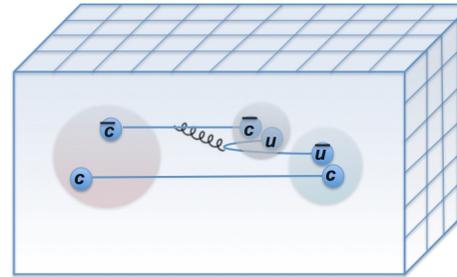


Conventional and exotic charmonia from lattice QCD

$\bar{c}c$, $\bar{c}q\bar{q}c$



Sasa Prelovsek University of Ljubljana & Jozef Stefan Institute, Slovenia

(guest professor at Regensburg and member of SFB, 2016-2021)

QCD on and off the lattice, Regensburg University, 18th September, 2023

Sara Collins, Daniel Mohler, M. Padmanath, Stefano Piemonte, SP

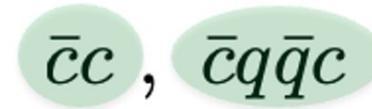
1811.04116, PRD 2019 : with **Andreas Schafer** and Simon Weishaeupl

1905.03506, PRD2019

2011.02542, JHEP 2021

2111.02934 : review

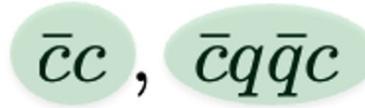
Outline : Conventional and exotic charmonia



- motivation
- glimpse of the results
- path to the results
- results (again)
comparison to exp, implications
-
- challenges & how we addressed them
- challenges that remain open

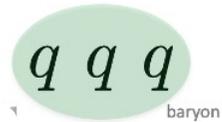
Motivation

Charmonium(like) states

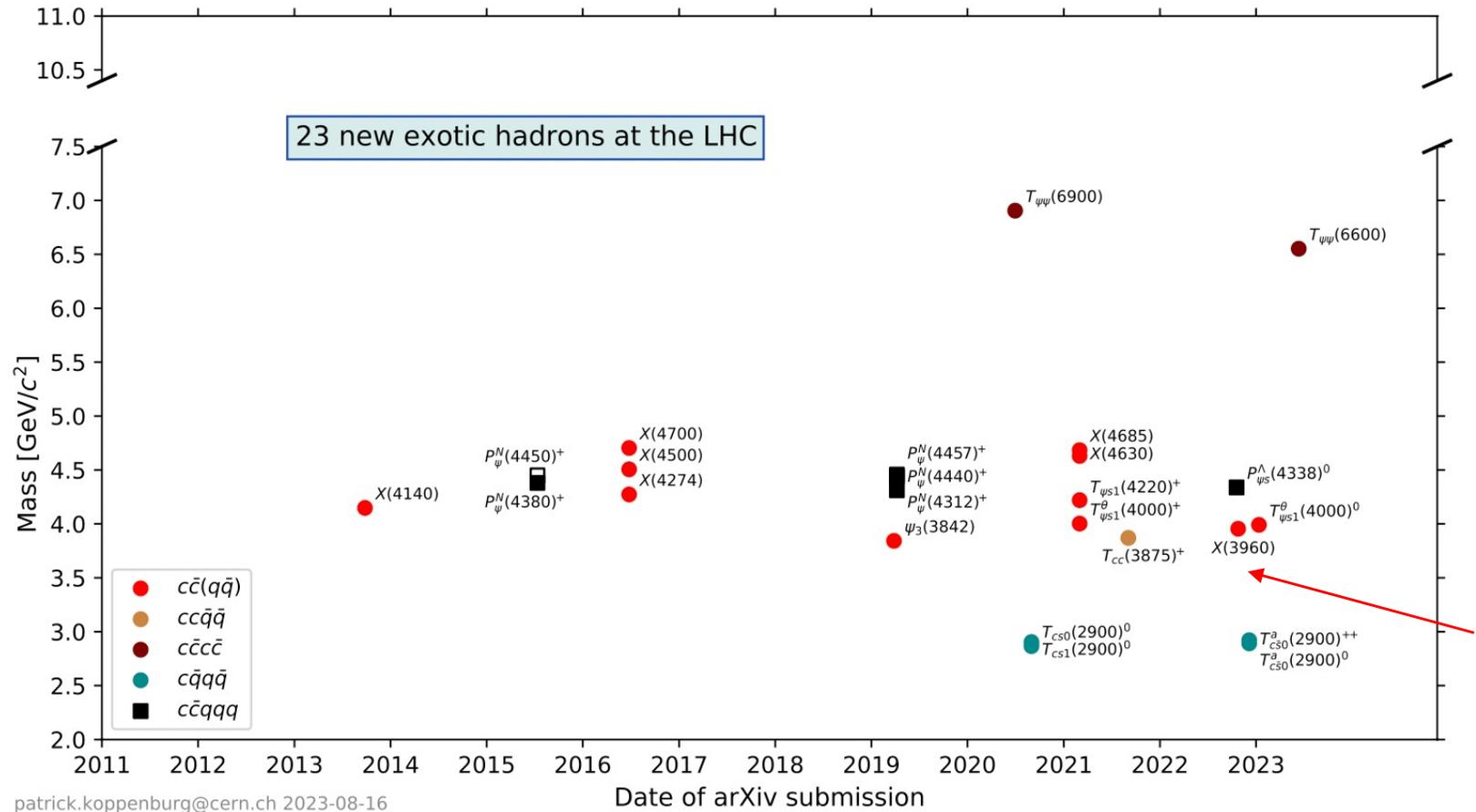
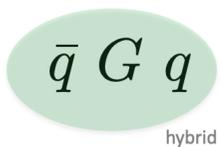
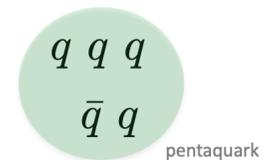
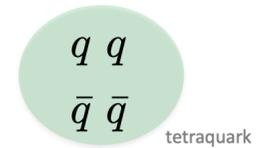


majority of exotic hadrons contain $c\bar{c}$

conventional hadrons



exotic hadrons



<https://www.nikhef.nl/~pkoppenb/particles.html>

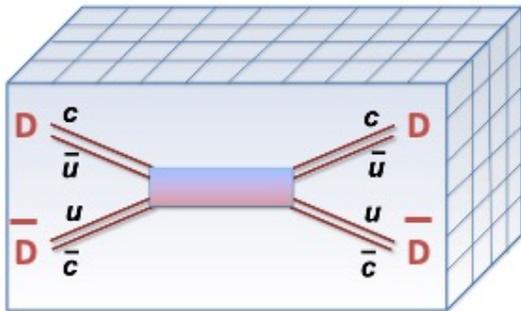
Charmonium(like) states

$$\bar{c}c\bar{d}u$$

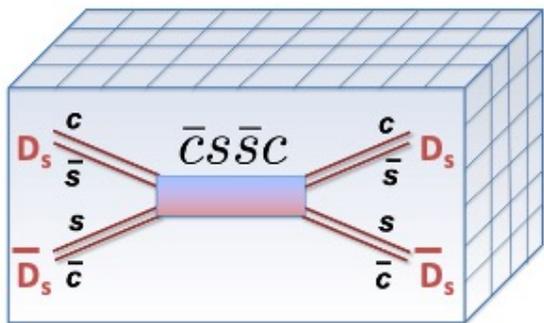
$$I=1$$

$$\bar{c}c, \bar{c}c\bar{q}q \quad (\bar{q}q = \bar{u}u + \bar{d}d, \bar{s}s)$$

$$I=0$$



$D\bar{D} - D_s\bar{D}_s$ scattering

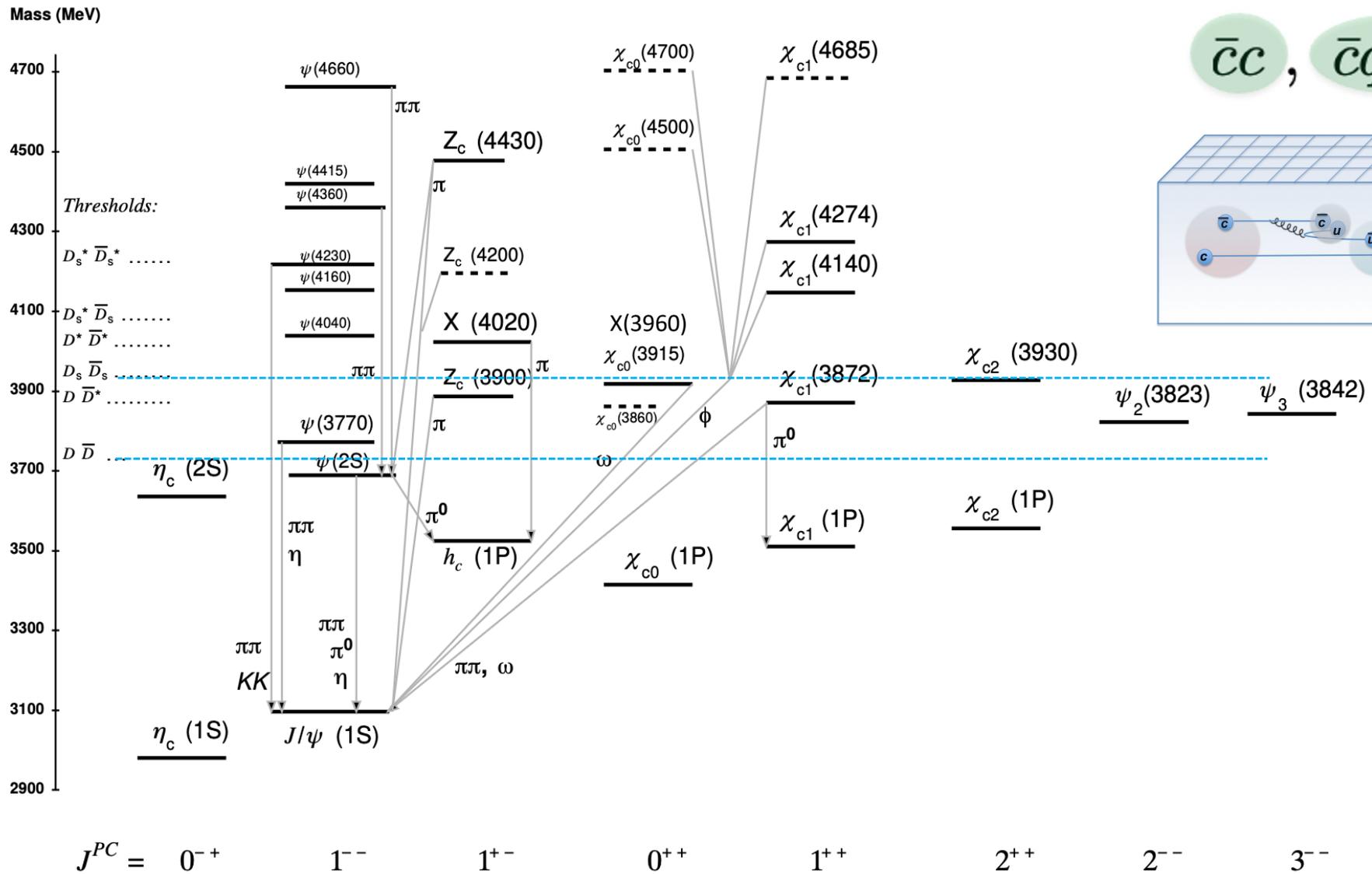


$$I=0, J^{PC}=0^{++}, 1^{--}, 2^{++}, 3^{--}$$

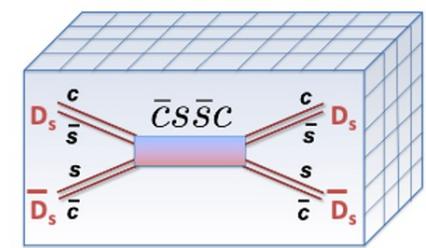
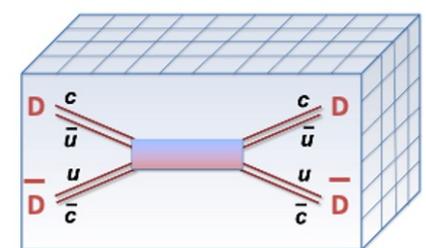
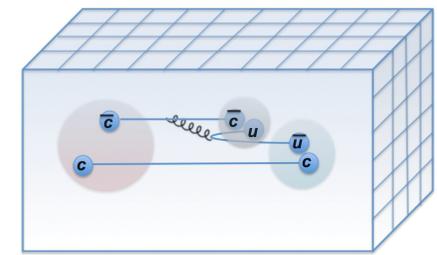
discussed in this talk

Charmonium(like) system: experimental status (PDG)

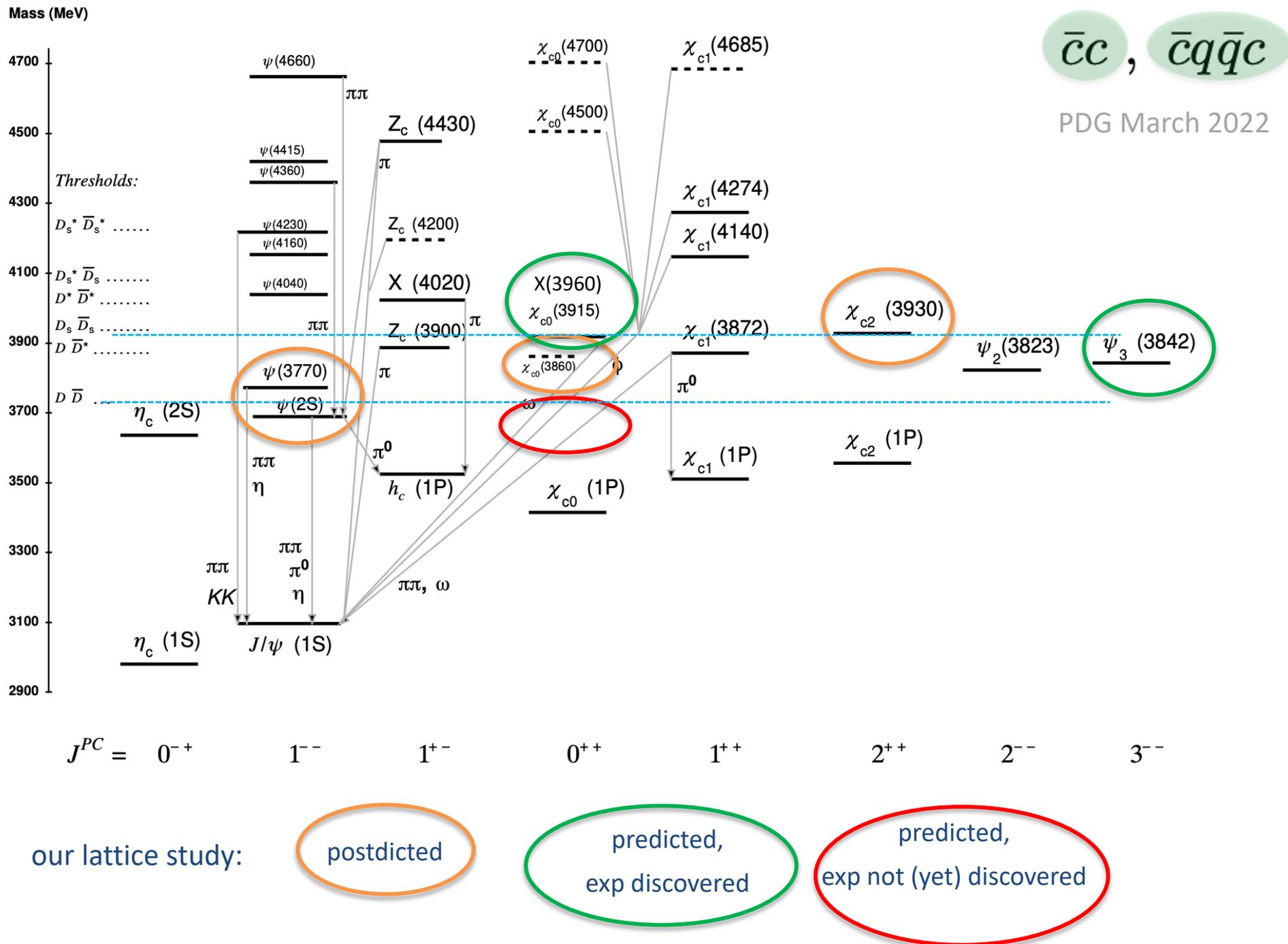
PDG March 2022



$\bar{c}c$, $\bar{c}q\bar{q}c$



Charmonium(like) system: experimental status (PDG) and summary of our lattice results



