High-density nuclear symmetry energy extracted from observations of neutron stars and gravitational waves

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NSF

Office of Science
U.S. DEPARTMENT OF ENERGY

Outline

Introduction: EOS of dense neutron-rich nuclear matter

Goal: to determine the high-density behavior of nuclear symmetry energy

Approach: Solving the neutron star inverse-structure problem: given an observable, find the necessary EOS(s)

1.Exact Inversion in 3D: Inverting exactly the TOV equation in 3D EOS parameter space

2.Statistical Inversion in Multi-D: Bayesian inference of the Probability Distribution Functions (PDFs) of ALL EOS parameters

Conclusion: E_{sym} below twice the saturation density is reasonably well constrained by the available radius data of canonical neutron stars from GW170817 and X-rays

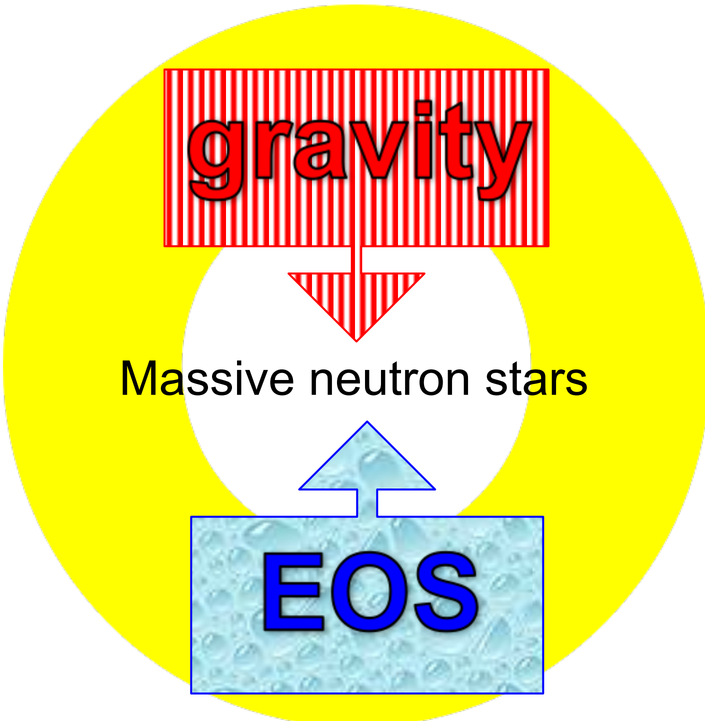
Connecting Quarks with the Cosmos:
Eleven Science Questions for the New Century
National Research Council (2003)

- What is the dark matter?
- What is the nature of the dark energy?
- How did the universe begin?
- What is gravity?
- Are there additional spacetime dimensions?
- What are the masses of the neutrinos, and how have they shaped the evolution of the universe?
- How do cosmic accelerators work and what are they accelerating?
- Are protons unstable?
- Are there new states of matter at exceedingly high density and temperature?
- How were the elements from iron to uranium made?
- Is a new theory of matter and light needed at the highest energies?

Compact stars are natural laboratories to test some of these

Gravity-EOS Degeneracy in massive neutron stars

Strong-field gravity: GR or Modified Gravity?



GR+[Modified Gravity]

Action $S=S_{gravity}+S_{matter}$

Matter+[Dark Matter]+[Dark Energy]

Contents and stiffness of the EOS of super-dense matter

At high-densities, cold neutron-rich nucleonic matter, the most uncertain part of the EOS is the nuclear symmetry energy besides possible phase transitions

W.A. Zajc, Nucl. Phys. A 967 (2017) 265

The QCD phase diagram

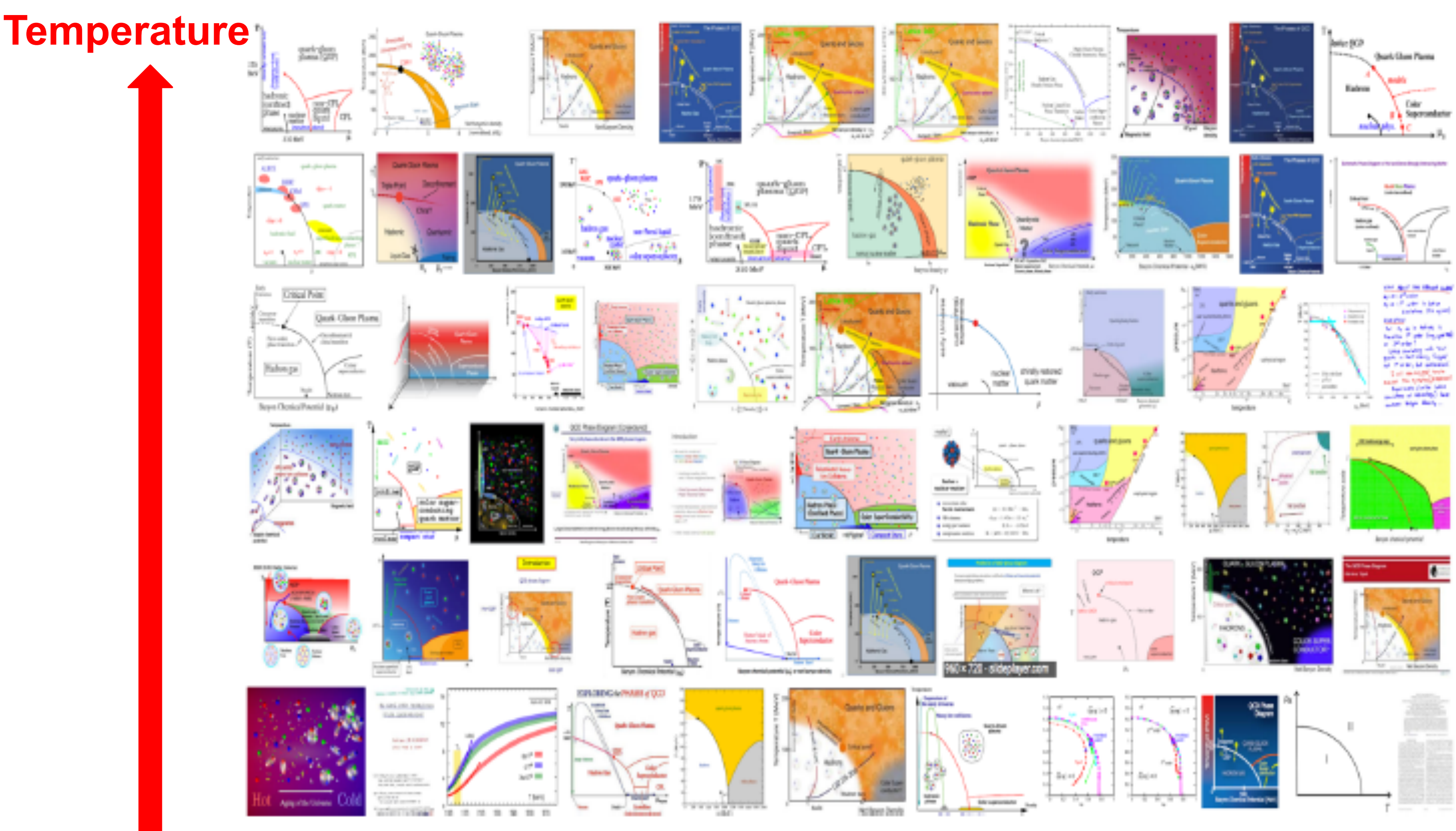


Fig. 1: A selection of representations of the QCD phase diagram in the (μ_B, T) plane.

Empirical parabolic law of the EOS of cold, neutron-rich nucleonic matter

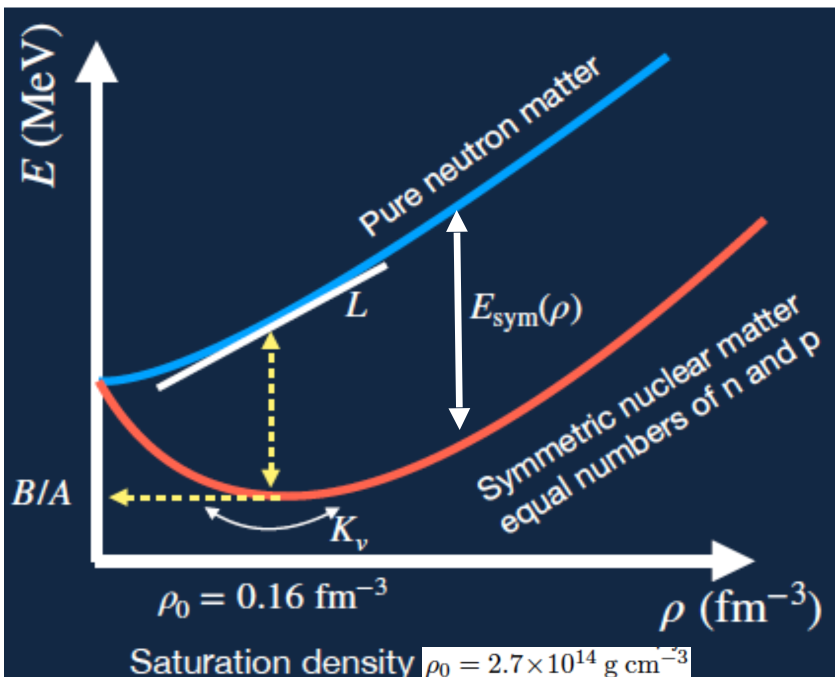
symmetry energy

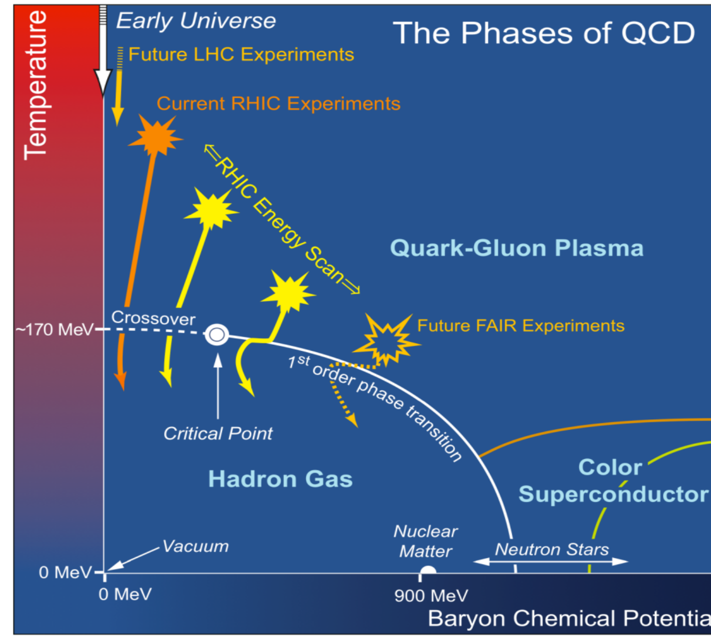
Isospin asymmetry δ

$$E(\rho_n,\rho_p) = E_0(\rho_n=\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy in asymmetric nucleonic matter





density

New opportunities

Isospin asymmetry

$\delta = (\rho_n - \rho_p)/\rho$

Single-nucleon (Lane) potential in isospin-asymmetric matter

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\cdot\delta + U_{sym2}(k,\rho)\cdot\delta^2 + o(\delta^3)$$

Isovector

According to the Hugenholtz-Van Hove (HVH) theorem:

J. Dabrowski and P. Haensel, PLB 42, (1972) 163.
S. Fritsch, N. Kaiser and W. Weise, NPA. A750, 259 (2005).
C. Xu, B.A. Li, L.W. Chen, Phys. Rev. C 82 (2010) 054607.

$$E_{sym}(\rho) = \frac{1}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} + \frac{1}{2}U_{sym,1}(\rho,k_F),$$

Potential

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k^2}{2m_0^*}|_{k_F} - \frac{1}{6}\left(\frac{\hbar^2 k^3}{m_0^{*2}}\frac{\partial m_0^*}{\partial k}\right)|_{k_F} + \frac{3}{2}U_{sym,1}(\rho,k_F) + \frac{\partial U_{sym,1}}{\partial k}|_{k_F}\cdot k_F + 3U_{sym,2}(\rho,k_F),$$

Kinetic

Nucleon effective mass in isospin symmetric matter $m_0^*(\rho,k) = \dfrac{m}{1+\frac{m}{\hbar^2 k}\frac{\partial U_0(\rho,k)}{\partial k}},$

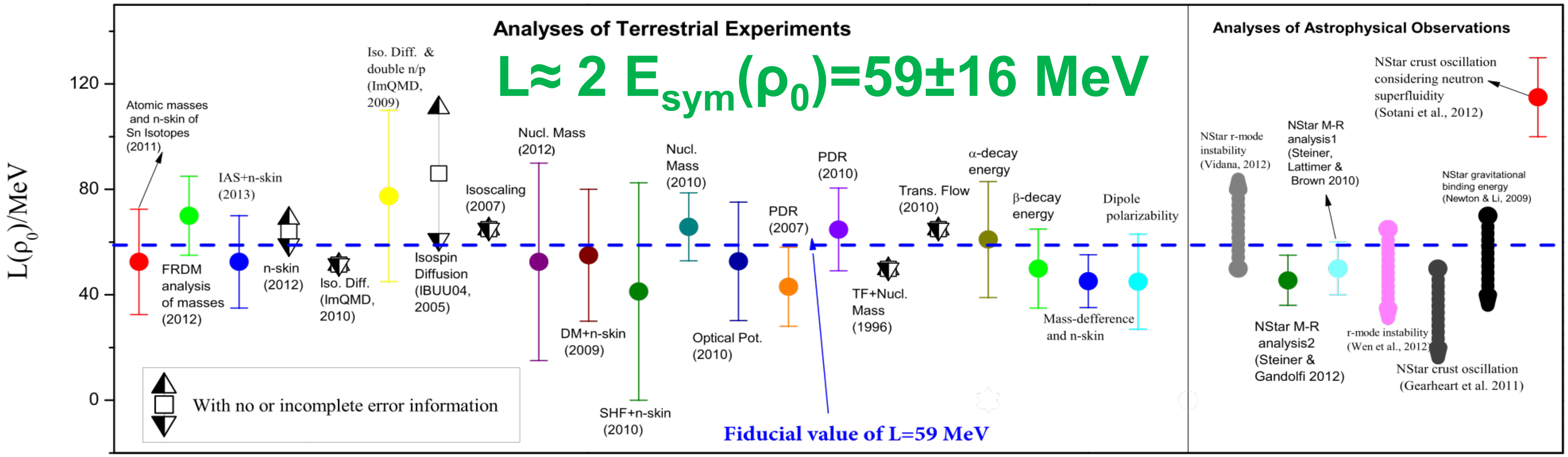
Neutron-proton effective mass splitting in neutron-rich matter

$$m^*_{n-p} \approx 2\delta\frac{m}{\hbar^2 k_F}\left[-\frac{dU_{sym,1}}{dk} - \frac{k_F}{3}\frac{d^2U_0}{dk^2} + \frac{1}{3}\frac{dU_0}{dk}\right]_{k_F}\left(\frac{m_0^*}{m}\right)^2$$

$$\approx 2\delta\left(\frac{M_s^*}{M}\right)^2\left[\frac{M}{M_v^*} - \frac{M}{M_s^*}\right]$$

B.A. Li et al., Prog. in Part. and Nucl. Phys. 99 (2018) 29.

