Double parton scattering

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QCD on and off the lattice, 20 September 2023

Plan of this talk

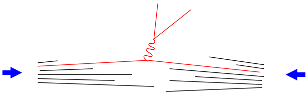
- introduction to double parton scattering
- some theory results
- lattice studies

much of this work done in collaboration with Andreas and others in this room
from 2011 to now

Hadron-hadron collisions

- standard description based on factorisation formulae

cross sect = parton distributions × parton-level cross sect



- factorisation formulae are for inclusive cross sections $pp \to A + X$
where A = produced by parton-level scattering, specified in detail
X = summed over, no questions asked

Hadron-hadron collisions

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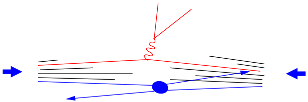
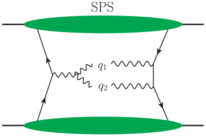


- factorisation formulae are for inclusive cross sections $pp \to A + X$
  where $A =$ produced by parton-level scattering, specified in detail
  $X =$ summed over, no questions asked
- spectator interactions
  - cancel in inclusive cross sections thanks to unitarity
  - can be soft $\rightsquigarrow$ part of underlying event
  - ... or hard $\rightsquigarrow$ multiparton scattering
- double parton scattering: $pp \to A_1 + A_2 + X$ with scales $Q_1, Q_2 \gg \Lambda$
  - have factorisation formula with double parton distributions

Single vs. double hard scattering

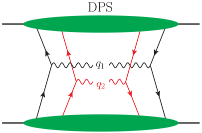
- example: two gauge bosons with transverse momenta $\vec{q}_1$ and $\vec{q}_2$



single scattering (SPS)

$|\vec{q}_1|$ and $|\vec{q}_2| \sim$ hard scale Q

$|\vec{q}_1 + \vec{q}_2| \ll Q$



double scattering (DPS)

both $|\vec{q}_1|$ and $|\vec{q}_2| \ll Q$

- for transverse momenta $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{SPS}}}{d^2\vec{q}_1\, d^2\vec{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\vec{q}_1\, d^2\vec{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but SPS populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \;\gg\; \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double hard scattering

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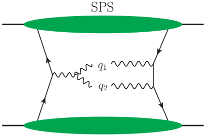

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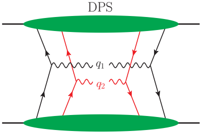


double scattering (DPS)

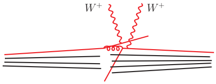
both $|\vec{q}_1|$ and $|\vec{q}_2| \ll Q$

- DPS can be enhanced by
  - small parton mom. fractions x because of parton luminosity
    roughly, $\sigma_{\text{SPS}} \sim \text{PDF}^2$ and $\sigma_{\text{DPS}} \sim \text{PDF}^4$
  - large rapidity separation ΔY between systems A_1 and A_2
    large invariant mass of overall system $\rightsquigarrow$ large x in SPS
  - parton type (quarks vs. gluons), coupling constants, etc.

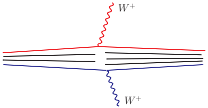
Many experimental observations at Tevatron and LHC

- like-sign W pairs

$\sigma_{\text{SPS}} \propto \mathcal{O}(\alpha_s^2)$ with ≥ 2 jets



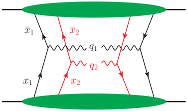
$\sigma_{\text{DPS}} \propto \mathcal{O}(\alpha_s^0)$



observed by CMS in run 2

- jets and gauge bosons:
  4 jets, γ + 3 jets, γ + 2 jets, W + 2 jets
- heavy flavours:
  $W + J/\Psi$, $J/\Psi + J/\Psi$, $J/\Psi + \Upsilon$, $\Upsilon\Upsilon$, double open charm, ...

