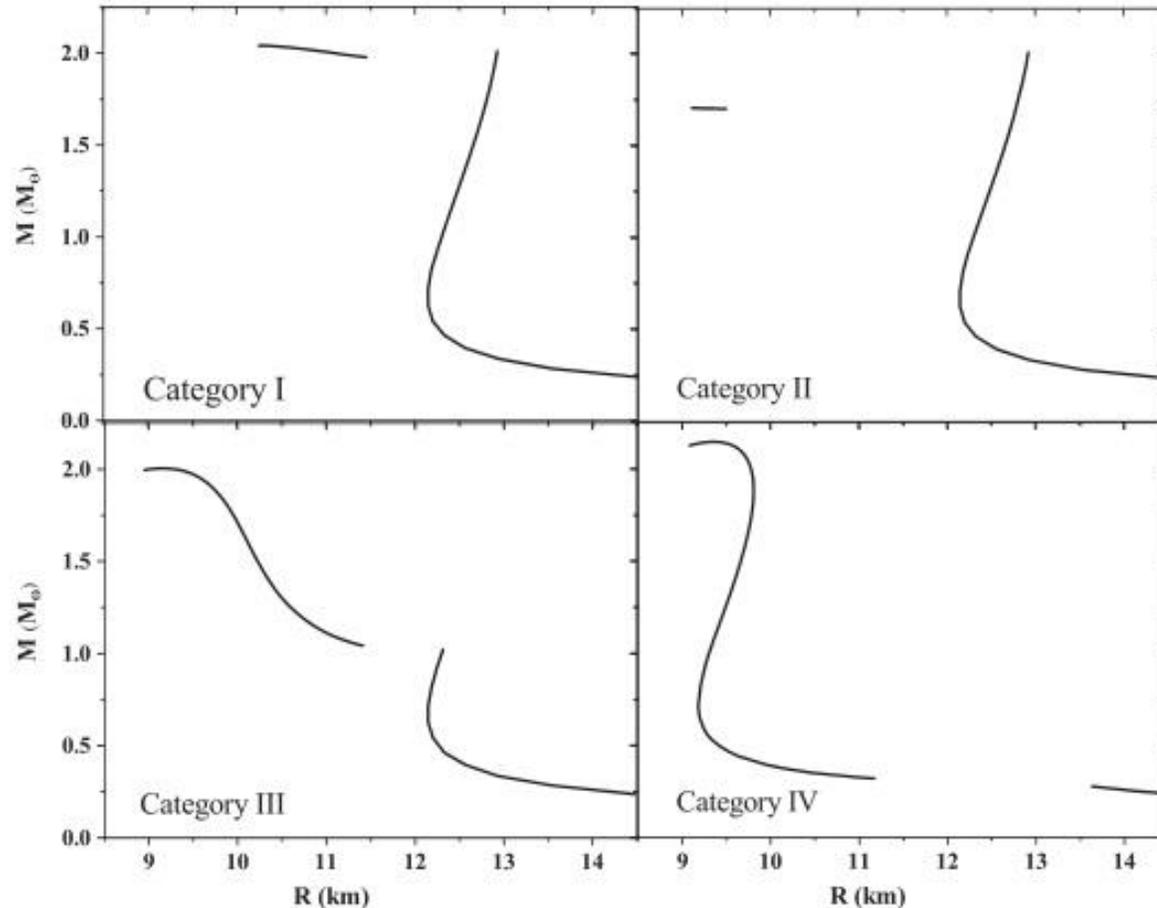
$$c_{QM} = 1$$

Seidov condition: For large discontinuity in energy density at the transition, the star undergoes instability when central pressure equals transition pressure. This will happen whenever the value of $\Delta\varepsilon$ is equal or greater than the so called Seidov limit:

$$\frac{\Delta\varepsilon_{\text{crit}}}{\varepsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\text{trans}}}{\varepsilon_{\text{trans}}}$$

M. G. Alford, G. F. Burgio, S. Han, G. Taranto, and D.Zappala, Phys. Rev. D **92**, 083002 (2015).
Z. F. Seidov, Sov. Astron. **15**, 347 (1971).

