Decoherence and Thermalization in Heavy Ion Collisions

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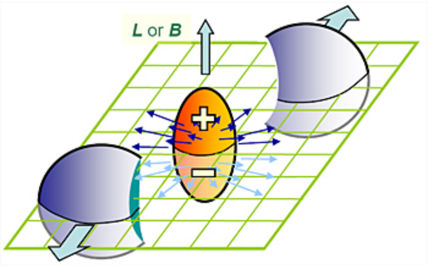
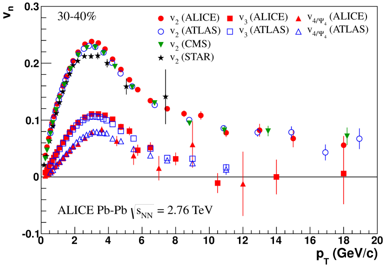
- One of the main tasks of Quantum Information Theory is to understand decoherence and thermalization of many-body quantum systems.
- We discuss this from the point of view of heavy ion physics.

Key questions of relativistic heavy ion physics: Does the quark gluon plasma realy thermalize? Is “hydrodynamization” equivalent to thermalization? Does thermal Lattice QCD describe Experiment?





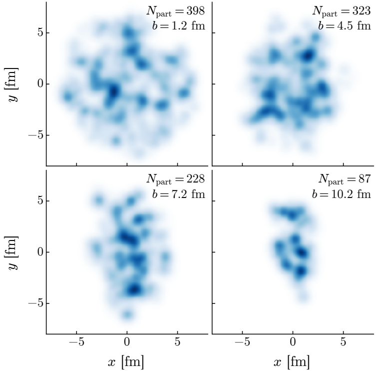
Observable: Elliptic flow $v_n \sim \cos(n\phi)$ with $n = 2$

How can transverse communication happen in less than 1fm/c?

$\gamma(Pb) > 2500$ giving it a width of $11\text{fm}/2500 = 0.004\text{fm}$

In QCD the transverse color coherence length is of order $1/Q_s < 0.2$ fm which is much smaller than the transverse size.

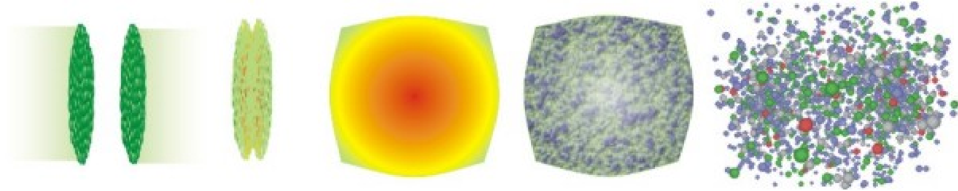
arXiv:1605.03954



Also: Entropy cannot be produced because QCD is T-invariant! The apparent hydrodynamization must be observable dependent.⇒ ETH "Eigenstate Thermalization Hypothesis"

⇒ Focus on anomalies

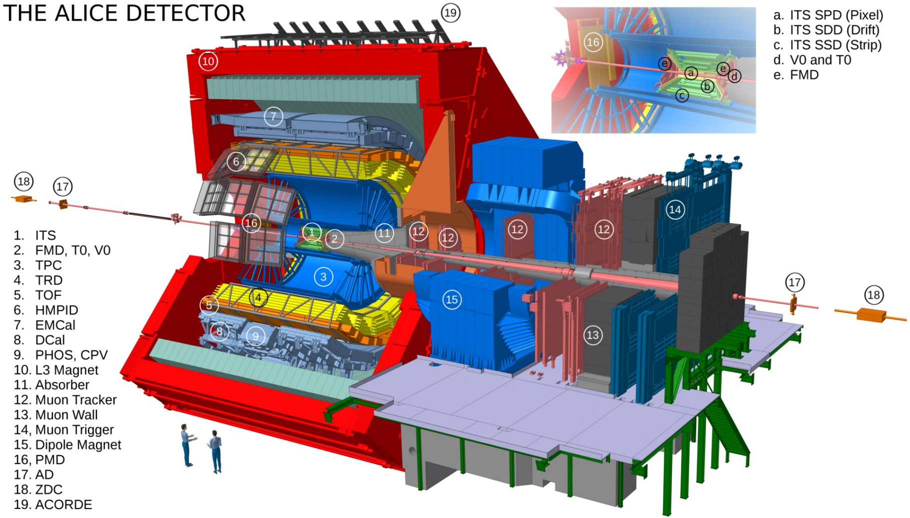
Just one example, the hadron yields: arXiv:1809.04681, ALICE, CERN

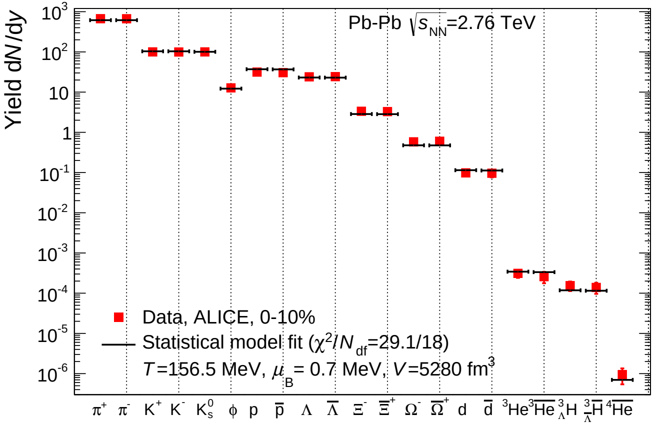


AdS/CFT clarified that hydrodynamization (local obervables) is fast.

