Calculation of the kinetic coefficients of arbitrary degenerate electrons in magnetized dense matter

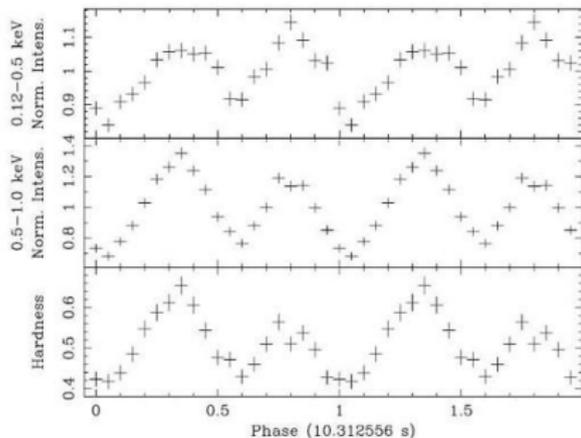M.V. Glushikhina, G. S. Bisnovatyi-Kogan

Introduction

Fig. 3. Pulse profile of RBS1223 in the 0.12–0.5 keV (soft) and 0.5–1.0 keV (hard) energy bands, together with the ratio hard/soft, obtained from the EPIC-pn data of the Jan. 2003 observation.

X-ray observations of thermal emission show periodic variabilities in single neutron stars, indicating to the anisotropic temperature distribution. This results in that the geometry of the magnetic field in the interior of the NS leaves an observable imprint at the surface, potentially allows us to study the internal structure of the magnetic field through modelling of the spectra and pulse profile of thermally emitting NSs.

M.V. Glushikhina, G. S. Bisnovatyi-Kogan

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Introduction

- Among all neutron stars with thermal radiation, only X-ray single neutron stars have a surface blackbody spectrum without the addition of radiation from an accretion disk, a surrounding nebula or supernova remnant. Observations of thermal radiation from the surface of a neutron star are extremely important, they can provide information about the mass and radius of the star, which is necessary to determine the equation of state of its matter.

- To simulate thermal radiation from the surface of a neutron star, it is necessary to know the transport properties in magnetized degenerate matter.

- Kinetic coefficients obtained phenomenologically with the mean free path theory are used, for temperature distribution simulation on the surface of the star.

Introduction

- Isotherm modelling in isolated magnetized neutron star was performed by many authors (Geppert, Kuker, Page 2004, Perez-Azorin, Miralles, Pons 2005 and others)

- In most works considering the diffusion and heat conductivity in the astrophysical objects, in the neutron stars in particular, the influence of the magnetic field on the heat flux and electroconductivity was taken into account phenomenologically using the coefficient: $\dfrac{\sigma_\perp}{\sigma_\parallel} = \dfrac{1}{1 + (\omega\tau)^2}$ Flowers, Itoh (1976); $\dfrac{\lambda_\perp}{\lambda_\parallel} = \dfrac{1}{1 + (\omega\tau)^2}$, Yakovlev and Urpin (1980). Here ω is electron cyclotron frequency, τ is the time between collisions.

Introduction

- The purpose of this work is to find the kinetic coefficients of arbitrarily degenerate electrons in a magnetized dense matter. The following tasks were performed:

- Solving the Boltzmann equation by the Chapman-Enskog method of succecive approximations using an expansion in three polynomials generalizing the Sonin (Laguerre) polynomials for arbitrarily degenerate electrons in a magnetic field. Calculations of the coefficients of the tensors of thermal conductivity, thermal diffusion, diffusion, and diffusion thermal effect for the approximation of nondegenerate electrons are made explicitly.

$$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial r_i} - \left[\frac{e}{m_e} (E_i + \frac{1}{c} \epsilon_{ikl} c_k B_l) - F_i \right] \frac{\partial f}{\partial c_i} + J = 0, \tag{1}$$

- Solving the Boltzmann equation in the Lorentz approximation for strongly degenerate electrons in a magnetic field and obtaining explicitly the components of the four tensors.

M.V. Glushikhina, G. S. Bisnovatyi-Kogan

Chapman-Enskog method

- This method is used here for conditions, when distribution functions are close to their values in thermodynamic equilibrium,and deviations are considered in a linear approximation. The zeroth approximation to the electron distribution function is a Fermi-Dirac distribution.