Categories of twin stars

Twin stars, **consisting of neutron stars with similar masses but different sizes, originate from the EoS with a strong first-order phase transition.**
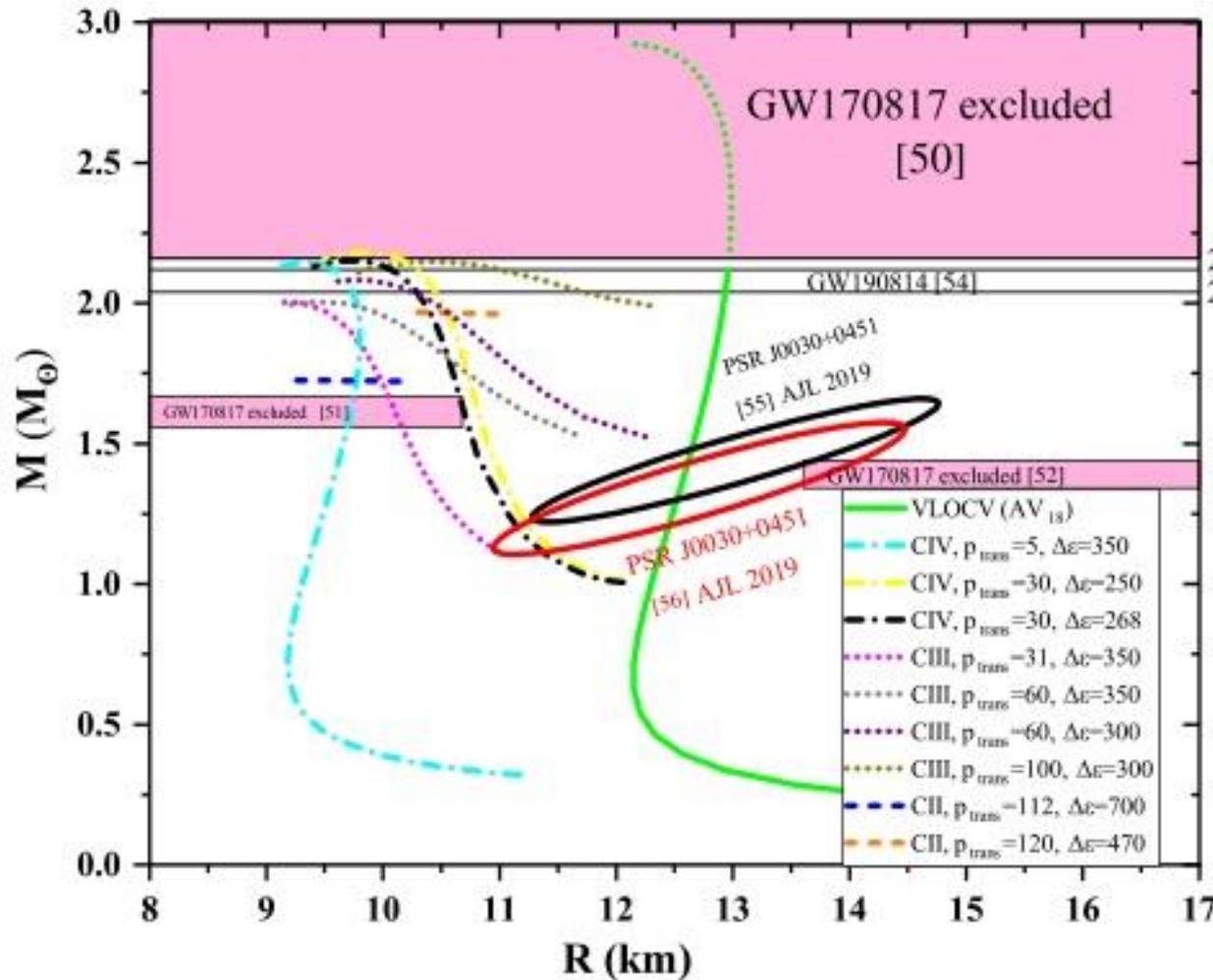

TOV equations:

$$\frac{dP}{dr} = -\frac{\varepsilon m}{r^2}\left(1 + \frac{P}{\varepsilon}\right)\left(1 + \frac{4\pi P r^3}{m}\right)\left(1 - \frac{2m}{r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

J.-E. Christian, A. Zacchi, and J. Schaffner-Bielich,Eur. Phys. J. A 54,28 (2018).