ETH requires much longer to apply, see below (system wide correlations).

There is very much high precision data, e.g. from ALICE.





But: R(rms , $^3_\Lambda H$)=10.6 fm~ $2R_{\rm Pb}$;

$-B = 0.4 MeV << 156$ MeV the yield should be suppressed

One has two convincingly motivated interpretations which seem to be contradictory

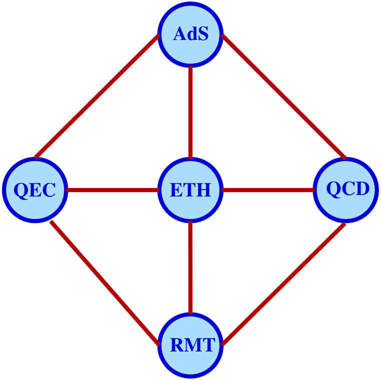
- Hundreds of detailed measurements support the fireball interpretation, i.e. entropy production, hydrodynamics etc.
- General T-invariance suggest a microcanonical picture with highly entangled many particle quark-gluon and hadronic states.

Proponents of both lines of argument seem to be correct. How can this be?

One needs two standard elements of quantum information theory: Page curve plus ETH. The Page curve reminds of the information problem of BH physics and in fact it is argued that both are very similar.

ETH could explain the ${}^{3}_{\Lambda}H$ puzzle.

ETH predicts that small probes thermalize fast, large probes thermalize slowly and probes of > half the system do not thermalize completely (no cloning theorem).



ETH: D'Alesio, Kafri, Polkovnikov, Rigol 1509.06411

$$O_{mn} = \langle m|\hat{O}|n\rangle = O(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{mn}$$

$\bar{E} = (E_m + E_n)/2$, $\omega = E_m - E_m)$, $S(\bar{E})$ thermodynamic entropy at energy $\bar{E}$, $O(\bar{E})$ and $f_O(\bar{E},\omega)$ are smooth functions, $O(\bar{E})$ is identical to the expectation value of the microcanonical ensemble at energy $\bar{E}$, and R_{mn} is a strongly fluctuating but not really a random matrix in the sense of RMT.

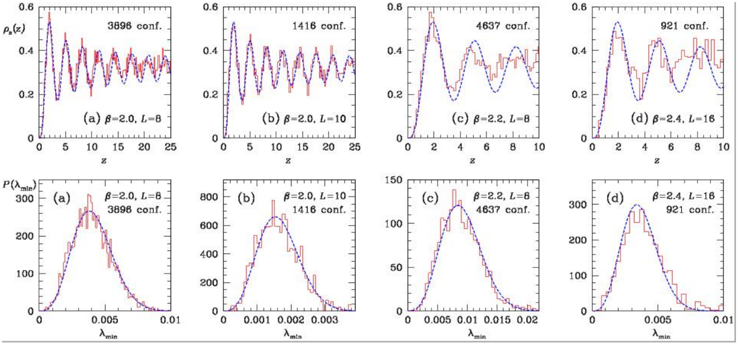
Questions: For which operators does ETH apply? To which QCD operators does it apply?

QCD is a prime example for an ergodic theory.

A HIC in the ultra vacuum of the LHC is a prime example for an isolated system.

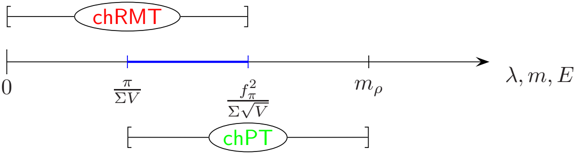
A long story: Berbenni-Bitsch, Meyer, AS, Verbaarschot and Wettig, “Microscopic universality in the spectrum of the lattice Dirac operator,” hep-lat/9704018

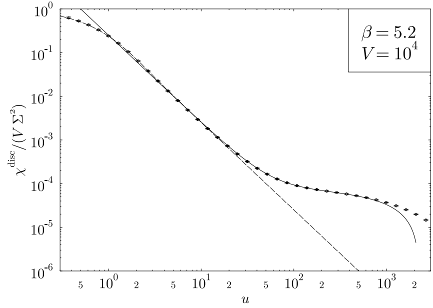
Comparison of microscopic level spacing for LQCD (red) and RMT(blue)