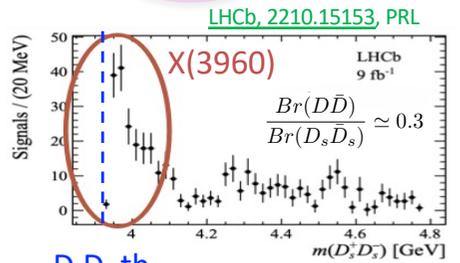
Outline of results:

$\bar{c}c$, $\bar{c}q\bar{q}c$ $q=u,d,s$ $I=0$

$$\left| \frac{c_{D\bar{D}}^2}{c_{D_s\bar{D}_s}^2} \right| = 0.02^{+0.02}_{-0.01}$$

$$T_{ij}(E_{cm}) \sim \frac{c_i c_j}{E_{cm}^2 - m^2}$$

$\bar{D}_s D_s$ $J^P=0^+$ state

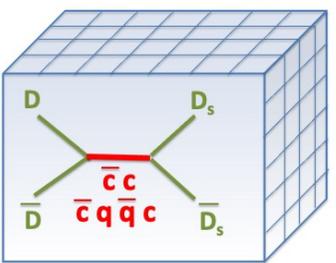
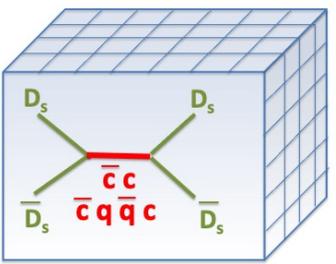
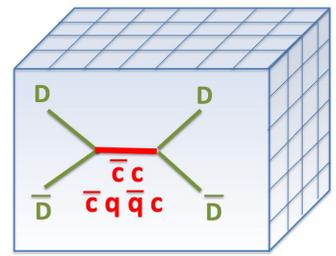


$\bar{D}D$ $J^P=0^+$ state

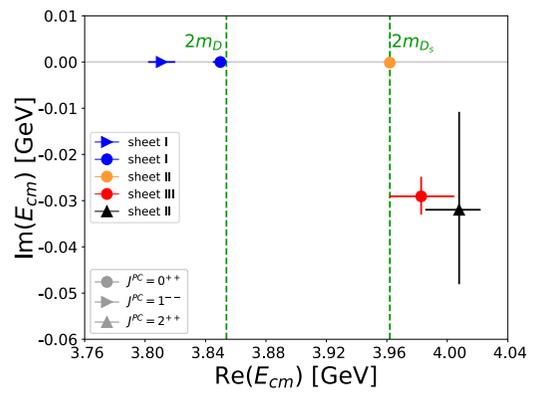
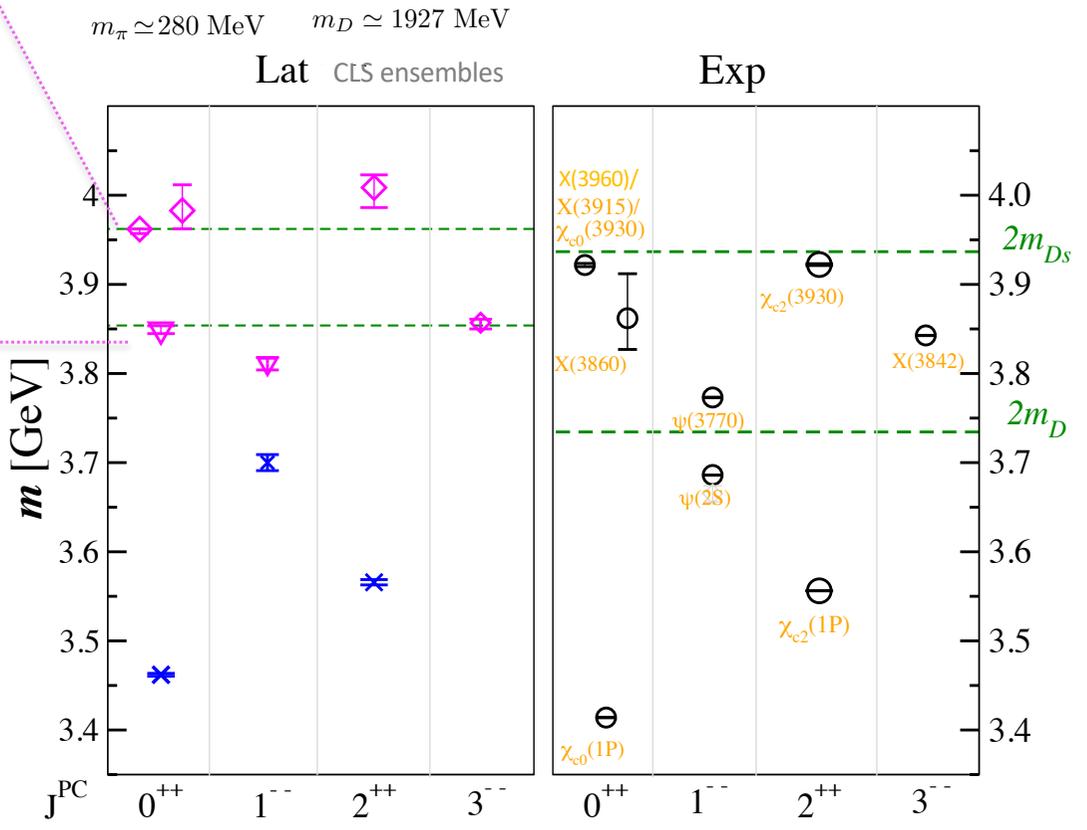
predicted in models [Oset et al, 0612179 PRD, Guo et al 2101.01021]

seen in re-analysis of exp. [Danilkin et al 2111.15033, Ji, F.K. Guo et al., 2212.00613]

+ expected conventional charmonia



$\bar{D}D - \bar{D}_s D_s$



Charmonium(like) resonances and bound states

Path to results

Lattice setup

CLS ensembles with u/d, s dynamical quarks

$a \simeq 0.086$ fm, $m_\pi = 280(3)$ MeV

$L = 2.1$ fm, 2.7 fm

U101 H105

lat exp

$$m_{u/d} > m_{u/d}^{\text{exp}}$$

$$m_s < m_s^{\text{exp}}$$

$$m_u + m_d + m_s = m_u^{\text{exp}} + m_d^{\text{exp}} + m_s^{\text{exp}}$$

two values of m_c (relativistic charm quarks)

	m_D [MeV]	$\frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$ [MeV]
lat ($m_c \gtrsim m_c^{\text{exp}}$)	1927(1)	3103(3)
lat ($m_c \lesssim m_c^{\text{exp}}$)	1762(1)	2820(3)
exp	1864.85(5)	3068.6(1)

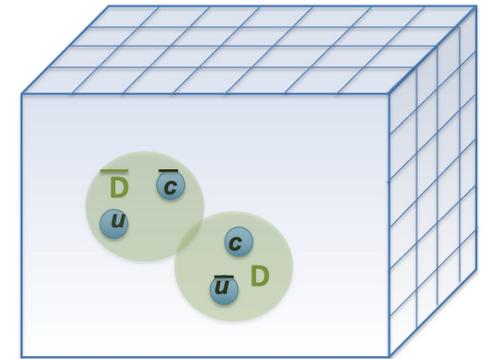
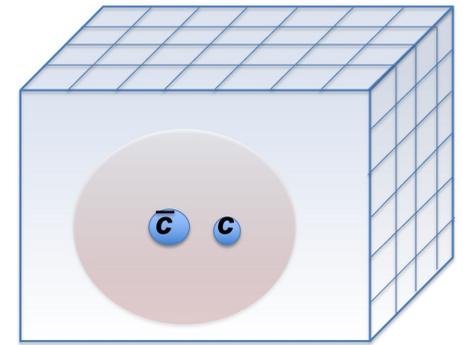
Quantity extracted from lattice: eigen-energy E_n

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- correlators evaluated using distillation method [Peardon et al, 2009]
- eigen-energies extracted using GeVP
- for strongly stable state well below threshold : $E_n(P=0) = m$
- in general :

$$E_n^{cm} \rightarrow T(E_n^{cm})$$

Luscher's relation



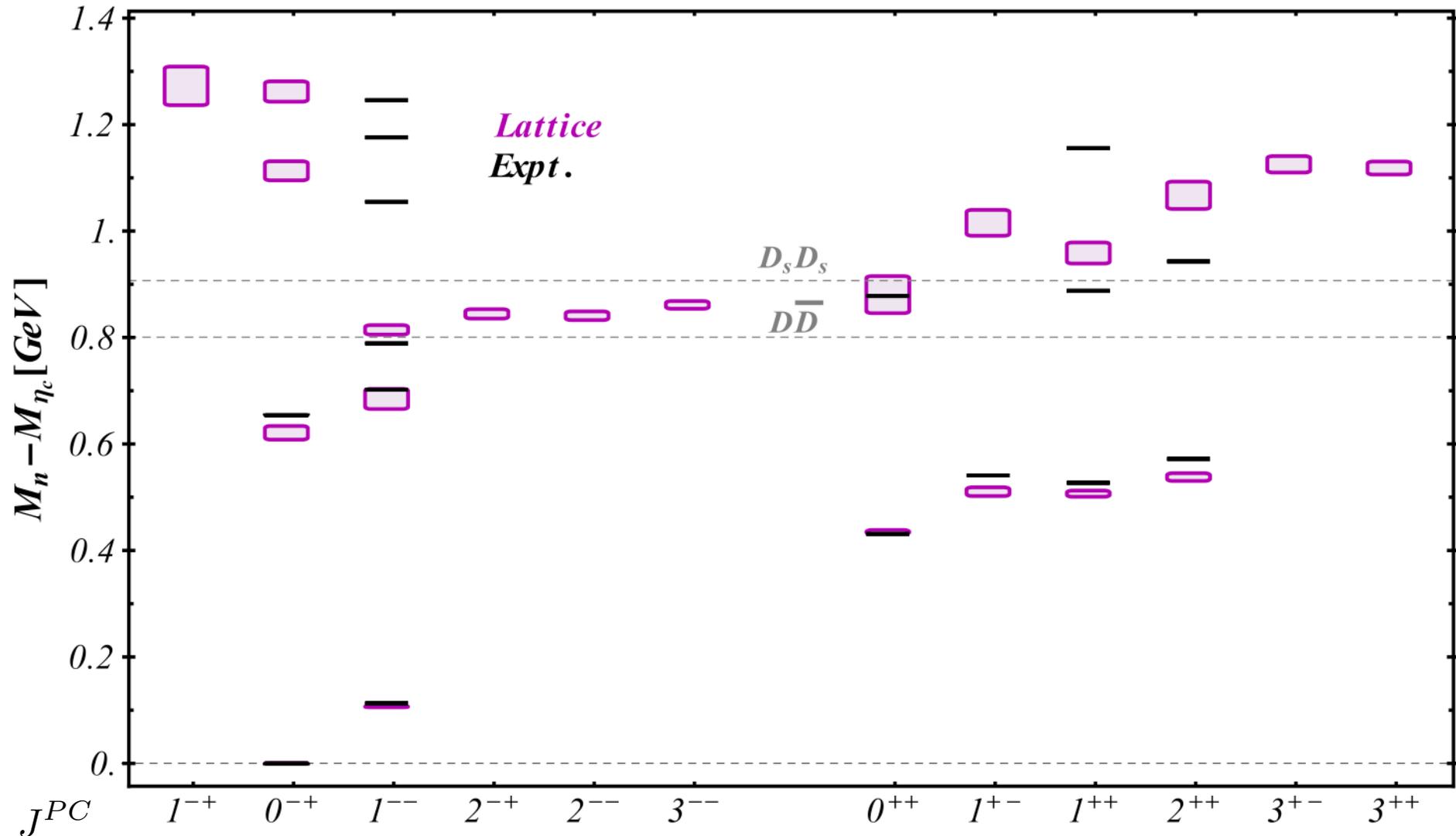
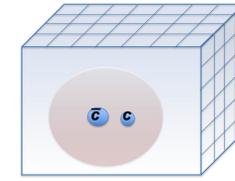
○ Initial step: consider charmonia to be strongly stable

$$\mathcal{O} : \quad \bar{c}(x)\Gamma c(x), \quad \bar{c}(x)\Gamma\vec{D}_j c(x), \quad \bar{c}(x)\Gamma\vec{D}_j\vec{D}_k c(x).$$

(up to 30 interpolators in given irrep)

➤ Charmonia at rest

$$E_n(P=0) = m$$



○ Initial step: consider charmonia to be strongly stable

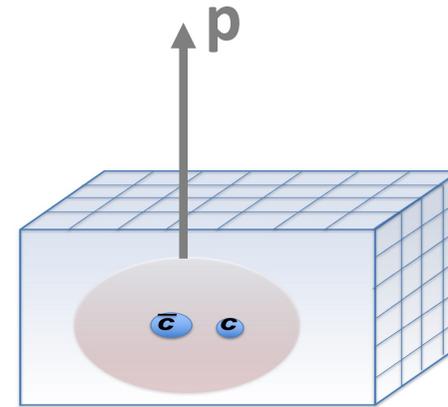
➤ Charmonia with momentum $p \neq 0$: Challenge

$$E_n^{cm} = \sqrt{(E_n^{cm})^2 - P^2}$$

Motivation to study $p \neq 0$: extraction of scattering matrix

$$E_n^{cm} \rightarrow T(E_n^{cm})$$

Challenge: identify J^P of charmonium in flight

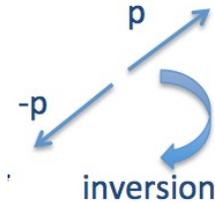


Symmetries: $p \neq 0$

continuum

C: good

P: NOT good



Rotations/reflections:

transformations that leave p invariant
rotations around p ; little group $U(1)$

spin J : not good

helicity : good $\lambda = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|}$

$\tilde{\eta}$: good (only for $\lambda=0$ states)

$$\Pi |p, J^P, \lambda\rangle = \tilde{\eta} |p, J^P, -\lambda\rangle$$

$$\tilde{\eta} \equiv P(-1)^J.$$

cubic lattice

C: good

P: NOT good

Rotations/reflections:

transformations that leave box and p invariant:

$p=(0,0,1)$: group Dic_4 , 8 elements

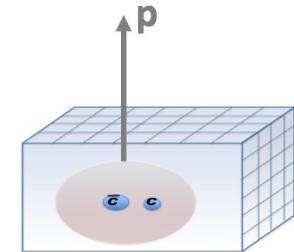


TABLE VII. Choice of representation matrices for the Dic_4 little group. I denotes the identify transformation, $R(\phi)$ denotes a rotation around the z -axis by ϕ and Π denotes a reflection in the yz plane ($x \rightarrow -x$).