- For the arbitrary degeneracy case, we need to seek for a solution in the form of an expansion in polynomials Q_n, similar to the Sonin polynomials with weight: $f_0(1 - f_0)x^{3/2}$:

- $Q_0(x) = 1, \quad Q_1(x) = \dfrac{5G_{5/2}}{2G_{3/2}} - x,$

  $Q_2(x) = \dfrac{35}{8}\dfrac{G_{7/2}}{G_{3/2}} - \dfrac{7}{2}\dfrac{G_{7/2}}{G_{5/2}}x + \dfrac{1}{2}x^2$

- $G_n = \dfrac{1}{\Gamma(n)}\int_0^\infty \dfrac{x^{n-1}dx}{1+exp(x-x_0)}, \ \ x_0 = \dfrac{\mu}{kT}$, here G_n is Fermi integral and μ is chemical potential.

Lorentz approximation

- The Lorentz approximation is applicable to solving the kinetic equation when the mass of light particles is much less than the mass of heavy ones, and electron-electron collisions can be neglected. In this approximation, the Boltzmann equation has an exact solution.

- The Lorentz approximation is useful for testing an polynomial solution, since it makes it possible to trace the convergence of an polynomial solution to an exact one with increasing degree of polynomials.

Lorentz approximation

- General expressions for the heat flux q_i and average directional (diffusion) electron velocity v_i by

$$q_i = -\lambda_{ij}\frac{\partial T}{\partial x_j} - n_e\frac{G_{5/2}}{G_{3/2}}\nu_{ij}d_j = q_i^{(A)} + q_i^{(D)}, \qquad (2)$$
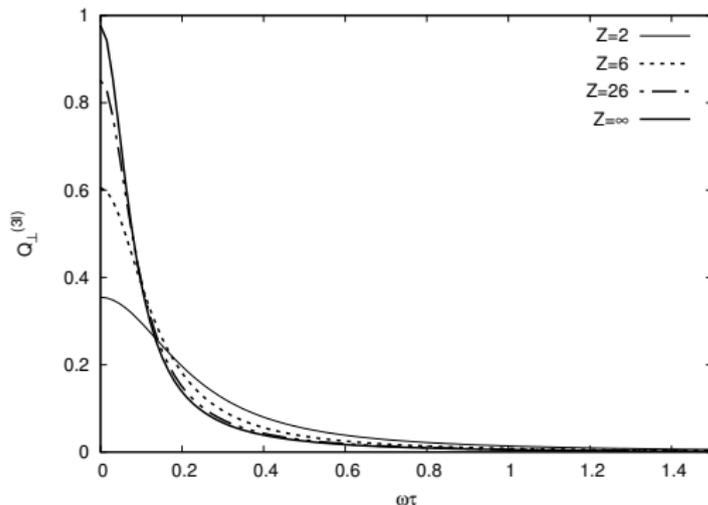
$$\langle v_i \rangle = -\mu_{ij}\frac{\partial T}{\partial x_j} - n_e\frac{G_{5/2}}{G_{3/2}}\eta_{ij}d_j = \langle v_i^{(A)} \rangle + \langle v_i^{(D)} \rangle, \qquad (3)$$

where λ_{ij} and ν_{ij} — are the thermal conductivity and diffusion thermoeffect tensors, respectively, while μ_{ij} and η_{ij} — are the thermal diffusion and diffusion tensors, respectively. The indices (A) and (D) correspond to the heat flux and diffusion velocity of electrons determined by the temperature gradient $\partial T/\partial x_j$, and diffusion vector d_j.

Lorentz approximation

- We consider only the heat flux connected with the temperature gradient at zero value of the diffusive vector d_i.

- In laboratory conditions, when the electrical conductivity is small and electrical current is damped rapidly, another limiting case is considered, where $j_i \sim v_i = 0$. When the thermal conductivity and diffusion are calculated in the same procedure, both heat and diffusion fluxes are calculated without any restrictions on the diffusion vector or diffusion velocity. Such consideration will be performed elsewhere.