DPS cross section: basic theory

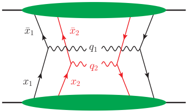


$$\frac{d\sigma^{A_1 A_2}_{\text{DPS}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{1+\delta_{A_1 A_2}}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})\, F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$\hat{\sigma}_i(x_i, \bar{x}_i) =$ parton-level cross section for $a_i + b_i \to A_i$
$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) =$ double parton distribution (DPD)
$\boldsymbol{y} =$ transverse distance between partons

- can extend $\hat{\sigma}_i$ to higher orders in α_s
- tree-level formula from Feynman graphs and kinematic approximations
  Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2011
- all-order factorisation proof for double Drell-Yan
  Manohar, Waalewijn 2012; Vladimirov 2016, 2017; MD, Buffing, Gaunt, Kasemets, Nagar, Ostermeier, Plößl, Schäfer, Schönwald 2011–2018
  requires modification of above formula ⇝ more later

DPS cross section: basic theory



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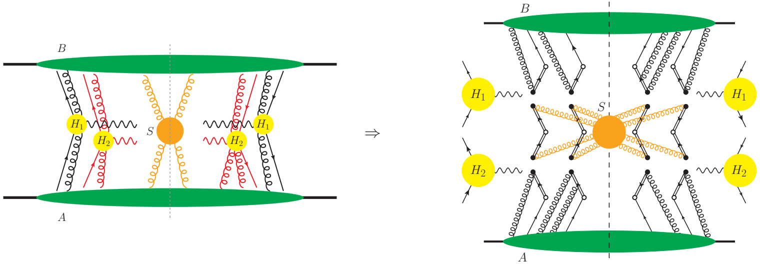
if assume $F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, G(\boldsymbol{y}) \;\Rightarrow$ pocket formula

$$\frac{d\sigma^{A_1 A_2}_{\text{DPS}}}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} = \frac{\sigma_{\text{eff}}^{-1}}{1+\delta_{A_1 A_2}}\, \frac{d\sigma^{A_1}_{\text{SPS}}}{dx_1\, d\bar{x}_1}\, \frac{d\sigma^{A_2}_{\text{SPS}}}{dx_2\, d\bar{x}_2} \qquad \text{with} \quad \sigma_{\text{eff}}^{-1} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$$

- straightforward generalisation to N independent scatters
  underlies implementations in event generators `PYTHIA`, `Herwig`, `Sherpa`
  with adjustments for conserving momentum and quark number
- underlies bulk of phenomenological estimates (with some exceptions)
- fails when the assumption on $F_{a_1 a_2}$ is invalid
  or when cross sect. formula misses important contributions $\rightsquigarrow$ more later

Factorisation

- generalise arguments for single to double Drell-Yan

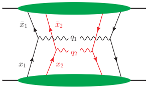


- basic steps:
  - identify leading graphs and momentum regions (power counting)
  - decouple collinear gluons ⇝ Wilson lines in DPD matrix elements
  - decouple soft gluons ⇝ DPS oft factor (vev of 4×2 Wilson lines)
  - show that Glauber gluons cancel (unitarity argument)

  MD, D Ostermeier, A Schäfer 2011; MD, J Gaunt, P Plößl, A Schäfer 2015

Parton correlations

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{\hat{\sigma}_1\, \hat{\sigma}_2}{1+\delta_{A_1 A_2}} \int d^2\boldsymbol{y}\; F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})\, F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



Factorisation formula includes parton correlations between

- x_1, x_2, and $\boldsymbol{y}$
- spins (even in an unpolarised proton)
  - parton spin correlations can affect final state distributions in DPS
    gauge boson pairs: Manohar, Waalewijn 2011; Kasemets, MD 2012
    double charm: Echevarria, Kasemets, Mulders, Pisano arXiv:1501.07291
  - evolution to high scales tends to wash out spin correlations
    unpol. densities evolve faster than polarised ones MD, Kasemets 2014
- colours
  - technically more involved, not discussed in this talk

What do we know about DPDs?

