# Parton Distributions from Boosted Fields in the Coulomb Gauge

LaMET 2023

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**JULY 24, 2023**

Xiang Gao, Wei-Yang Liu and Yong Zhao, arXiv: 2306.14960.

Argonne NATIONAL LABORATORY

# Outline

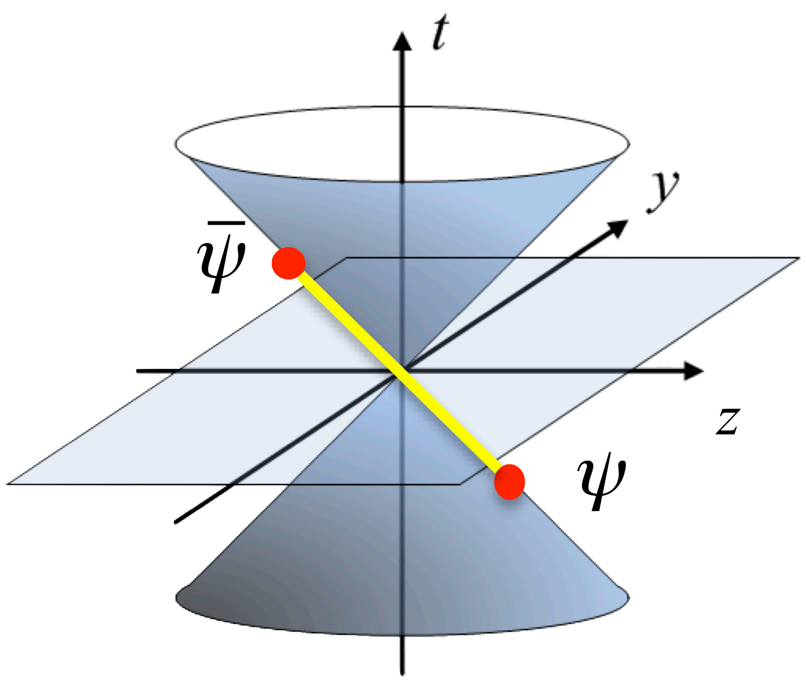
- **Methodology**
  - Large-Momentum Effective Theory
  - Universality class and quasi-PDF in the Coulomb gauge
  - Factorization
- **Lattice calculation**
  - Bare matrix elements at on- and off-axis momenta
  - Renormalization and matching
  - Comparison of final results
- **Outlook**

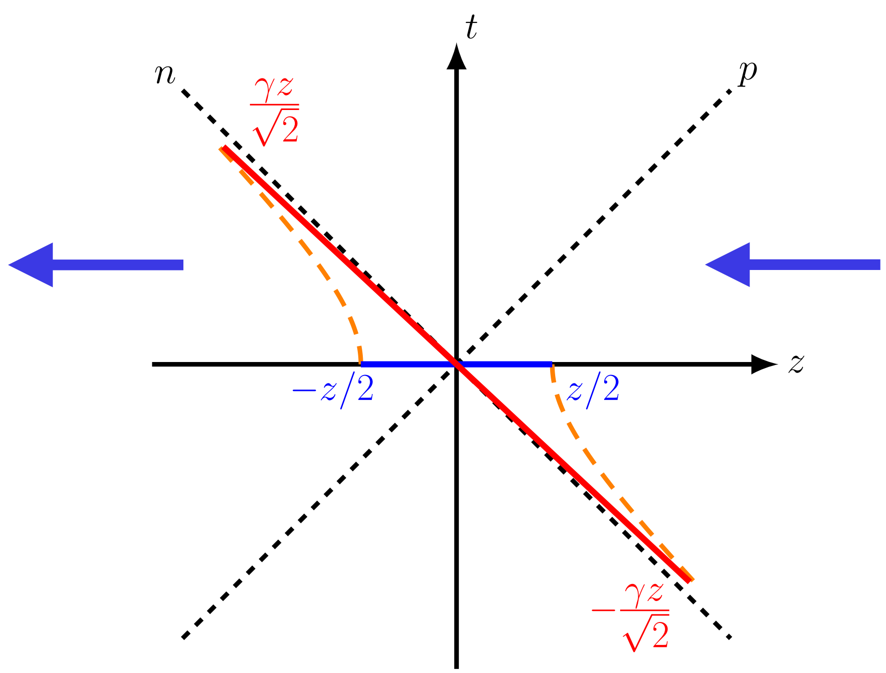
# Large-Momentum Effective Theory (LaMET)

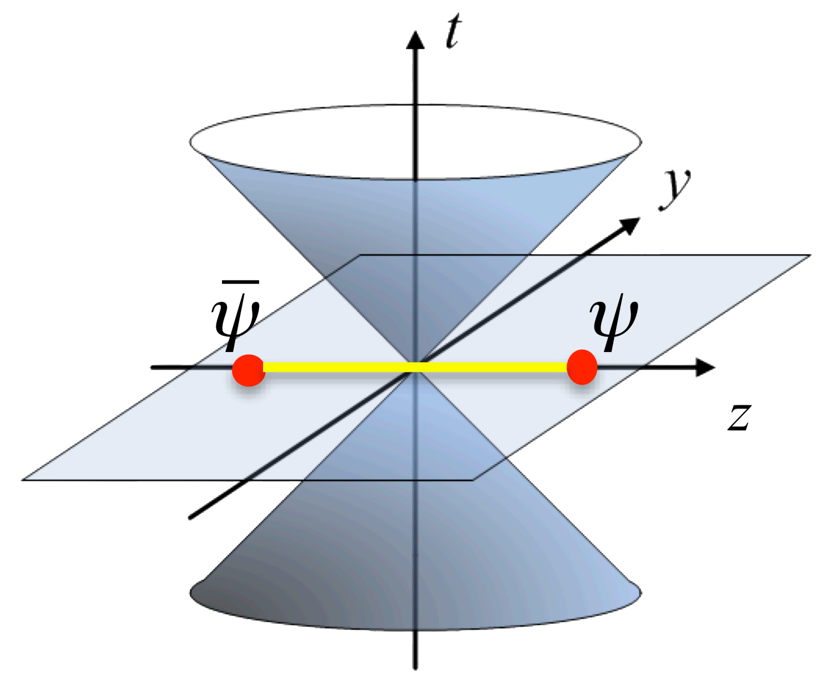
$z + ct = 0, \;\; z - ct \neq 0$

Related by Lorentz boost

$t = 0, \;\; z \neq 0$




PDF $f(x)$:
Cannot be calculated on the lattice

- X. Ji, PRL **110** (2013); SCPMA **57** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

$$f(x) = \int \frac{dz^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(z^-) \times \frac{\gamma^+}{2} W[z^-, 0] \psi(0) | P \rangle$$

$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{iz(xP^z)} \langle P | \bar{\psi}(z) \times \frac{\gamma^z}{2} W[z, 0] \psi(0) | P \rangle$$

# Large-Momentum Effective Theory (LaMET)

**Systematic calculation of *x*-dependence:**

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \ + \ \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Renormalization

Perturbative Matching

# Large-Momentum Effective Theory (LaMET)

## Systematic calculation of *x*-dependence:

$$f(x,\mu)=\int_{-\infty}^{\infty}\frac{dy}{|y|}C^{-1}\left(\frac{x}{y},\frac{\mu}{yP^z},\frac{\tilde{\mu}}{\mu}\right)\tilde{f}(y,P^z,\tilde{\mu})\ +\ \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2},\frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

**Renormalization**

$$O_B^{\Gamma}(z,a)=\bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0)$$
$$=e^{-\delta m(a)|z|}\,Z_O(a)O_R^{\Gamma}(z)$$

**"Hybrid scheme"** X. Ji, YZ, et al., NPB **964** (2021).

$$|z|\le z_s\,,\quad \frac{h(z,P^z,a)}{h(z,0,a)}$$

$$|z|>z_s\,,\quad e^{(\delta m(a)+\bar{m}_0)|z|}\frac{h(z,P^z,a)}{h(z_s,0,a)}$$

- **Subtraction of linear divergence**

$$\delta m(a)=\frac{m_{-1}}{a}+\mathcal{O}(\Lambda_{\rm QCD})$$ Renormalon ambiguity

  - Self renormalization — Y. Huo, et al. (LPC), NPB **969** (2021).
  - Static potential — X. Gao, YZ, et al., PRL **128** (2022).
  - …

- **Subtraction of leading renormalon ambiguity**

Matching to the OPE of $P^z$=0 matrix element:

$$C_0^{\overline{\rm MS}}(\mu,z)=C_0^{\rm LRR}(\mu,z)e^{-m_0^{\overline{\rm MS}}|z|}$$

Leading-renormalon resummation

See talk by R. Zhang

- Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
- Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023).