Irrep	I	$R(\pi)$	$R(3\pi/2)$	$R(\pi/2)$	Π	$R(\pi)\Pi$	$R(\pi/2)\Pi$	$R(3\pi/2)\Pi$
-------	-----	----------	-------------	------------	-------	-------------	---------------	----------------

$p=(1,1,0)$; group Dic_2 , 4 elements

Irrep	I	$R(\pi)$	Π	$R(\pi)\Pi$
-------	-----	----------	-------	-------------



irreps: good quantum numbers

helicity: not good

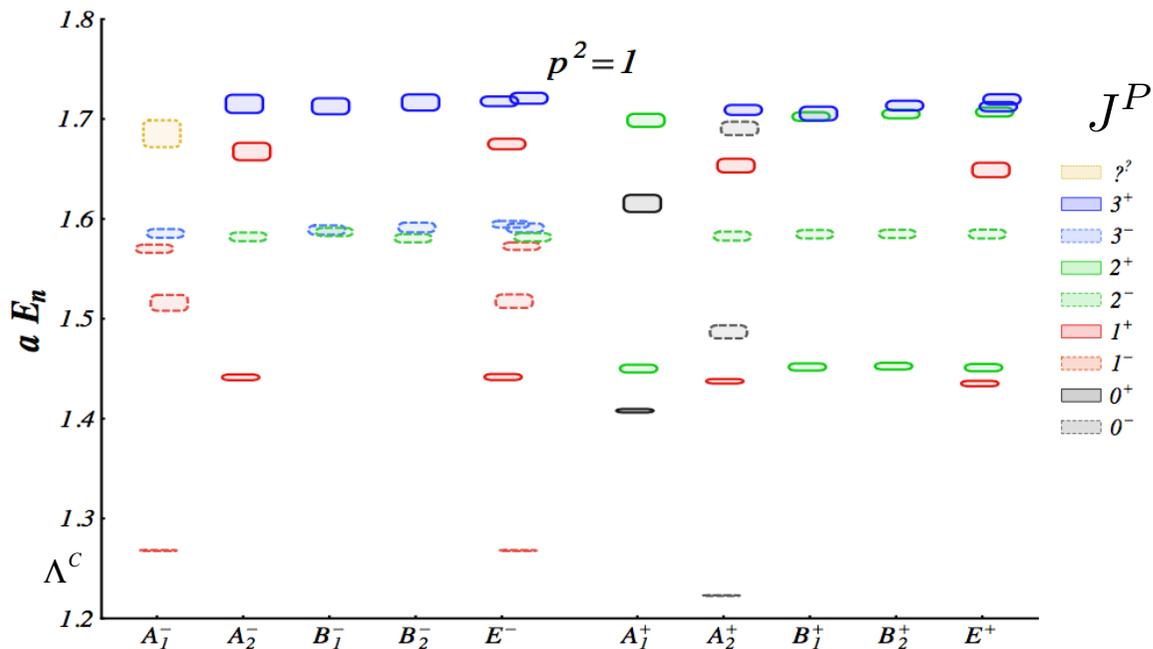
$\tilde{\eta}$: good (only for $\lambda=0$ states)

The challenge to determine J^P

$$|\lambda| \leq J$$

$$\lambda = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|}$$

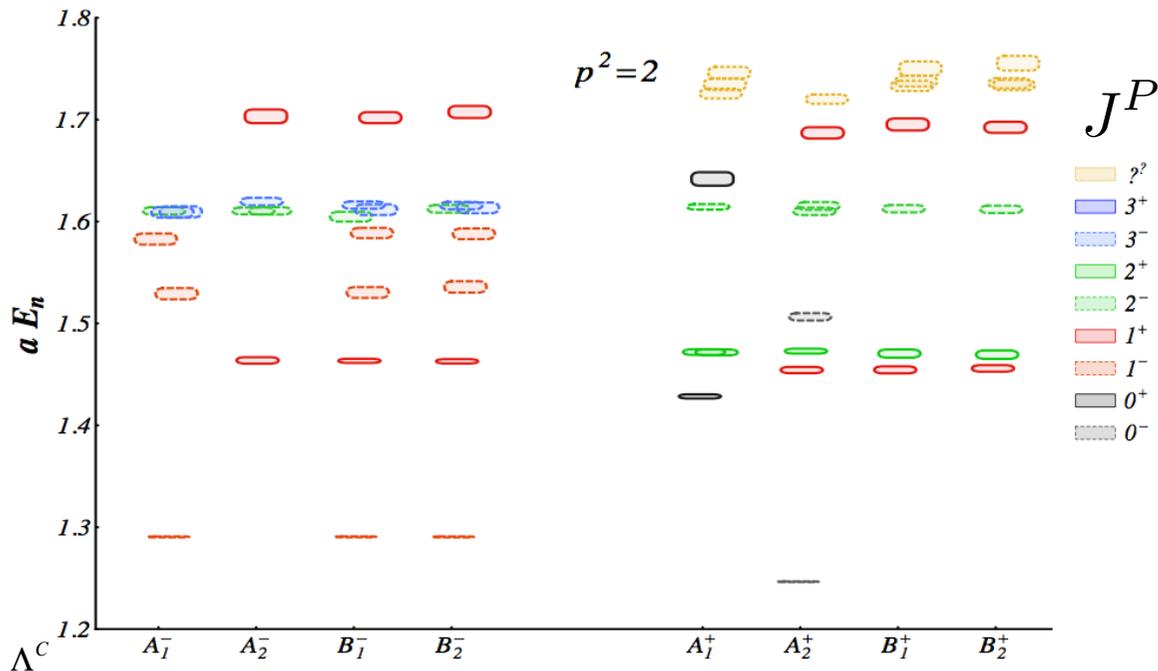
refers to J^P in charmonium's rest frame



$\mathbf{p} = (0, 0, 1), Dic_4$

Λ (<i>dim</i>)	$ \lambda ^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+, 1^-, 2^+, 3^-$
A_2 (1)	0^-	$0^-, 1^+, 2^-, 3^+$
E (2)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm
B_1 (1)	2	$2^\pm, 3^\pm$
B_2 (1)	2	$2^\pm, 3^\pm$

FIG. 11. I^P -identified charmonium spectrum in the moving frame with $\mathbf{p} = (0, 0, 1)$. Irreps Λ^C of group Dic_4 are presented



$\mathbf{p} = (1, 1, 0), Dic_2$

Λ (<i>dim</i>)	$ \lambda ^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+, 1^-, 2^+, 3^-$
	2	$2^\pm, 3^\pm$
A_2 (1)	0^-	$0^-, 1^+, 2^-, 3^+$
	2	$2^\pm, 3^\pm$
B_1 (1)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm
B_2 (1)	1	$1^\pm, 2^\pm, 3^\pm$
	3	3^\pm

state with J has $|\lambda| \leq J$

state with $|\lambda|$ can have $|\lambda| \leq J$

Identifying J^P for hadrons in flight

following strategy by HadSpec
employed for light mesons [PRD 2012](#)

$$\bar{c} \Gamma D \dots c$$

$$O_i^{J^{PC}, \lambda}(\mathbf{p}) = \sum_M \mathcal{D}_{M, \lambda}^{(J)*}(R) O_i^{J^{PC}, M}(\mathbf{p})$$

$$O_{\Lambda, \mu} = \sum_{R \in Dic_{2,4}} T_{\mu\mu}^\Lambda(R)^* R O$$

$$O_{i, \Lambda^C}^{[J^{PC}, |\lambda|]}$$

criteria :

$$\langle O_{i, \Lambda^C}^{[J^{PC}, |\lambda|]} | \mathbf{p}, J^{PC}, \lambda \rangle > \langle O_{i, \Lambda^C}^{[J^{PC}, |\lambda|]} | \mathbf{p}, J'^{P'C}, \lambda \rangle$$

$$\langle O_{i, \Lambda_1^C}^{[J^{PC}, |\lambda|]} | \mathbf{p}, J^{PC}, \lambda \rangle \simeq \langle O_{i, \Lambda_2^C}^{[J^{PC}, |\lambda|]} | \mathbf{p}, J^{PC}, \lambda \rangle$$

construction of operators

considering
overlaps of eigenstates

$$\langle O | n \rangle$$

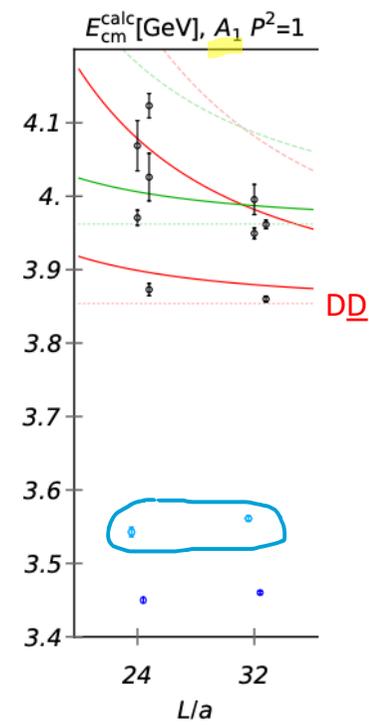
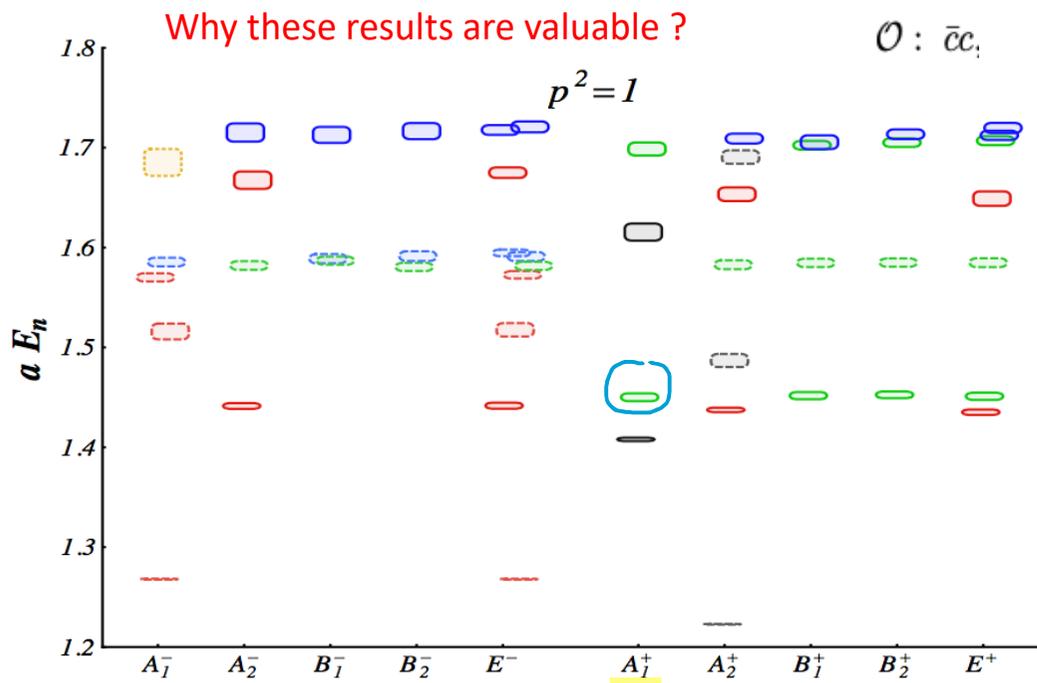
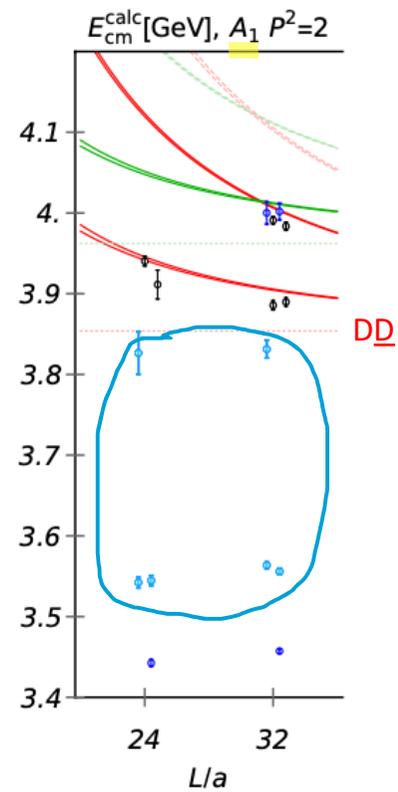
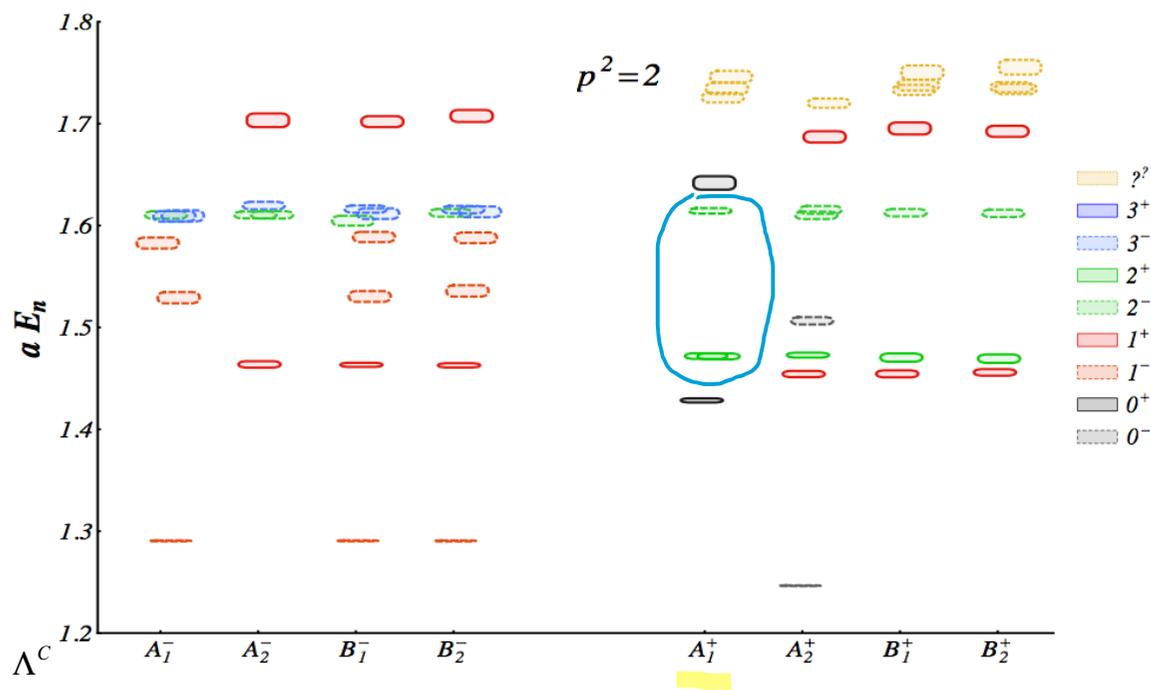
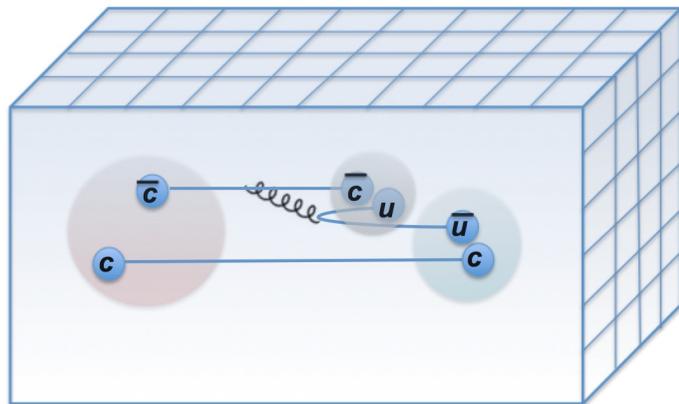