Properties of DPDs

- PDFs and DPDs are matrix elements of twist-two operators $\mathcal{O}_a(\boldsymbol{y}, \mu)$

$$f_a(x;\mu) \sim \langle p|\,\mathcal{O}_a(\mathbf{0};\mu)\,|p\rangle$$
$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\,\mathcal{O}_{a_1}(\mathbf{0};\mu_1)\,\mathcal{O}_{a_2}(\boldsymbol{y};\mu_2)\,|p\rangle$$

$\rightsquigarrow$ scale dependence described by DGLAP evolution equations

def. and evolution more complicated for DPDs with colour correlations

M Buffing, MD, T Kasemets 2018; MD, F Fabry, A Vladimirov 2022;
MD, F Fabry, P Plössl soon

- sum rules for momentum and quark number

integrals of DPDs over x_2 and $\boldsymbol{y}$ $\leftrightarrow$ PDF at x_1

  - provide constrains on model ansätze for DPDs
    J Gaunt, W Stirling 2009; K Golec-Biernat et al 2015, 2022;
    MD, J Gaunt, D Lang, P Plößl, A Schäfer 2020
  - formal proof: J Gaunt, PhD thesis 2012; MD, P Plößl, A Schäfer 2019

The small y limit

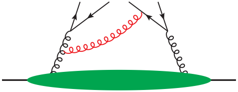
- for $y \ll 1/\Lambda$ can use operator product expansion

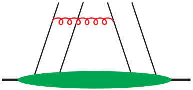
$$\mathcal{O}_{\text{twist 2}}(\mathbf{0})\, \mathcal{O}_{\text{twist 2}}(\boldsymbol{y}) = C_2(y) \otimes \mathcal{O}_{\text{twist 2}}(\mathbf{0}) + C_4(y) \otimes \mathcal{O}_{\text{twist 4}}(\mathbf{0}) + \ldots$$

  coefficient functions

  $C_2 \sim y^{-2}$ starts at $\mathcal{O}(\alpha_s)$
  $C_4 \sim y^{0}$ starts at $\mathcal{O}(\alpha_s^0)$

- splitting contribution: splitting kernel $\otimes$ PDF
- $\mathcal{O}(\alpha_s^2)$ kernels: MD, J Gaunt, P Plößl, A Schäfer 2019; MD, J Gaunt, P Plößl 2021
- quark mass effects: MD, R Nagar, P Plößl 2022





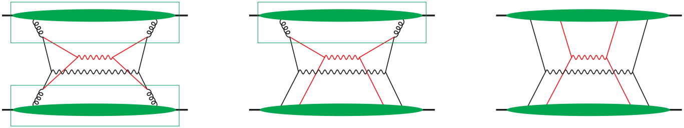
- intrinsic contribution
- subleading in $1/y^2$, but without α_s suppression
- expect stronger enhancement at small x_1, x_2

Splitting and double counting

- in small y limit: $F = F_{\text{spl}} + F_{\text{intr}}$ with $F_{\text{spl}} \propto y^{-2}$ and $F_{\text{intr}} \propto y^{0}$
- in DPS cross section this gives

$$\int d^2y \, \left[F_{\text{spl}}F_{\text{spl}} + F_{\text{spl}}F_{\text{intr}} + F_{\text{intr}}F_{\text{spl}} + F_{\text{intr}}F_{\text{intr}}\right]$$

1v1 + 1v2 + 2v1 + 2v2



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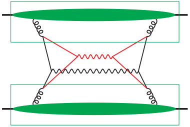

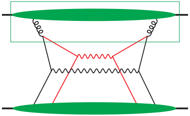

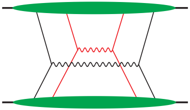

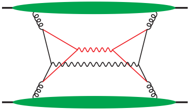

SPS (double box)

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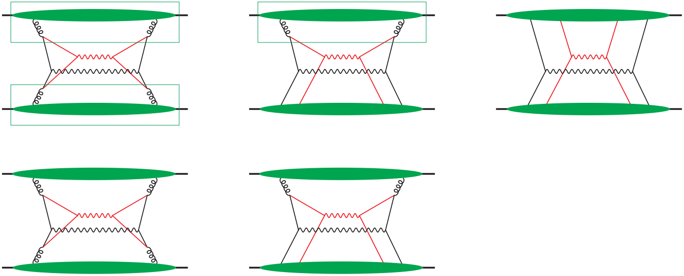