# Large-Momentum Effective Theory (LaMET)

## Systematic calculation of *x*-dependence:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

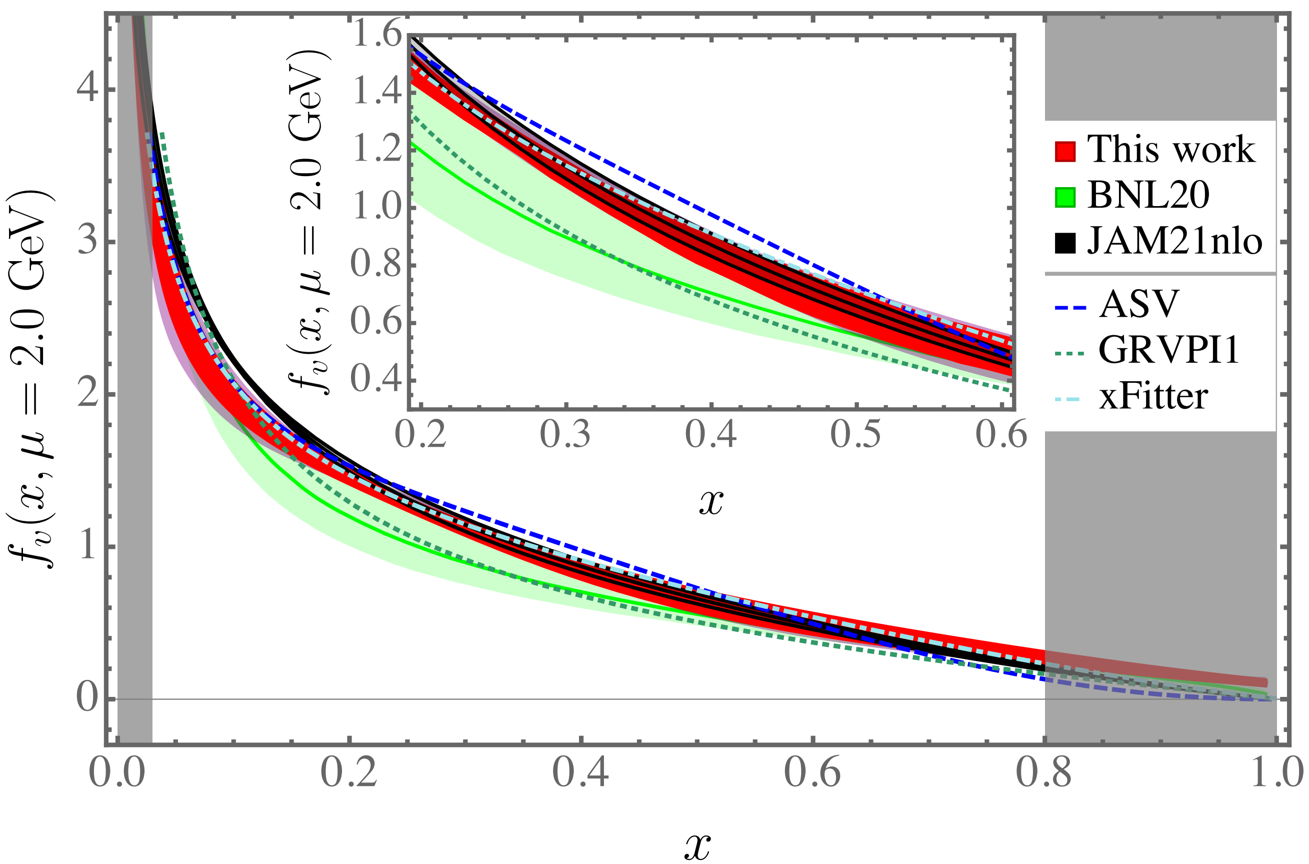
Perturbative Matching

- **Next-to-next-to-leading order (NNLO) kernel**
  - Chen, Zhu and Wang, PRL **126** (2021);
  - Li, Ma and Qiu, PRL **126** (2021).
- **Resummation of small-*x* logarithms $\alpha_s \ln \frac{\mu}{2xP^z}$ (DGLAP evolution)**
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - Y. Su, J. Holligan et al., NPB **991** (2023).
- **Subtraction of leading renormalon $C(x/y) \to C^{\rm LRR}(x/y)$**
  - Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
  - Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023).
- **Resummation of large-*x* (threshold) logarithms $\frac{\alpha_s \ln(1-x/y)}{(1-x/y)}$**
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - X. Ji, Y. Liu and Y. Su, arXiv:2305.04416.