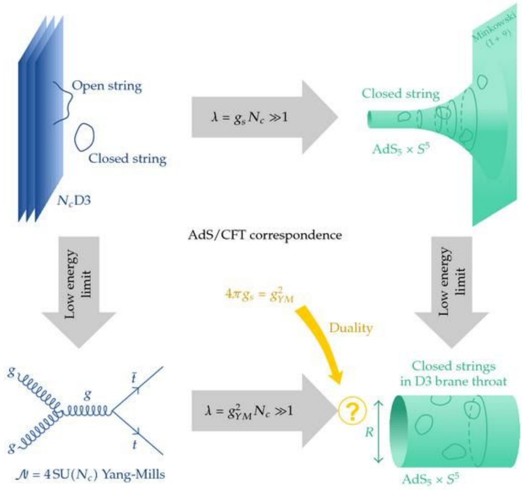
Simulations with quenched SU(3) Kogut-Susskind fermions

M. Göckeler, H. Hehl, P. Rakow, AS, T. Wettig hep-lat/0105011

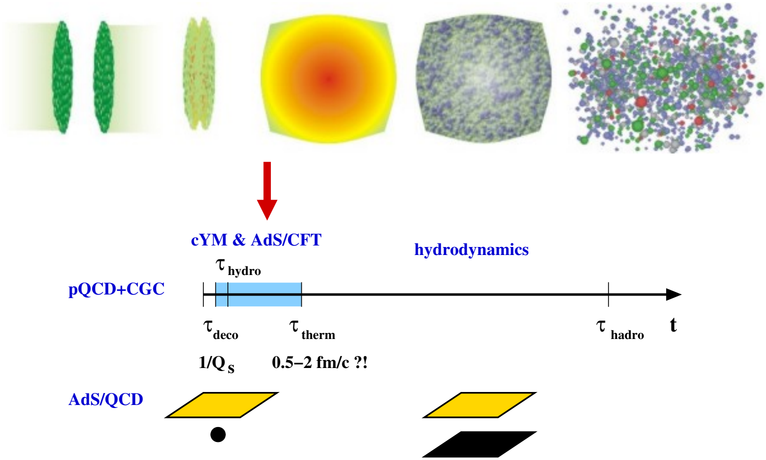




A naive sketch of AdS/CFT

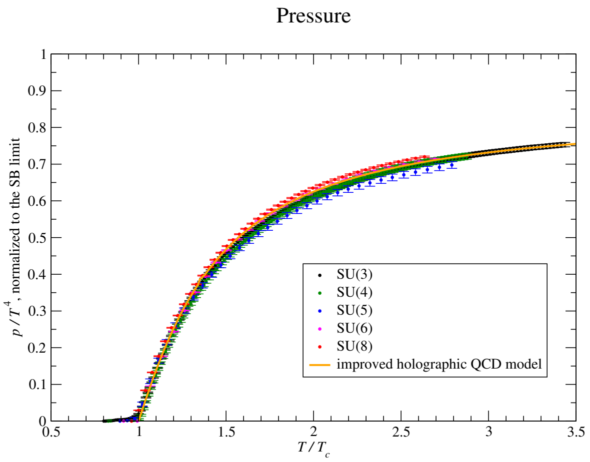


The AdS/CFT picture of HICs



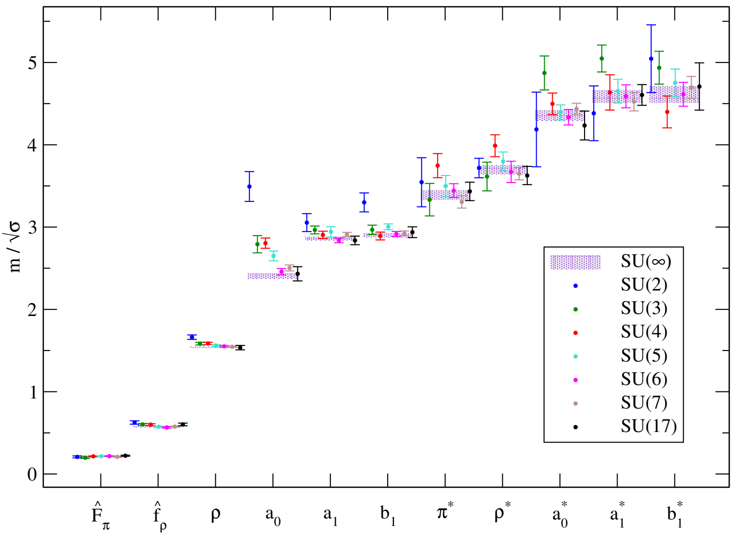
entropy production = information loss

We published various tests, including lattice ones.