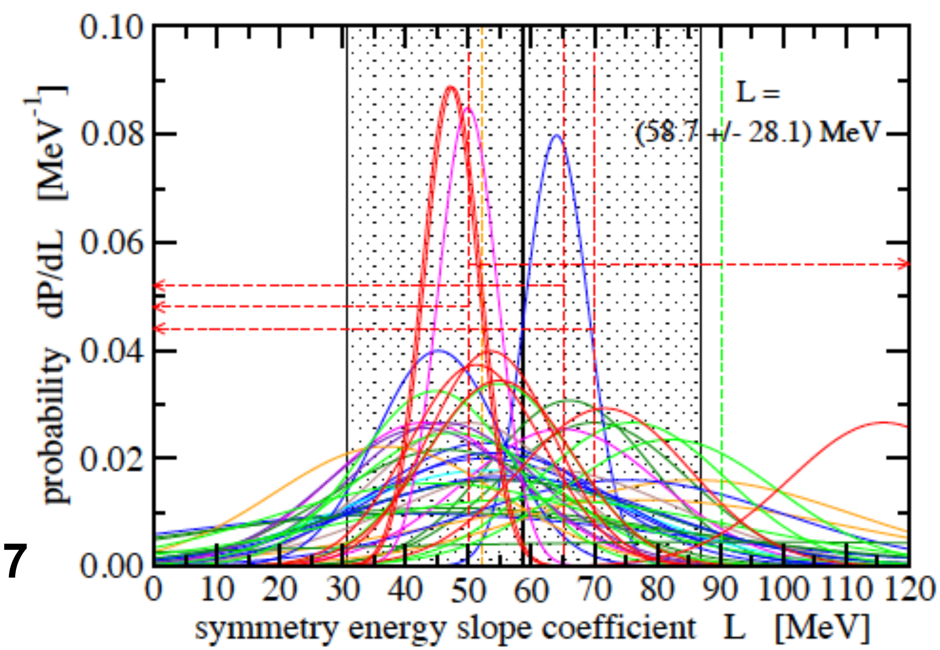
Constraints on L as of 2013 based on 29 analyses of data



Bao-An Li and Xiao Han, Phys. Lett. B727 (2013) 276

L=58.7±28.1 MeV

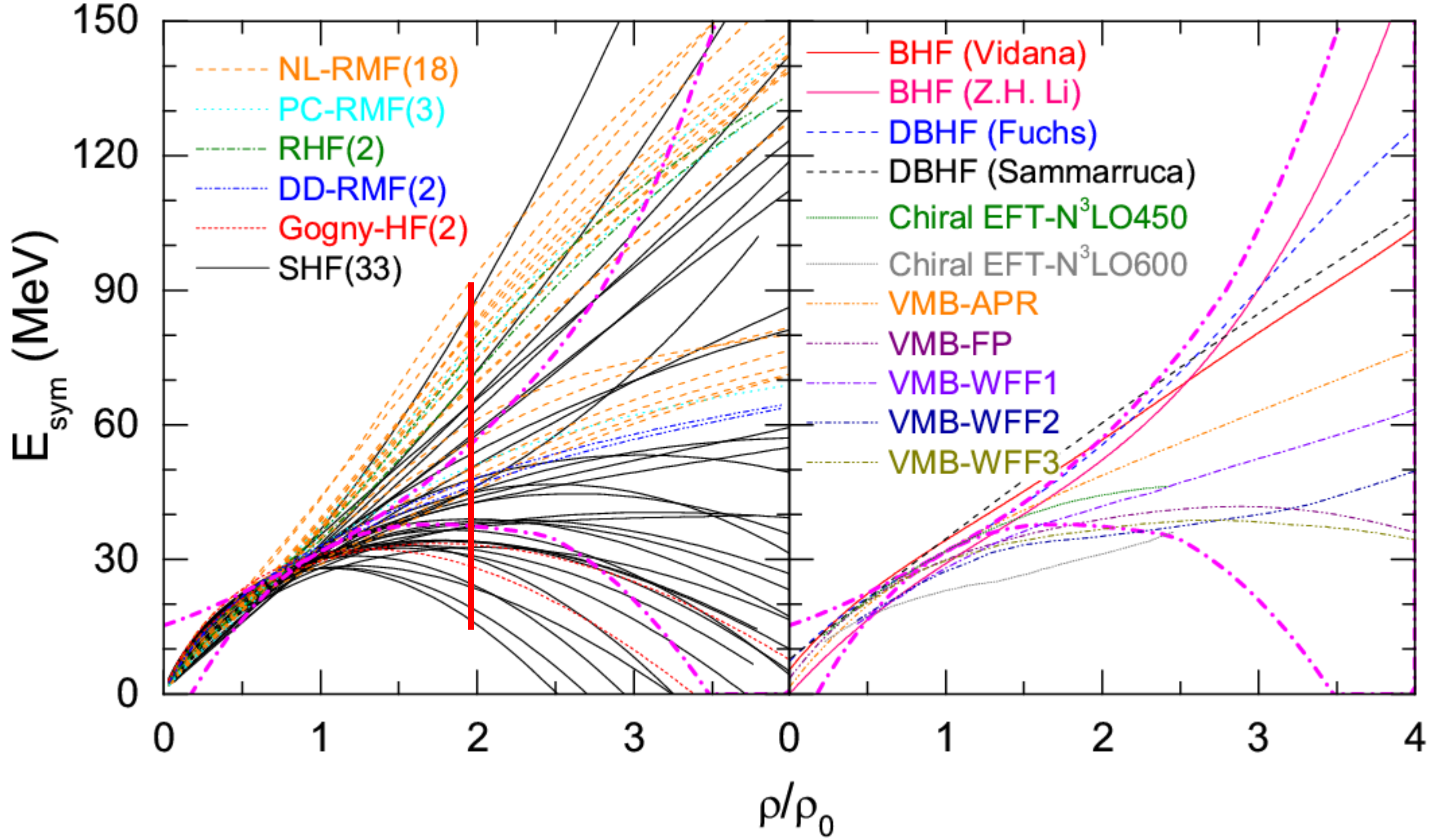
Fiducial value as of 2016 from surveying 53 analyses



M. Oertel, M. Hempel, T. Klähn, S. Typel

Review of Modern Physics 89 (2017) 015007

➢ Predicted $E_{sym}(2\rho_0)$ scatters between approximately 15 to 100 MeV



N.B. Zhang and B.A. Li, EPJA 55, 39 (2019)

➢ $E_{sym}(2\rho_0)$ is most relevant for determining the radii of canonical neutron stars

What are the fundamental physics behind the symmetry energy?

$$U_{n/p}(k,\rho,\delta) = U_0(k,\rho) \pm U_{sym1}(k,\rho)\cdot\delta + U_{sym2}(k,\rho)\cdot\delta^2 + o(\delta^3)$$

- **Isospin dependence of strong interactions and correlations**

Tensor force due to pion and ***ρ*** meson exchange MAINLY in the T=0 channel

$V_{T0} = V'_{np}$ (n-p pair in the T=0 state)

$V_{T1} = V_{nn} = V_{pp} = V_{np}$ (charge independence in the T=1 state)

$V_{np}(T0) \neq V_{np}(T1)$

In a simple interacting Fermi gas model:

Isospin-dependent correlation function

$$U_{sym}(k_F,\rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij})f^{T1}(r_{ij}) - V_{T0}(r_{ij})f^{T0}(r_{ij})]d^3 r_{ij}$$

Isospin-dependent effective 2-body interaction

M.A. Preston and R.K. Bhaduri, Structure of the Nucleus, 1975

Major issues relevant to high-density E_{sym}, heavy-ion reactions and neutron stars

- **Momentum dependence of the symmetry potential due to the finite-range of isovector int.**
- **Short-range correlations due to the tensor force in the isosinglet n-p channel**
- **Spin-isospin dependence of the 3-body force**
- **Isovector interactions of △(1232) resonances and their spectroscopy (mass and width)**
- **Possible sign inversion of the symmetry potential at high momenta/density**

Tensor force induced (1) high-momentum tail in nucleon momentum distribution and (2) isospin dependence of SRC

Theory of Nuclear matter
H.A. Bethe
Ann. Rev. Nucl. Part. Sci., 21, 93-244 (1971)

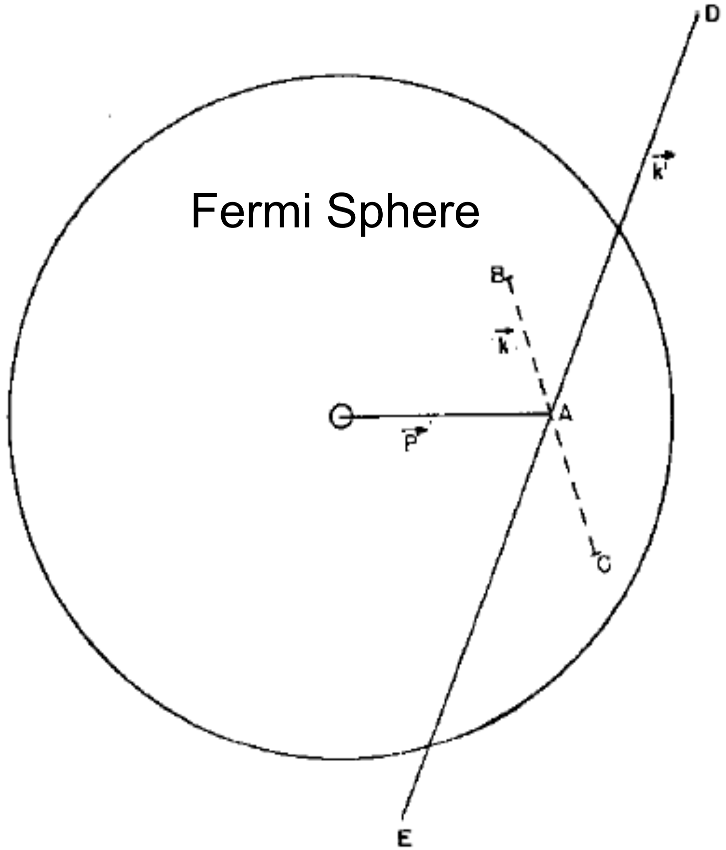
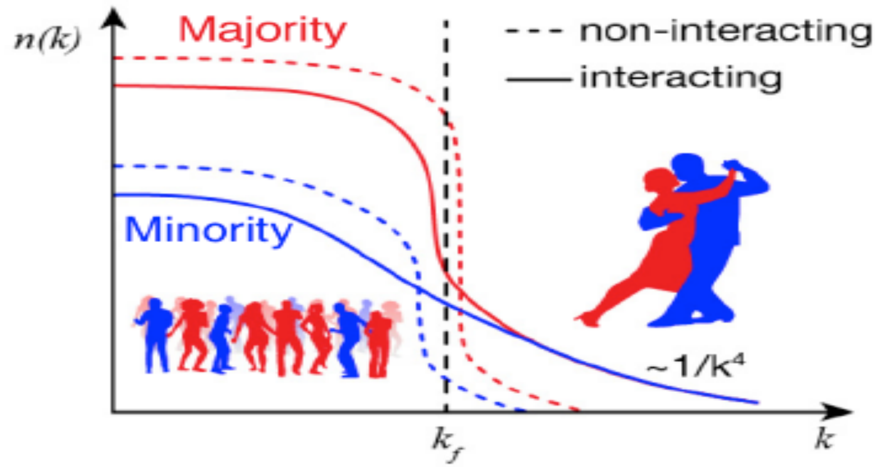
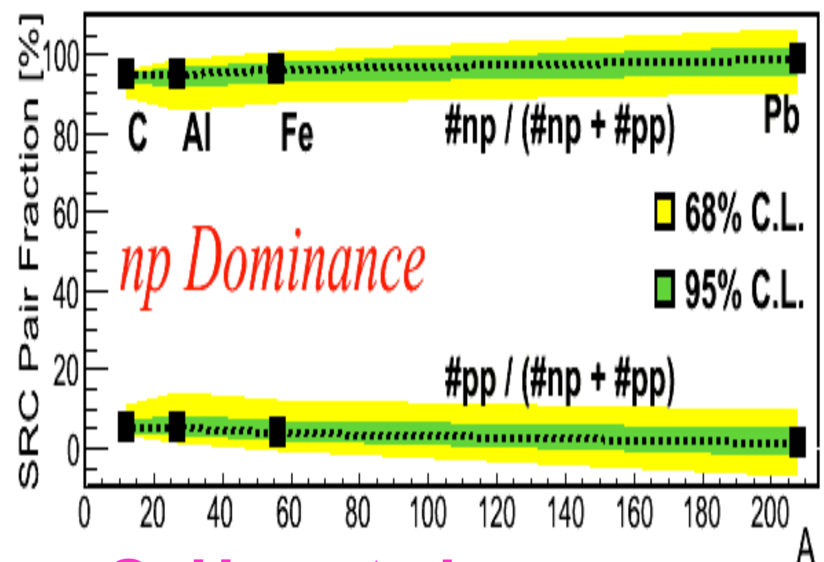


FIGURE 10. Two nucleons are initially in states B and C, having average momentum $\boldsymbol{P}$ and relative momentum $\boldsymbol{k}$. When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum $\boldsymbol{k}'$. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in DE.