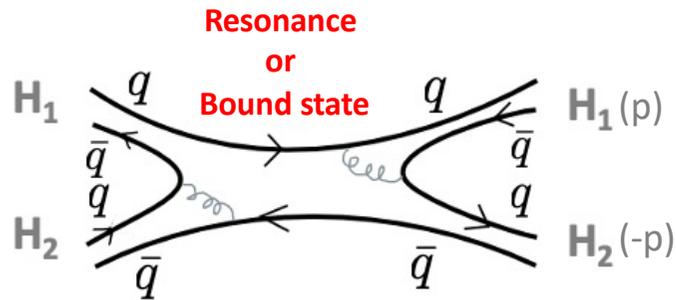
FIG. 11. J^P -identified charmonium spectrum in the moving frame with $\mathbf{p} = (0, 0, 1)$. Irreps Λ^C of group Dic_4 are presented. The colors indicate J^P of states according to the color-coding (21).



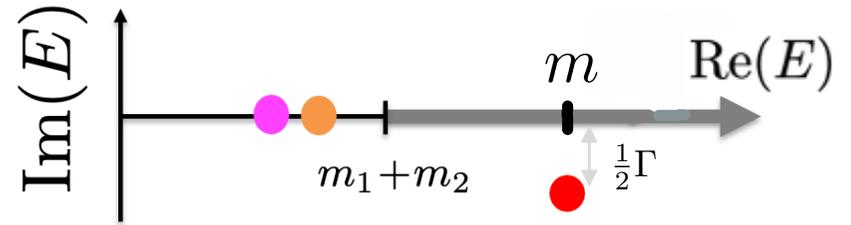
Taking strong decays into account



Extract resonances and (virtual) bound states from $H_1 H_2$ scattering

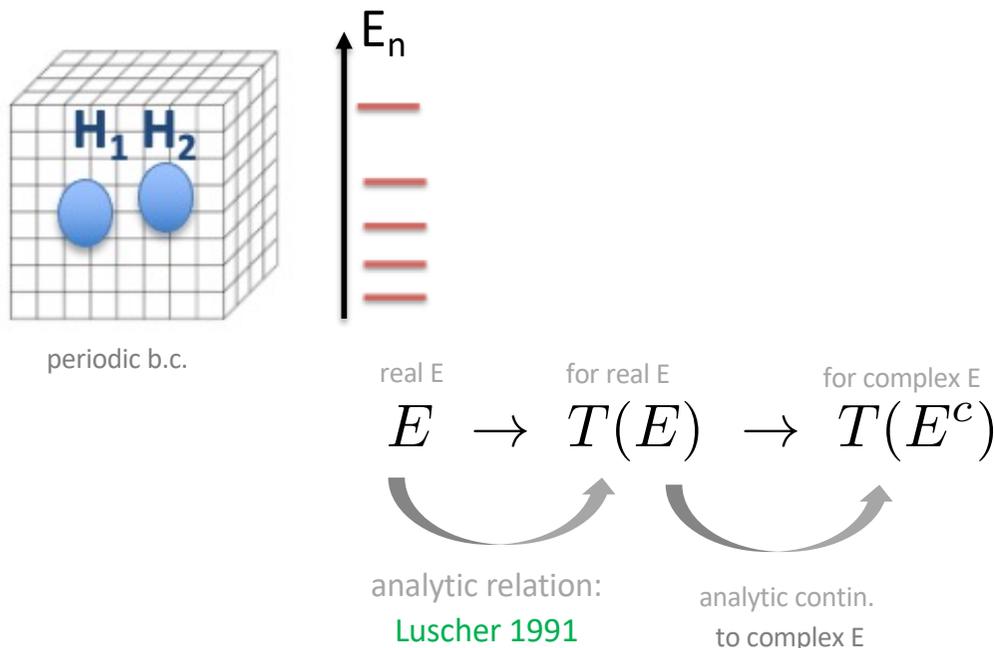


scattering matrix $T(E)$



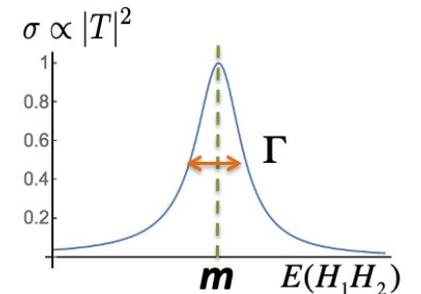
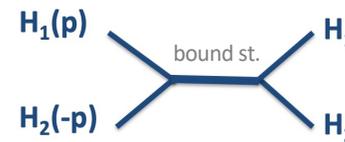
Virtual bound st. $p = -i |p|$ Bound st. $p = i |p|$ Resonance

Scattering matrix $T(E)$ from lattice QCD



$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$



Relation between E and $\delta(E)$, $T(E)$: 1D quantum mechanics

$$S(E) = e^{2i\delta(E)} = 1 + 2i \frac{2p}{E} T(E)$$

$$E \rightarrow T(E)$$

$V \neq 0$: outside the region of potential

$$E = p^2/2m$$

$$\psi(x) = A \cos(p|x| + \delta) = \begin{cases} A \cos(px + \delta) & x > R \\ A \cos(-px + \delta) & x < -\frac{R}{2} \end{cases}$$

- this form already ensures

$$\psi(L/2) = \psi(-L/2)$$

- the other BC:

$$\psi'(L/2) = \psi'(-L/2)$$

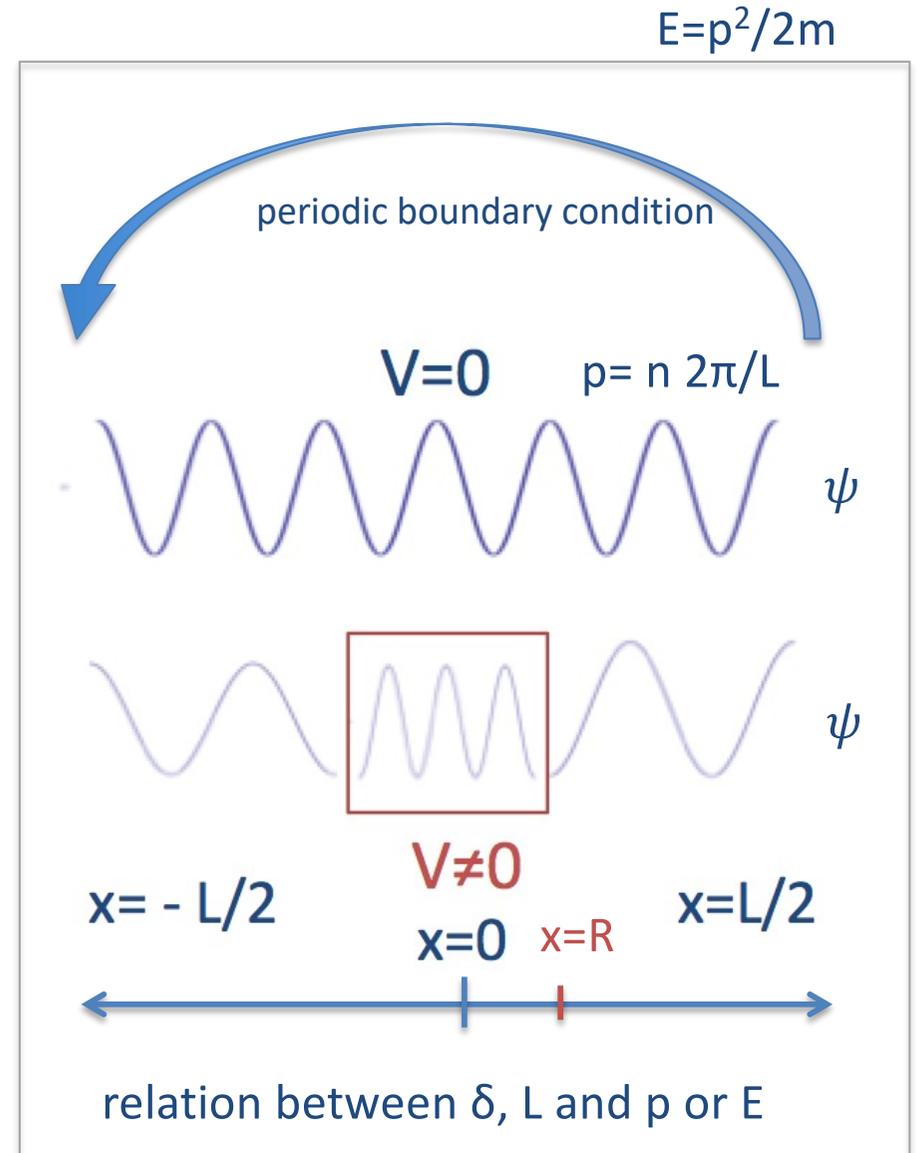
this requires

$$Ap \sin(p(\frac{L}{2}) + \delta) = -Ap \sin(-p(-\frac{L}{2}) + \delta)$$

$$\rightarrow \psi'(L/2) = 0, \sin(p\frac{L}{2} + \delta) = 0$$

$$p\frac{L}{2} + \delta = m\pi \quad \boxed{p = m\frac{2\pi}{L} - \frac{2}{L}\delta}$$

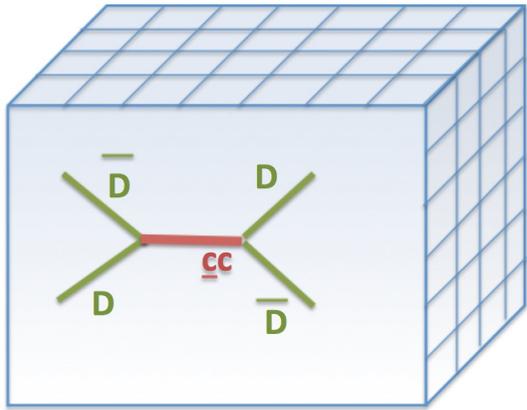
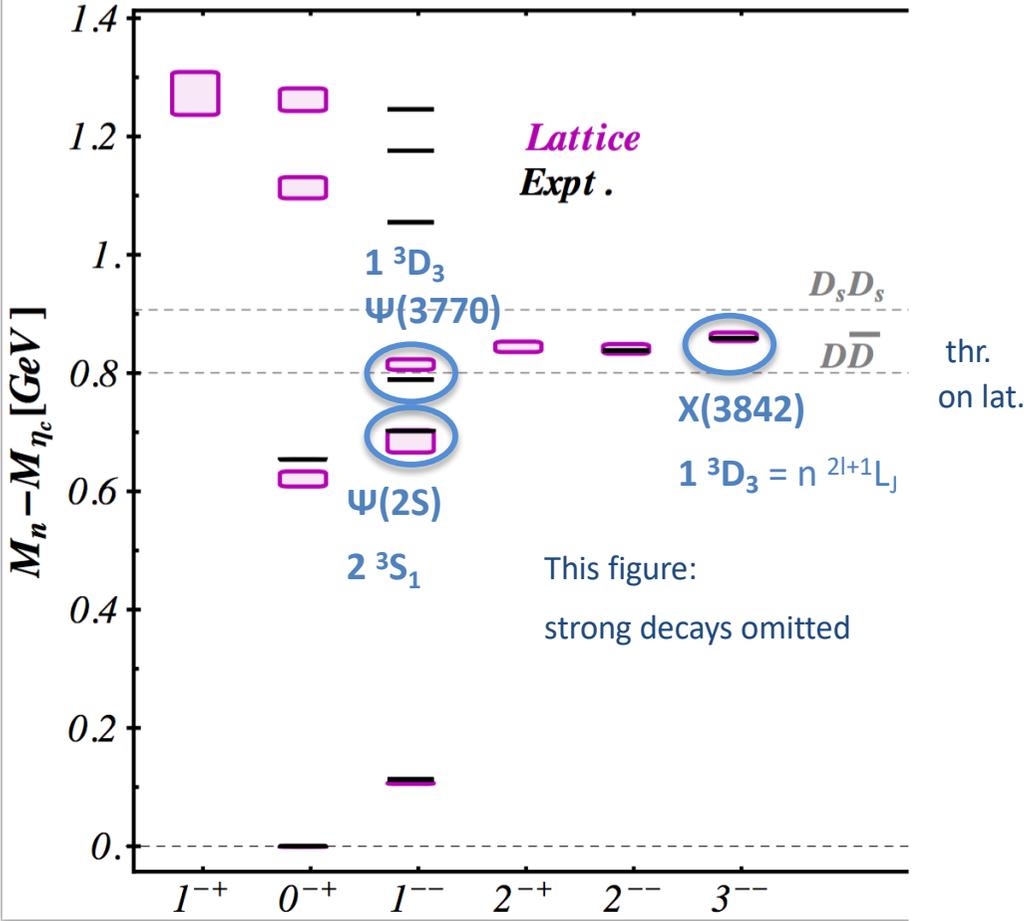
relation between δ, L