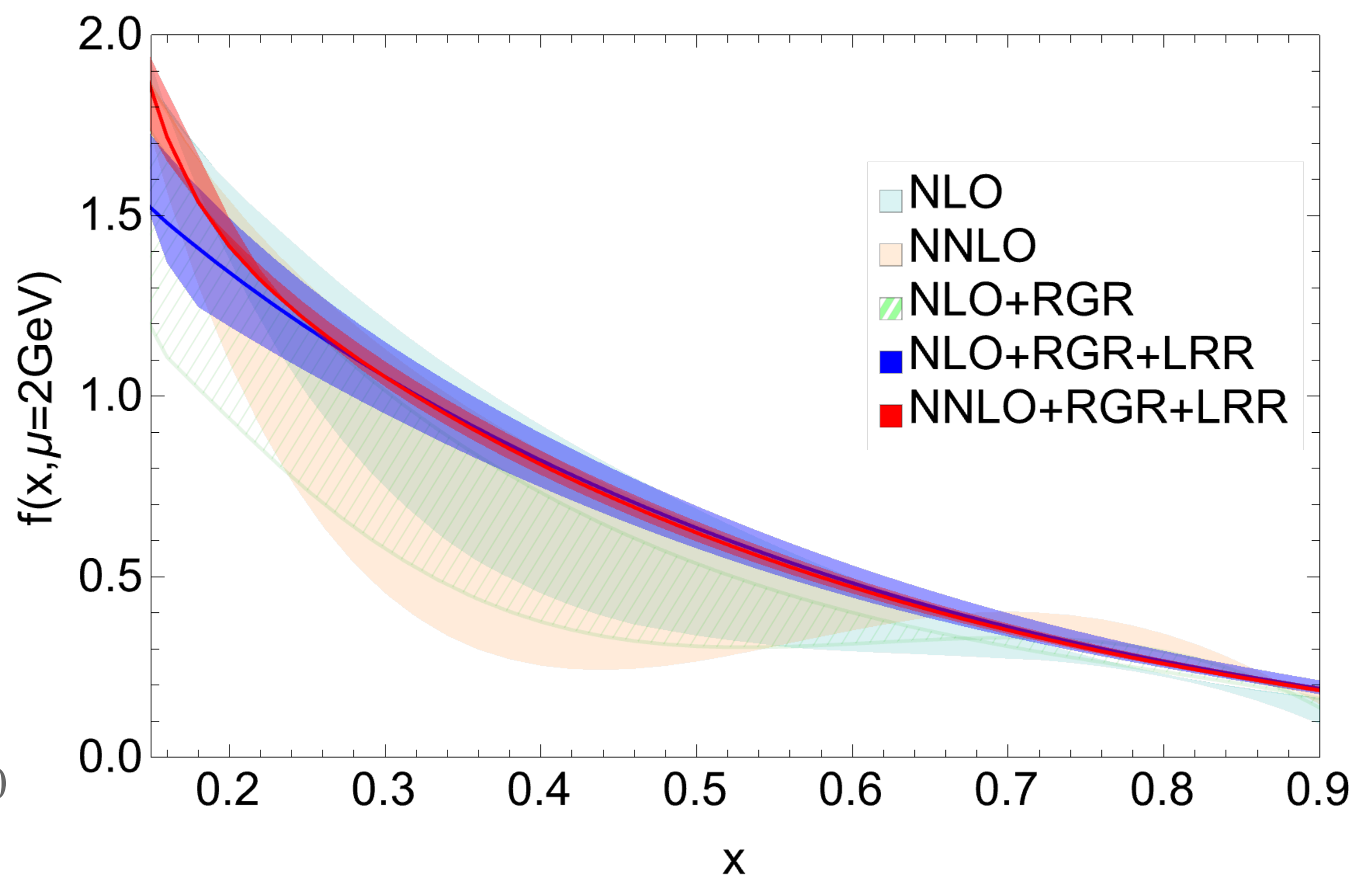
See talk by Y. Liu

# State-of-the-art calculation of pion PDF

Gao, YZ et al. (BNL-ANL21), **128** (2022).




Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023)




| | Hybrid scheme | Leading-renormalon resummation | NNLO | Small-*x* resummation | Threshold resumation | Subleading renormalon | Discretization effects |
|---|---|---|---|---|---|---|---|
| BNL-ANL21 | ✔ | | ✔ | | | | |
| ZJHS23 | ✔ | ✔ | ✔ | ✔ | | | |

See talk by X. Ji

# Universality in LaMET

- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

**Gauge-invariant bilinear**

$$\bar{\psi}(z)\Gamma W[z,0]\psi(0)$$

**Current-current correlator**

$$J^{\mu}(z)J^{\nu}(0)$$

- Detmold and Lin, PRD **73** (2006);
- Braun and Müller, EPJC **55** (2008);
- A Chambers et al. (QCDSF), PRL **118** (2017)
- Ma and Qiu, PRL **120** (2018).

**Free bilinear in a physical gauge**

$$\bar{\psi}(z)\Gamma\psi(0)\Big|_{G(A)=0} \qquad G(A) = A^0, A^z, \nabla\cdot\mathbf{A}$$

X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

**Light-cone bilinear**

$$\bar{\psi}(\xi^-)\gamma^+ W[\xi^-,0]\psi(0)$$

**Or**

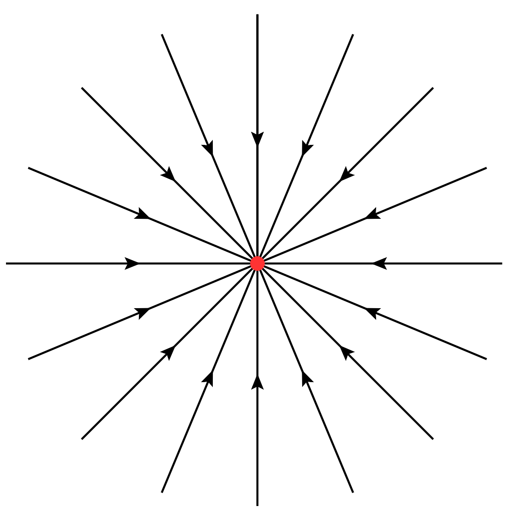
$$\bar{\psi}(\xi^-)\gamma^+\psi(0)\Big|_{A^+=0}$$

# Quasi-PDF in the Coulomb gauge

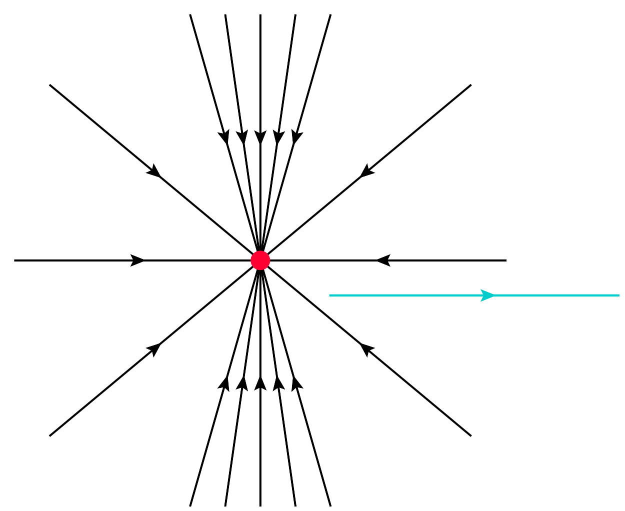
$$\tilde{h}(\vec{z},\vec{p},\mu) = \frac{1}{2p^t}\langle p\,|\,\bar{\psi}(\vec{z})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|p\rangle\,, \quad \vec{z}\ //\ \vec{p}$$

$$\tilde{f}(x,|\vec{p}\,|,\mu) = |\vec{p}\,|\int_{-\infty}^{\infty}\frac{d|\vec{z}\,|}{2\pi}\,e^{ix\vec{p}\cdot\vec{z}}\,\tilde{h}(\vec{z},\vec{p},\mu)$$

Static charge



Moving charge



**First proposed in the lattice calculation of gluon helicity**

$$\Delta G = \langle P_\infty\,|\,(\mathbf{E}\times\mathbf{A})^3\,|_{\nabla\cdot\mathbf{A}=0}\,|P_\infty\rangle$$

- X. Ji, J.-H. Zhang and YZ, PRL **111** (2013);
- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, J.-H. Zhang and YZ, PLB **743** (2015);
- Y.-B. Yang, R. Sufian, YZ, et al. PRL **118** (2017).

# Quasi-PDF in the Coulomb gauge

$$\tilde{h}(\vec{z},\vec{p},\mu) = \frac{1}{2p^t}\langle p\,|\,\bar{\psi}(\vec{z})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|\,p\rangle\,, \quad \vec{z}\ //\ \vec{p}$$

$$\tilde{f}(x,|\vec{p}\,|,\mu) = |\vec{p}\,|\int_{-\infty}^{\infty}\frac{d\,|\vec{z}\,|}{2\pi}\;e^{ix\vec{p}\cdot\vec{z}}\;\tilde{h}(\vec{z},\vec{p},\mu)$$

| | **Momentum direction** | **Renormalization** | **Gribov copies** | **Power corrections** | **Mixing** | **Higher-order corrections** |
|---|---|---|---|---|---|---|
| Gauge-invariant (GI) | $(0,0,n_z)$<br>$(n_x,0,0)$<br>$(0,n_y,0)$ | Linear divergence + vertex and wave function renormalization | N/A | $\Lambda^2_{\rm QCD}/P_z^2$<br>with renormalon subtraction | Lorentz symmetry | Available at NNLO now |
| Coulomb gauge (CG) | $(n_x, n_y, n_z)$<br>🙂 | Wave function renormalization<br>🙂 | Affecting IR (long range) region<br>🧐 | $\Lambda^2_{\rm QCD}/\vec{p}^{\,2}$ | 3D rotational symmetry<br>🙁🧐 | Difficult to go beyond NLO<br>🧐 |