SPS (double box) twist 2 × twist 4

- scheme to remove double counting: MD, J Gaunt, K Schönwald 2017

DPDs at large $\boldsymbol{y}$: lattice studies

- Mellin moments of quark DPDs in x_1 and x_2
  $\rightsquigarrow$ matrix element $\langle p|J_1(y)J_2(0)|p\rangle$
  with local operators $J_1, J_2 =$ vector/axial vector/tensor current
- separation y between currents is spacelike
  $\rightsquigarrow$ can evaluate matrix element in Euclidean spacetime at $y^4 = 0$
- subtlety: DPD matrix element includes integral $\int_{-\infty}^{\infty} d(py)$
  but with $y^4 = 0$ have $|py| = |\vec{p}\,\vec{y}\,| \leq |\vec{p}\,||\vec{y}\,|$
  $\rightsquigarrow$ need lattice simulations for large hadron momenta $\vec{p}$
  similar paradigm as for quasi-PDFs, quasi-TMDs, ...
  $\Rightarrow$ can almost compute moments of DPDs on the lattice
- DPDs of the pion (2020) and the nucleon (2021)
  G Bali, L Castagnini, MD, J Gaunt, A Schäfer, Ch Zimmermann, et al
  follow-up studies in progress with D Reitinger

Different direction: quasi-DPDs on the lattice (no simulations yet)
M Jaarsma, R Rahn, W Waalewijn 2023; J-H Zhang 2023

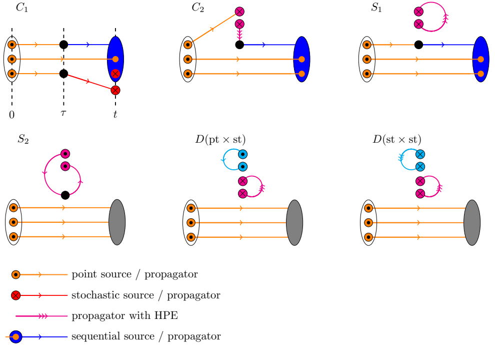
Lattice study for the nucleon

arXiv:2106.03541

- use 990 configurations from CLS ensemble H102:

id	β	a[fm]	$L^3 \times T$	κ_l	κ_s	$m_{\pi,K}$[MeV]	$m_\pi La$
H102	3.4	0.0856	$32^3 \times 96$	0.136865	0.136549339	355, 441	4.9

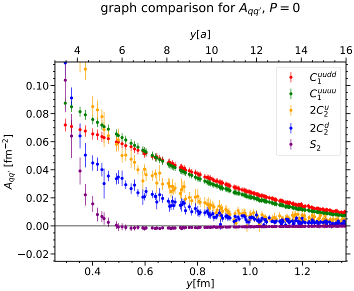
- all relevant combinations of two currents V, A, and T (unpolarised, longitudinally, and transversely polarised quarks)
- all relevant contractions, using various techniques

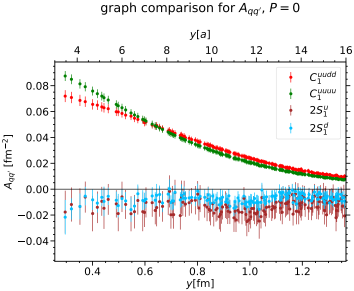


Lattice study for the nucleon

arXiv:2106.03541

- good signal for all graphs except doubly disconnected one
- connected graphs C_1, C_2 generally dominate
  S_2 may become important at small y,
  but in a region where have indications for discretisation effects





- following plots show sum of C_1 and C_2 contributions

Lattice study for the nucleon

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$$\int_{-\infty}^{\infty} d(py)\, A_{qq'}(py, y^2) = \int dx_1\, dx_2\, F_{(q-\bar{q})\,(q'-\bar{q}')}(x_1, x_2, \boldsymbol{y})$$