SU(*N*) pure gauge theory in 1+3 dimensions

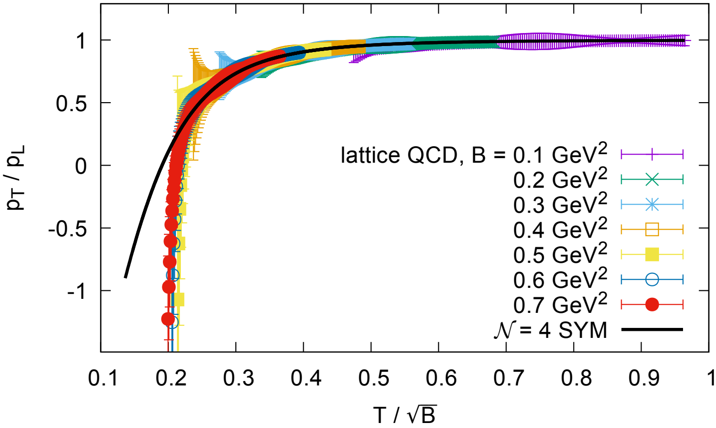
M. Panero, 0907.3719



T = 0 meson spectrum and decay constants

G. Bali et. al, 1304.4437

Another test: QCD has no conformal symmetry (e.g scale anomaly, Λ_{QCD}) AdS is $\Rightarrow$ What happens if you break conformal symmetry explicitly by a background magnetic field? Endrodi, Kaminski, A.S, Wu and Yaffe, [arXiv:1806.09632].



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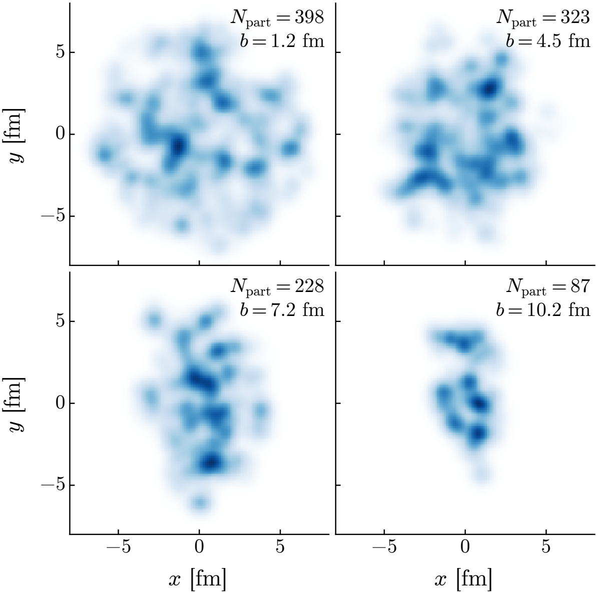
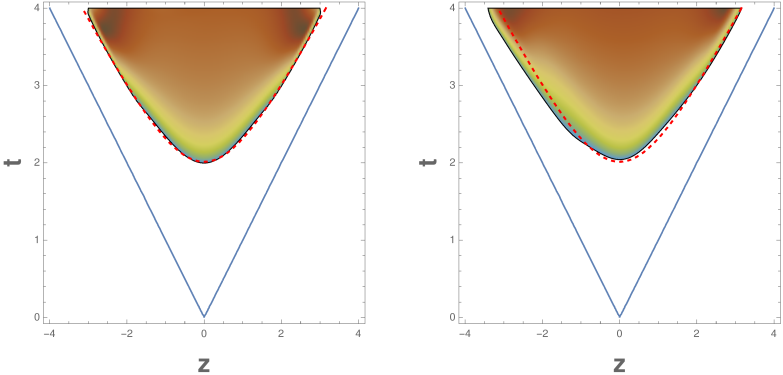

$N_{\text{part}} = 398$
$b = 1.2$ fm
$N_{\text{part}} = 323$
$b = 4.5$ fm
$N_{\text{part}} = 228$
$b = 7.2$ fm
$N_{\text{part}} = 87$
$b = 10.2$ fm
y [fm]
x [fm]
5
0
−5

Also this can be described by AdS/CFT 1906.05086 Waeber, Yaffe et al.



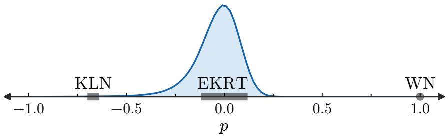
answer: Hydrodynamization occurs at **fixed eigenzeit** ⇒ basically not boost dependent, geometric mean

criterium: $\Delta = \frac{1}{p}\sqrt{\delta T^{\mu\nu}\ \delta T_{\mu\nu}} < 0.15$ with $\delta T^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{\mathrm{hydro}}$

Bernhard, Moreland, Bass Liu, Heinz arXiv:1605.03954 Fit result: parameterization of combined entropy density:

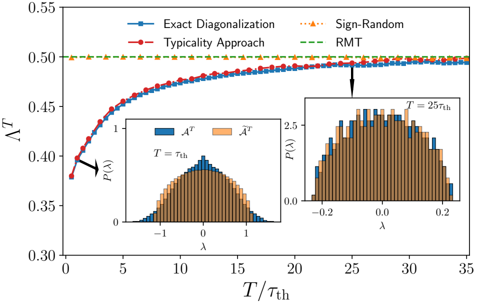
$$s \sim \left(\frac{\tilde{T}_A^p + \tilde{T}_B^p}{2}\right)^{1/p} \quad , \quad -\infty \leq p \leq \infty$$

$$\tilde{T}(x_{perp}) = \sum_{i=1}^{N_{part}} \frac{\gamma_i}{2\pi w^2} \exp\left(-\frac{(x_\perp - x_{i,\perp})^2}{2w^2}\right)$$



By construction the hydro initialization time must be identical for each transverse pixel. Both features are reproduced by AdS/CFT 1906.05086

But ETH does NOT apply in all situations (also not for OTOCs).
Wang, Lamann, Richter, Steinigeweg, Dymarsky 2110.04085
The time needed to establish ETH behavior depends on the observable. Here for an Ising spin chain. It can take much longer than a HIC.