# Factorization

- Large-momentum factorization:

$$\tilde{f}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{|y|P^z}\right) f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(1-x)^2 P_z^2}\right)$$

$$C\left(\xi, \frac{\mu}{p^z}\right) = \delta(\xi - 1) + \frac{\alpha_s C_F}{2\pi} C^{(1)}\left(\xi, \frac{\mu}{p^z}\right) + \mathcal{O}(\alpha_s^2)$$

$$C^{(1)}\left(\xi, \frac{\mu}{p^z}\right) = C^{(1)}_{\rm ratio}\left(\xi, \frac{\mu}{p^z}\right) + \frac{1}{2|1-\xi|} + \delta(1-\xi)\left[-\frac{1}{2}\ln\frac{\mu^2}{4p_z^2} + \frac{1}{2} - \int_0^1 d\xi' \frac{1}{1-\xi'}\right]$$

$$C^{(1)}_{\rm ratio}\left(\xi, \frac{\mu}{p^z}\right) = \left[P_{qq}(\xi)\ln\frac{4p_z^2}{\mu^2} + \xi - 1\right]_{+(1)}^{[0,1]}$$

$$+\left\{P_{qq}(\xi)\Big[\mathbf{sgn}(\xi)\ln|\xi| + \mathbf{sgn}(1-\xi)\ln|1-\xi|\Big] + \mathbf{sgn}(\xi) + \frac{3\xi - 1}{\xi - 1}\frac{\tan^{-1}\left(\frac{\sqrt{1-2\xi}}{|\xi|}\right)}{\sqrt{1-2\xi}} - \frac{3}{2|1-\xi|}\right\}_{+(1)}^{(-\infty,\infty)}$$

$$\xrightarrow{\xi\to\infty} \frac{1}{\xi^2}$$

# Factorization

- Short-distance factorization:

$$\tilde{h}(z, P^z, \mu) = \int du\ \mathscr{C}(u, z^2\mu^2) h(u\tilde{\lambda}, \mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

$$\mathscr{C}\left(u, \frac{\mu}{p^z}\right) = \delta(u-1) + \frac{\alpha_s C_F}{2\pi}\mathscr{C}^{(1)}\left(u, \frac{\mu}{p^z}\right) + \mathcal{O}(\alpha_s^2)$$

$$\mathscr{C}^{(1)}(u, z^2\mu^2) = \mathscr{C}^{(1)}_{\rm ratio}(u, z^2\mu^2) + \frac{1}{2}\delta(1-u)\left(1 - \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)$$

$$\mathscr{C}^{(1)}_{\rm ratio}(u, z^2\mu^2) = \left[-P_{qq}(u)\ln\frac{z^2\mu^2 e^{2\gamma_E}}{4} - \frac{4\ln(1-u)}{1-u} + 1 - u\right]^{[0,1]}_{+(1)}$$

$$+\left[\frac{3u-1}{u-1}\frac{\tan^{-1}\left(\frac{\sqrt{1-2u}}{|u|}\right)}{\sqrt{1-2u}} - \frac{3}{|1-u|}\right]^{(-\infty,\infty)}_{+(1)} \qquad \xrightarrow{u\to\infty} \frac{1}{u^2}$$

# Lattice setup

Wilson-clover valence fermion on 2+1 flavor HISQ gauge configurations (HotQCD).

| $\vert\vec{p}\vert$ (GeV) | $\vec{n}$ | $\vec{k}$ | $t_s/a$ | (#ex,#sl) |
|---|---|---|---|---|
| 0 | (0,0,0) | (0,0,0) | 8,10,12 | (1, 16) |
| 1.72 | (0,0,4) | (0,0,3) | 8 | (1, 32) |
| | | | 10 | (3, 96) |
| | | | 12 | (8, 256) |
| 2.15 | (0,0,5) | (0,0,3) | 8 | (2, 64) |
| | | | 10 | (4, 128) |
| | | | 12 | (8, 256) |
| 2.24 | (3,3,3) | (2,2,2) | 8 | (1, 32) |
| | | | 10 | (2, 64) |
| | | | 12 | (4, 128) |

$a = 0.06$ fm

$m_\pi = 300$ MeV

$L_s^3 \times L_t = 48^3 \times 64$

$N_{\rm cfg} = 109$

- T. Izubuchi, L. Jin et al., PRD **100** (2019);
- X. Gao, N. Karthik, YZ et al., PRD **102** (2020).

#ex and #sl: numbers of exact and sloppy inversions per configuration

For $n_z$=(3,3,3): half the statistics for $n_z$=(0,0,5)

# Coulomb gauge fixing

- Find the gauge transformation $\Omega$ of link variables $U_i(t, \vec{x})$ that minimizes:

$$F[U^\Omega] = \frac{1}{9V} \sum_{\vec{x}} \sum_{i=1,2,3} \left[ -\textbf{re Tr } U_i^\Omega(t, \vec{x}) \right]$$

**Precision ~ $10^{-7}$**

- Gauge-variant correlations may differ in different Gribov copies.

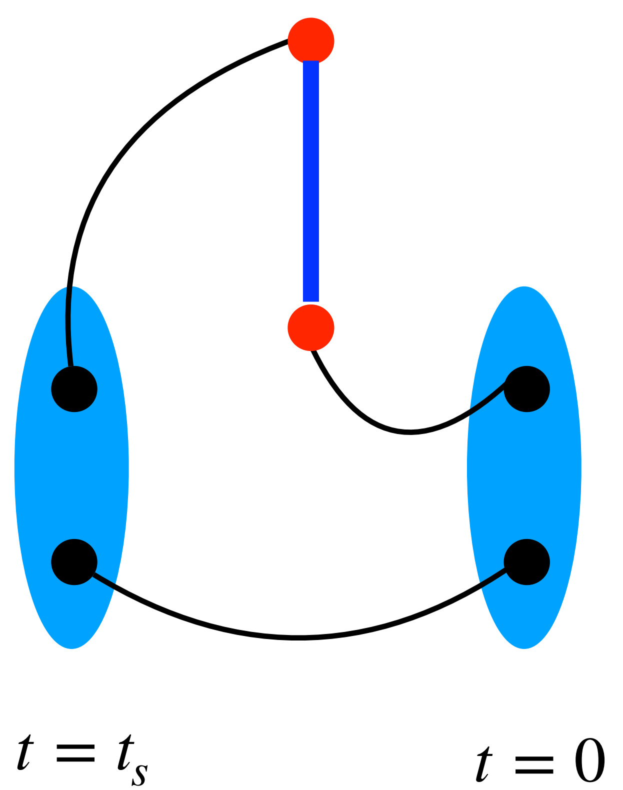
- In SU(2) Yang-Mills theory, different Gribov copies only affects the gluon propagator at far infrared region $|p| \lesssim 0.2\ \mathrm{GeV}$, though the ghost propagator are more sensitive to them.

A. Mass, Annals. Phys. 387 (2017).

- 🧐: Gribov copies only affect long-range correlations in physical states, or PDF at small *x*.

