What's next? Novel computational frontiers in QCD

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QCD on & off the lattice, *Regensburg, September 2023*

Computational frontiers

- Lattice gauge theory:
  - Bigger, faster, clever algorithms, ...
  - More fully controlled observables
  - Higher precision

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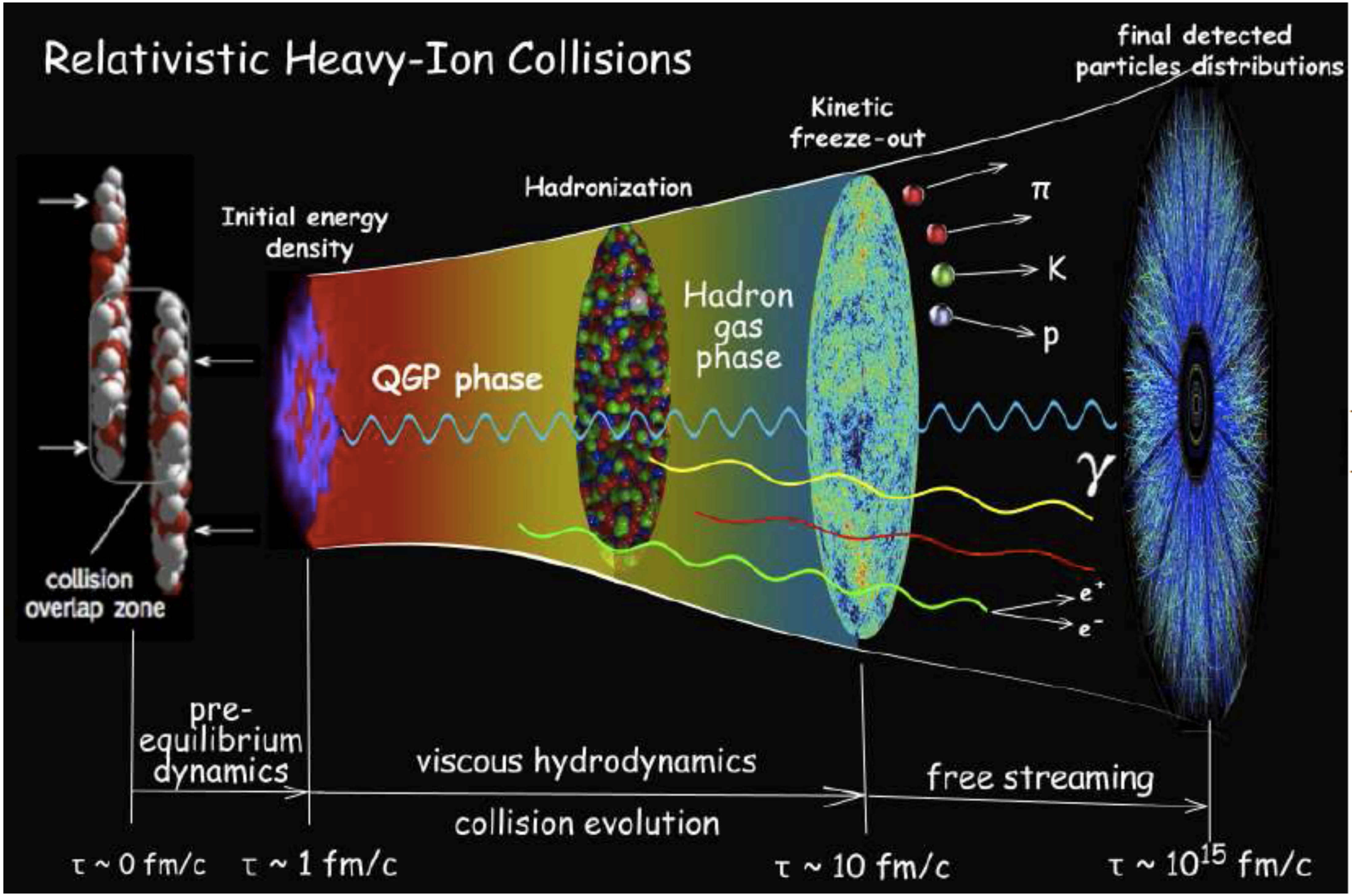
Computational frontiers

- Numerical holography
  - Real-time dynamics: pre-hydrodynamic evolution in heavy ion collisions
  - Better justified than alternative weak-coupling models
  - Computationally challenging
- Large-N QCD
  - Nested classical limits
  - Requires decent truncation of infinite-dimensional phase space
  - Worth the trouble?