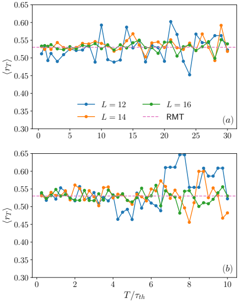
$$\Lambda^T = \frac{\mathcal{M}_2^2}{\mathcal{M}_4}; \qquad \mathcal{M}_k = \mathrm{Tr}[(\mathcal{O}_c^T)^k]/d; \qquad \mathcal{O}_c^T = \mathcal{O}^T - \mathrm{Tr}(\mathcal{O}^T)/d \quad (1)$$

energy window $\left[-\frac{\pi}{T}, \frac{\pi}{T}\right]$

the mean ratio of adjacent level spacings

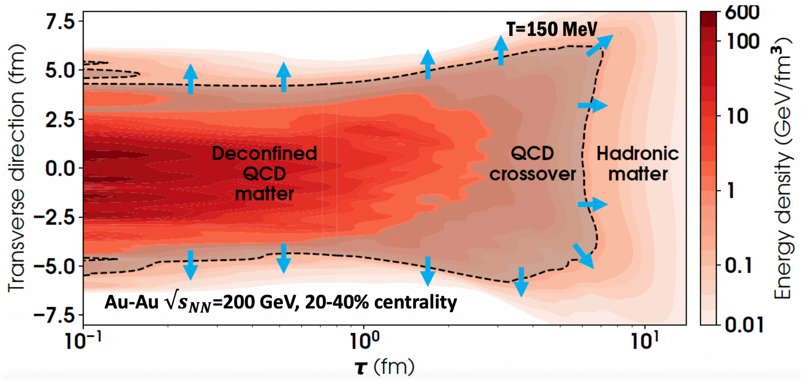
$$\langle r_T \rangle = \frac{1}{d} \sum_{\alpha} \frac{\min(\Delta_\alpha, \Delta_{\alpha+1})}{\max(\Delta_\alpha, \Delta_{\alpha+1})}$$

gap between two adjacent eigenvalues $\Delta_\alpha = |\lambda^T_{\alpha+1} - \lambda^T_\alpha|$ of $\mathcal{O}^T$



We do the same for SU(2).

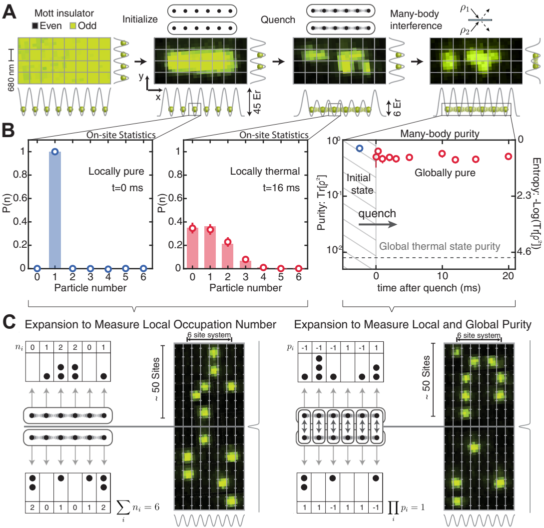
The long times dynamics of HICs is complicated. Gale et al., arXiv:2009.07841 (80% final freeze-out, 20 % hadron radiation)

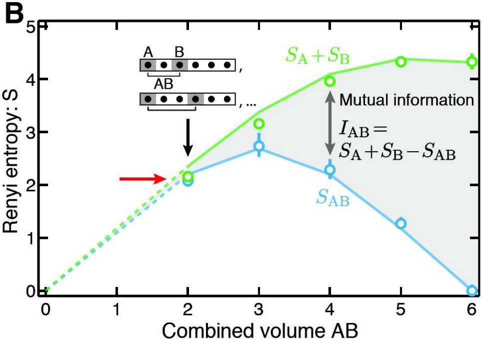


So: How does τ_{ETH} compare for hadron production?