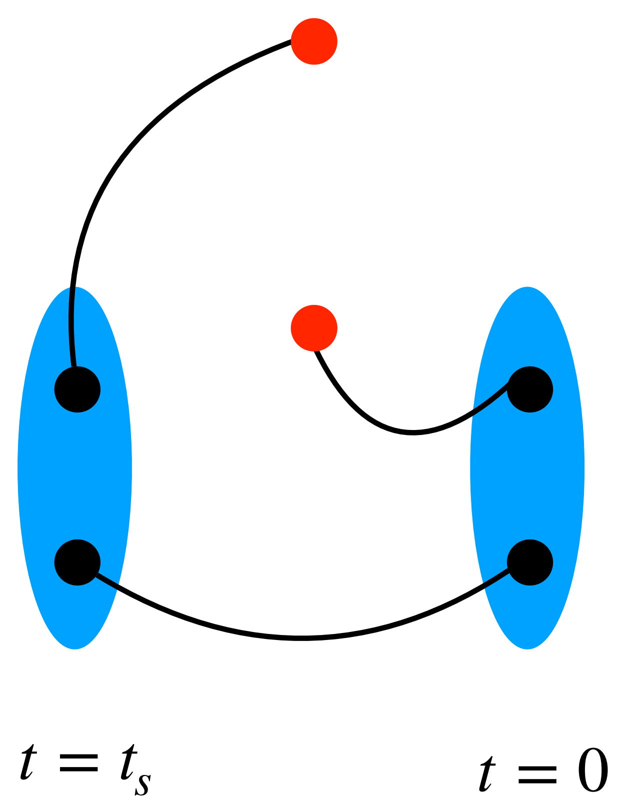
# Bare matrix elements

GI




1-step hypercubic smeared Wilson line

CG

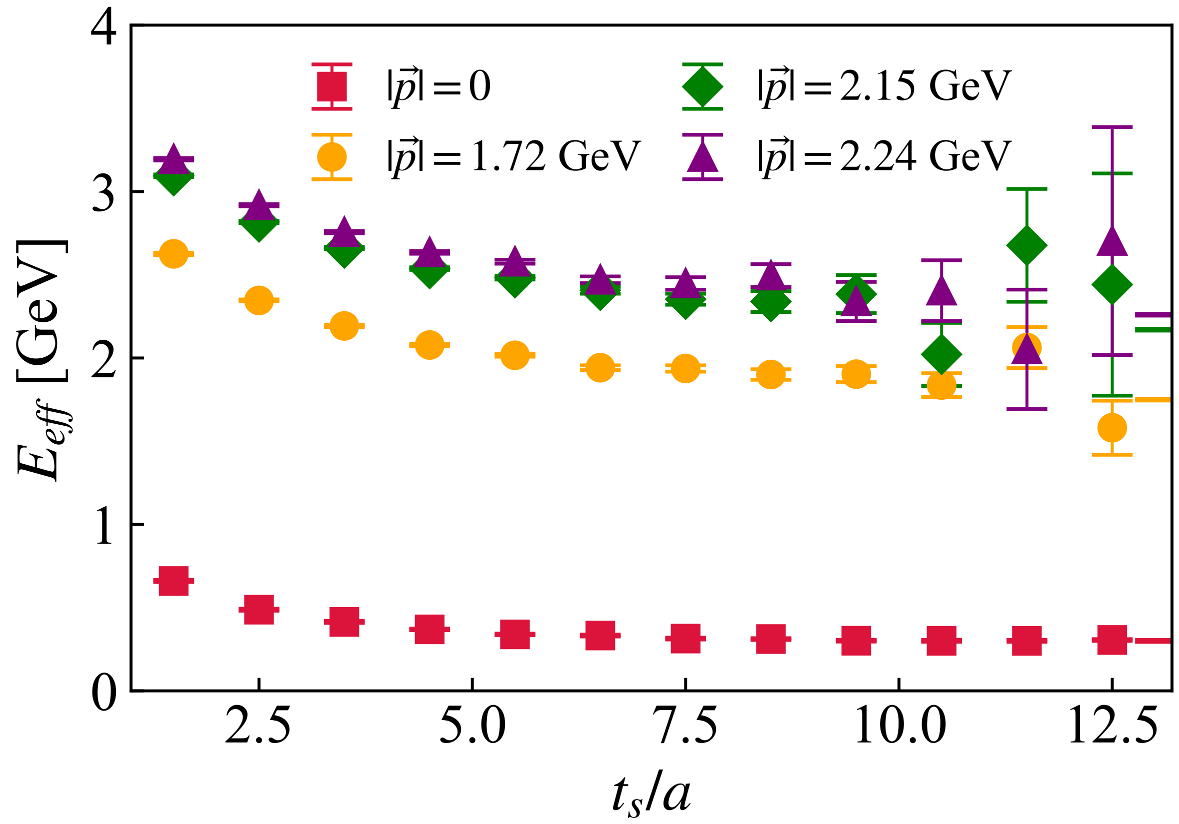



No Wilson line

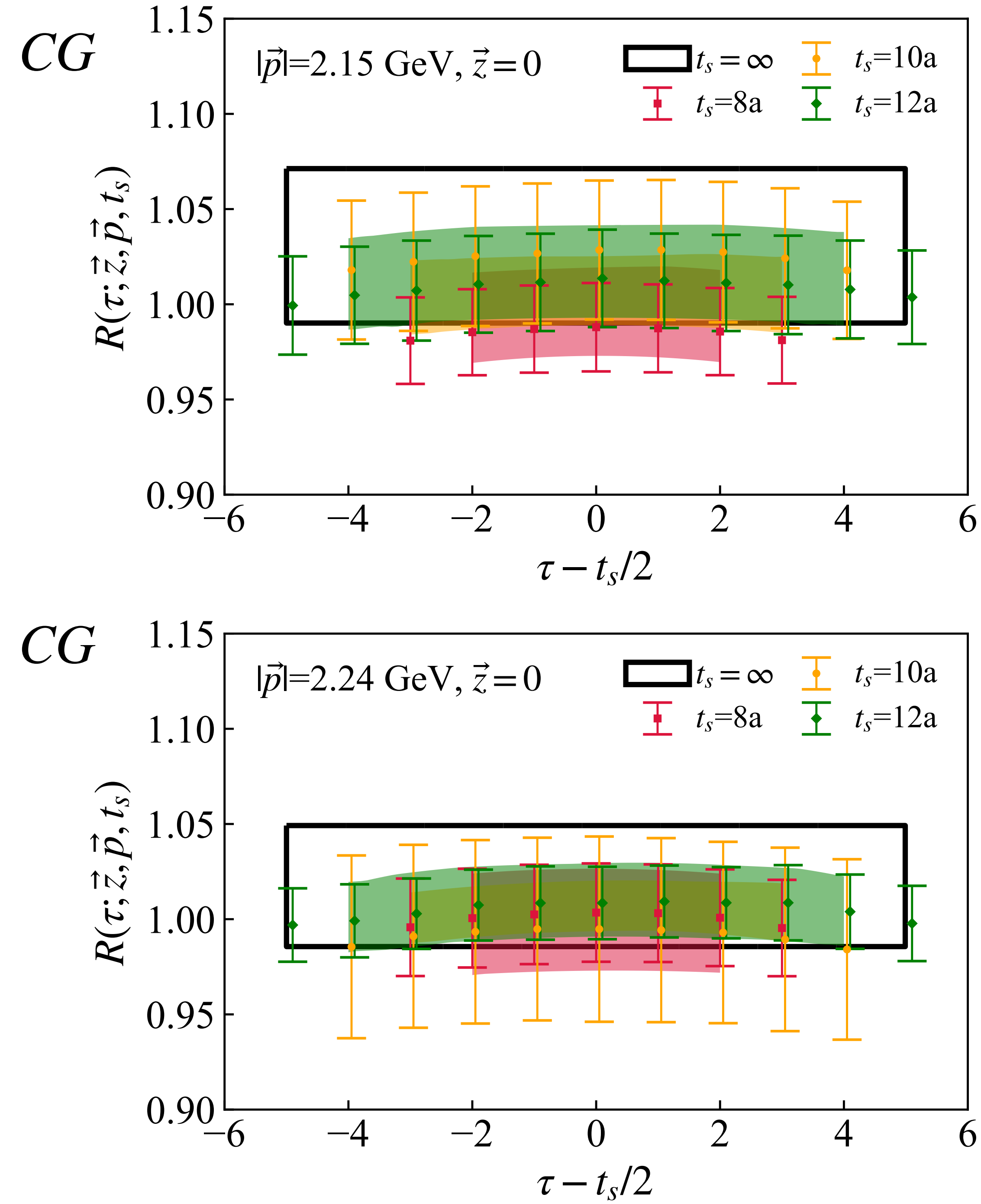
**Same quark propagators!