Heavy ion collisions



Shen & Heinz, 1507.01558

Heavy ion collisions

- Accessible quark-gluon plasma:
  - Low viscosity, $\eta/s \approx 0.1$
  - Effective temperatures T_{eff} = few $\times T_c$, not $\ggg T_c$
  - Effective coupling $\propto 1/\ln(T_{\text{eff}}/T_c)$ not at all small!
  - Substantial thermal masses, $m_{\text{th}}/T = O(1)$, not $\ll 1$
  - Near-conformal, $(\epsilon - 3p)/\epsilon$ small except very close to T_c

➡ Accessible QGP = strongly coupled plasma, not weakly coupled!

- Color-glass condensate (IP-Glasma) modeling of initial state:
  - Beautiful picture of asymptopia: arbitrarily weak coupling, highly collinear gluon dynamics, elaborate hierarchy of scales, logarithmic evolution, …
  - Asymptopia is very, very far from accessible QGP!

➡ Instantaneous switch from weak-coupling to strong coupling (fluid) description is inherently inconsistent!

Holographic modeling

- Complementary model:

Early-stage QGP = strongly coupled, near-conformal non-Abelian plasma ≈ strongly coupled, maximally supersymmetric ($\mathcal{N} = 4$) Yang-Mills plasma

hot QCD	$\mathcal{N} = 4$ SYM
- non-Abelian plasma	- non-Abelian plasma
- neutral fluid hydro	- neutral fluid hydro
- weak dependence on N_c	- weak dependence on N_c
- strongly coupled	- fixed, arbitrary coupling
- near-conformal prior to hadronization	- conformal

- Use gauge/gravity duality to solve (honestly) pre-hydrodynamic evolution of initial states in strongly coupled $\mathcal{N} = 4$ SYM which resemble real colliding nuclei

Holographic modeling

- Large N, strong coupling dynamics of $\mathcal{N} = 4$ SYM correctly described by classical 5D asymptically-AdS gravitational dynamics

- Solve gravitational dynamics to model early stage heavy-ion collisions:

  - Incoming projectile energy density = realistic model of nuclear energy density, Lorentz contracted (γ = O(100)). Uniquely determines bulk geometry corresponding to incident projectile.

  - Superpose well-separated projectiles, transform to infalling coordinates ➨ gravitational initial data.

  - Solve 5D Einstein equations with asymptotically-AdS boundary conditions & extract boundary stress-energy tensor $\langle T^{\mu\nu} \rangle$.

  - Evolve to onset of hydrodynamic regime, holographic $\langle T^{\mu\nu} \rangle$ ➨ initial data for further hydrodynamic evolution.