Comparison of Lorentz approximation solution and solution obtained by polynomial expansion; non-degenerate case

The plots of the value $Q_\perp^{(3l)}$ as a function of $\omega\tau$ in 3 polynomial approximation are presented for nondegenerate plasma of helium (Z=2), carbon (Z=6), iron (Z=26), for comparison with the Lorentz plasma, formally corresponding to $Z = \infty$. The deviations from the Lorentz plasma are connected with the input of electron-electron collisions.

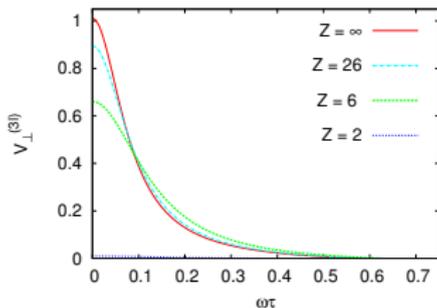$$Q_\perp^{(3l)} = \frac{\lambda_\perp^{(3)}}{\lambda_\parallel^{lor}}$$
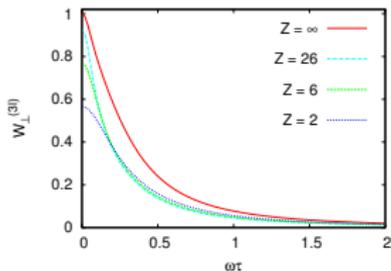
Рис.: Diffusion

Рис.: Diffusional thermal effect

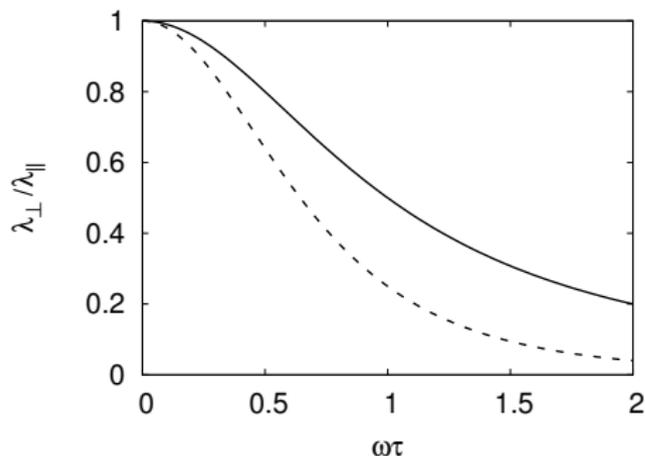Рис.: Thermaldiffusion

M.V. Glushikhina, G. S. Bisnovatyi-Kogan

Partially degenerate electrons

- To examine the effect of degeneracy on the convergence of the polynomial approximation to the exact value, we compare the coefficients calculated by the method of successive approximations with the coefficients obtained as an exact solution in the Lorentz approximation. In a partially degenerate plasma, it is enough to simply calculate collision integral for electron-ion at $\mu = 0$ - chemical potential with the degeneracy level $DL = \epsilon_{fe}/kT = 1.011$

- When compared with the exact solutions for heat conductivity and thermaldiffusion, obtained by the Lorentz method, the 3-polynomial solution for thermal conduction differs from the exact solution by about 13%, for thermal diffusion - by 7%.

- For non-degenerate electrons in the 3-polynomial approximation, the coeff. thermal conductivity differs by 2.2%, and thermal diffusion by 1.3%.

- For partial degeneracy, it is found that the 3-polynomial solution for diffusion differs from the exact one by 2.6%. For diffusion thermal effect: 3-polynomial by about 8%.

- For the nondegenerate case, the 3-polynomial approximation differs by 0.14% from the exact one for diffusion, and by 0.38% for the diffusion thermal effect.

- The accuracy of approximation by a series of orthogonal functions decreases with an increase in the degree of degeneracy.