**

# Bare matrix elements

**Effective mass**

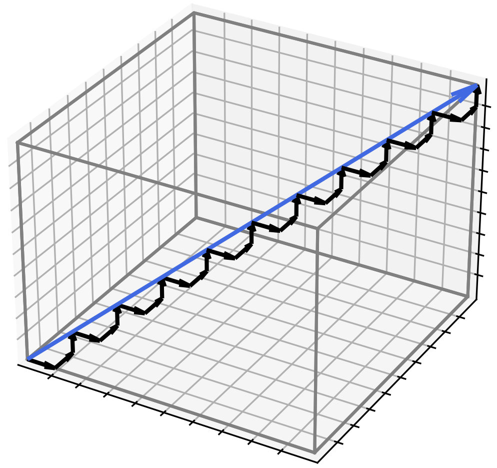



**3pt/2pt ratio**



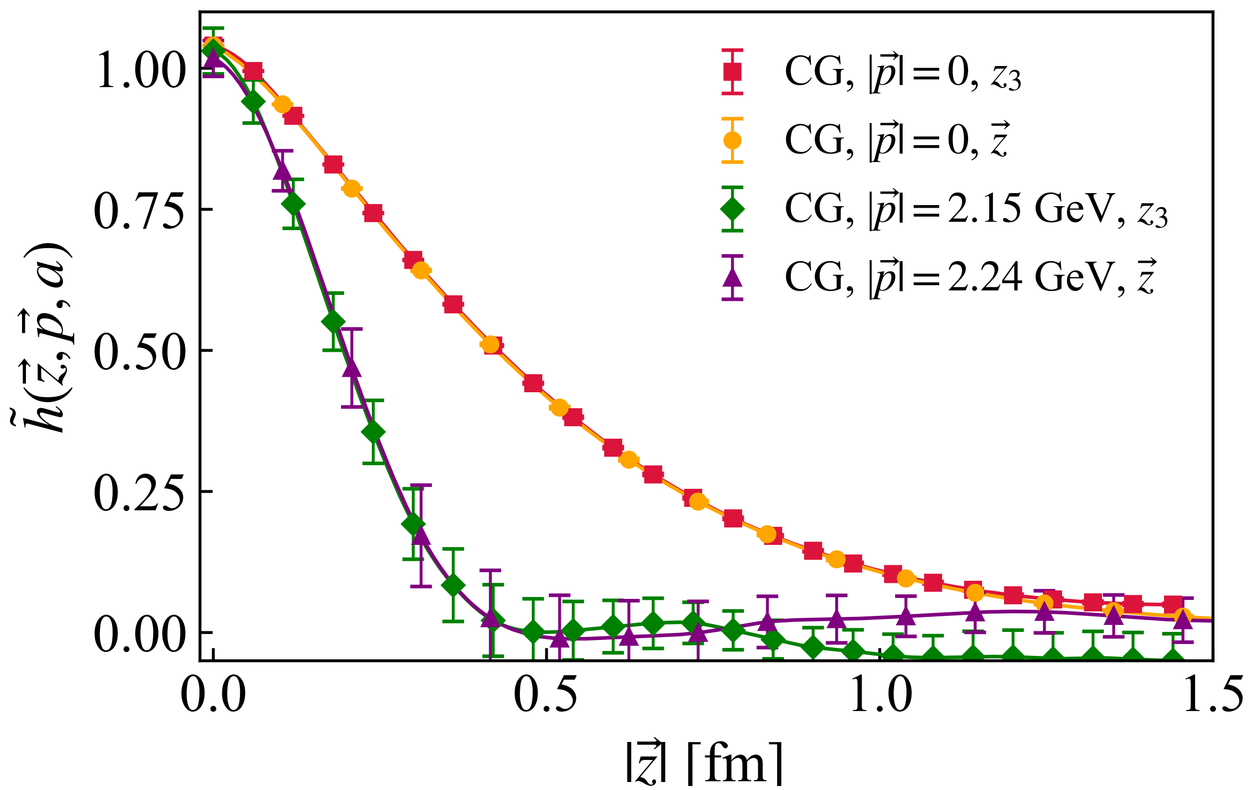
# Bare matrix elements

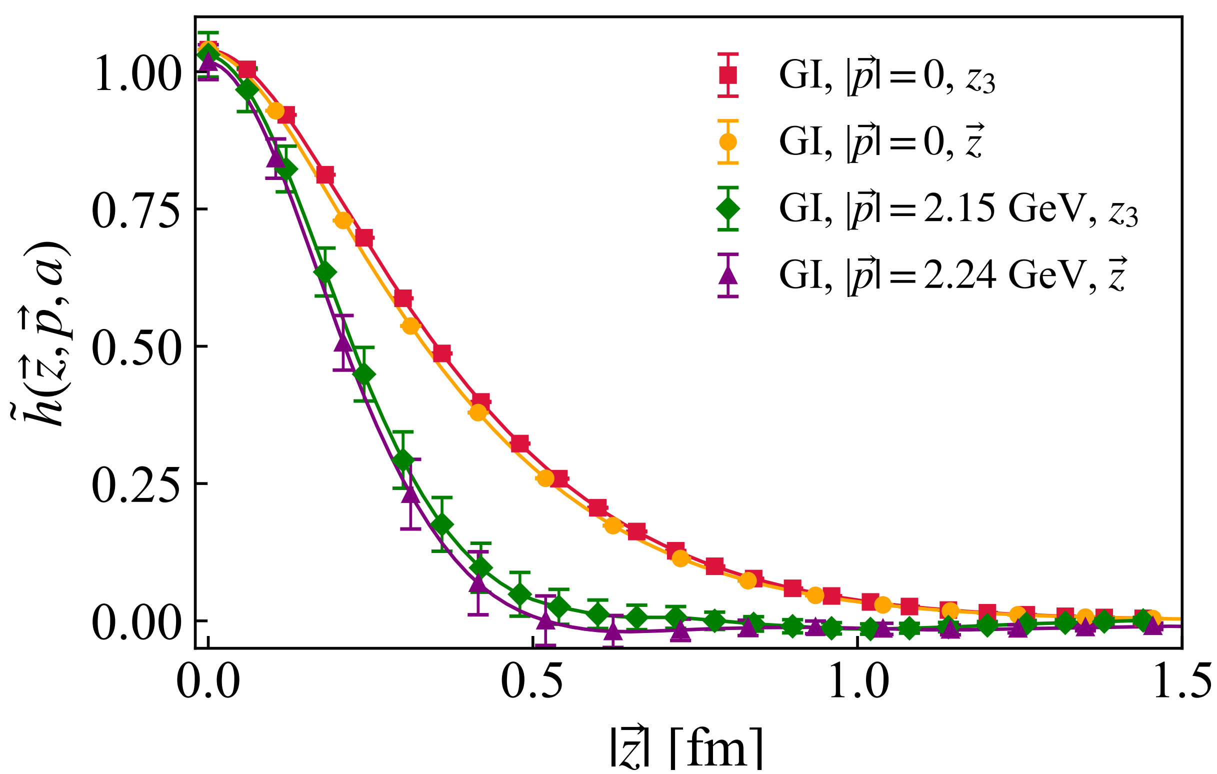
Rotational symmetry: on-axis V.S. off-axis momenta