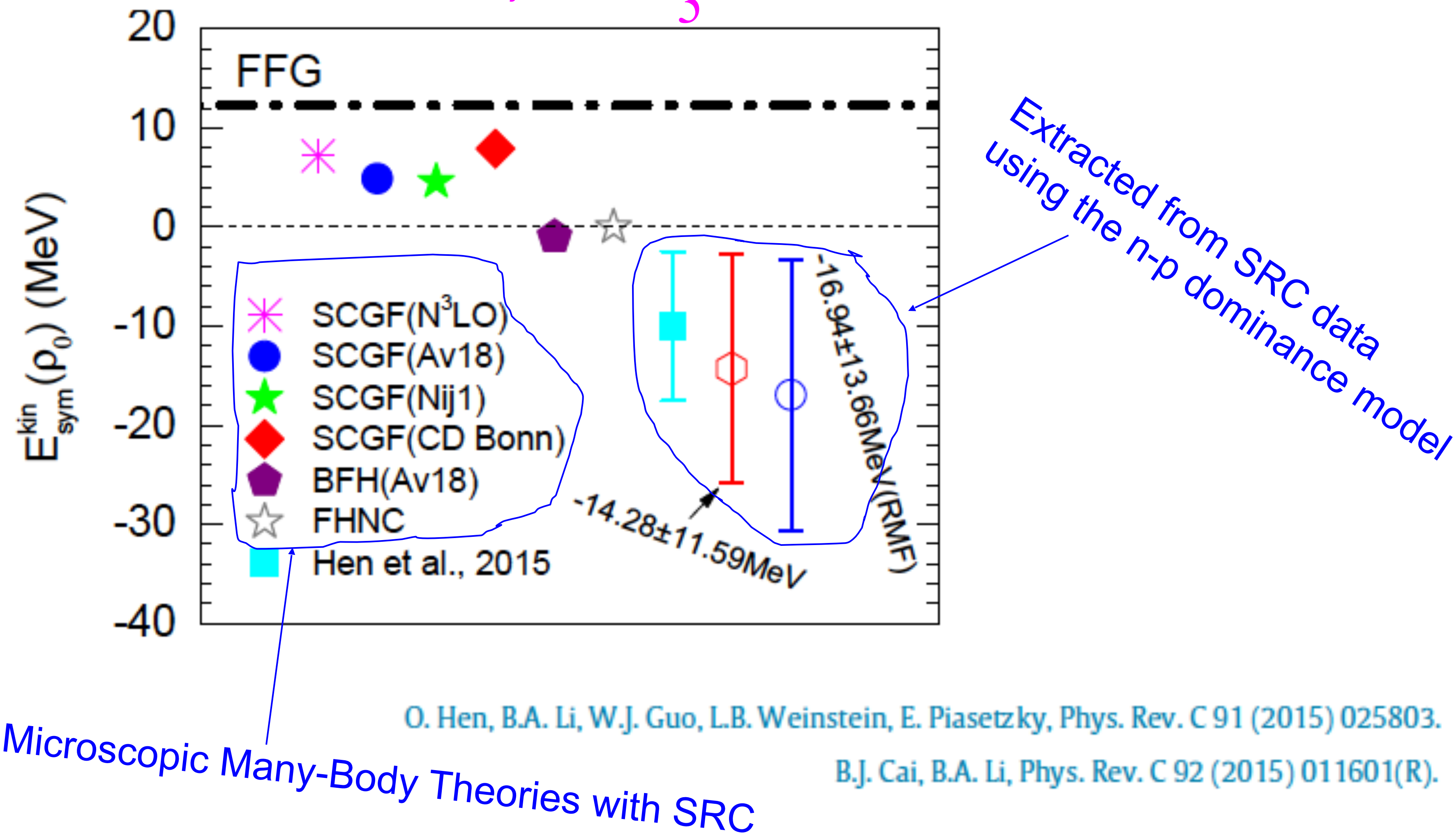




O. Hen et al.,
Science 346, 614 (2014)

Reduced Kinetic symmetry energy of quasi-nucleons due to the isospin dependence of SRC

Free-Fermi Gas (FFG): $E_{sym}^{kin}(\rho) = \frac{1}{3} E_F(\rho_0)(\rho/\rho_0)^{2/3} \approx 12.5 \text{MeV at } \rho_0$



O. Hen, B.A. Li, W.J. Guo, L.B. Weinstein, E. Piasetzky, Phys. Rev. C 91 (2015) 025803.
B.J. Cai, B.A. Li, Phys. Rev. C 92 (2015) 011601(R).

EOS of dense neutron-rich matter is a major scientific motivation of
(1) High-energy rare isotope beam facilities around the world
(2) Various x-ray satellites
(3) Various gravitational wave detectors

Among the promising observables of high-density symmetry energy:

- **π^-/π^+ and n/p spectrum ratio, neutron-proton differential flow and correlation function in heavy-ion collisions at intermediate energies**
- **Radii of neutron stars**
- **Neutrino flux of supernova explosions**
- **Tidal polarizability in neutron star mergers, strain amplitude of gravitational waves from deformed pulsars, frequency and damping time of neutron star oscillations**

EPJA, Vol. 50, No. 2 (2014)

How does the symmetry energy affect NS observables?

(1) For npe matter at beta equilibrium, its proton fraction is determined by the $E_{sym}(\rho)$:

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$

(2) The pressure in the npe matter at beta equilibrium:

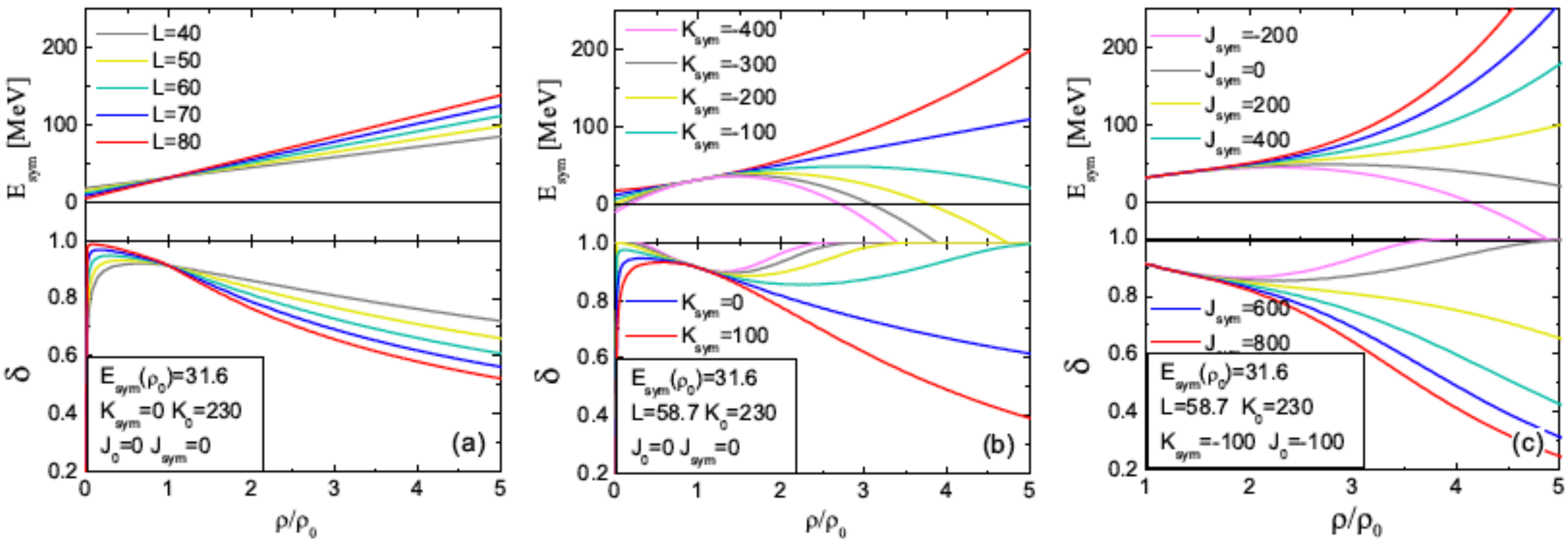
δ=1-2x

$$P(\rho,\delta) = \rho^2[\frac{dE_0(\rho)}{d\rho} + \frac{dE_{sym}(\rho)}{d\rho}\delta^2] + \frac{1}{2}\delta(1-\delta)\rho E_{sym}(\rho)$$

(3) The crust-core transition density and pressure is determined by:

$$K_\mu = \rho^2\frac{d^2E_0}{d\rho^2} + 2\rho\frac{dE_0}{d\rho} + \delta^2\left[\rho^2\frac{d^2E_{sym}}{d\rho^2} + 2\rho\frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1}(\rho\frac{dE_{sym}}{d\rho})^2\right]$$

How neutron rich the neutron stars can be?



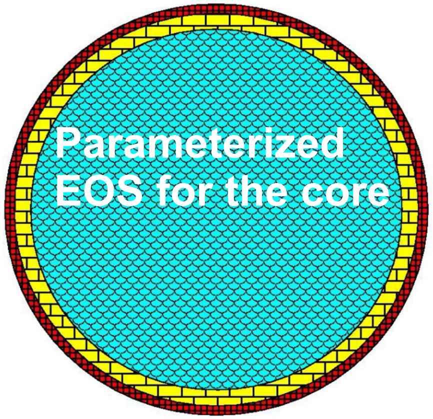
Symmetry energy and the isospin asymmetry at beta equilibrium

$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{\rm sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{\rm sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3$$

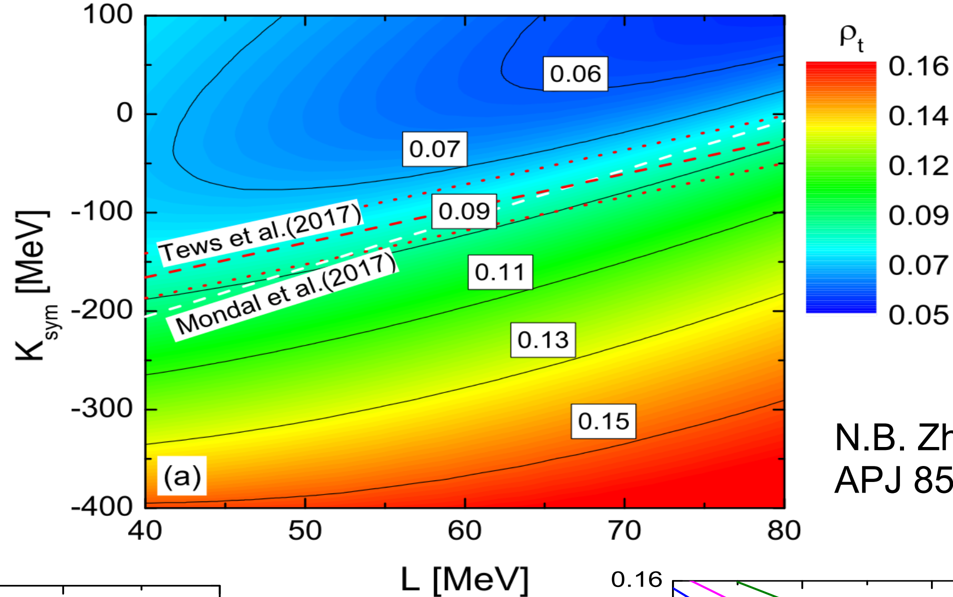
The proton fraction x:

$$x = 0.048[E_{sym}(\rho)/E_{sym}(\rho_0)]^3(\rho/\rho_0)(1-2x)^3$$

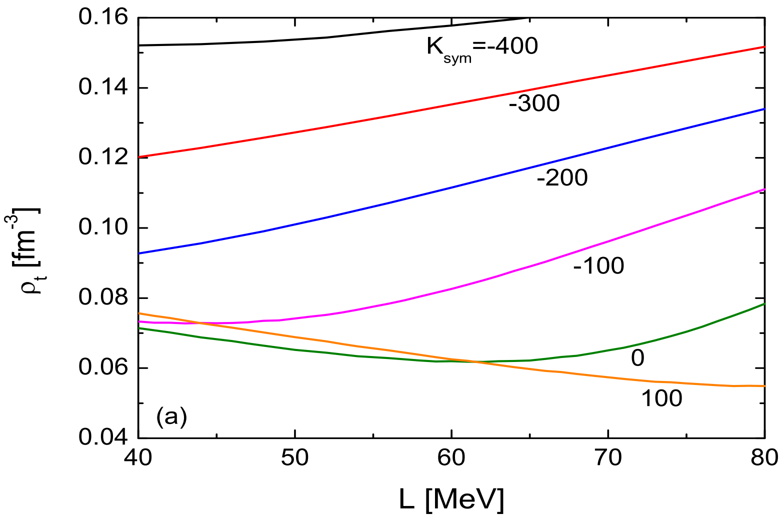
Effects of symmetry energy on the crust-core transition density

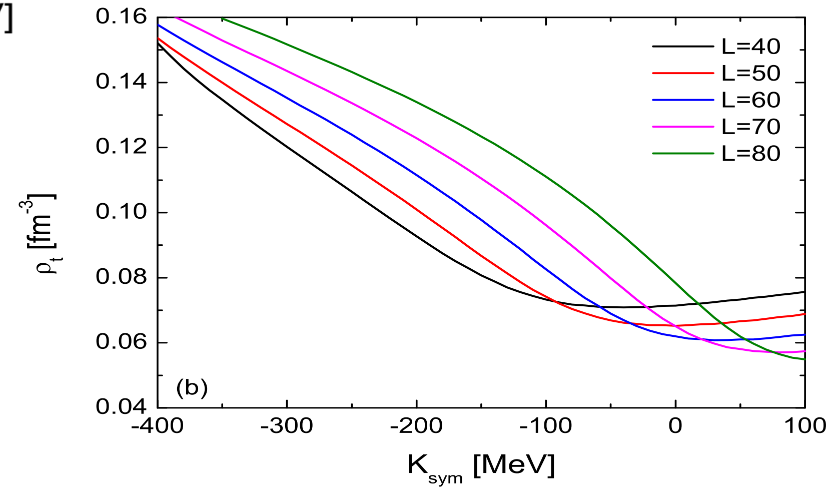


NV+BPS EOS
for the crust



N.B. Zhang, B.A. Li and J. Xu,
APJ 859, 90 (2018)



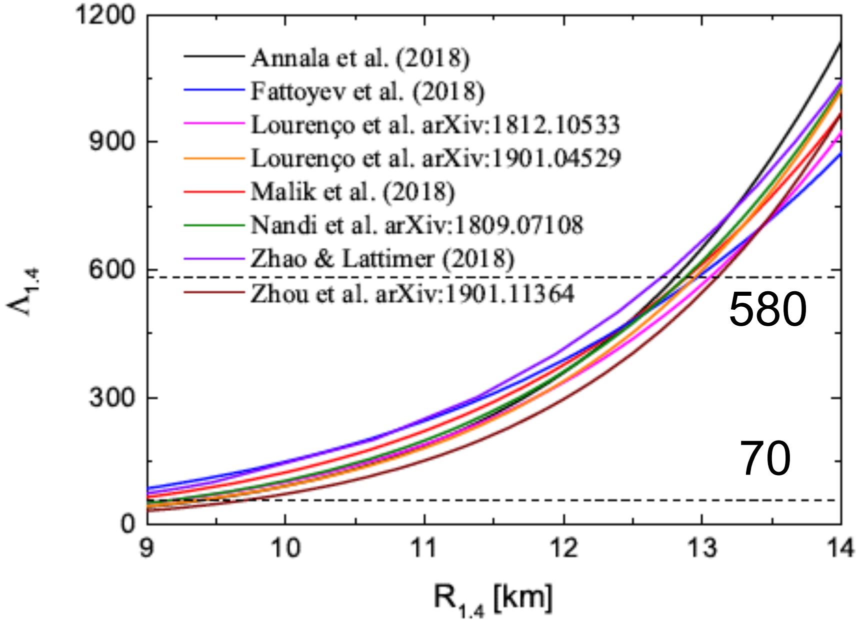


At the crust-core transition: Incompressibility in neutron stars at β equilibrium = 0