M.V. Glushikhina, G. S. Bisnovatyi-Kogan

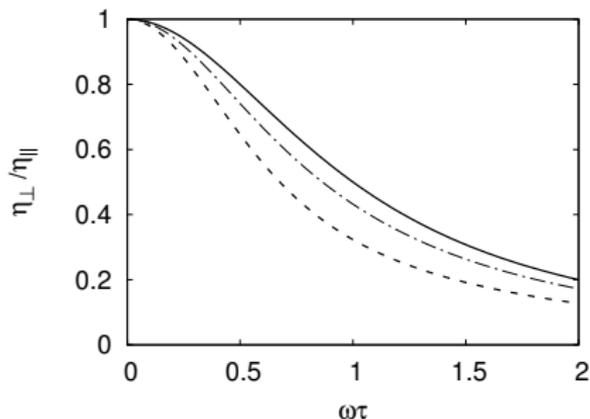## Solution method for strong degeneracy

- For strongly degenerate case, collision integral for neutron-neutron collisions had been found analytically in (Tomonaga, 1938). They were calculated for strongly degenerate neutrons in a nuclear matter, and for the neutrons in the inner crust of a neutron star, with many free neutrons. It was found in the last case that in presence of non-degenerate heavy nuclei, and strongly degenerate neutron, the input of collisions between them in the heat transfer and diffusion coefficients is negligibly small, in comparison with neutron-nuclei collisions.

- The same situation we have for the strongly degenerate electrons. Therefore for strongly degenerate electrons the Lorentz approximation, with account of collisions between light and heavy particles only, is asymptotically exact.
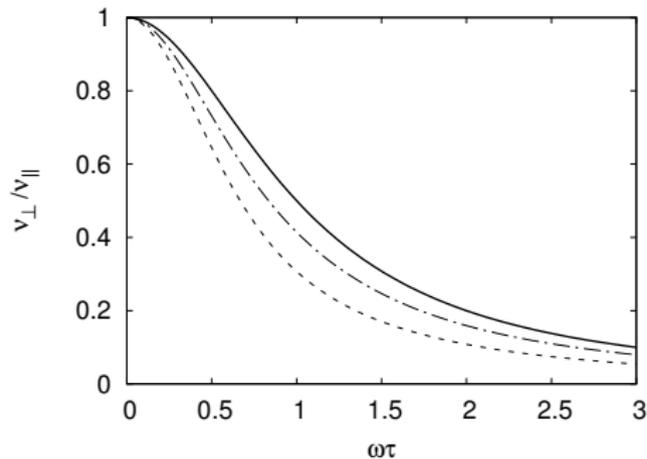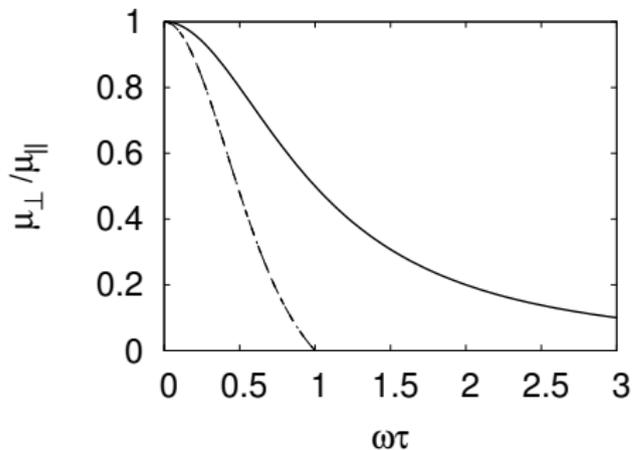
Heat conductivity

Plots of the ratio $\lambda_\perp/\lambda_\parallel$ as function of $\omega\tau$ are presented for comparison of the thermal conductivity coefficients obtained by the phenomenological method (solid curve) with asymptotic thermal conductivity coefficients obtained from the solution of the Boltzmann equation (dashed curve).

Diffusion

The ratio of $\eta_\perp/\eta_\parallel$ depending on $\omega\tau$. For diffusion, the curves obtained by the phenomenological method and from the solution of the Boltzmann equation coincide at strong degeneracy. If the exact solution takes into account the terms of the order of $1/x_0^2$ smallness, the solution obtained from the Boltzmann equation will be different from the phenomenological one. The solid curve corresponds to the phenomenological solution and to the asymptotic solution obtained from the Boltzmann equation; the dot-dashed curve corresponds to the solution taking into account small terms at $kT = 0.(11)E_f, (x_0 = \mu/kT = 9)$, the dashed curve at $kT = 0.2E_f, (x_0 = \mu/kT = 5)$.