The second crucial feature besides ETH: The Page curve

The experiment arXiv:1603.04409 “Quantum thermalization through entanglement in an isolated many-body system”

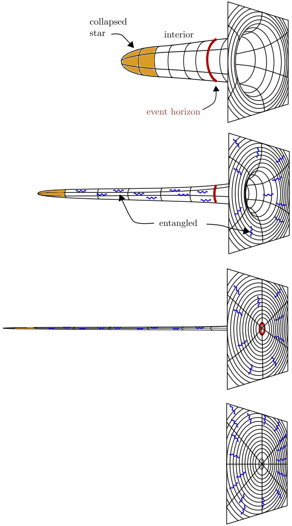


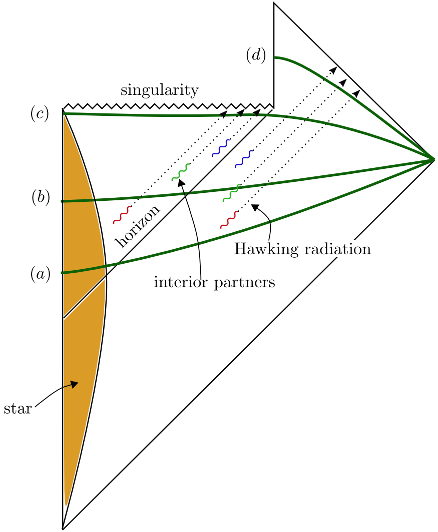


To describe HICs one needs a hadronization mechanism which produces a Page curve.

Why not copy ideas from black hole physics?

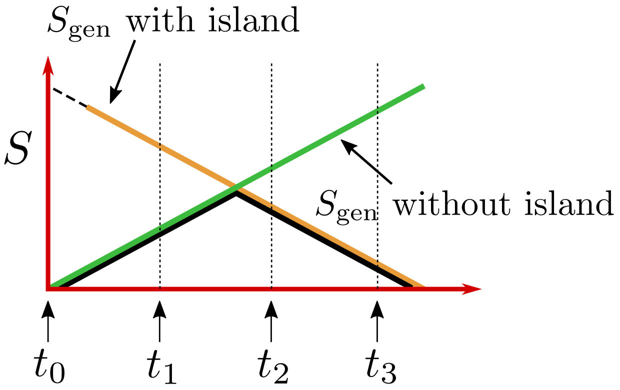
A new description of BH evaportion Almheiri et al. 2006.06872



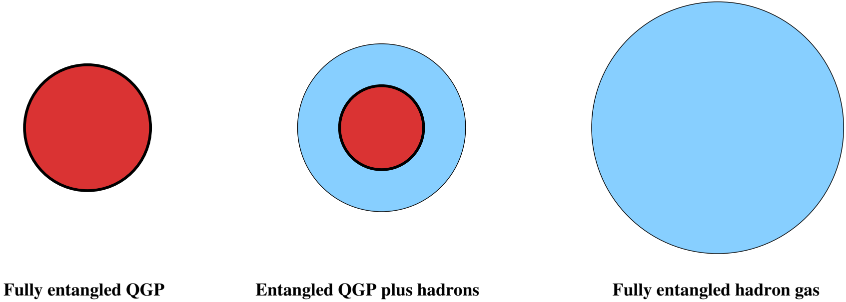


green: spatial slices

The Hawking radiation is entangled with an “island”.
This results in the Page curve

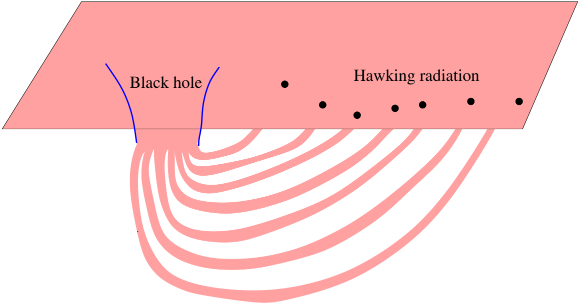


An analogy:



There is a marked difference: BH infalling particles move balistically
QGP: Infalling holes in a medium → rather inward propagating entanglement wave with $v_E \leq c$.

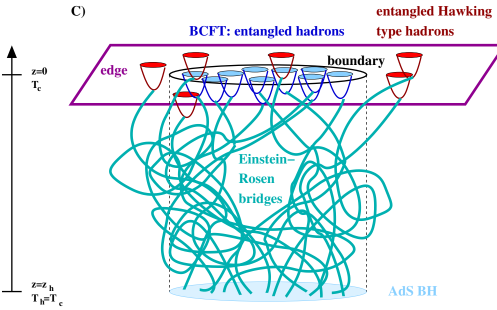
entangled CFT's in the boundary = Einstein-Rosen bridges in the holographical dual (EPR=ER). This idea was already in Maldacena and Susskind 1306.0533



Proving entanglement with Bell inequalities is supposedly a NP-complete problem. The only chance is to translate this problem with AdS/CFT into a purely geometric one.

ER=EPR

Tractable calculations are only possible in low dimension → JT-gravity. For example: Anderson et al. 2103.14746



Conclusions

- ETH, decoherence and thermalization of isolated quantum systems are topics of universal interest.
- Heavy Ion Collisions in the ultra-high vacuum of, e.g. the LHC, offer an ideal situation to study them. There are many Pbyte of data, the question is how to interpret it.
- There exist many technically different approaches (classical nonlinear dynamics, RMT and ETH, AdS/CFT, QCD phenomenology, pQCD, hydrodynamics, quantum computing ...) which are expected to provide compatible pieces of this puzzle.
- The most interesting research fields are the various overlap regions of these fields.