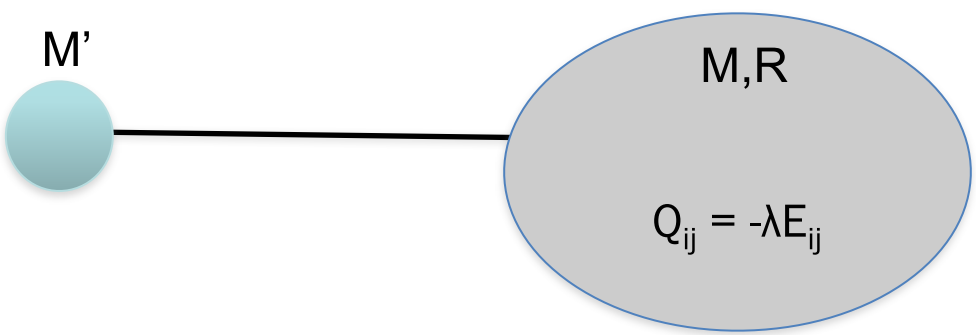
$$K_\mu = \rho^2 \frac{d^2 E_0}{d\rho^2} + 2\rho \frac{dE_0}{d\rho} + \delta^2 \left[\rho^2 \frac{d^2 E_{sym}}{d\rho^2} + 2\rho \frac{dE_{sym}}{d\rho} - 2E_{sym}^{-1} (\rho \frac{dE_{sym}}{d\rho})^2 \right]$$

Lattimer & Prakash, Phys. Rep., 442, 109 (2007)

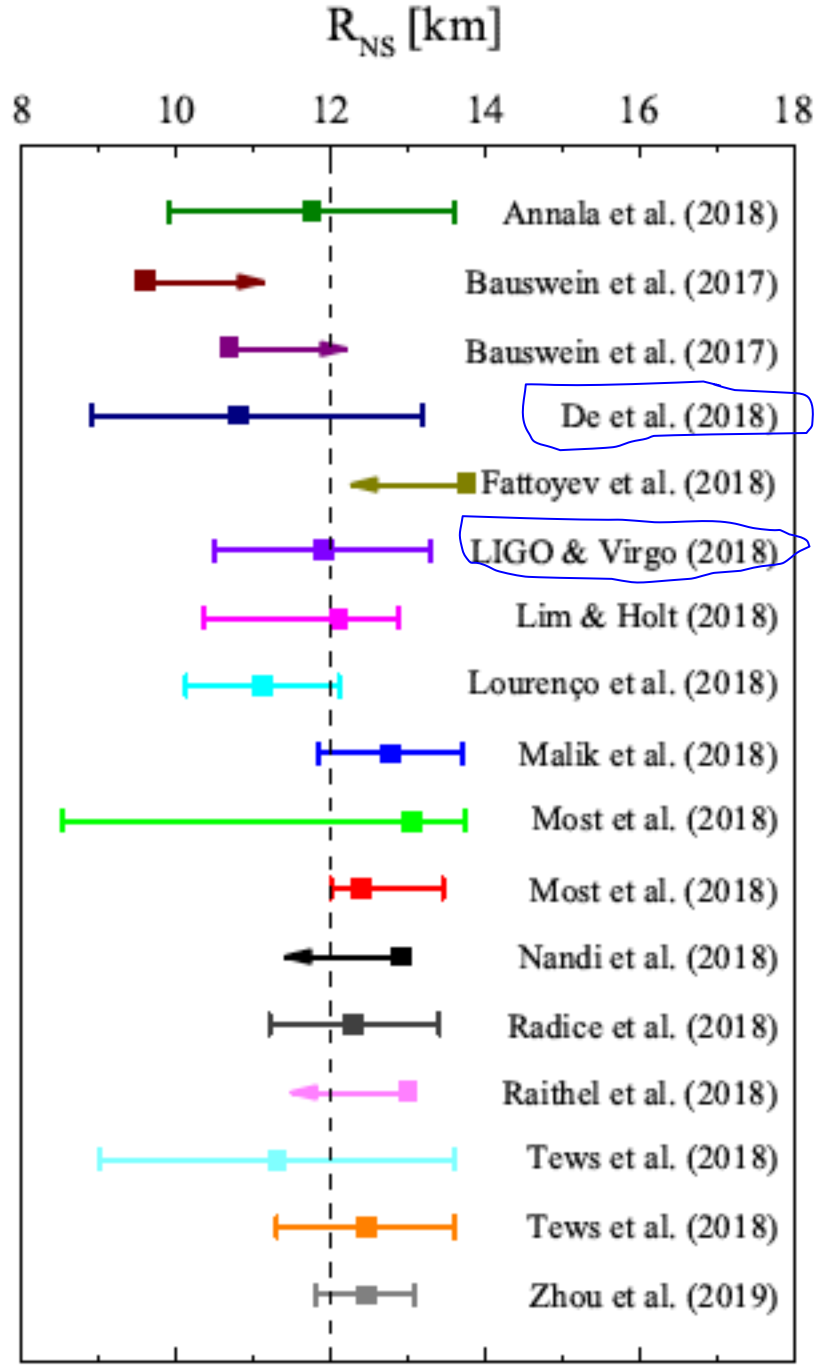
Tidal deformability and radius from GW170817



$$\Lambda = \frac{2}{3}\frac{k_2}{\beta^5}$$



B.A. Li et al., EPJA 55, 117 (2019)



Solving the NS inverse-structure problems by calling the TOV solver within 3 Do-Loops: Given an observable-→ Find ALL necessary EOSs

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \quad (2.15)$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 \quad (2.16)$$

$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\cdot\delta^2$$

Fix the saturation parameters $E_0(\rho_0)$, $E_{sym}(\rho_0)$ and L at their most probable values currently known

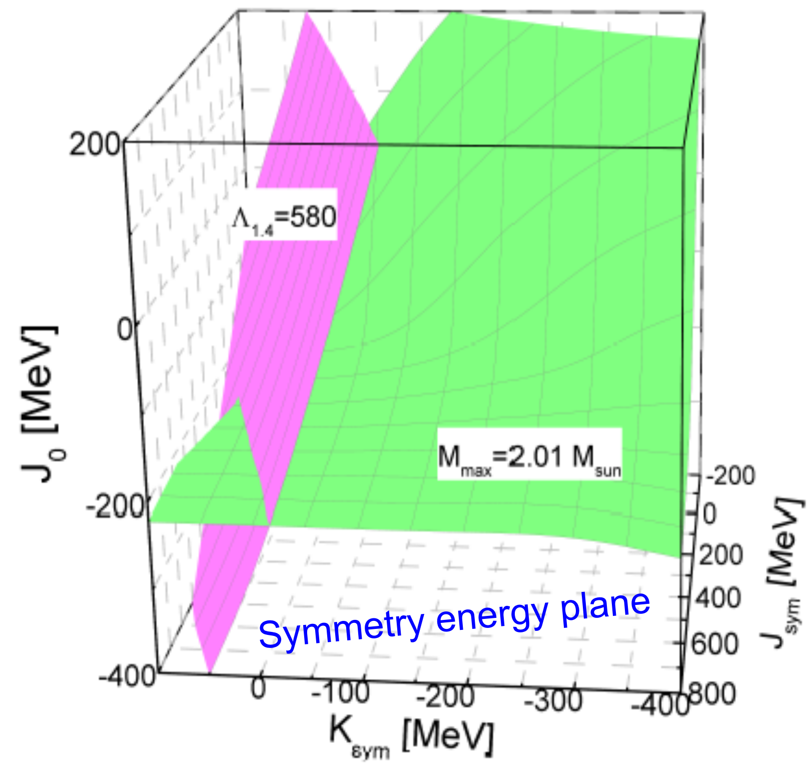
Example:

J_0=-189 is found given M_{max}=2.01M_{sun}

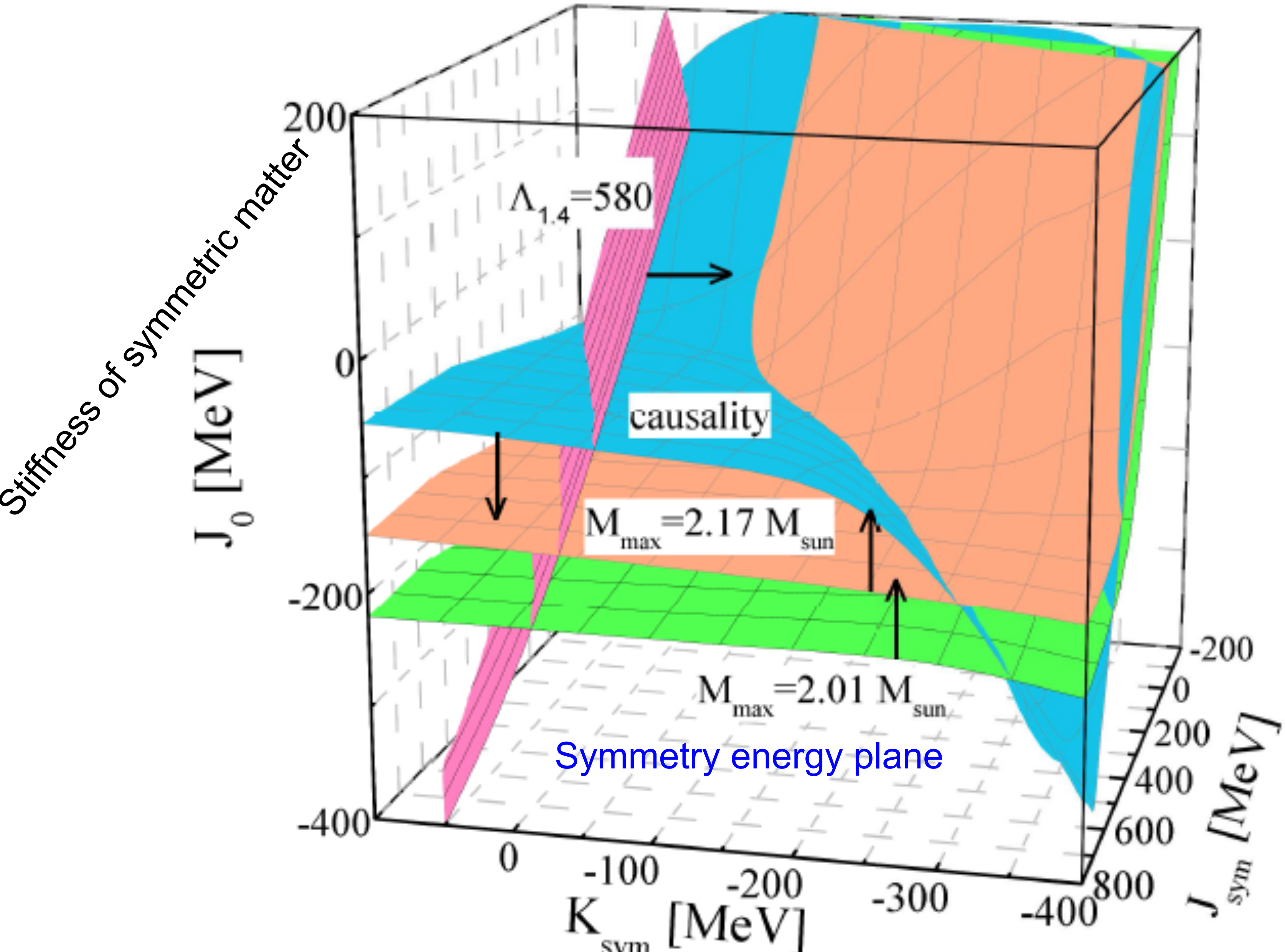
J_0 loop

TOV

at K_{sym}=-200 & J_{sym}=400 inside the K_{sym} and J_{sym} loops



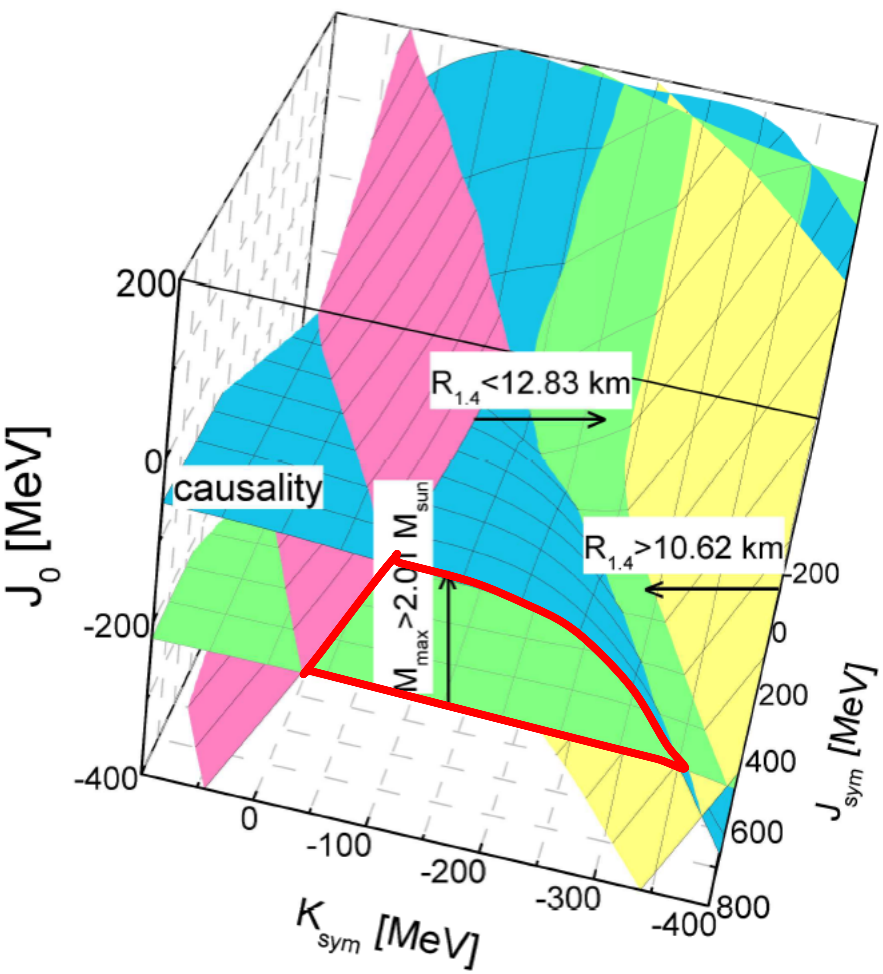
N.B. Zhang, B.A. Li and J. Xu,
The Astrophysical Journal 859, 90 (2018)