Zig-zagged Wilson line for GI bilinear

B. Musch et al., PRD **83** (2011).

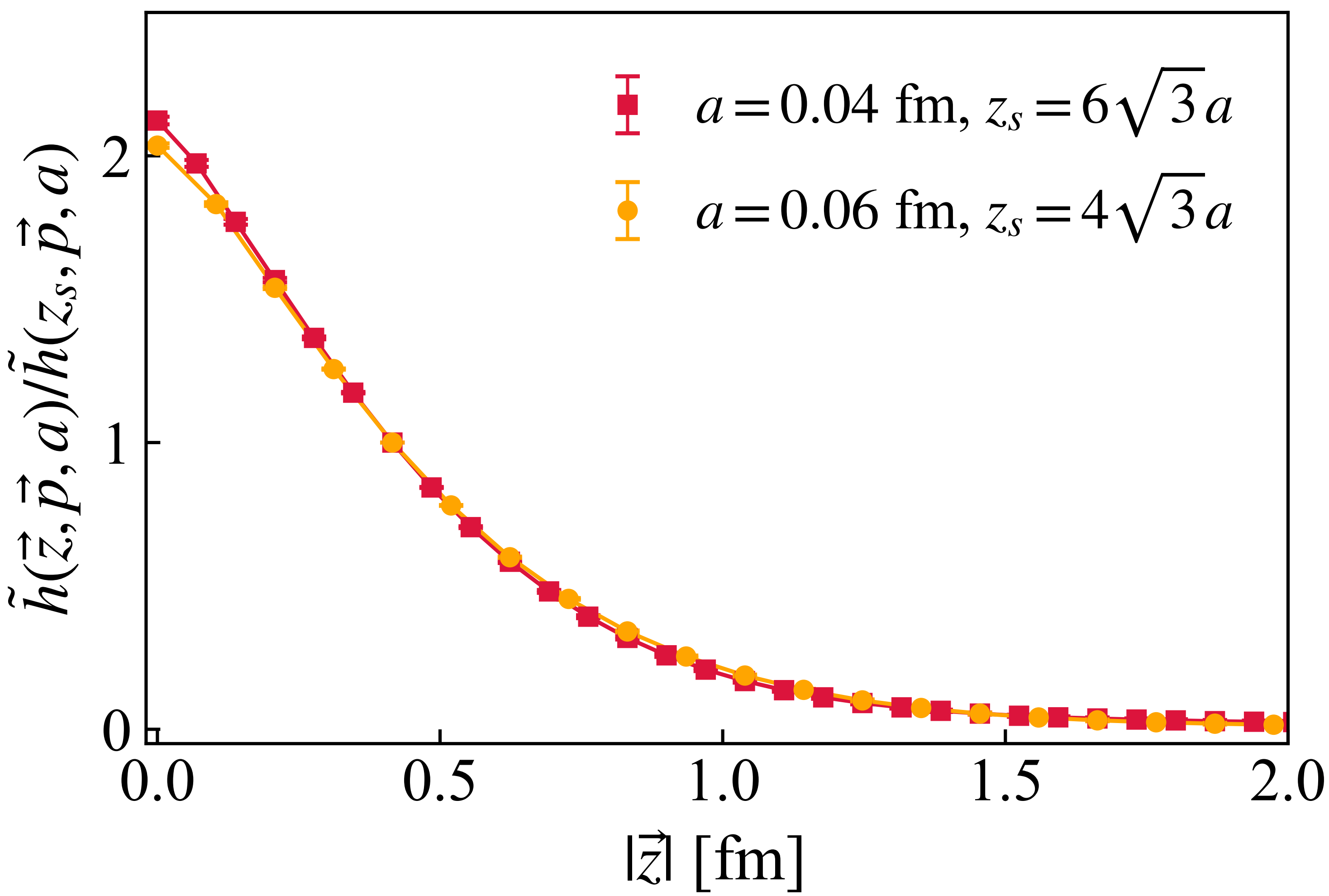



**CG matrix elements precisely preserve the 3D rotational symmetry, which is broken for GI matrix elements with a zig-zagged Wilson line**

# Renormalizability

$$\bar{\psi}_0(z)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z)\Gamma\psi(0)\right]_R \quad \Rightarrow \lim_{a\to 0} \frac{\tilde{h}(z,0,a)}{\tilde{h}(z_s,0,a)} = \text{finite}$$




**Comparison with a finer lattice with**

$a = 0.04$ fm

$m_\pi = 300$ MeV

$L_s^3 \times L_t = 64^4$

$N_{\text{cfg}} = 12$

$\vec{z} = (1,1,1)z$

**No remaining *a*-dependence except for the discretization effects at *z~a* !**

# Consistency at short distance

Double ratio:

$$\mathscr{M}(z, P^z, a) = \frac{\tilde{h}(z, P^z, a)}{\tilde{h}(z,0,a)} \frac{\tilde{h}(0,0,a)}{\tilde{h}(0,P^z, a)}$$

K. Orginos et al., PRD **96** (2017).

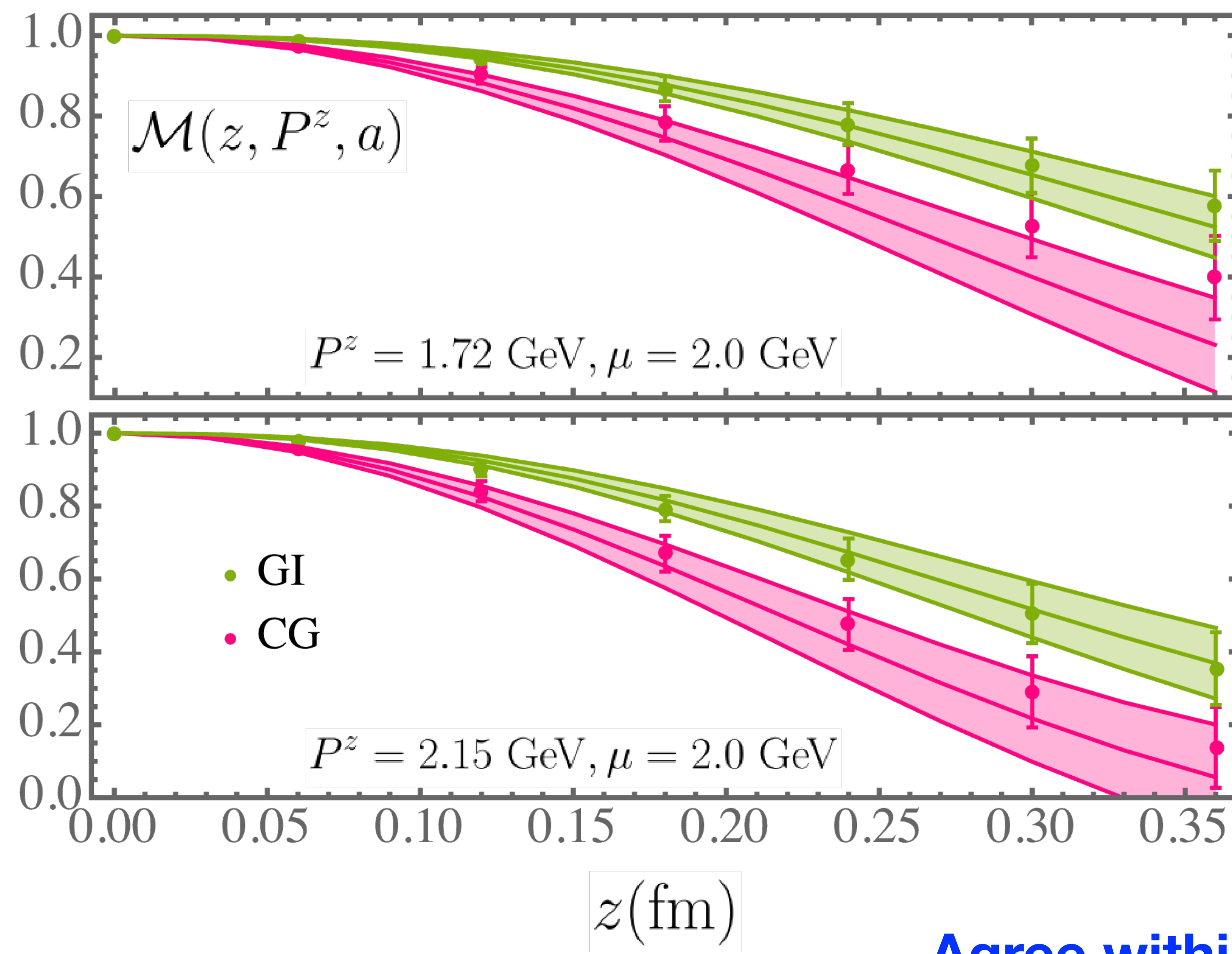
**Parameterize PDF**
$f_v(x) \sim x^\alpha (1-x)^\beta$