Categories of twin stars

VLOCV (AV_{18})	Low p_{trans}	High p_{trans}	Low $\Delta\varepsilon$	High $\Delta\varepsilon$
CI	108	130	380	430
CII	108	127	420	700
CIII	31	107	250	415
C IV	5	30	250	430
LOCV (AV_{18} + TNI)				
CIII	40	41	250	273
C IV	4	39	250	445

Mass-Radius relation: VLOCV (AV$_{18}$)

$$2.01^{+0.04}_{-0.04} \leq M_{TOV} / M_\odot \leq 2.16^{+0.17}_{-0.15}$$

$$R_{1.6 M_\odot} > 10.68^{+0.15}_{-0.04} \, km$$

$$R_{1.4 M_\odot} \leq 13.6 \, km$$

The desirable EoS models fulfilling all these limits are the ones with
$$p_{trans} \approx 30-100 \, MeV / fm^3 \text{ having}$$
$$\Delta\varepsilon \leq 300 \, MeV / fm^3$$

[50] L. Rezzolla, E.R. Most Elias, and R. Weih Lukas, Astrophys. J. Lett. **852**, L25 (2018).
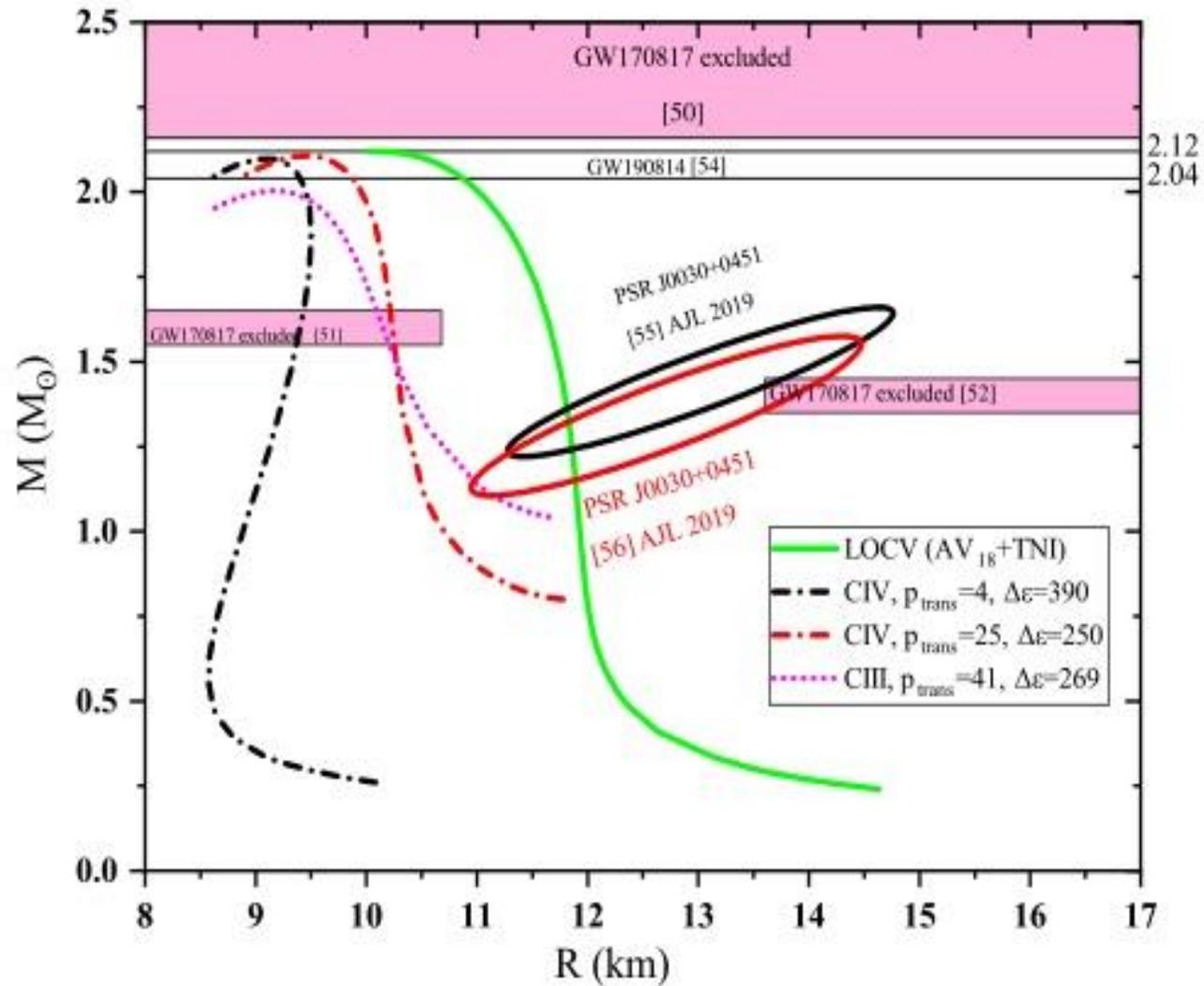[51] A. Bauswein, O. Just, H. T. Janka, and N. Stergioulas, Astrophys. J. Lett. **850**, L34 (2017).
[52] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, Phys. Rev. Lett. **120**, 172703 (2018).
[54] E.R. Most, L. J. Papenfort, L. R. Weih, and L. Rezzolla, Mon. Not. R. Astron. Soc. 499, L82 (2020).
[55] M.C. Miller etal., Astrophys. J. Lett. **887**, L24 (2019).
[56] T.E. Riley et al., Astrophys. J. Lett. **887**, L21 (2019).