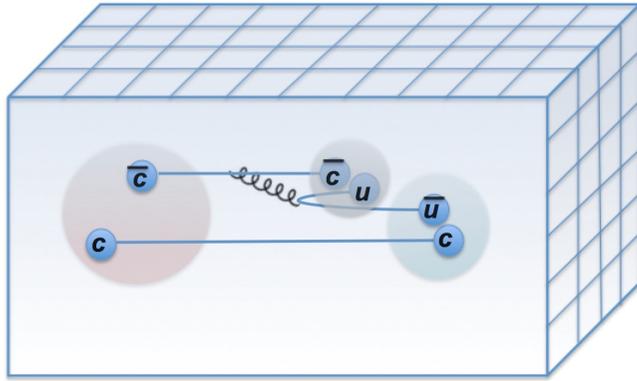
Charmonia with $J^{PC}=1^{--}$ and 3^{-}

from one-channel DD scattering

Charmonia with $J^{PC}=1^{--}$ and 3^{--}
from one-channel $D\bar{D}$ scattering

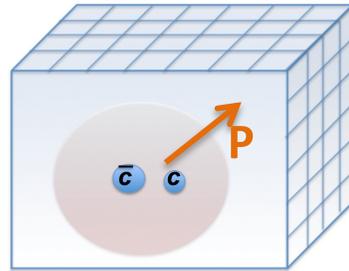


strong decay

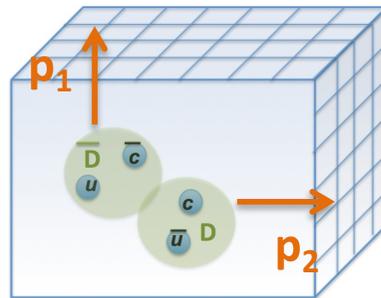


Operators

$$\mathcal{O}^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$



$$\begin{aligned} \mathcal{O}^{\bar{D}D} &= (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2} \\ &= \bar{D}(\vec{p}_1) D(\vec{p}_2) \end{aligned}$$



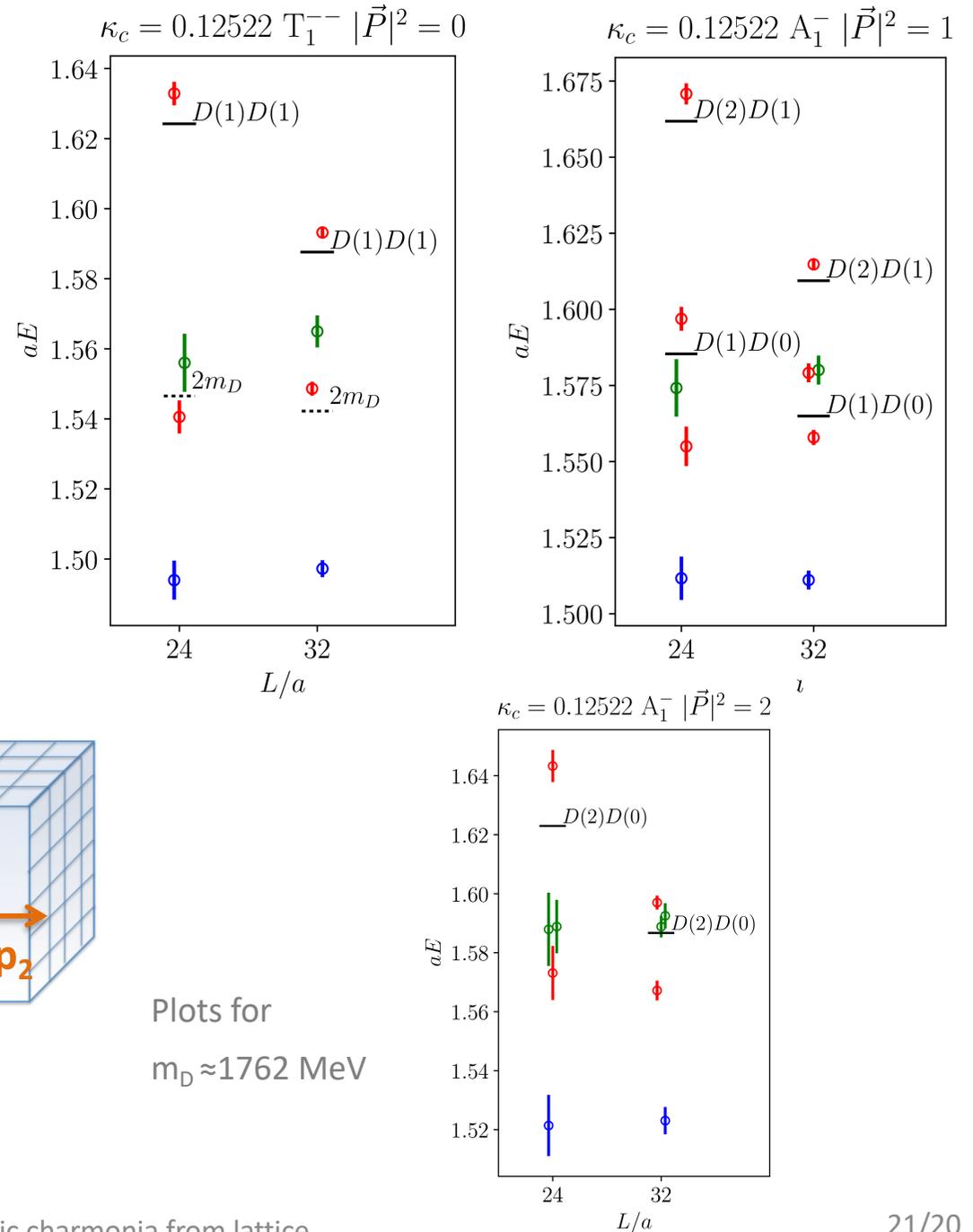
$$\begin{aligned} \vec{P} &= \vec{p}_1 + \vec{p}_2 & \mathbf{P}: 0 \\ & & (0,0,1) \ 2\pi/N_L \\ & & (1,1,0) \ 2\pi/N_L \end{aligned}$$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

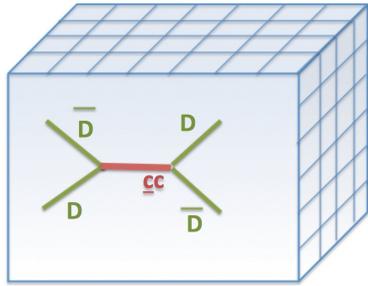
Sasa Prelovsek

Conventional and exotic charmonia from lattice

Eigen-energies E_n



Charmonia with $J^{PC}=1^{--}$ and 3^{--} from one-channel $\underline{D}\underline{D}$ scattering



p = D-meson momentum in CMF

$$\frac{p^{2l+1} \cot(\delta_l)}{\sqrt{s}} = \frac{m^2 - s}{G^2} \quad \text{Breit Wigner}$$

$$l=1 \quad \frac{p^3 \cot(\delta_1)}{\sqrt{s}} = \left(\frac{G_1^2}{m_1^2 - s} + \frac{G_2^2}{m_2^2 - s} \right)^{-1}$$

$$l=3 \quad \frac{p^7 \cot(\delta_3)}{\sqrt{s}} = \frac{m_3^2 - s}{g_3^2}$$

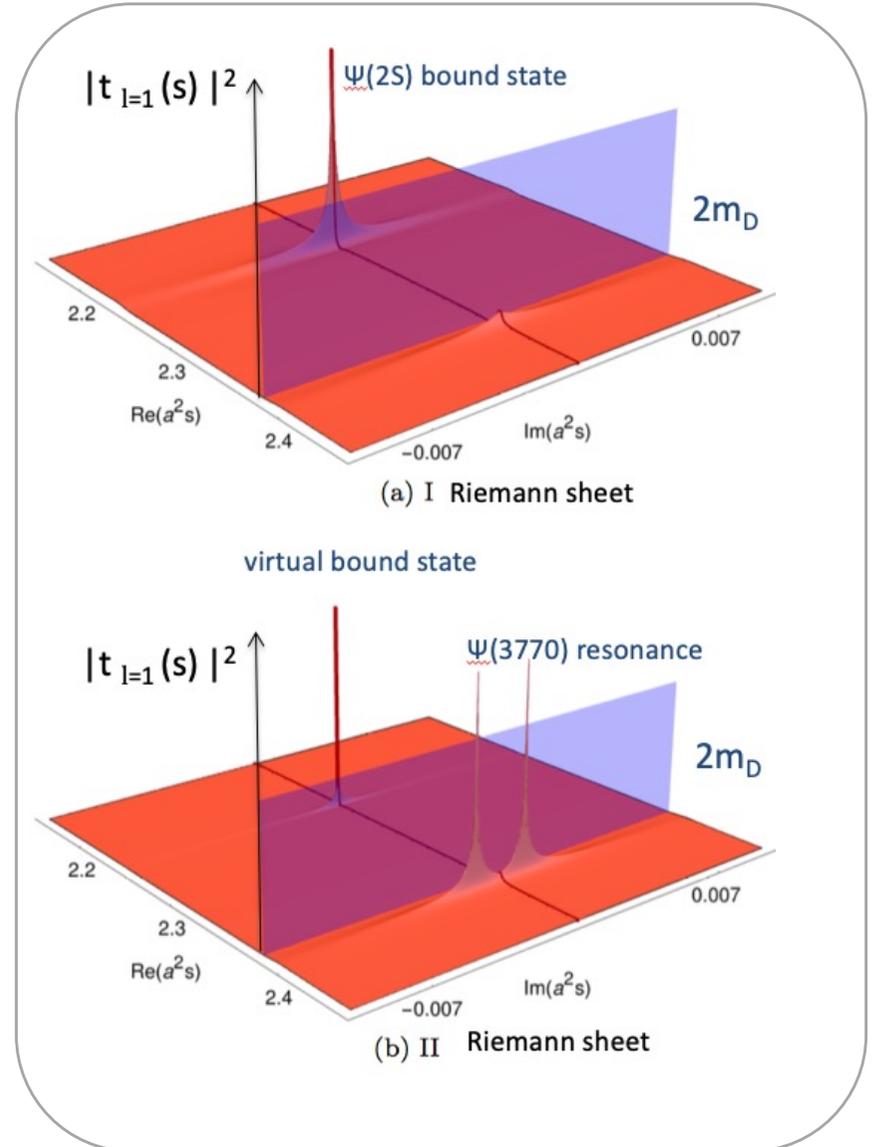
X(3842)

$$\Gamma = \frac{g^2 p^3}{6\pi s}$$

$\Psi(3770)$

	g
lat	$16.0^{+2.1}_{-0.2}$
exp	18.7 ± 0.9

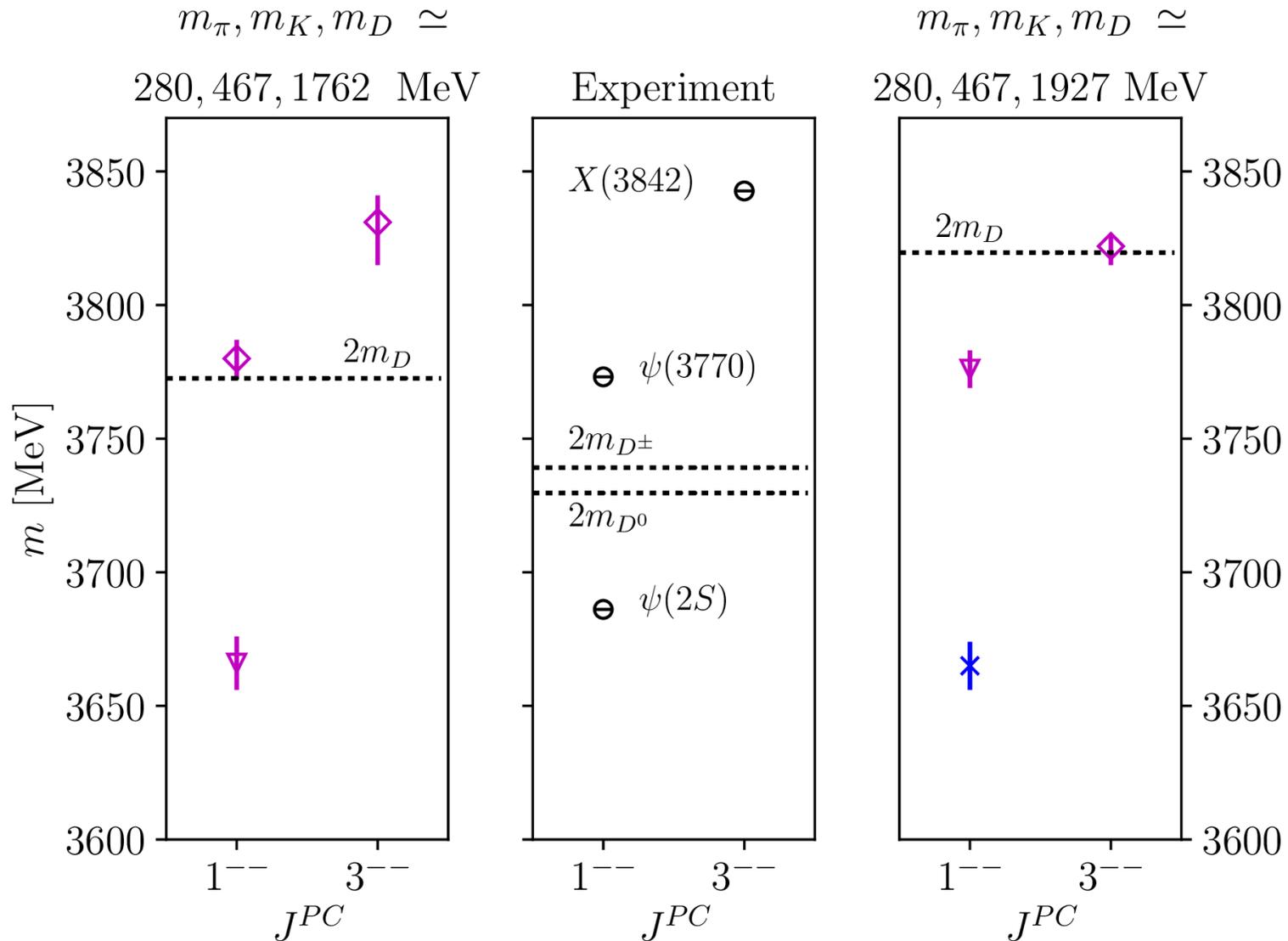
$l=1$ and $m_D \approx 1762$ MeV



Masses of charmonium resonances and bound states

$$M_{\text{av}} = \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$$

$$m = m^{\text{lat}} - M_{\text{av}}^{\text{lat}} + M_{\text{av}}^{\text{exp}}$$

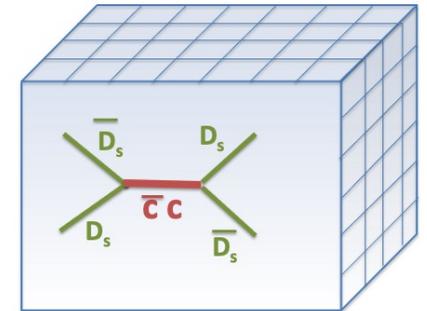
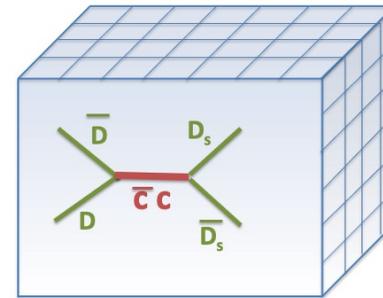
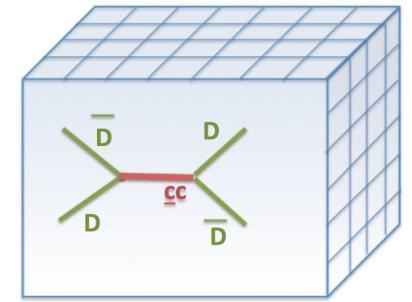