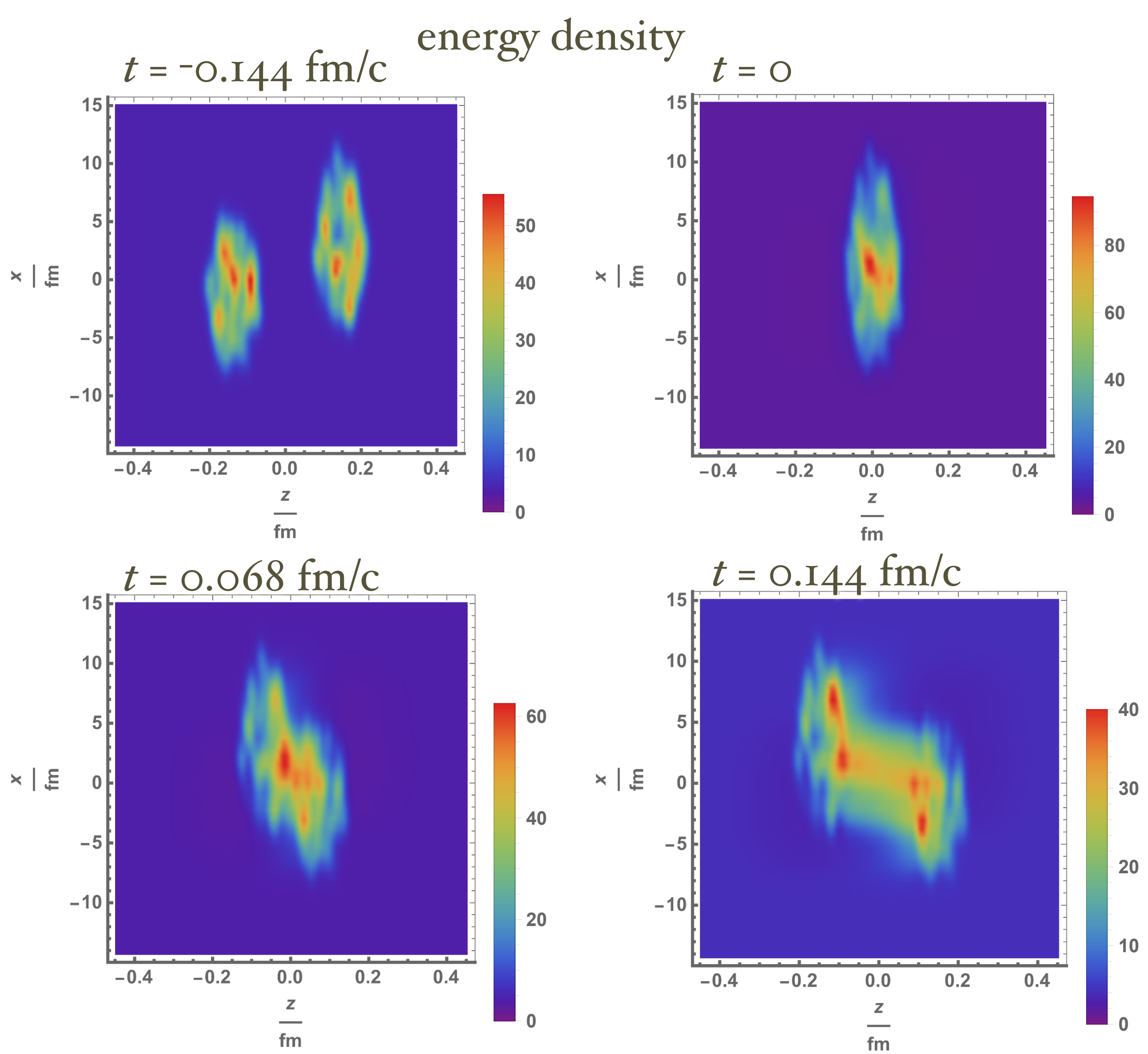
- Earlier work: planar collisions, smooth projectiles, "pixel-by-pixel" phenomenology