Transition density, Central density and maximum mass of twin star branch

VLOCV (AV$_{18}$)	ρ_t (fm^{-3})	M$_{max}$ (M$_\odot$)	ρ_c (fm^{-3})
CIV, $p_{trans} = 5$, $\Delta\varepsilon = 350$	0.21	2.15	0.77
CIV, $p_{trans} = 30$, $\Delta\varepsilon = 250$	0.33	2.18	0.90
CIV, $p_{trans} = 30$, $\Delta\varepsilon = 268$	0.33	2.15	0.92
CIII, $p_{trans} = 31$, $\Delta\varepsilon = 350$	0.33	2.004	0.98
CIII, $p_{trans} = 60$, $\Delta\varepsilon = 350$	0.38	2.004	0.99
CIII, $p_{trans} = 60$, $\Delta\varepsilon = 300$	0.38	2.08	0.96
CIII, $p_{trans} = 100$, $\Delta\varepsilon = 300$	0.42	2.15	0.90
CII, $p_{trans} = 112$, $\Delta\varepsilon = 700$	0.43	1.72	1.03
CII, $p_{trans} = 120$, $\Delta\varepsilon = 470$	0.44	1.97	0.90
LOCV (AV$_{18}$ + TNI)			
CIV, $p_{trans} = 4$, $\Delta\varepsilon = 390$	0.19	2.1	0.77
CIV, $p_{trans} = 25$, $\Delta\varepsilon = 250$	0.36	2.11	0.97
CIII, $p_{trans} = 41$, $\Delta\varepsilon = 269$	0.42	2.003	1.08

A combined analysis of GW170817 and GW190425 results in a range of densities of the neutron star core in between:

$$0.48 - 0.96 \, fm^{-3}$$

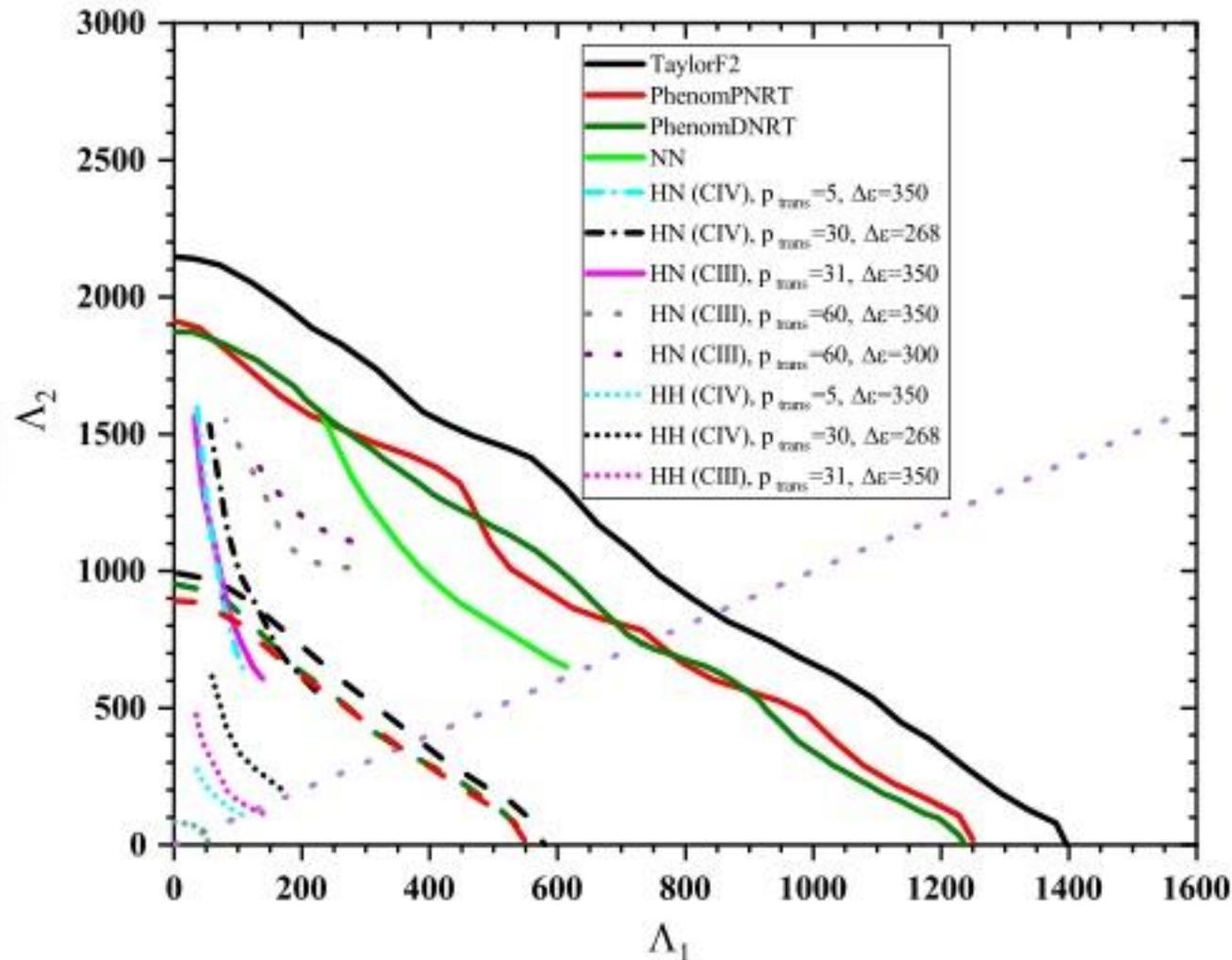B.P. Abbott et al., Astrophys. J. Lett. **892**, L3 (2020).

The compact star central values for our models are compatible with this information.