Charmonium(like) states with $J^{PC}=0^{++}$ and 2^{++}

from coupled-channel scattering $D\bar{D} - D_s\bar{D}_s$

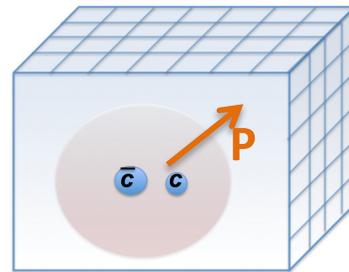
the only available coupled-channel study for charmonia with $I=0$

Charmonia with $J^{PC}=0^{++}$ and 2^{++}
 from coupled-channel scattering $D\bar{D} - D_s\bar{D}_s$

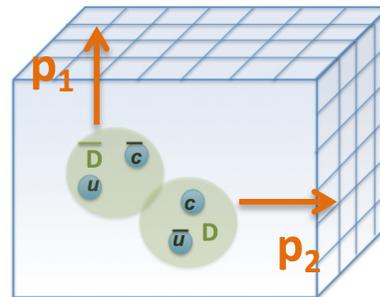


Operators

$$\mathcal{O}^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$



$$\begin{aligned} \mathcal{O}^{\bar{D}D} &= (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2} \\ &= \bar{D}(\vec{p}_1) D(\vec{p}_2) \end{aligned}$$

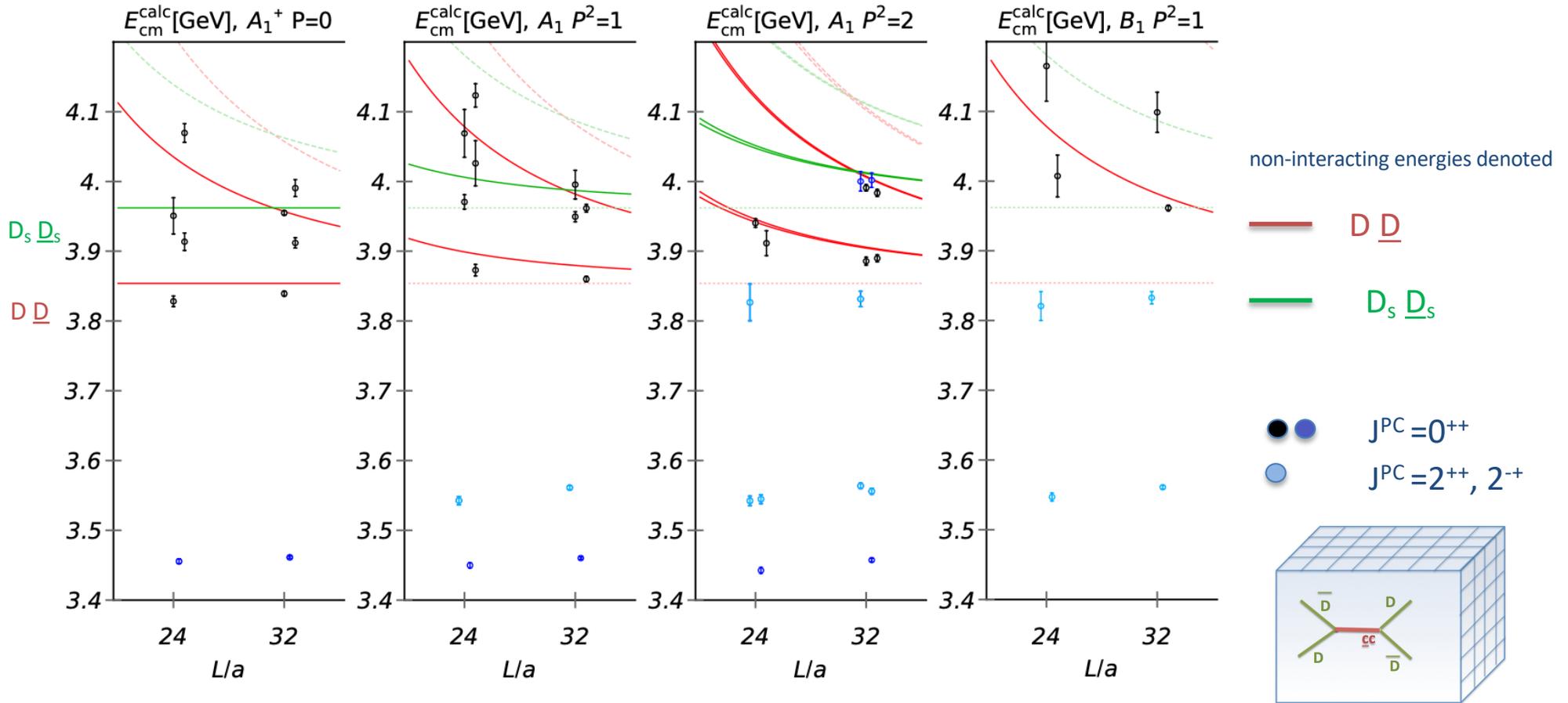


$$\begin{aligned} \mathcal{O}^{\bar{D}_s D_s} &= (\bar{c}\Gamma_1 s)_{\vec{p}_1} (\bar{s}\Gamma_2 c)_{\vec{p}_2} \\ &= \bar{D}_s(\vec{p}_1) D_s(\vec{p}_2) \end{aligned}$$

$$\begin{aligned} \vec{P} &= \vec{p}_1 + \vec{p}_2 & \text{P: } 0 \\ & & (0,0,1) \ 2\pi/N_L \\ & & (1,1,0) \ 2\pi/N_L \end{aligned}$$

$$N_L=24,32$$

Energies of eigen-states E_n in irreps that contain $J^{PC}=0^{++}, 2^{++}$



$$E \not\leftrightarrow t(E)$$

$$S_{ij}(E_{cm}) = 1 + 2i \rho t_{ij}(E_{cm})$$

Extraction of matrix $t(E)$: NOT straightforward !

$$\det[1 + i t(E_{cm}) F(E_{cm})] = 0$$

known 2x2 matrix

$$t(E_{cm}) = \begin{vmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{vmatrix}$$

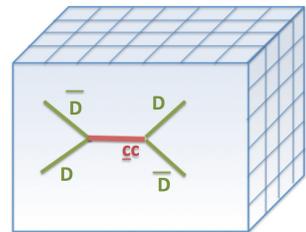
one equation, three unknowns (at each E_{cm})

$$(t^{-1})_{ij} = \frac{2}{E_{cm} p_i^l p_j^l} (\tilde{K}^{-1})_{ij} - i \rho_i \delta_{ij}$$

$$\rho_i \equiv 2p_i/E_{cm}$$

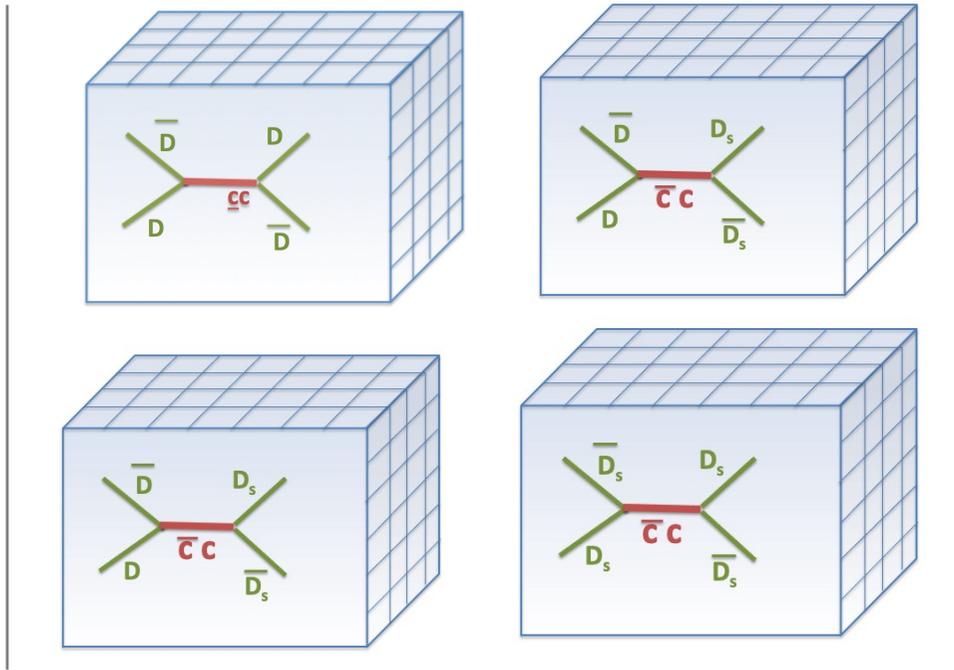
$$\frac{\tilde{K}_{ij}^{-1}(s)}{\sqrt{s}} = a_{ij} + b_{ij}s$$

$$s = E_{cm}^2$$



1: $\underline{D}\underline{D}$, 2: $\underline{D}_s\underline{D}_s$

Challenge: determine coupled-channel scattering matrix



1: \underline{DD} , 2: $\underline{D_s D_s}$

$$t(E_{cm}) = \begin{vmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{vmatrix}$$

Challenge: parametrization of scattering matrix

$$\rho_i \equiv 2p_i/E_{cm}$$

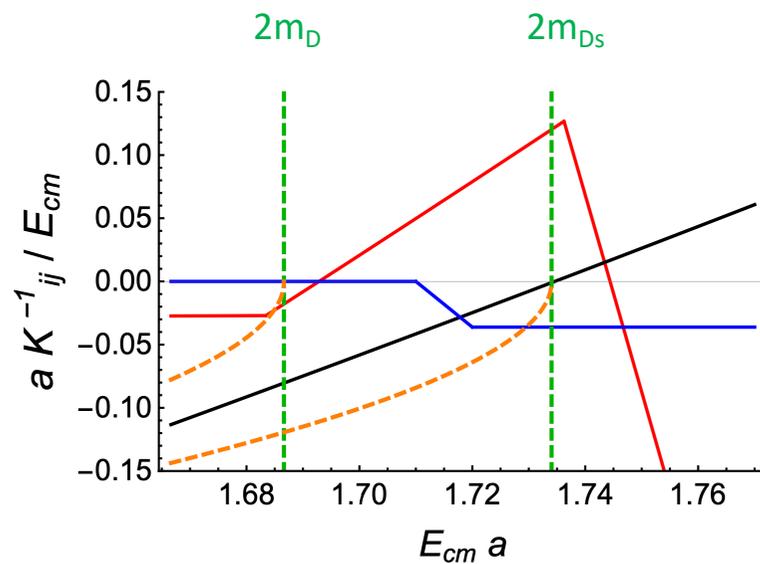
$$s = E_{cm}^2$$

$$t(E_{cm}) = \begin{pmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{pmatrix}$$

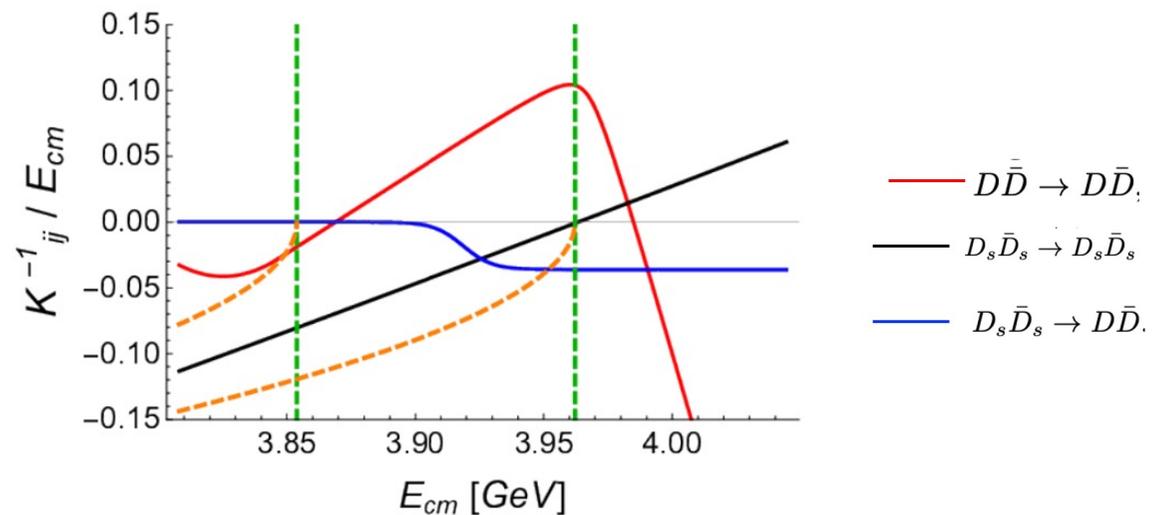
$$(t^{-1})_{ij} = \frac{2}{E_{cm} p_i^l p_j^l} (\tilde{K}^{-1})_{ij} - i \rho_i \delta_{ij}$$