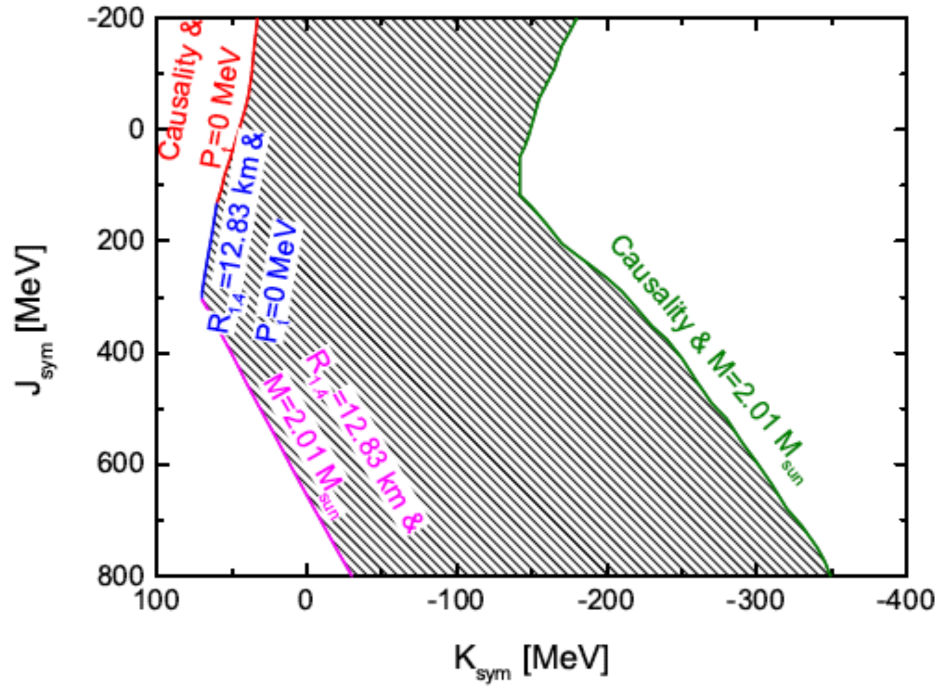
Effect of the observed maximum mass of neutron stars
$2.14^{+0.1}_{-0.09}$ (PSR J0740+6620)
H.T. Cromartie et al., Nature Astronomy
Sept. 16, 2019

N.B. Zhang and Bao-An Li, ApJ 879, 99 (2019)
Using 2.17 +/- 0.1 in their preprint

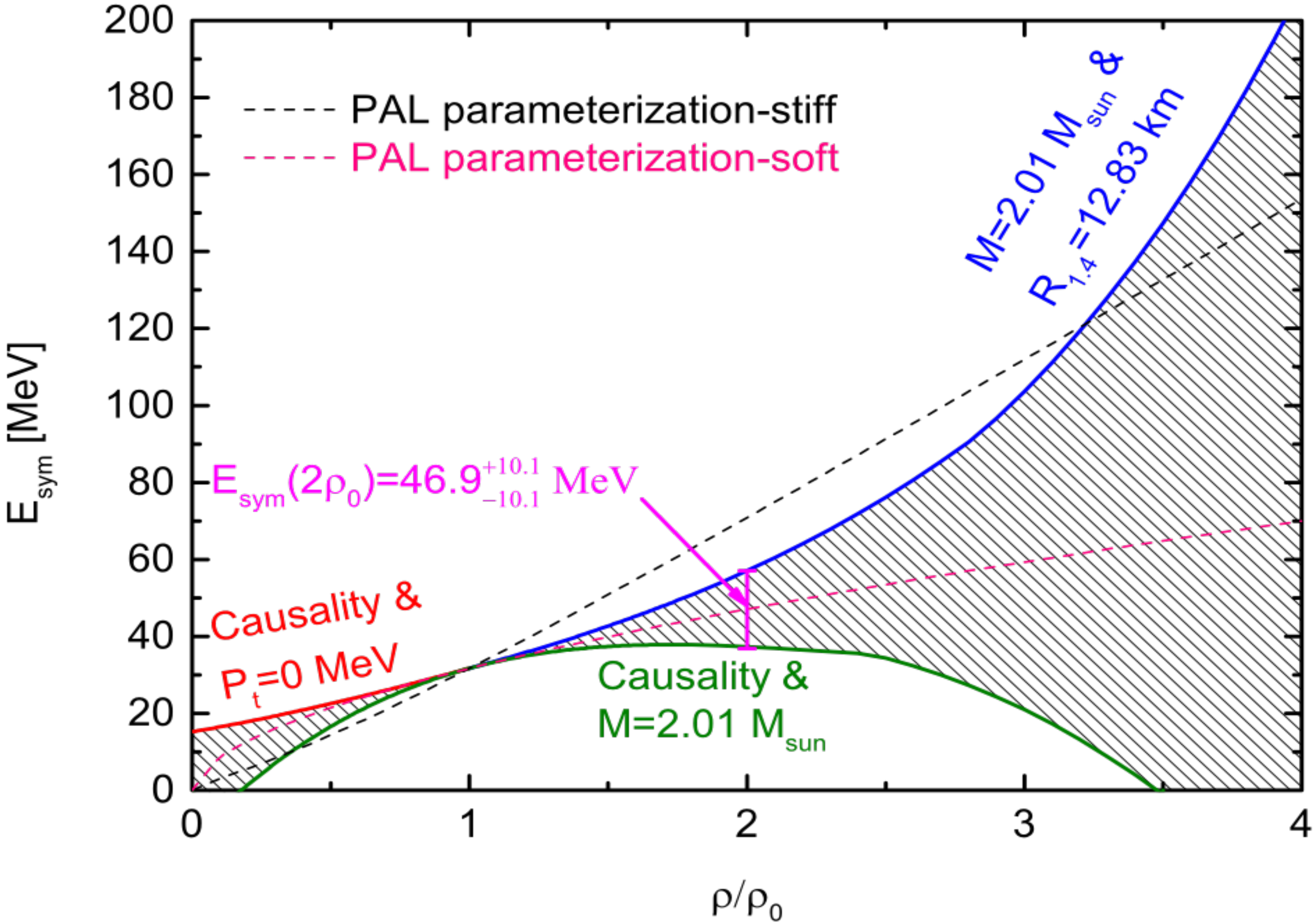
Restricted high-density EOS parameter space by (1) M_{max}>2.01, (2) 10.62<$R_{1.4}$<12.83 km and (3) causality



Boundaries in the plane of high-density E_{sym} from crossings of any two conditions



N.B. Zhang and B.A. Li
EPJA 55, 39 (2019)



With L fixed at its currently known most probable value of 60 MeV

Table 1. The radius $R_{1.4}$ data used in this work.

Radius $R_{1.4}$ (km) (90% confidence level)	Source	Reference
$11.9^{+1.4}_{-1.4}$	GW170817	(Abbott et al. 2018)
$10.8^{+2.1}_{-1.6}$	GW170817	(De et al. 2018)
$11.7^{+1.1}_{-1.1}$	QLMXBs	(Lattimer & Steiner 2014)
$11.9 \pm 0.8, 10.8 \pm 0.8, 11.7 \pm 0.8$	Imagined case-1	this work
11.9 ± 0.8	Imagined case-2	this work

Posterior probability distribution $$P(\mathcal{M}|D) = \frac{P(D|\mathcal{M})P(\mathcal{M})}{\int P(D|\mathcal{M})P(\mathcal{M})d\mathcal{M}},$$ **(Bayes' theorem)**

Likelihood: $$P[D(R_{1,2,3})|\mathcal{M}(p_{1,2,\cdots 6})] = \prod_{j=1}^{3} \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{obs},j}} \exp[-\frac{(R_{\mathrm{th},j} - R_{\mathrm{obs},j})^2}{2\sigma^2_{\mathrm{obs},j}}],$$

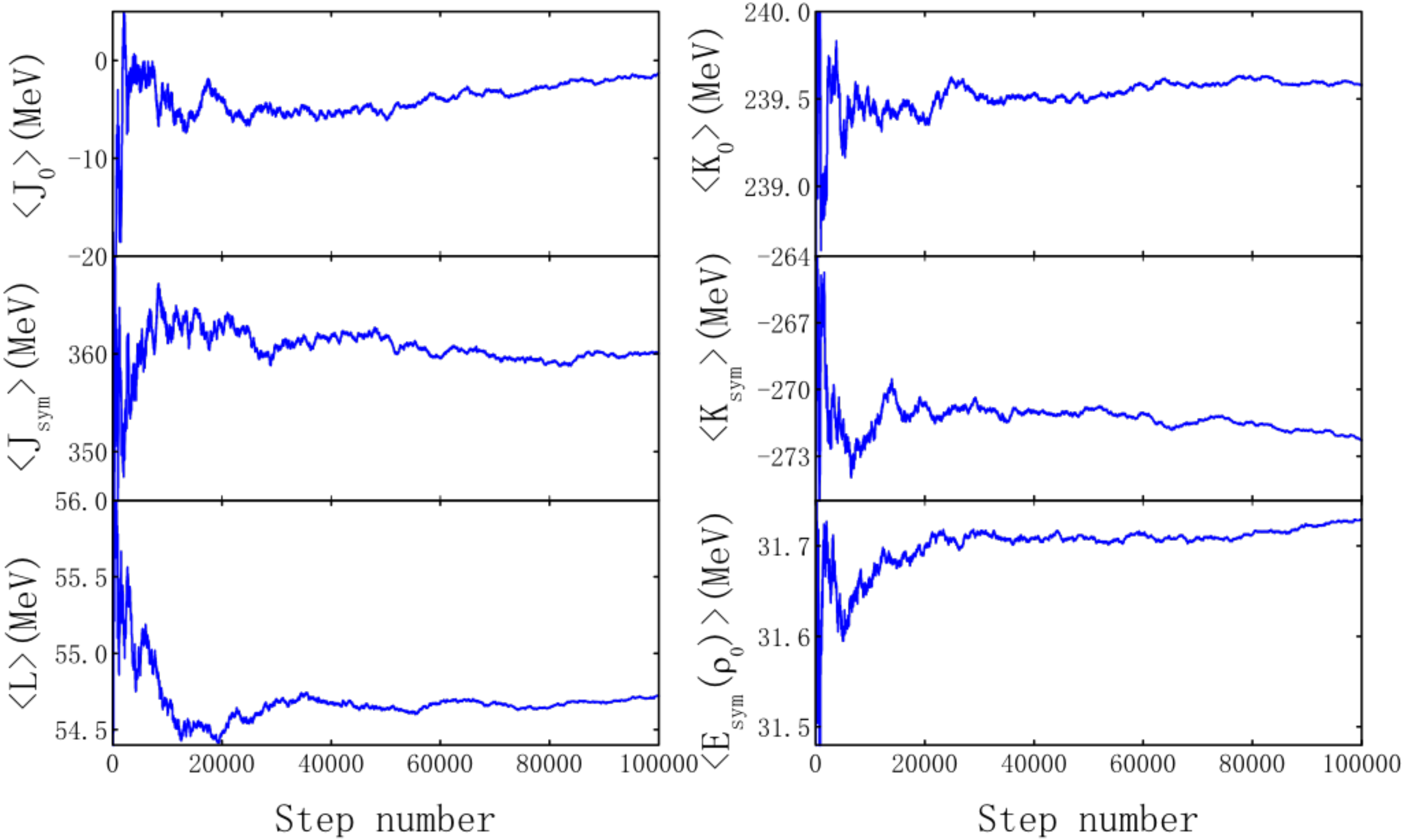
Uniform prior distribution P(M) in the ranges of

Table 2. Prior ranges of the six EOS parameters used

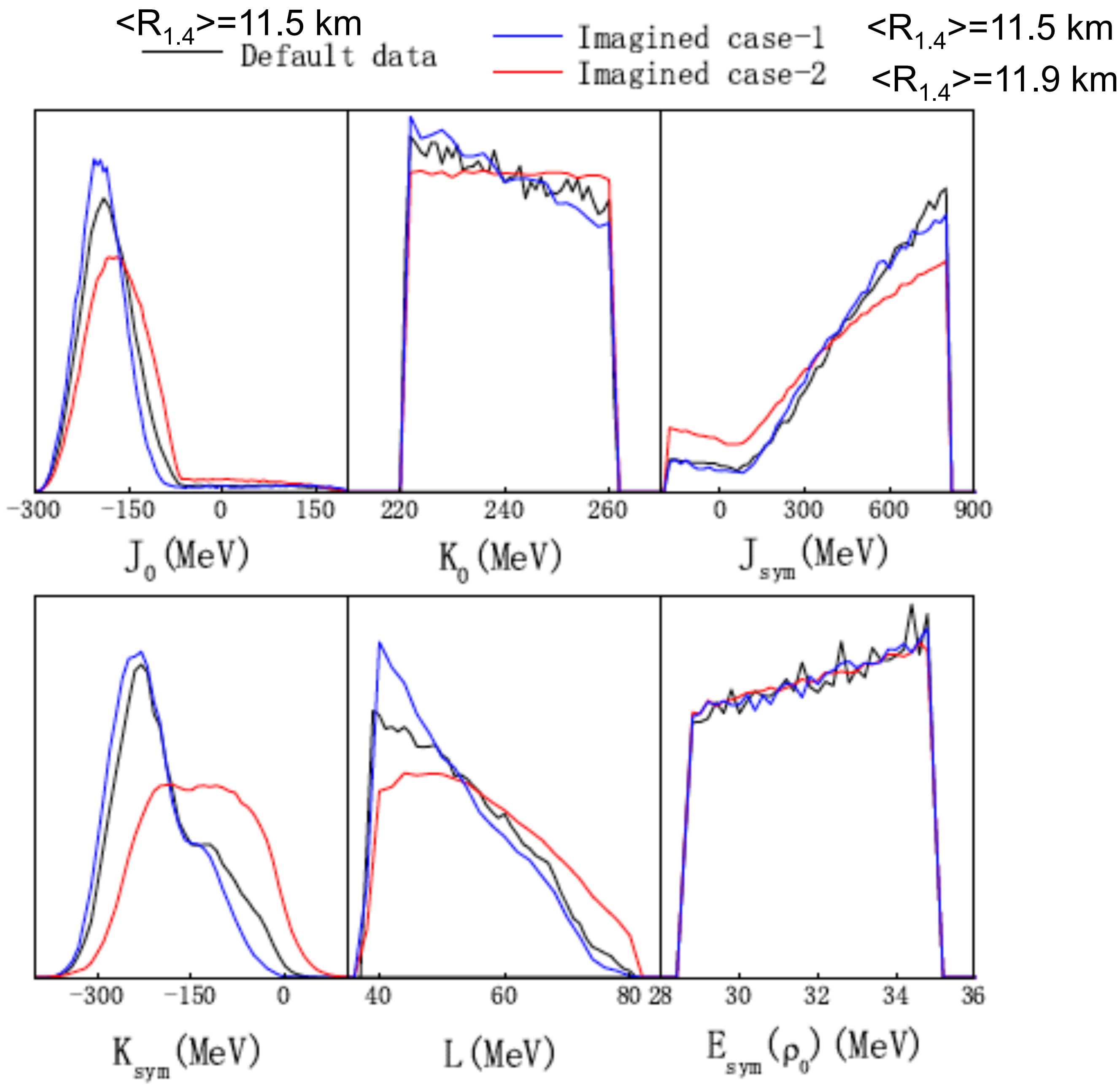
Parameters	Lower limit	Upper limit (MeV)
K_0	220	260
J_0	-800	400
K_{sym}	-400	100
J_{sym}	-200	800
L	30	90
$E_{\mathrm{sym}}(\rho_0)$	28.5	34.9

Bayesian inference of high-density E_{sym} from the radii $R_{1.4}$ of canonical neutron stars in 6D EOS parameter space

Markov Chain Monte Carlo (MCMC) sampling of the posterior probability distribution functions (PDFs) of EOS parameters and their correlations



Posterior probability distribution function of 6 EOS parameters



W.J. Xie and B.A. Li
arXiv:1907.10741 , APJ (2019) in press.