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Coefficients of thermaldiffusion and diffusional thermal effect

M.V. Glushikhina, G. S. Bisnovatyi-Kogan

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Conclusion

- The solution of the Boltzmann equation is obtained in the Lorentz approximation for a plasma in a magnetic field with strongly degenerate nonrelativistic electrons and nondegenerate nuclei. This approximation, which neglects electron-electron collisions, is asymptotically accurate for a plasma with highly degenerate electrons. The components of four tensors of kinetic coefficients in a non-quantizing magnetic field are calculated in the Lorentz approximation.

- With the Chapman – Enskog method, a solution of the Boltzmann equation for plasma in a magnetic field with arbitrary degeneracy of electrons and nondegenerate nuclei is obtained using the expansion in the first three generalized Sonin polynomials. The influence of degeneracy on the convergence of a 3-polynomial solution to an exact one is investigated. It is shown, with the Lorentz approximation as an example, that the accuracy of approximation decreases with an increase in the degree of degeneracy.

- It is shown that the kinetic coefficients obtained from the solution of the Boltzmann equation more accurately take into account the influence of the magnetic field on the conductivity of arbitrarily degenerate electrons than the approximate coefficients obtained phenomenologically using the free path theory. The influence of the magnetic field turns out to be stronger and more complex.
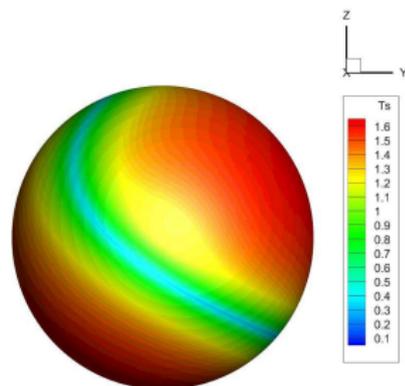
Illustrations

Рис.: $Tcore = 2*10^8 K$,
$B_q = 0.5*10^{13}$ G, $B_d = 10^{13}$ G,
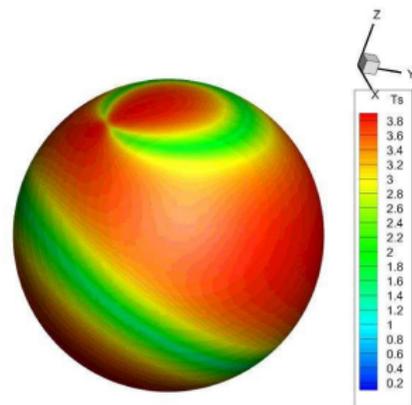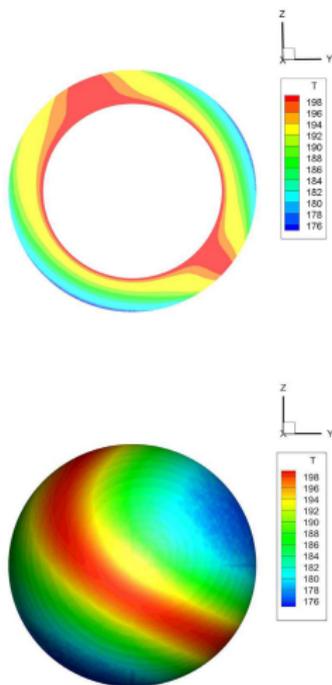angle between axis$= pi/4$,
$Tmax/Tmin \sim 5.3$.

Рис.: $Tcore = 1*10^9 K$,
$B_q = 1*10^{13}$ G, $B_d = 10^{13}$ G, anlge
between axis $= pi/6$,
$Tmax/Tmin \sim 2.5$.

Illustrations

Temperature distribution (in units of 10^6 K) in the NS crust for quadrupolar and dipolar magnetic fields with polar inductions $B_{pq} = 5 \times 10^{12}$ G and $B_{pd} = 10^{13}$ G correspondingly. Magnetic axes are rotated from each other to the angle $\theta = \pi/4$, hereinafter a quadrupolar axis if fixed along Z-axis, and dipolar component is rotated on plots. The core temperature is $T_{core} = 2 \times 10^8$. Upper picture – cross-section in Z–Y plane (the thickness of the crust is four times stretched for better visualization), lower one – the NS crust surface.
link: I. A. Kondratyev et al. MNRAS 2020

Thank you for attention!

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