- following plots show matrix elements at $py = 0$

  same qualitative behaviour seen for Mellin moments reconstructed with a model ansatz for py dependence

Lattice study for the nucleon

arXiv:2106.03541

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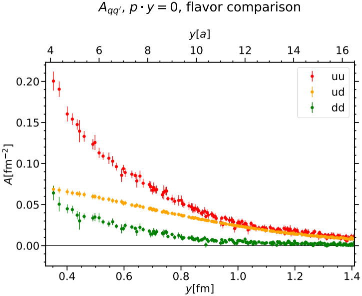
- see clear difference in y dependence between different flavours



incompatible with ansatz $F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, G(\boldsymbol{y})$
needed for DPS pocket formula

Lattice study for the nucleon

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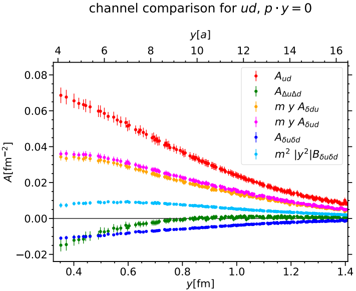
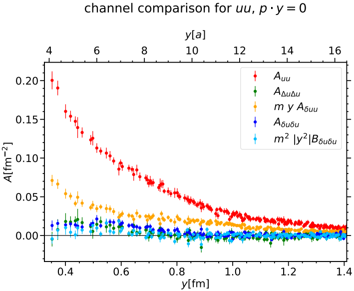
- spin dependence:





- find only small spin-spin correlations

note: model with static SU(6) symmetric uud wave function predicts

$$F_{\Delta u \Delta d}/F_{ud} = -2/3, \quad F_{\Delta u \Delta u}/F_{uu} = +1/3$$

Lattice study for the nucleon

arXiv:2106.03541

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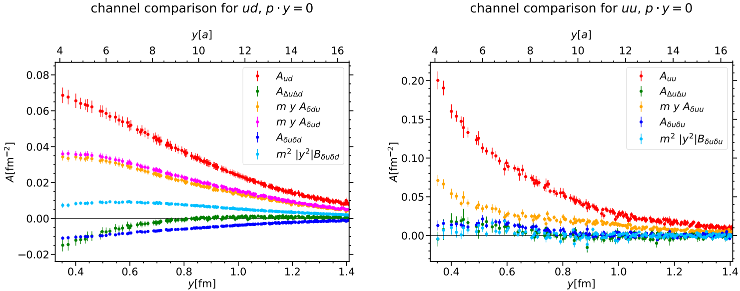

- largest spin effect: correlation between transverse polarisation of one quark and direction of $\boldsymbol{y}$, modulation $\propto (\vec{s}_q \times \vec{y}) \cdot \vec{p}$

Lattice study for the nucleon

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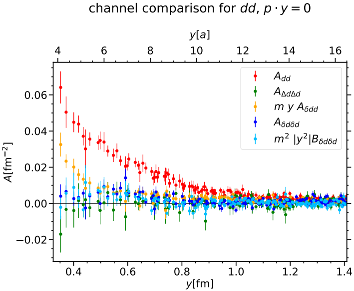
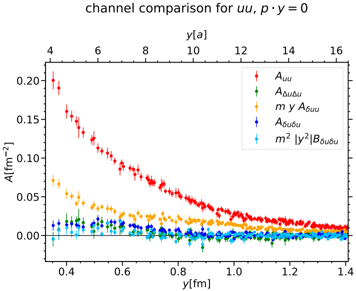