Prior EOS parameter ranges:

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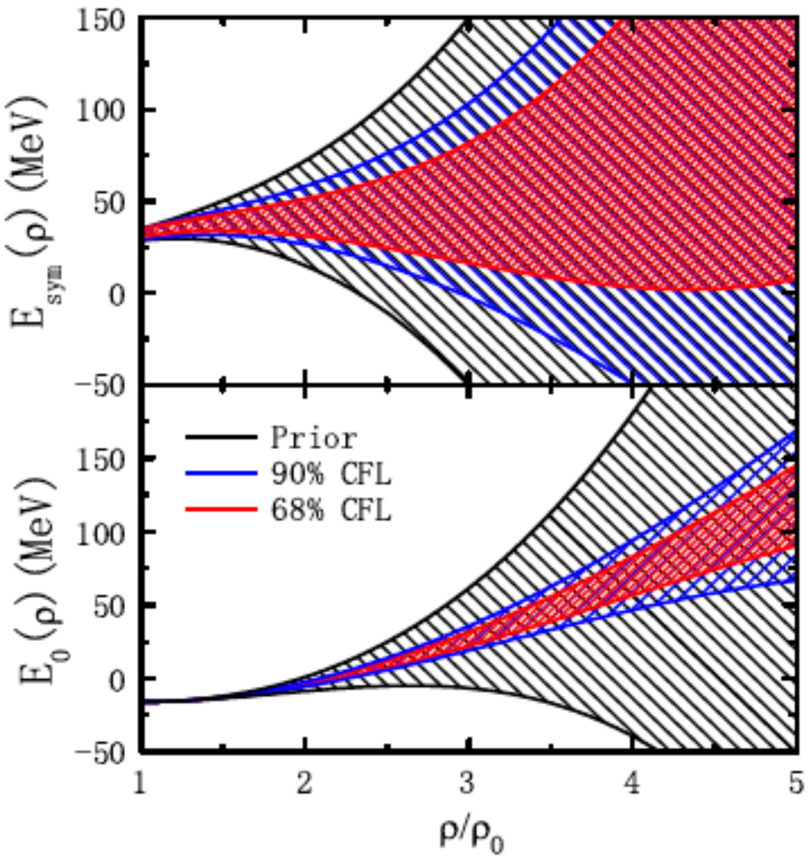
$E_{sym}(\rho_0)$	28.5	34.9

Significant improvement of our exiting knowledge about the EOS of dense neutron-rich matter



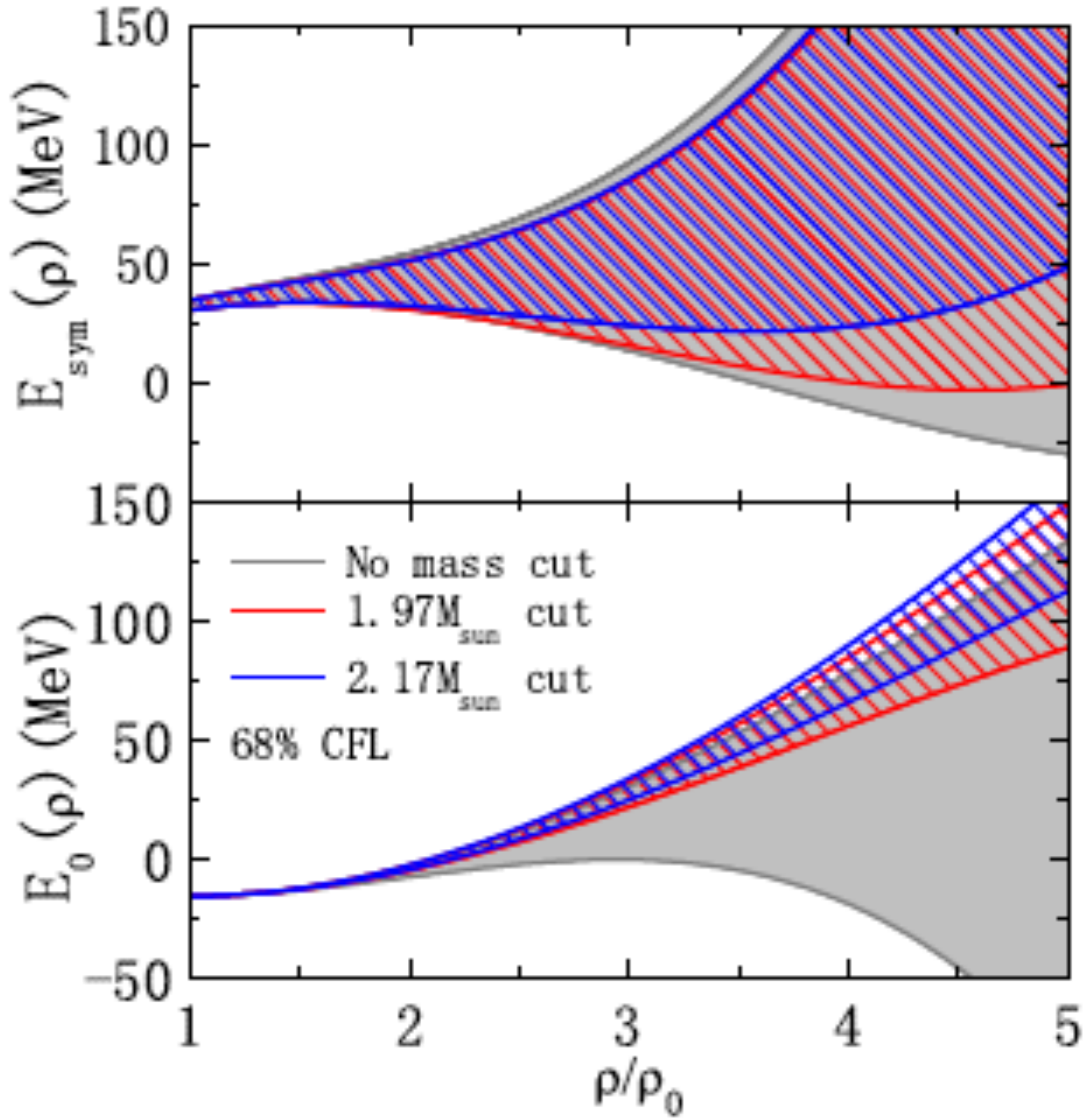
Posterior EOS parameter inferred:

Most probable values of the EOS parameters and their 68%, 90% confid

Parameter (MeV)	68% boundaries	90% boundaries
J_0	-190^{+40}_{-40}	-190^{+80}_{-70}
K_0	222^{+26}_{-0}	222^{-0}_{+35}
J_{sym}	800^{+0}_{-360}	800^{+0}_{-600}
K_{sym}	-230^{+90}_{-50}	-230^{+160}_{-70}
L	39^{+19}_{-0}	39^{+28}_{-1}
$E_{sym}(\rho_0)$	$34^{+0.8}_{-4.8}$	$34^{+0.8}_{-3.2}$

Effect of the observed maximum mass of neutron stars

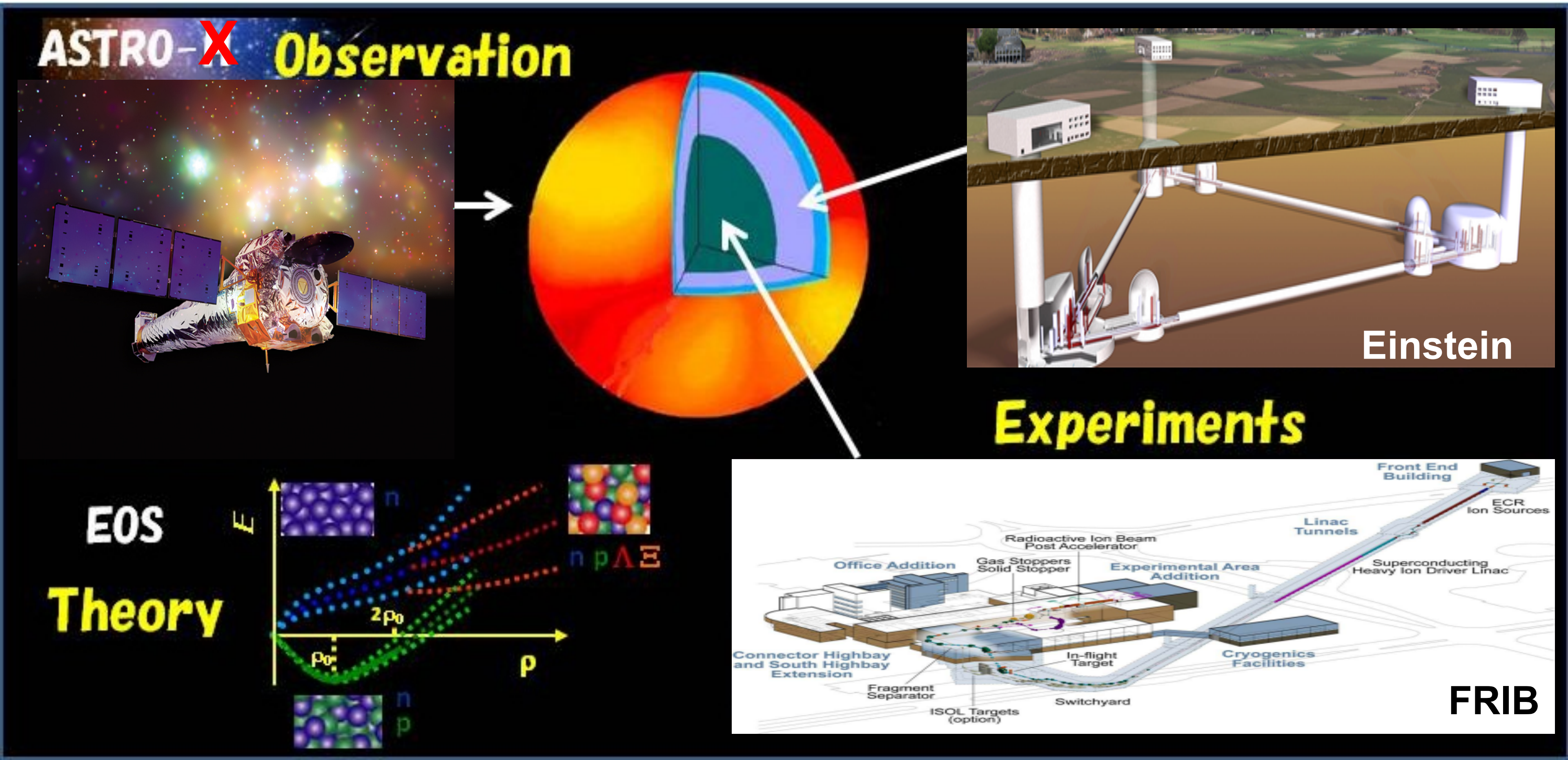
from 1.97 to 2.17 (PSR J0740+6620)