$$\frac{\tilde{K}_{ij}^{-1}(s)}{\sqrt{s}} = a_{ij} + b_{ij}s$$

linear parametrization
for low and high energy regions

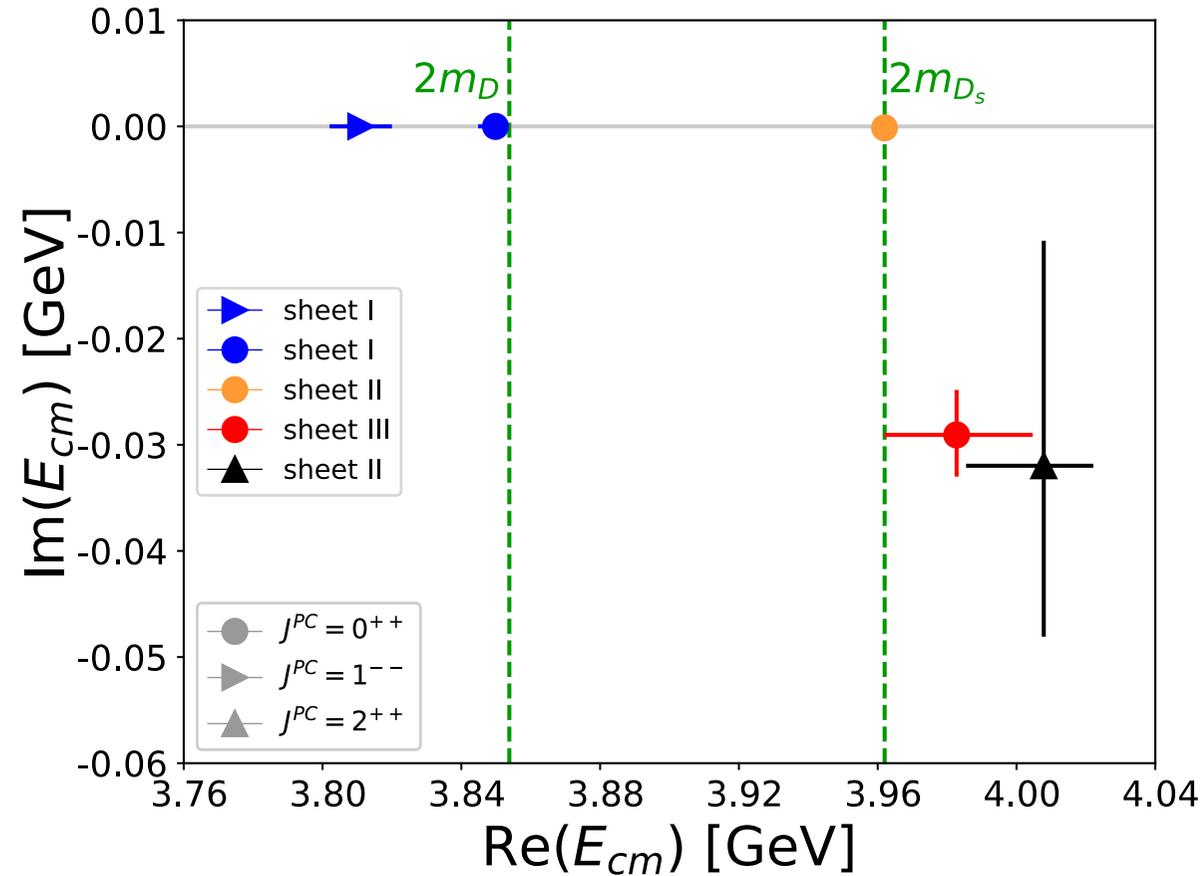


“gluing” via hyperbolas



Poles of the scattering amplitude $D\bar{D} - D_s\bar{D}_s$

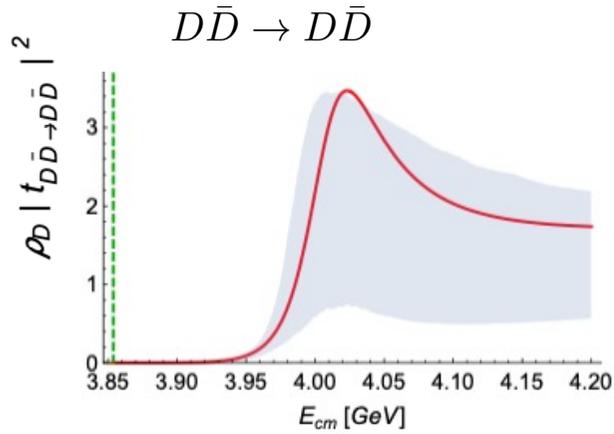
$\bar{c}c$, $\bar{c}q\bar{q}c$ $q=u,d,s$ $I=0$



sheet I : $\text{Im}(\rho_D) > 0, \text{Im}(\rho_{D_s}) > 0$, sheet II : $\text{Im}(\rho_D) < 0, \text{Im}(\rho_{D_s}) > 0$, $(\rho_i = 2p_i/E_{cm})$
 sheet III : $\text{Im}(\rho_D) < 0, \text{Im}(\rho_{D_s}) < 0$, sheet IV : $\text{Im}(\rho_D) > 0, \text{Im}(\rho_{D_s}) < 0$.

$J^{PC}=2^{++}$: conventional resonance

D-wave ($L=2, J^{PC}=2^{++}$)



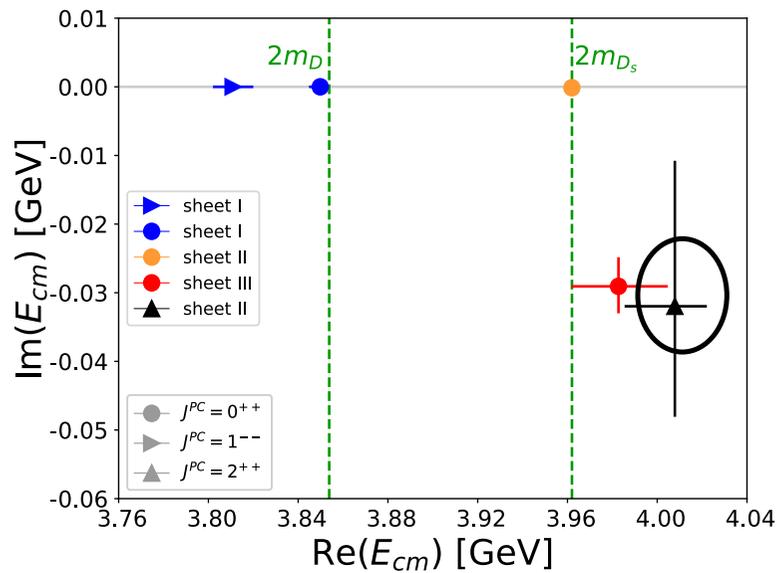
- 2^{++} resonance

$$\Gamma \equiv g^2 p_D^{2l+1} / m^2$$

$$lat : m = 3973_{-22}^{+14} \text{ MeV} \quad g = 4.5_{-1.5}^{+0.7} \text{ GeV}^{-1}$$

$$\chi_{c2}(3930) : m = 3923 \pm 1 \text{ MeV} \quad g = 2.65 \pm 0.12 \text{ GeV}^{-1}$$

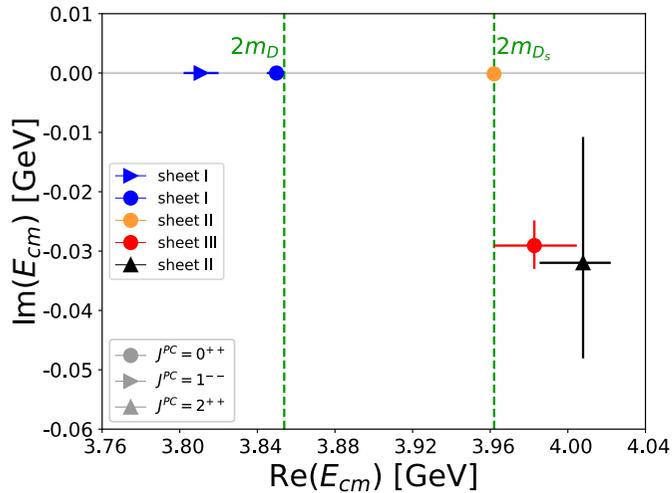
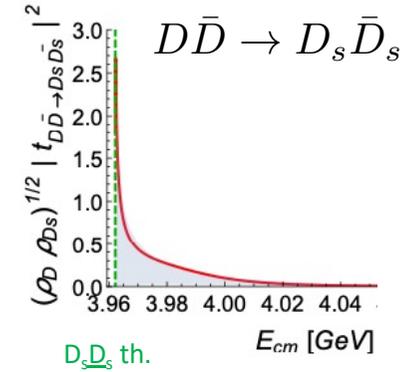
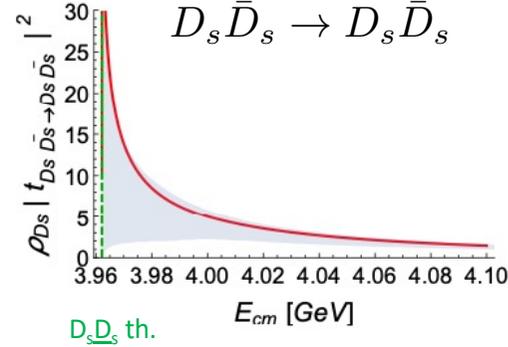
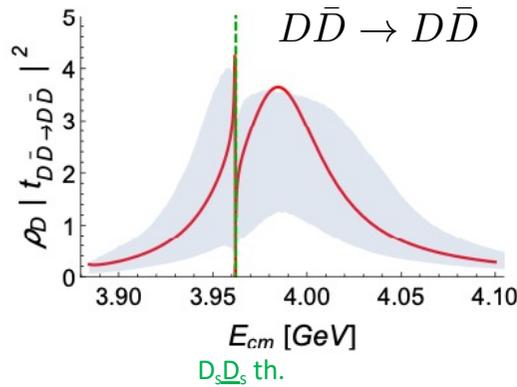
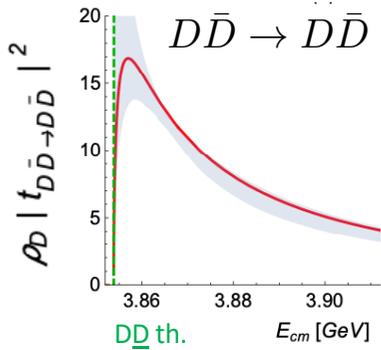
PDG



$J^{PC}=0^{++}$: some expected and unexpected states found

$$D\bar{D} - D_s\bar{D}_s$$

S-wave ($L=0, J^{PC}=0^{++}$)



- broad resonance coupling mostly to $D\bar{D}$

candidate for $\chi_{c0}(2P)$

$$\text{lat} : m = 3949_{-20}^{+28} \text{ MeV} \quad g = 1.35_{-0.08}^{+0.04} \text{ GeV}$$

$$X(3860) : m = 3862_{-35}^{+48} \text{ MeV} \quad g = 2.5_{-0.9}^{+1.2} \text{ GeV} \quad \Gamma \equiv g^2 p_D^{2l+1} / m^2$$

Belle 2017

- state near $D_s\bar{D}_s$ threshold coupling mostly to $D_s\bar{D}_s$

$$\frac{|c_{D\bar{D}}^2|}{|c_{D_s\bar{D}_s}^2|} = 0.02_{-0.01}^{+0.02}$$

$$\text{lat} : m - 2m_{D_s} = -0.2_{-4.9}^{+0.16} \text{ MeV}, \quad g = 0.10_{-0.03}^{+0.21} \text{ GeV}$$

- state near $D\bar{D}$ threshold

near pole
$$t_{ij} \sim \frac{c_i c_j}{(E_{cm}^p)^2 - E_{cm}^2}$$

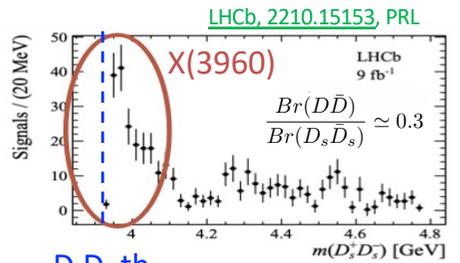
Charmonium(like) states: poles and masses

$\bar{c}c$, $\bar{c}q\bar{q}c$ $q=u,d,s$ $I=0$

$$\frac{|c_{D\bar{D}}^2|}{|c_{D_s\bar{D}_s}^2|} = 0.02^{+0.02}_{-0.01}$$

$$T_{ij}(E_{cm}) \sim \frac{c_i c_j}{E_{cm}^2 - m^2}$$

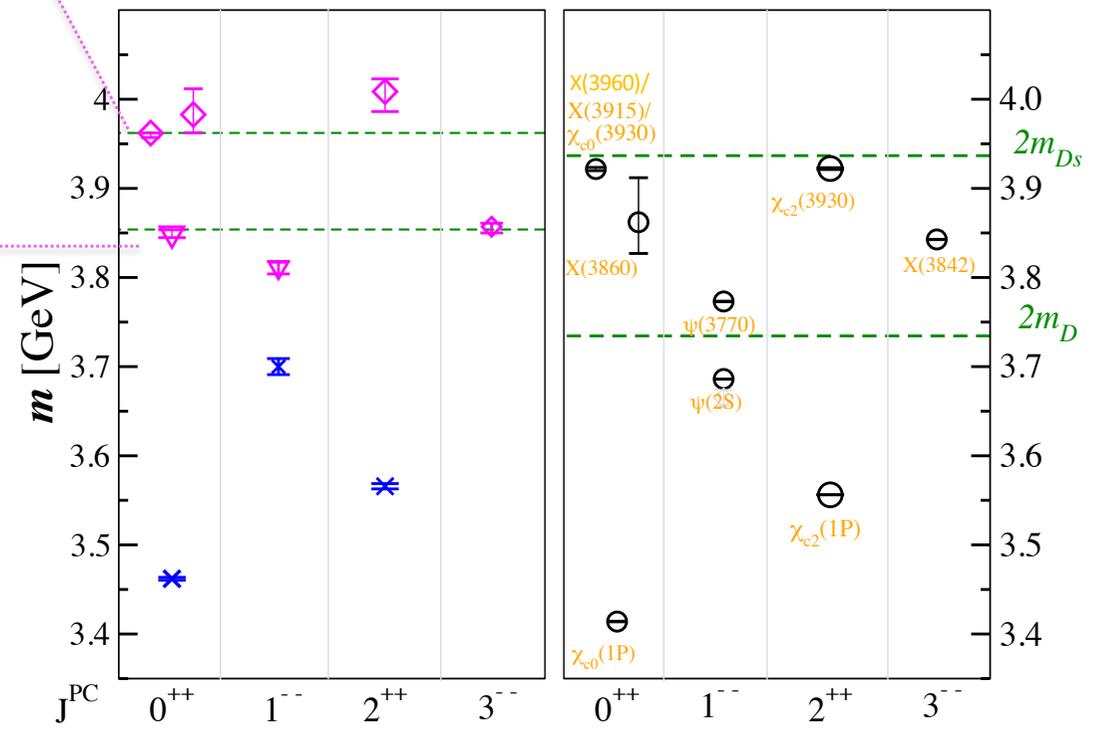
$\bar{D}_s D_s$ $J^P=0^+$ state



$m_\pi \simeq 280$ MeV $m_D \simeq 1927$ MeV

Lat CLS ensembles

Exp



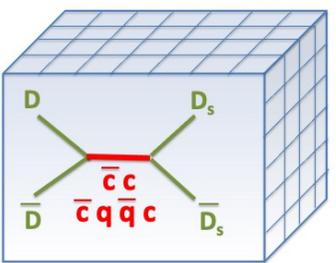
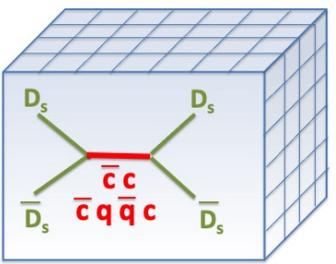
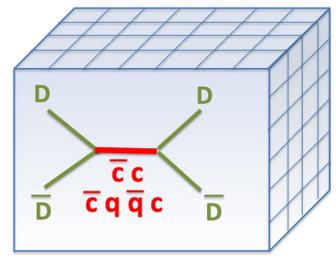
$\bar{D}D$ $J^P=0^+$ state

predicted in models [Oset et al, 0612179 PRD, Guo et al 2101.01021]

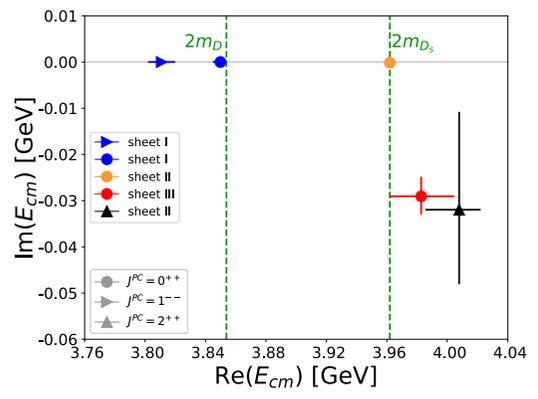
seen in re-analysis of exp.