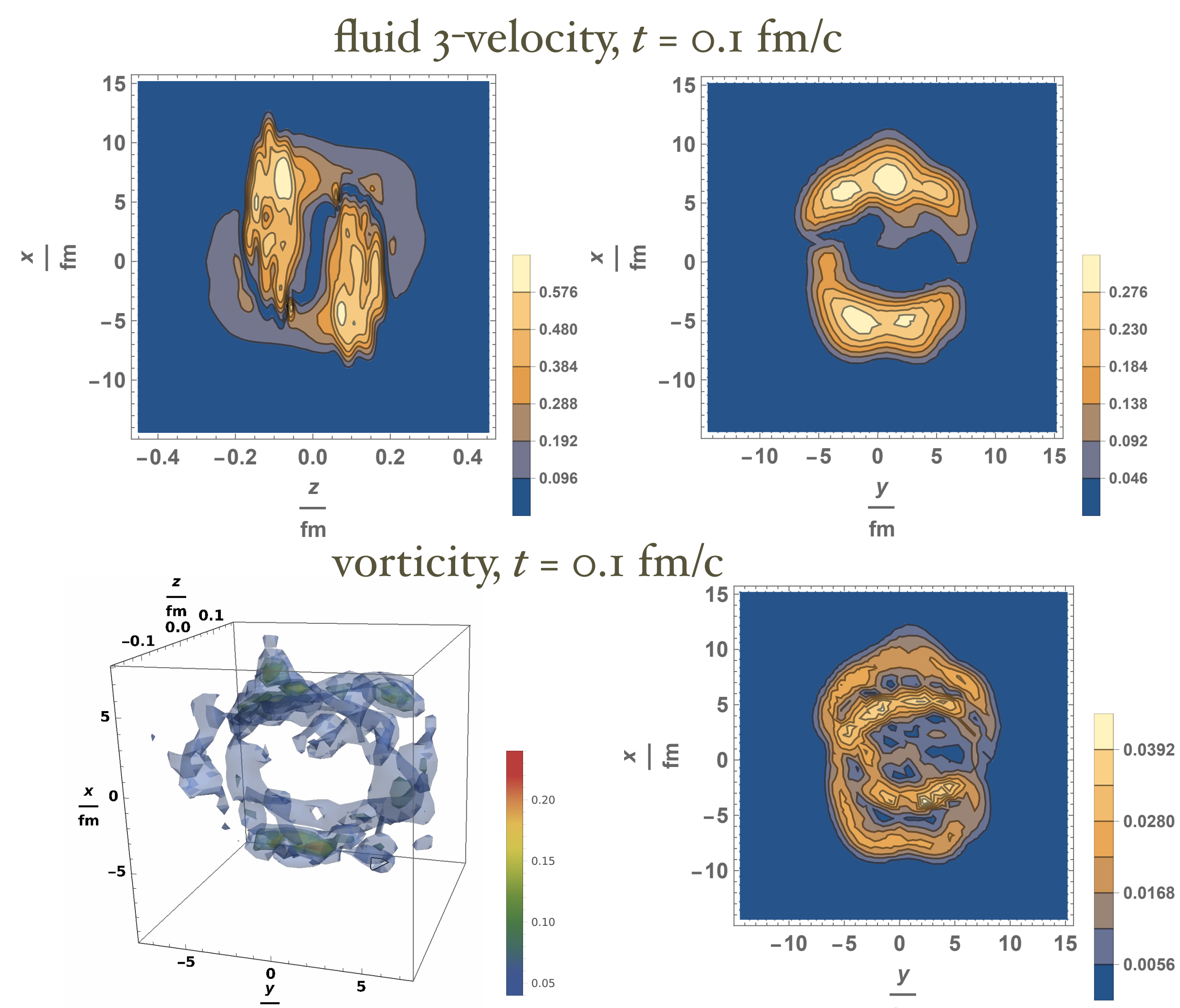
w. Paul Chesler, Berndt Müller, Andreas Schäfer, Sebastian Waeber

Gravitational dynamics

- Einstein GR = complicated, coupled 5D PDEs! But feasible:
  - Infalling coordinates ➨ nested linear equations
  - Spectral methods ➨ allow relatively coarse numerical grid
  - Transverse derivative expansion ➨ simplifies equations, O(10) speed-up & memory reduction

work w. Sebastian Waeber
2206.01819, 2211.09190





Gravitational dynamics

- Localized collisions, work to-date:
  - Mathematica implementation on multi-core workstation w. 128 Gb memory & unified memory architecture
  - Spectral methods ➨ long-range derivative discretizations, large matrices
  - Running time ≈ few weeks
- Needed improvements:
  - Faster! Plug-in module for use with hydro codes
  - Coding in C++?
  - Efficient implementation on distributed clusters? GPUs?
  - Great opportunity for someone with computational skills looking for new challenge!