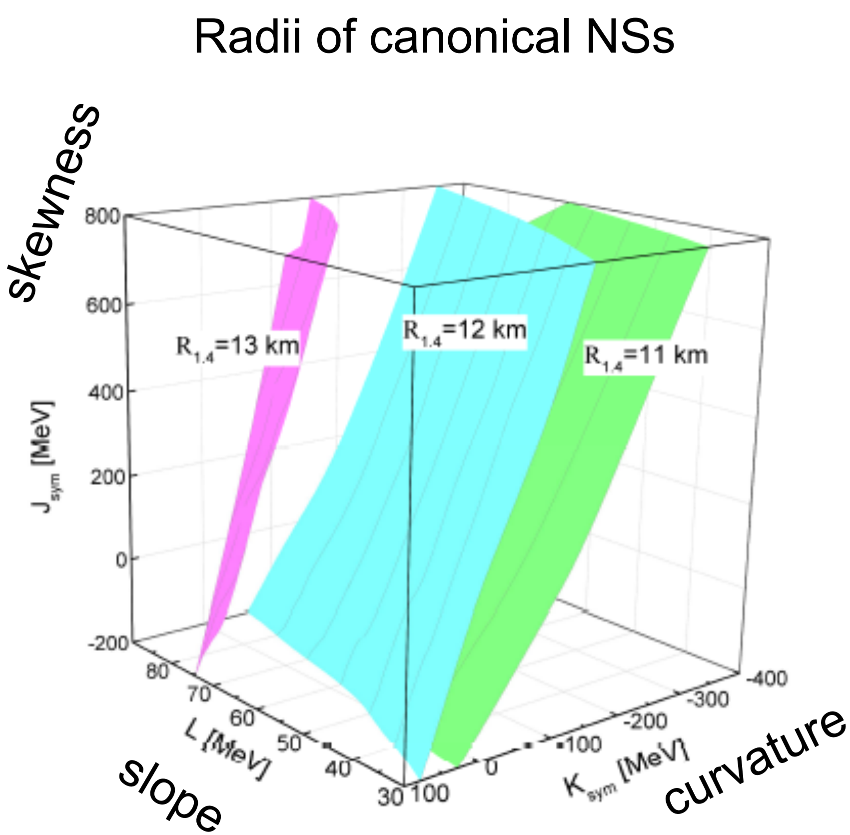


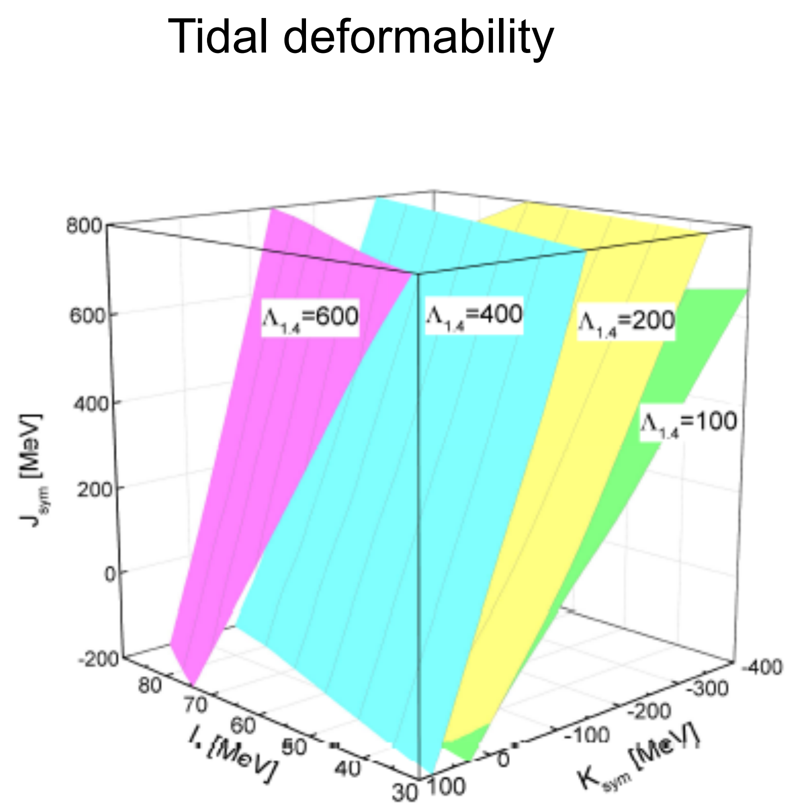
Summary

(1) Significant progress has been made in fixing nuclear symmetry energy below twice the saturation density

(2) Truly multi-messenger approach to probe the EOS of dense neutron-rich matter = astrophysical observations + terrestrial experiments + theories + …







Inverting the radius and tidal deformability in 3D symmetry energy parameter space at J_0=-180 MeV

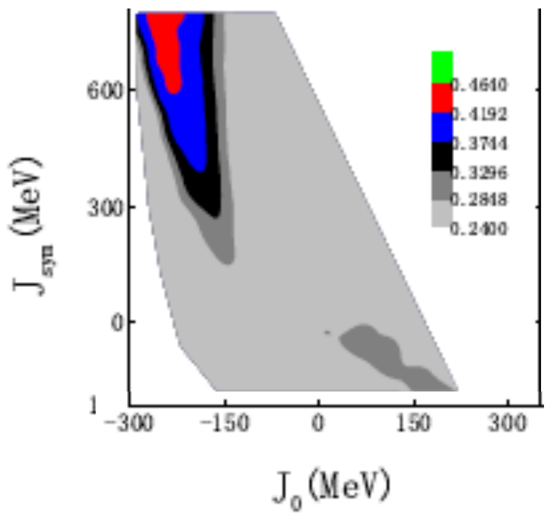
N.B. Zhang and Bao-An Li, JPG 46, 014002 (2019)

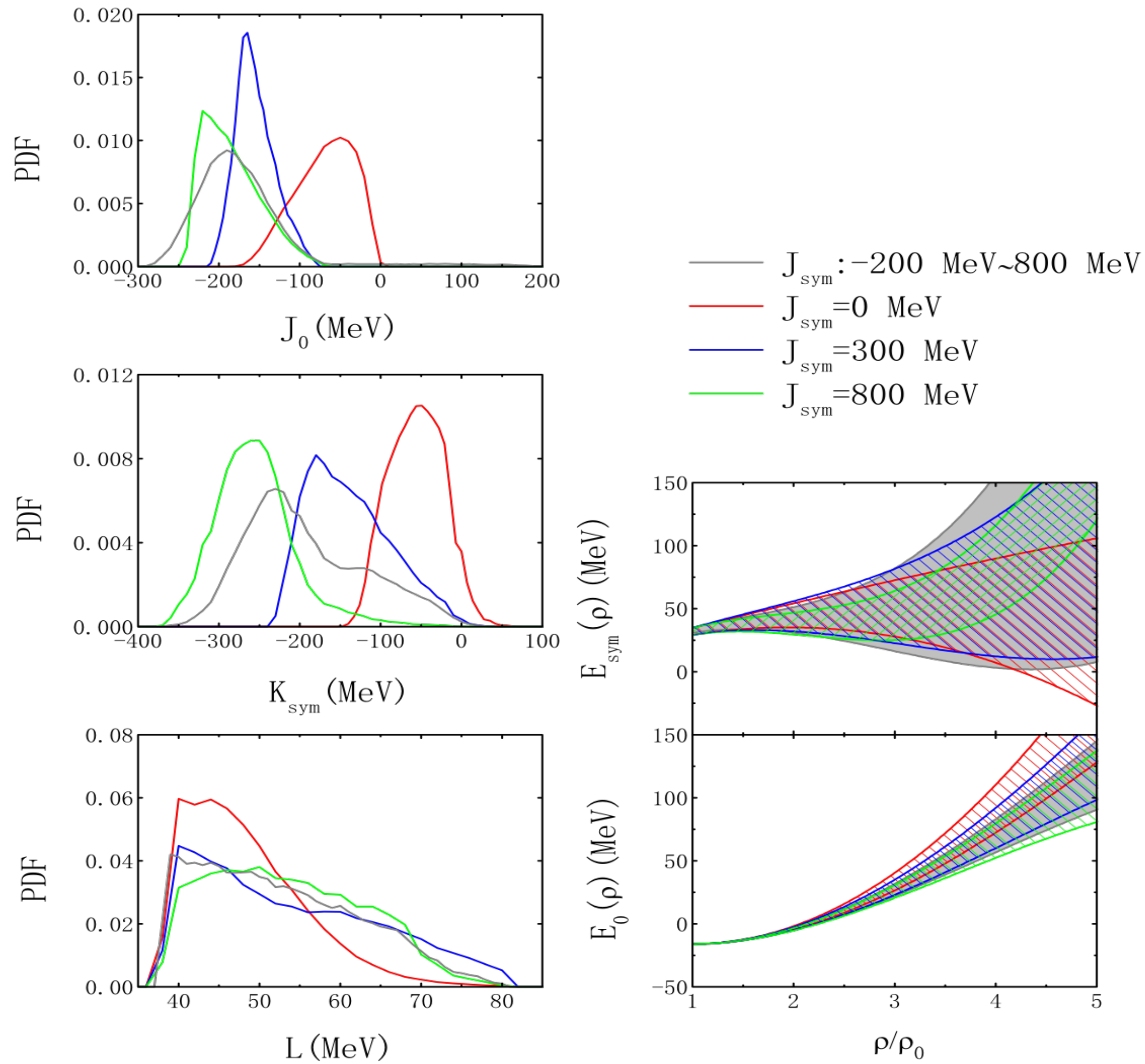
$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3, \quad (2.15)$$

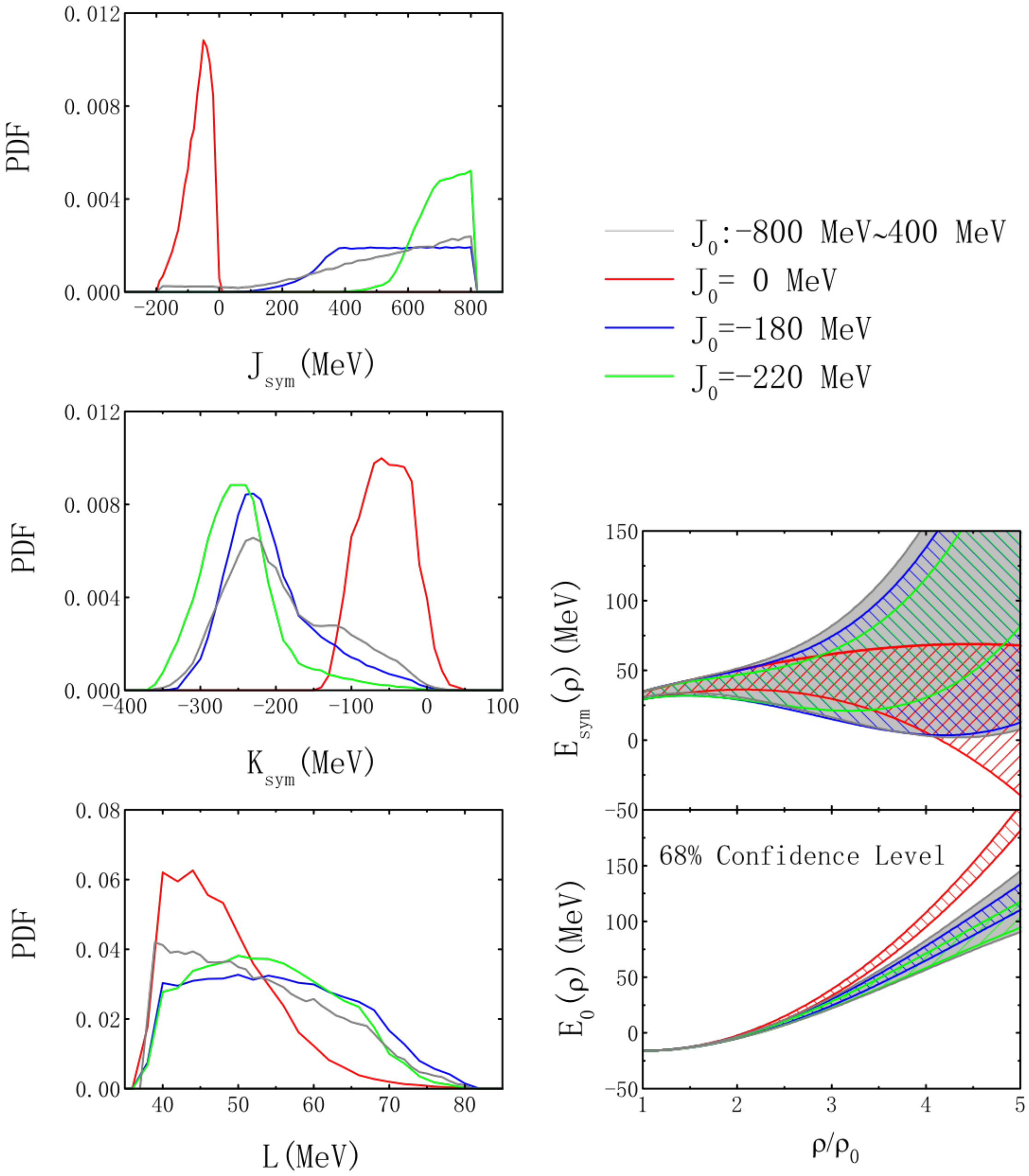
$$E_{sym}(\rho) = E_{sym}(\rho_0) + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{J_{sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 \quad (2.16)$$

$$E(\rho_n,\rho_p) = E_0(\rho_n=\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + o(\delta^4)$$

The high-density E_{sym} and symmetric nuclear matter EOS are anti-correlated

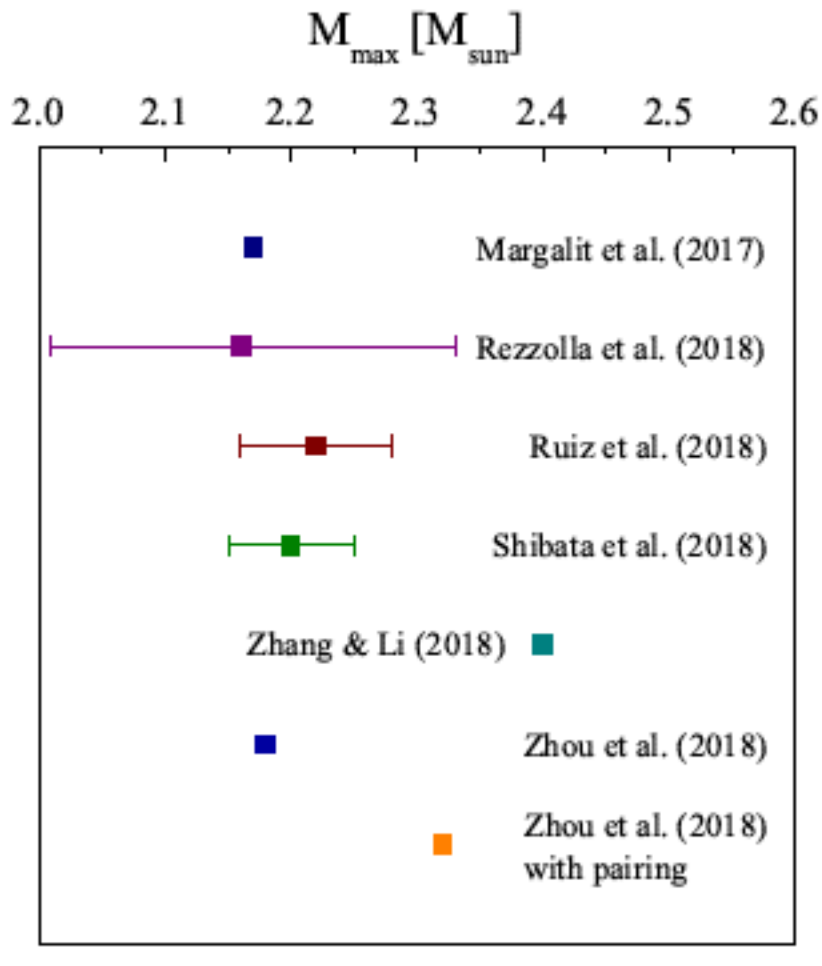






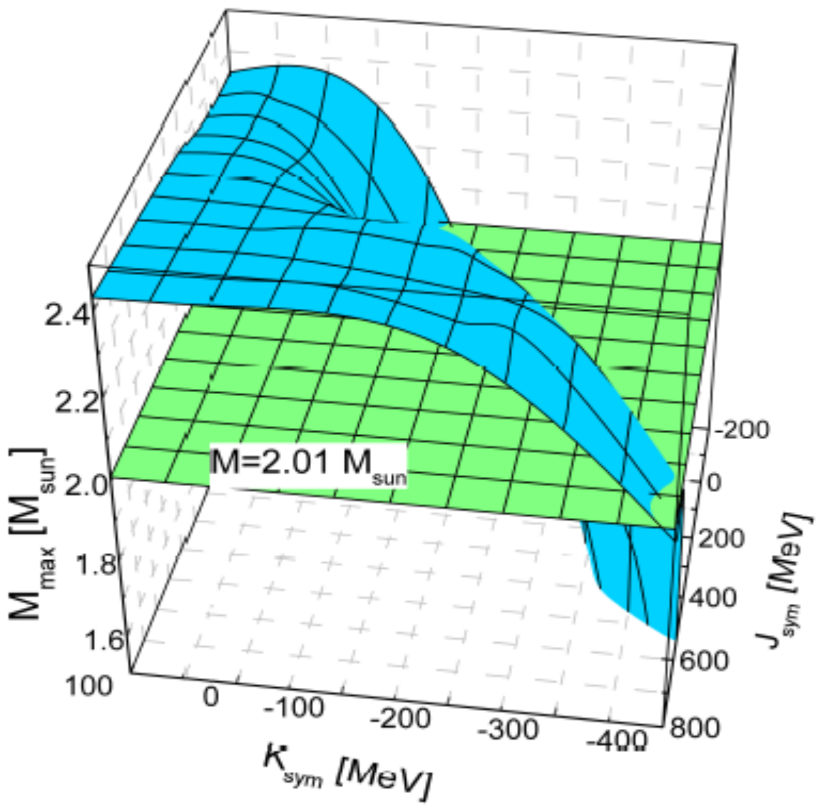
PDF
J_sym (MeV)
K_sym (MeV)
L (MeV)
J_0:-800 MeV~400 MeV
J_0= 0 MeV
J_0=-180 MeV
J_0=-220 MeV
E_sym(ρ) (MeV)
E_0(ρ) (MeV)
68% Confidence Level
ρ/ρ_0