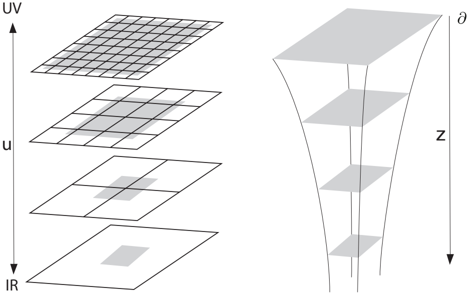
The ideas behind AdS/CFT nice review: Ramallo 1310.4319 renormalization flow of a SU(N) vertex function on ever coarser lattices

$$V(x,a) \rightarrow V(x,2a) \rightarrow V(x,4a) \rightarrow \ldots$$

$$u = a,\ 2a,\ 4a$$

$$\frac{\partial}{\partial \log u} g(u) = \beta(u)$$

$$\mathrm{J}|_{\mathrm{UV}} = \Phi|_{\partial}$$



geometric interpretation of new coordinate called z

$$ds^2 = \Omega^2(z)\,[dt^2 - dx^i dx^i - dz^2]$$

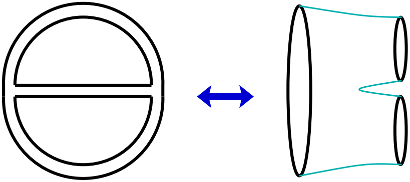
The properties of the renormalization flow is only simple for conformal theories.

$$\begin{aligned} z &\rightarrow \lambda z \\ \Omega(z) &= \frac{L}{z} \rightarrow \lambda^{-1}\Omega(z) \\ ds^2 &= \frac{L^2}{z^2}\,[dt^2 - dx^i dx^i - dz^2] \quad \text{AdS} - \text{metric} \end{aligned}$$

$SU(N), \mathcal{N} = 4$ is conformal

quantum corrections $\sim \left(\frac{\ell_{Pl}}{L}\right)^8 = \frac{\pi^4}{2N^2}$ are small for large N.

string theory formulation $AdS_5 \times S^5$; $R_S^4/(\alpha')^2 = 4\pi\sqrt{\lambda N}$.

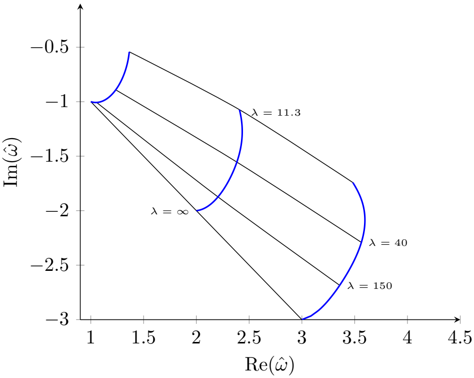


planar amplitude **string amplitude**

Thus the crucial questions are:

- is $N = 3$ already large ?
  answer: dedicated lattice calculations
- Are the effects of non-conformality of QCD calculable ?
  answer: yes, e.g. with conformal perturbation theory.

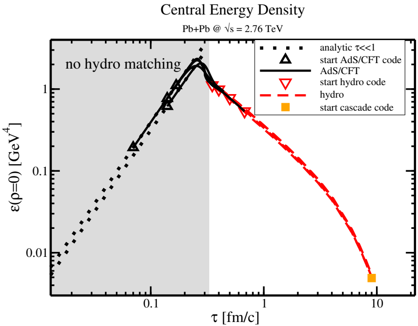
Finite coupling (QFT) corrections correspond to weak coupling quantum corrections in string theory
Waeber and AS, arXiv:1804.01912, The Quasi Normal Mode (QNM) spectrum.

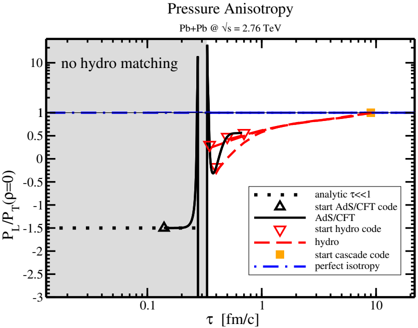


$\tau_{hydro} = -1/(Im\,\omega_{QNM})$ confirms earlier result and makes it more precise ($\tau_{hydro} \sim 0.02$ fm/c)

The AdS gravity equations result in a smooth transition to hydrodynamics. Viscous relativistic hydrodynamics is a gradient expansion which fails at early times. The late time behavior seems to be very stable and confirms perfect thermal and hydrodynamic behavior from 1fm/c on.