$$\Lambda = \frac{2}{3} \frac{k_2}{\beta^5} = \frac{\lambda}{m^5}$$
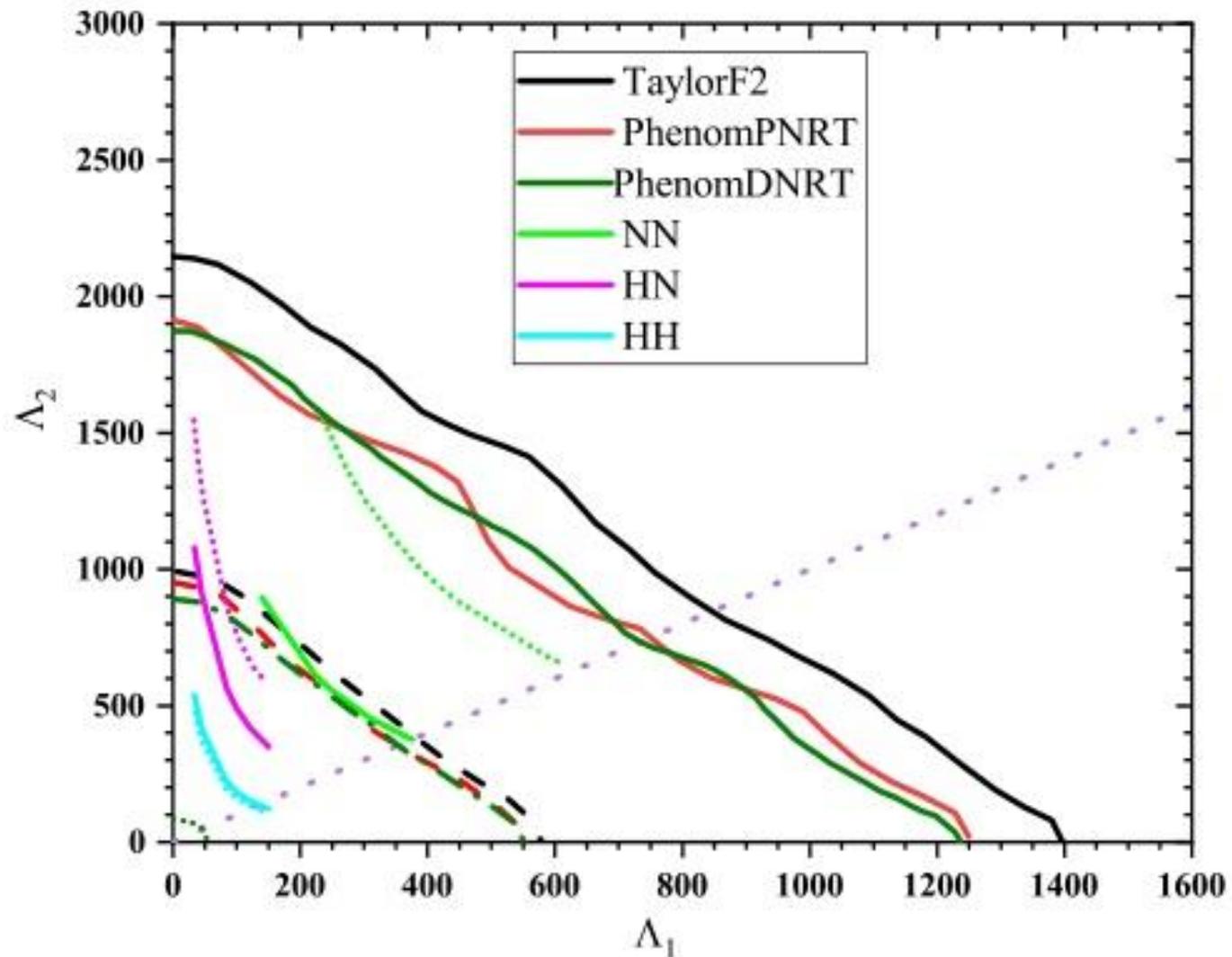
Chirp mass:

$$\mathcal{M} = 1.186 \, M_\odot$$

The combined dimensionless tidal deformability with the chirp mass of 1.44 $M_\odot$ related to the GW190425: $\tilde{\Lambda} \leq 600$

Our result:

$\tilde{\Lambda} : (83 - 138) \; with$

$p_{trans} = 60 \, and \; \Delta\varepsilon = 300 \; MeV \, / \, fm^3$

Legend:
- TaylorF2
- PhenomPNRT
- PhenomDNRT
- NN
- HN (CIV), p_{trans}=5, $\Delta\varepsilon$=350
- HN (CIV), p_{trans}=30, $\Delta\varepsilon$=268
- HN (CIII), p_{trans}=31, $\Delta\varepsilon$=350
- HN (CIII), p_{trans}=60, $\Delta\varepsilon$=350
- HN (CIII), p_{trans}=60, $\Delta\varepsilon$=300
- HH (CIV), p_{trans}=5, $\Delta\varepsilon$=350
- HH (CIV), p_{trans}=30, $\Delta\varepsilon$=268
- HH (CIII), p_{trans}=31, $\Delta\varepsilon$=350

B.P. Abbott et al., Phys. Rev. X **9**, 011001 (2019).
B.P. Abbott et al., Astrophys. J. Lett. 892, L3 (2020).

Conclusion

1. Depending on the parameter values, transition pressure $\approx$ 30–100 MeV/fm^3, and the energy density discontinuity $\Delta\varepsilon \lesssim$ 300 MeV/fm^3, both CIII and CIV of the VLOCV model can satisfy all the observational constraints, however, CIV has more limitations.

2. For both models, VLOCV (AV$_{18}$) and LOCV (AV$_{18}$+TNI), the merger of neutron stars (NN) is possible in the light of the GW170817 event, while neither of the other two combinations (HN and HH) is allowed for the LOCV (AV18+TNI) model considering the Bauswein constraint in the mass-radius diagrams.

3. The HN and HH combinations of the VLOCV EoS with low values of P_{trans} (having $\Delta\varepsilon \gtrsim$ 300 MeV/fm^3) for both categories of III and IV are also ruled out by this constraint.

4. In the case of the LOCV (AV18+TNI) approach, neither CIII nor CIV can meet all the mentioned limits.