Gravitational wave constraints on the high-density EOS parameter space



Causality surface: V_{sound}=C reached at the central density of the most massive NS

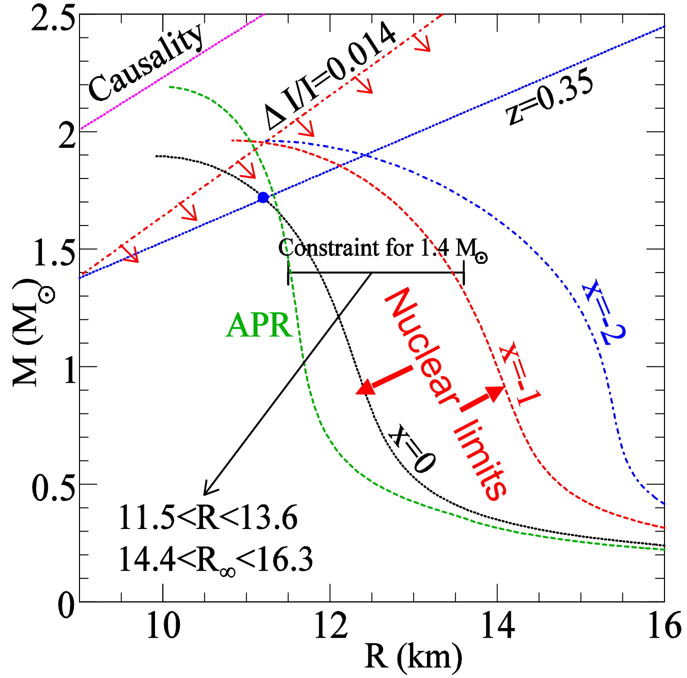
The absolutely maximum mass: The maximum mass on the causality surface M_{TOV} = 2.4M_{sun}



N.B. Zhang and B.A. Li, EPJA 55, 39 (2019)

Constraining the radii of neutron stars with terrestrial experiments

Bao-An Li and Andrew W. Steiner, Phys. Lett. B 642, 436 (2006)



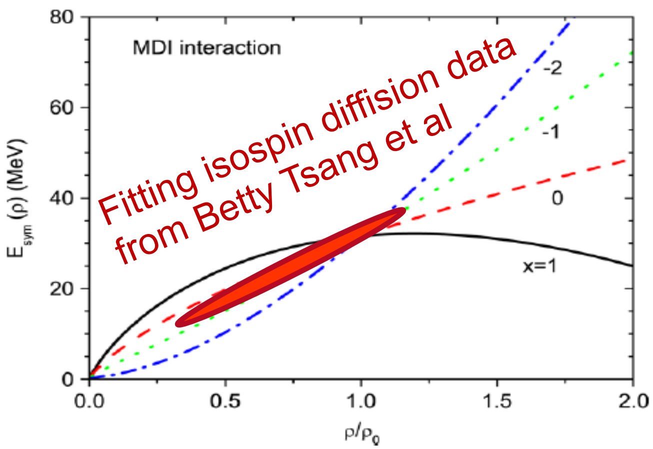
APR: K_0=269 MeV.

Radii of neutron stars inferred from observations

(a)thermal emissions from quiescent neutron star low-mass X-ray binaries (qLMXBs)
(b)photospheric radius expansion (PRE) bursts with H and/or He atmosphere models

Model	$R_{1.4}$ 90% confidence range
Alt/H+He QLMXB; $z = 0$ PRE	11.13 – 12.33
$z = 0$ PRE only	11.56 – 12.64
Base, QLMXB only	11.01 – 11.94
Alt, QLMXB only	10.62 – 11.50
H+He, QLMXB only	11.29 – 12.83
Alt/H+He, QLMXB only	11.24 – 12.59

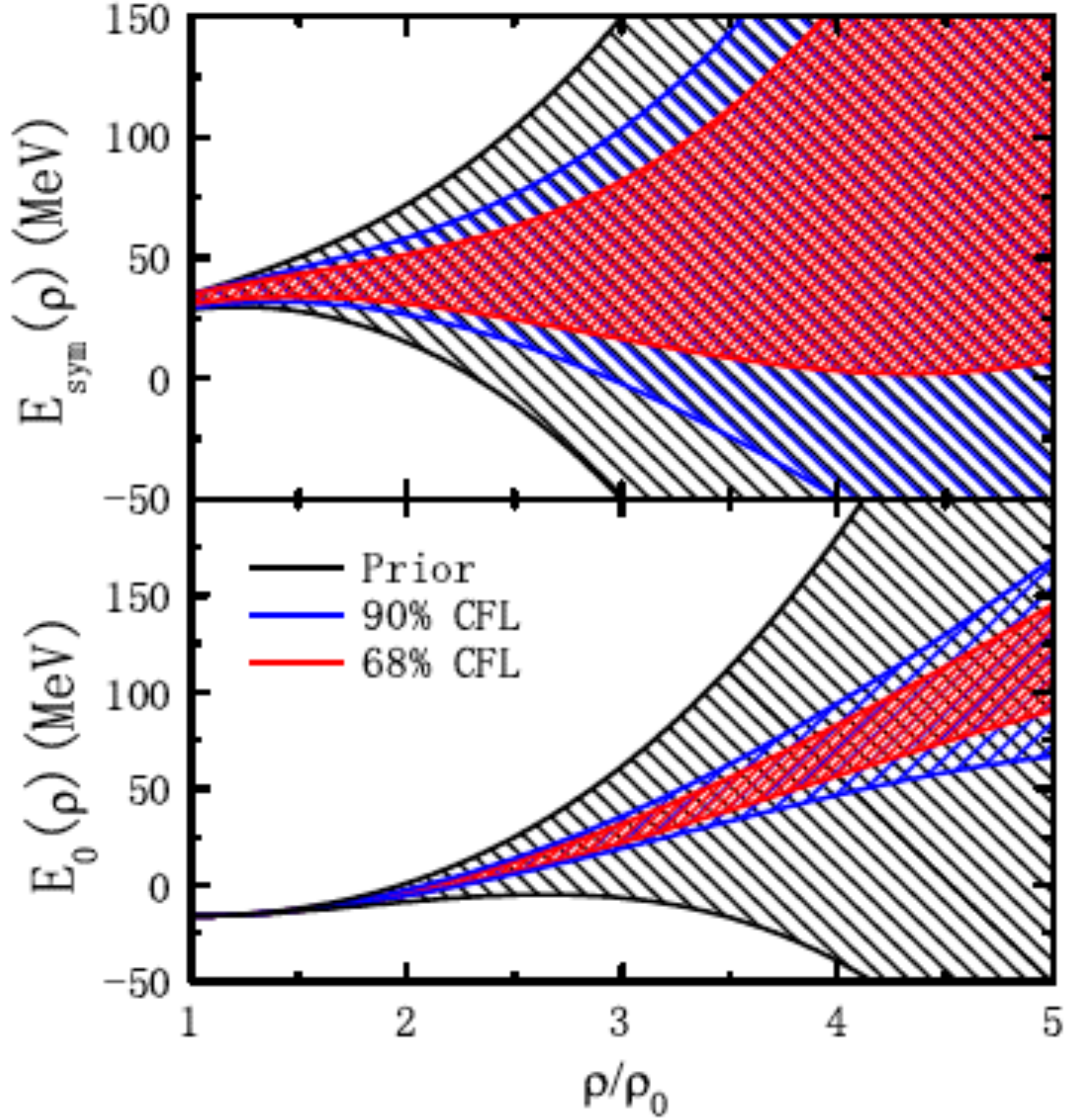
J.M. Lattimer and A.W. Steiner,
European Physics Journal A50, 40 (2014)



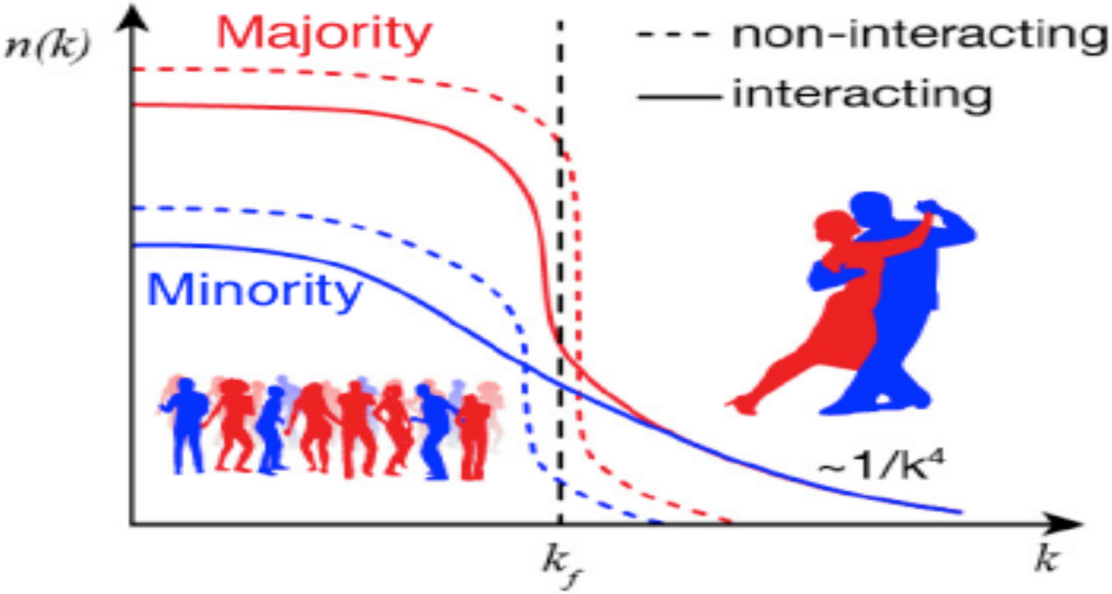
(C) Essentially ALL radii extracted from GW170817 by various groups are consistent with the prediction made in 2006 based on analyzing the terrestrial data

L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. Lett 94, 32701 (2005)

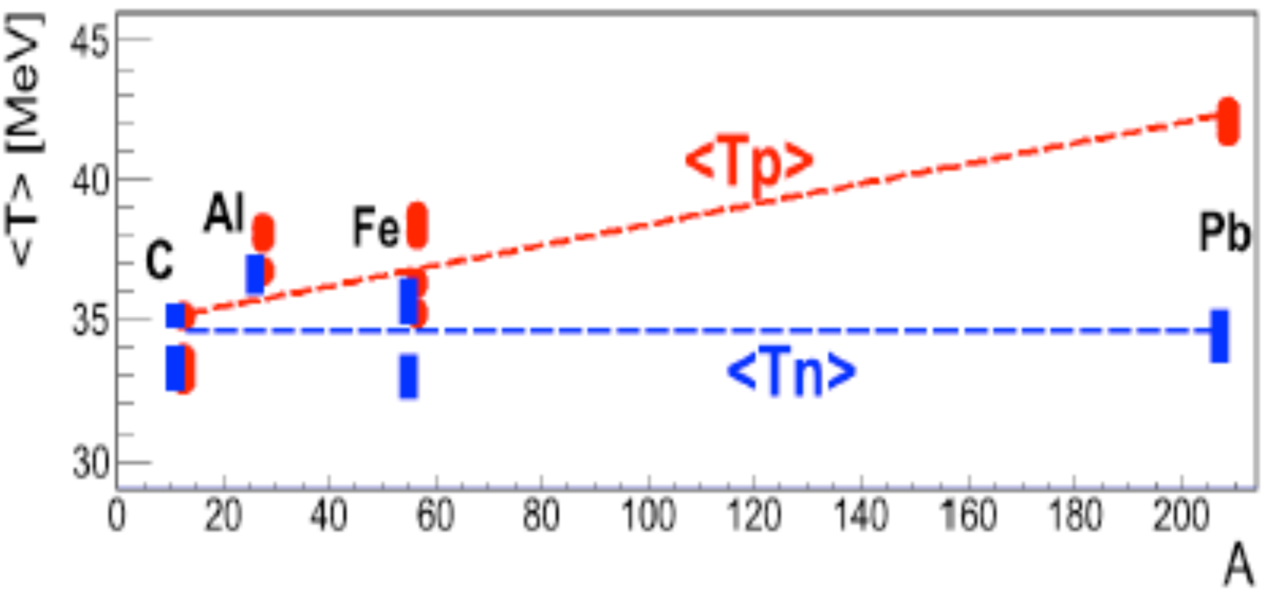
Conclusion: The radii $R_{1.4}$ from GW170817 and X-rays improve our knowledge about the high-density E_{sym} significantly



How does the SRC affect nuclear symmetry energy $E_{sym}(\rho)$?



Eli Piasetzky's Dancing Model



O. Hen et al. (Jlab CLAS), Science 346, 614 (2014)