Large-N QCD

- Planar diagrams dominate
- Vanishing meson & glueball widths
- Scattering amplitudes $\sim$ $(1/N)^{\#\,\text{particles} - 2}$
- Baryons $\sim$ solitons
- Factorization: $\langle AB \rangle \to \langle A \rangle \langle B \rangle$ for suitable observables
- Volume independence in confining phase
- Closed algebraic equations for Wilson loop expectations

Migdal & Makeenko '79

$$W_\Gamma \equiv \langle \tfrac{1}{N} \mathrm{tr}\, \mathscr{P}\, e^{\oint_\Gamma A\cdot dx} \rangle \qquad W_\Gamma = \sum_{\Gamma'} a_\Gamma^{\Gamma'} W_{\Gamma'} + \sum_{\Gamma',\Gamma''} b_\Gamma^{\Gamma'\Gamma''} W_{\Gamma'} W_{\Gamma''}$$

Large-N QCD

- Large-N limit = classical limit

LGY '82

  - Large N coherent states $\{\,|u\rangle\}$ = coadjoint orbit of ∞-dim Lie group = classical phase space

  - "Decent" quantum operators → classical observables: $a(u) = \lim_{N\to\infty} \langle u|A|u\rangle$

  - Vanishing overlaps: $\langle u|u'\rangle \sim e^{-N^2 f(u,u')}$ ➨ factorization

$$\lim_{N\to\infty} \langle u|AB|u\rangle = \langle u|A|u\rangle\langle u|B|u\rangle = a(u)\,b(u)$$

  - Classical action: $S_{\rm cl}[u(t)] \equiv \lim_{N\to\infty} \frac{1}{N^2}\int dt\, \langle u|i\partial_t - \hat{H}|u\rangle$

    ➨ ground state properties, spectrum, scattering amplitudes, ..

  - Fundamental representation quarks ➨ nested classical limits

    - $O(N^2)$ action = gluon dynamics, subleading $O(N)$ action = fermion dynamics

Numerical solution of $N = \infty$ QCD?

- Multiple approaches:

  - Solve Euclidean loop equations ➨ Wilson loop expectation values

  - Minimize Euclidean free energy $F[\rho] \equiv E[\rho] - TS[\rho]$ within restricted space of factorizing "coherent" density matrices $\{\rho[\{W_\Gamma\}]\}$ ➨ Wilson loop expectations & correlators

  - Minimize classical Hamiltonian $h_{\text{glue}}(u) \equiv \lim_{N\to\infty} N^{-2}\langle u\,|\,\hat{H}\,|\,u\rangle$ ➨ equal time Wilson loop expectations, then:

    - expand $S_{\text{cl}}[u]$ about minimum, small oscillation frequencies ➨ glueball masses, ...
    - minimize $h_{\text{quark}}[v;u] \equiv \lim_{N\to\infty} \frac{1}{N}\langle u, v\,|\,\hat{H}_{\text{quark}}\,|\,u, v\rangle$ over fermion coherent states ➨ fermion bilinear expectations
    - expand associated $N = \infty$ fermion classical action ➨ meson masses, scattering amplitudes

- All approaches require truncation of infinite dimensional space to some finite dimensional approximation, e.g., $W_{\text{big complicated loop}} \approx f[\{W_{\text{smaller loops}}\}]$

Numerical solution of $N = \infty$ QCD?

- "Naive" truncation explored in mid 80's: set all but selected loops to zero
- Better truncation schemes needed, e.g, factorization of self-intersecting loops, including loops with one or two electric field insertions
- Tricky programming to make efficient: loop decomposition, canonicalization, commutation of loops w. E-field insertions
- Is it worth the trouble?
  - Just for large-N Wilson loop expectations — maybe not?
  - But to do meson (& glueball) spectra, decay widths, …?
- Great opportunity for someone with excellent programming skills looking for new challenge!

Conclusion

- Want to explore new directions?
- Don't want to follow the crowd?
- Sign-up today !!!