- largest spin effect: correlation between transverse polarisation of one quark and direction of $\boldsymbol{y}$, modulation $\propto (\vec{s}_q \times \vec{y}) \cdot \vec{p}$

  less clearly seen also for dd

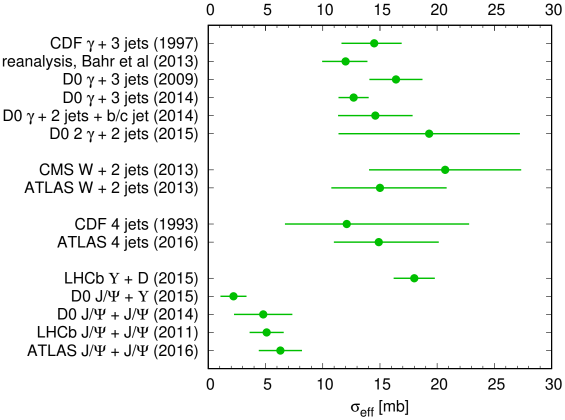
Summary

- after $\gtrsim 1$ decade of theory work have detailed understanding of factorisation for double parton scattering
- factorisation formula including correlations between two partons (kinematics, distance, quantum numbers)
- many (but not all) perturbative calculations for DPS at NLO accuracy
- information on moments of DPDs from lattice QCD
  $\rightsquigarrow$ flavour and spin dependence at large distances

Backup slides

Experimental investigations (incomplete)

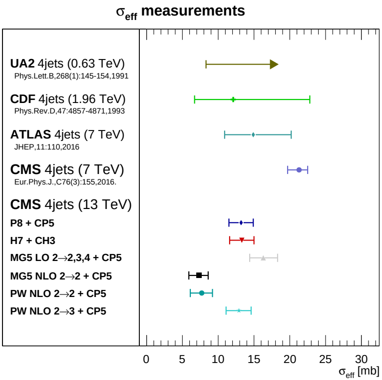
- a compilation of σ_{eff} values see also similar overview in PoS DIS2019 (2019) 258



- but cannot capture the physics of DPS in just one number σ_{eff} : cross sections, differential distributions

Experimental investigations (incomplete)

- extraction of σ_{eff} can have significant theory uncertainties
- example: 4 jets, CMS, arXiv:2109.13822
  different values for 13 GeV differ by adopted theory description of SPS

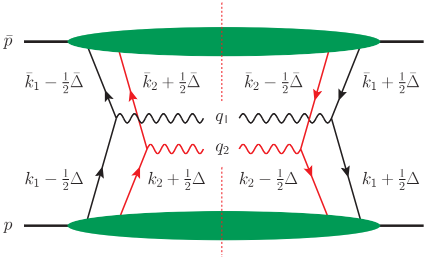


Experimental investigations (incomplete)

more LHC studies:

- run 1
  - double open charm $(D^0, D^+, D_s^+, \Lambda_c^+)$ and $J/\Psi+$ open charm — LHCb 2012
  - the same in p-Pb collisions — LHCb 2020
  - $\Upsilon + \Upsilon$ $(\sigma_{\text{eff}} \approx 2.2 \div 6.6\,\text{mb})$ — CMS 2016
  - $W + J/\Psi$ — ATLAS 2014, 2019
  - $Z + J/\Psi$ (limit on σ_{eff}) — ATLAS 2014
  - 4 leptons (limit on σ_{eff}) — ATLAS 2018
  - same-sign WW (limit on σ_{eff}) — CMS 2017
- run 2
  - $J/\Psi + J/\Psi$ $(\sigma_{\text{eff}} \approx 8.8 \div 12.5\,\text{mb})$ — LHCb 2016
  - Z + jets — CMS 2021
  - same-sign WW (observation) — CMS 2019
  - 4 jets (range of σ_{eff} values) — CMS 2022

Feynman graphs: momentum vs. distance



$\bar{\Delta} = -\Delta$

- large (plus or minus) momenta of partons fixed by final state
  $\rightsquigarrow$ equal in amplitude $\mathcal{A}$ and conjugate amplitude $\mathcal{A}^*$
- transverse parton momenta not equal in $\mathcal{A}$ and in $\mathcal{A}^*$
  cross section $\propto \int d^2\boldsymbol{\Delta}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{\Delta}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, -\boldsymbol{\Delta})$
- Fourier trf. to impact parameter: $F(x_i, \boldsymbol{k}_i, \boldsymbol{\Delta}) \to F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$
  cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  = equal in both colliding protons