[Danilkin et al 2111.15033, Ji, F.K. Guo et al., 2212.00613]

+ expected conventional charmonia

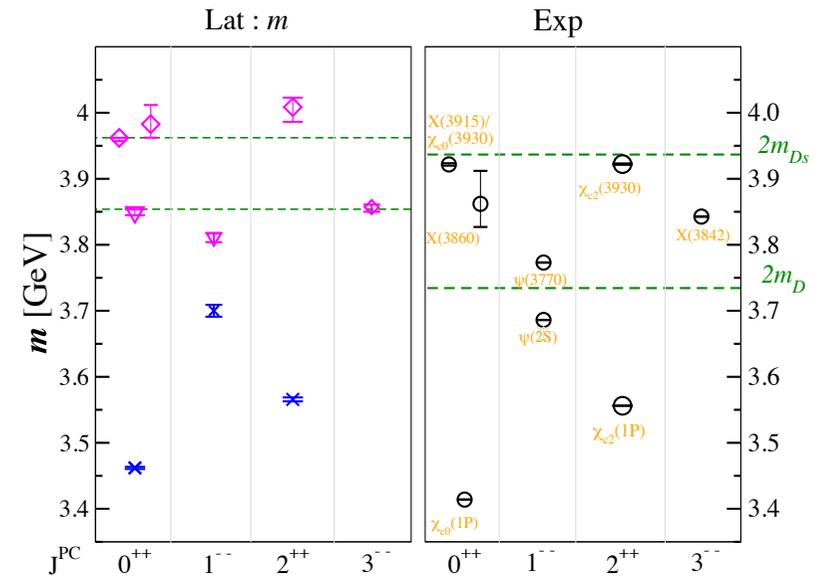
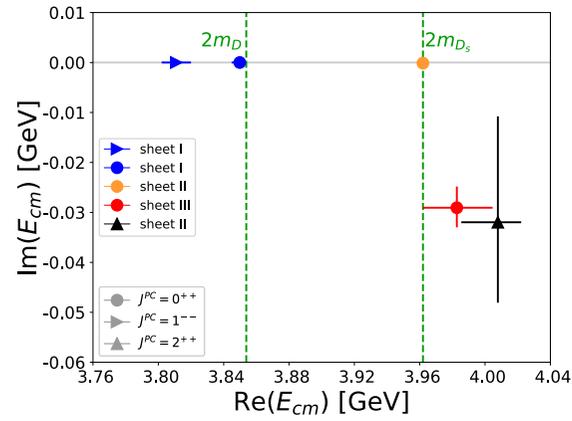


$\bar{D}D - \bar{D}_s D_s$

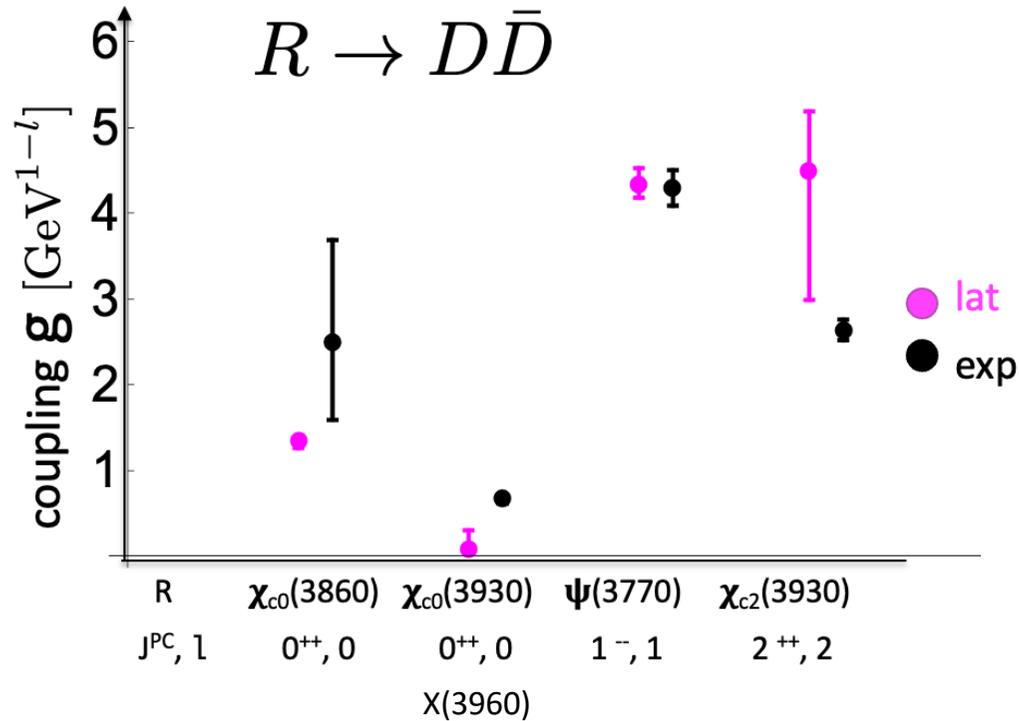


Charmonium(like) resonances and bound states

Charmonium(like) states: widths and masses



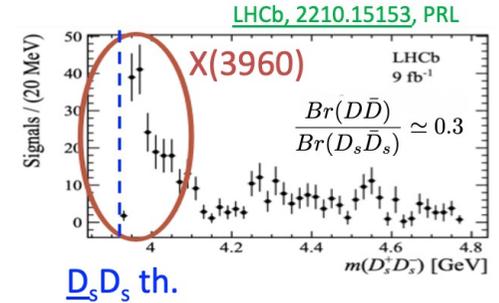
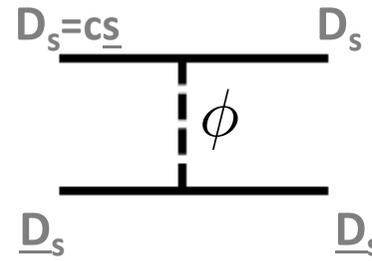
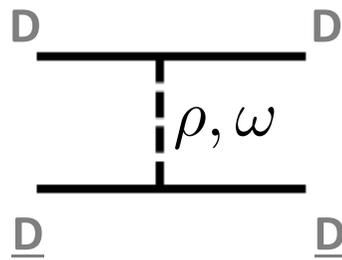
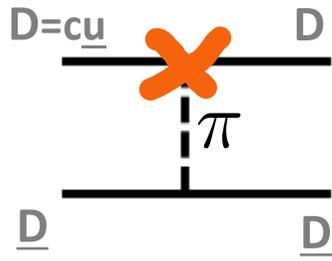
$$\Gamma \equiv g^2 \frac{p_D^{2l+1}}{m^2}$$



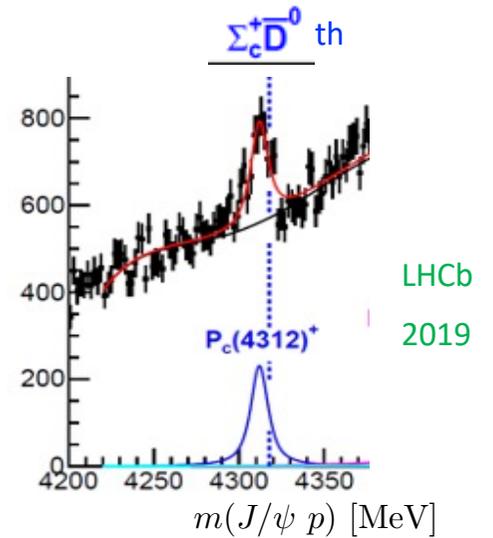
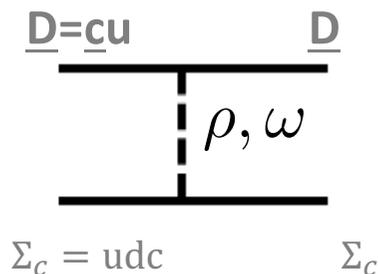
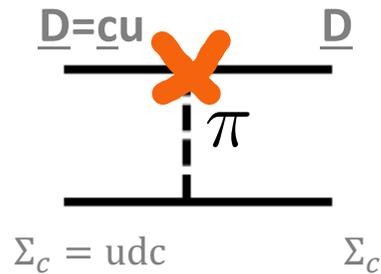
Possible interpretation of some near-threshold states:

a number of pheno studies
 Oset et al, 0612179 PRD,
 Wu, Molina, Oset, Zou, 1007.0573, PRL
 Guo et al, 2101.01021,...

“molecules” attracted by V exchange



now support also from lattice



currently to challenging for lattice

Challenges, caveats, ...

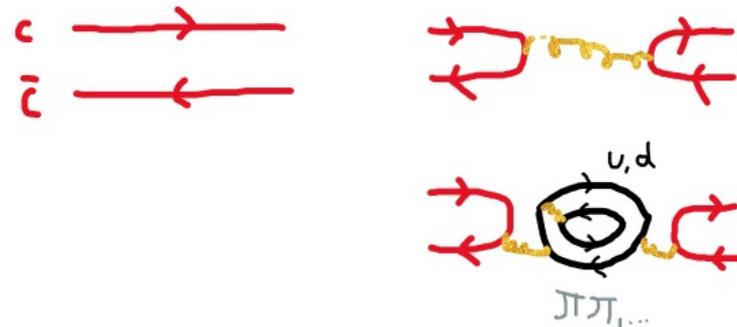
see: Assumptions and simplifications (Section 5, [2011.02542, JHEP 2021](#))

➤ caveat: lattice simulation at unphysical quark masses and single lattice spacing

➤ further channels:

- incorporated with interpolators, treated as decoupled: $J/\psi \ \omega, \ D^* \bar{D}^*$
- omitted $\eta_c \ \eta$

➤ charm annihilation omitted



Instead of conclusions

$$\bar{c}c, \bar{c}c\bar{q}q \quad (\bar{q}q = \bar{u}u + \bar{d}d, \bar{s}s) \quad I = 0$$

this talk

$$\bar{c}c\bar{d}u \quad I = 1$$

further exciting studies with Regensburg group

$$cc\bar{d}\bar{u}$$

....

Thanks to prof. Andreas Schafer for
involvement, support, effort, patience, enthusiasm, !

Backup

$J^{PC}=0^{++}$

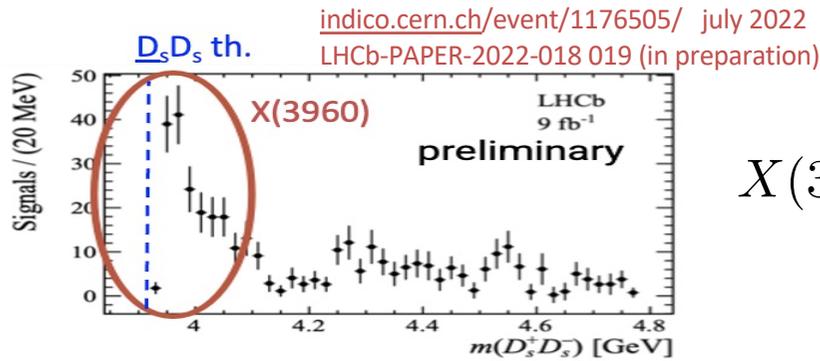
$\bar{c}s\bar{s}c$

likely related to $X(3915)$ / $\chi_{c0}(3930)$ / $X(3960)$

$$\text{lat: } \frac{|c_{D\bar{D}}^2|}{|c_{D_s\bar{D}_s}^2|} = 0.02^{+0.02}_{-0.01}$$

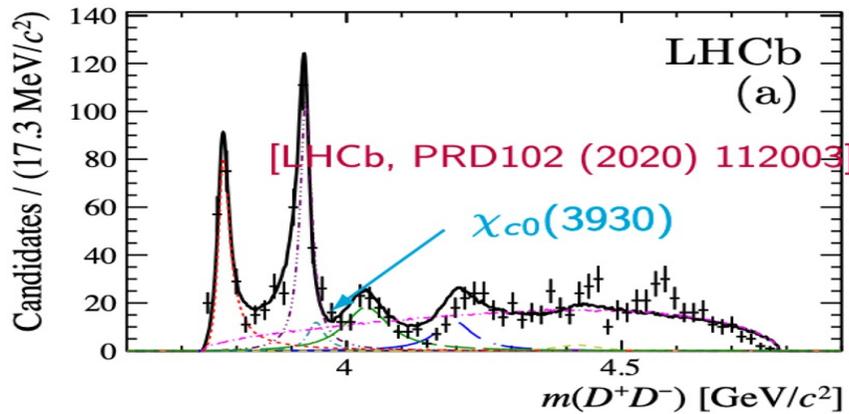
all three likely the same state
currently named $\chi_{c0}(3914)$ in PDG

talk by
Chen Chen
today

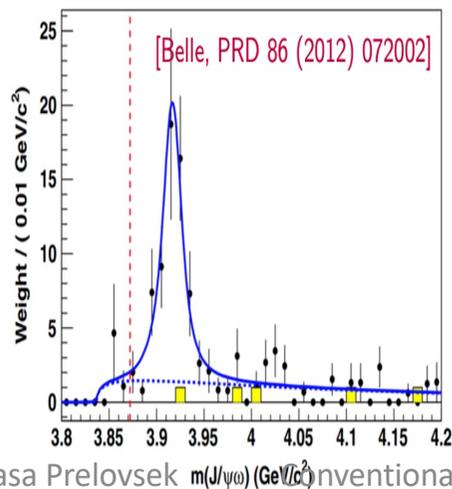


$$X(3960) \rightarrow D_s \bar{D}_s$$

$$\text{exp: } \frac{Br(D\bar{D})}{Br(D_s\bar{D}_s)} \simeq 0.3$$



$$\chi_{c0}(3930) \rightarrow D\bar{D}$$



$$X(3915) \rightarrow J/\psi \omega$$

$$(\Delta E^{\text{lat}}) = (E^{\text{lat}}) - (E_{H_1(\vec{p}_1)}^{\text{lat}}) - (E_{H_2(\vec{p}_2)}^{\text{lat}}) ,$$

$$(E^{\text{calc}}) = (\Delta E^{\text{lat}}) + (E_{H_1(\vec{p}_1)}^{\text{cont}}) + (E_{H_2(\vec{p}_2)}^{\text{cont}}) ,$$