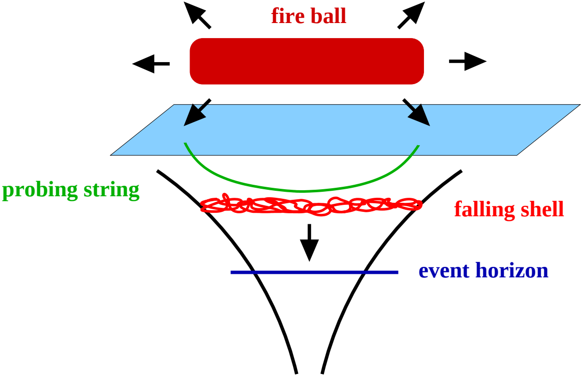
Hydrodynamics must, in fact, already apply at 1 fm/c to describe v_2 etc. This can be explained by AdS/CFT: Schee, Romatschke, Pratt 1307.2539





Equilibration times from AdS/CFT

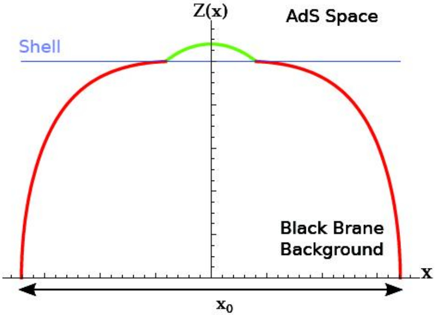
Idea: Probe black brane formation with a string or membrane, breaking conformal invariance by a “quench”
PRL: 1012.4753 PRD: 1103.2683



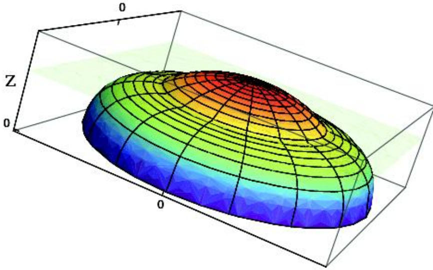
We solved analytically and numerically different cases:

$AdS_3 \sim CFT(1+1)$, $AdS_4 \sim CFT(1+2)$, $AdS_5 \sim CFT(1+3)$

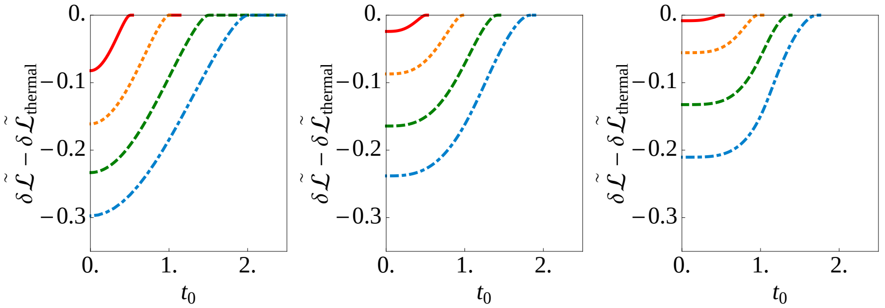
and analyzed how the length of the geodesic/the area of the surface approaches its thermal value, as a function of ℓ and t_0.



(A)



(B)

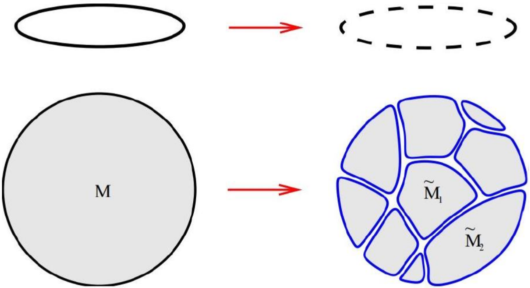


$\delta\tilde{\mathcal{L}} - \delta\tilde{\mathcal{L}}_{\text{thermal}}$ ($\tilde{\mathcal{L}} \equiv \mathcal{L}/\ell$) for $d = 2, 3, 4$ (left,right, middle) and $\ell = 1, 2, 3, 4$ (top to bottom curve).

Thermalization is approached as fast as compatible with causality. But the initial state (“Null dust”) has already entropy. Only valid for local observables, which is sufficient for hydrodynamization.

To reproduce ETH a more comprehensive description is needed, which also has to cover much longer time scales.

We need the holographic dual: van Raamsdonk Science 370 (2020) 6513, 198; Mary and Van Raamsdonk 2011.14258



A system of entangled BCFT-bits (Boundary CFT) is nearly holographically equivalent to a CFT (examples for 1+1 and 2+1 dimensions). QCD → BCFT.

Questions

B. Müller and A. Schäfer, “Quark-Hadron Transition and Entanglement,” arXiv:2211.16265

- Can one treat SU(2) lattice gauge theory in 1+1 and 1+2 and 1+3 dimensions? next talk
- Numerical AdS/CFT calculations cover only the first 1fm/c. How can one obtain the second half of the Page curve (assuming that as for black holes no information gets lost)?
- Can one find a holographic dual to hadronization a la Hawking radiation?
- Can one generalize JT calculation to many BHs and holographic tensor networks?
- Can one calculate the differences between QCD and CFT also for hadronization?