↓

**Fit $\alpha, \beta$ from the GI matrix elements**

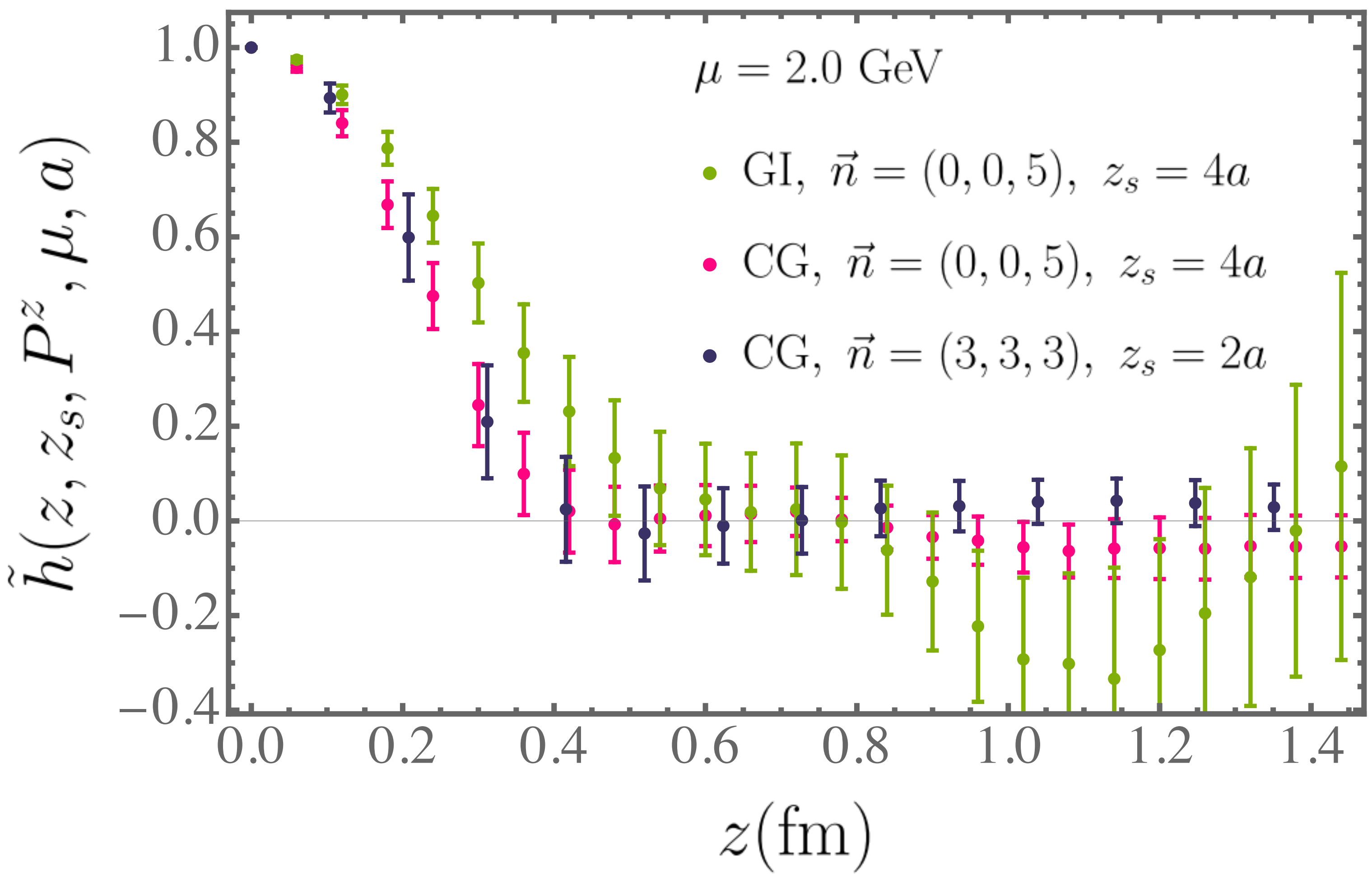
↓

**Match fitted PDF to the CG matrix elements**




**Agree within $1\sigma$ !**

# Hybrid scheme renormalization



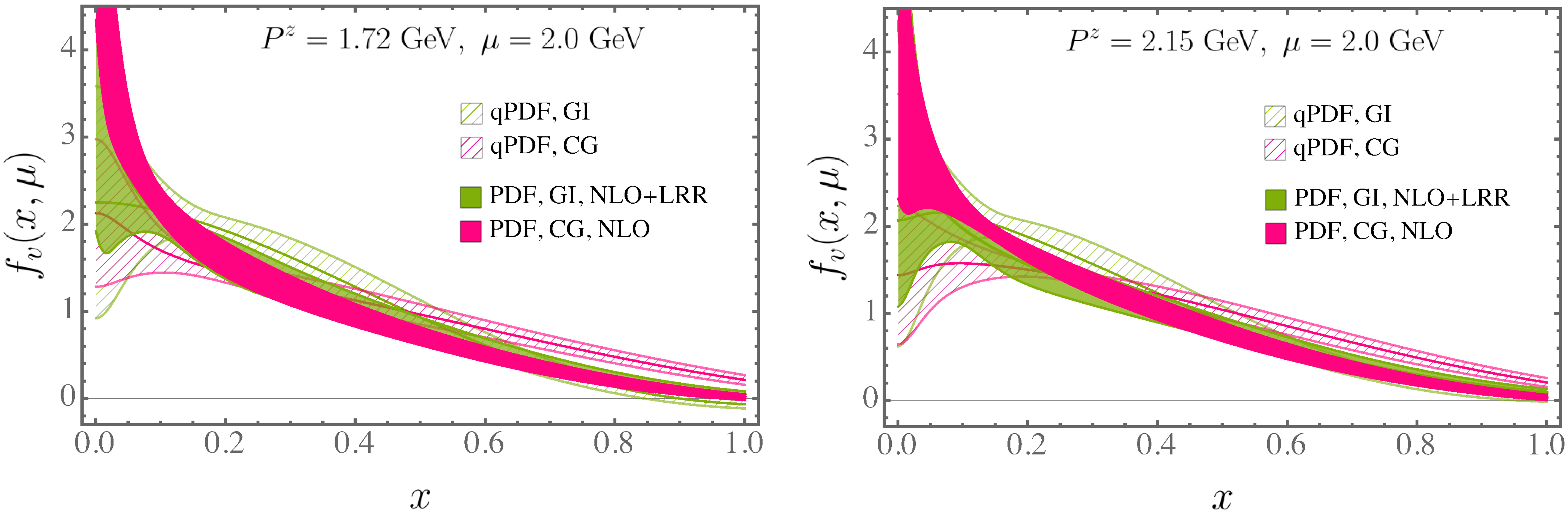

**For GI matrix elements: LRR coefficient at NLO and $\mu = 2$ GeV.**

- Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
- Zhang, Ji, Holligan and Su (ZJHS23), PLB **844** (2023).

- **Both CG matrix elements and their errors remain small at large |z|, which leads to better controlled Fourier transform;**
- **Off-axis and on-axis momenta matrix elements are at similar precision, despite half the statistics for the former.**

# Perturbative matching

Comparison of the GI and CG quasi-PDF methods:

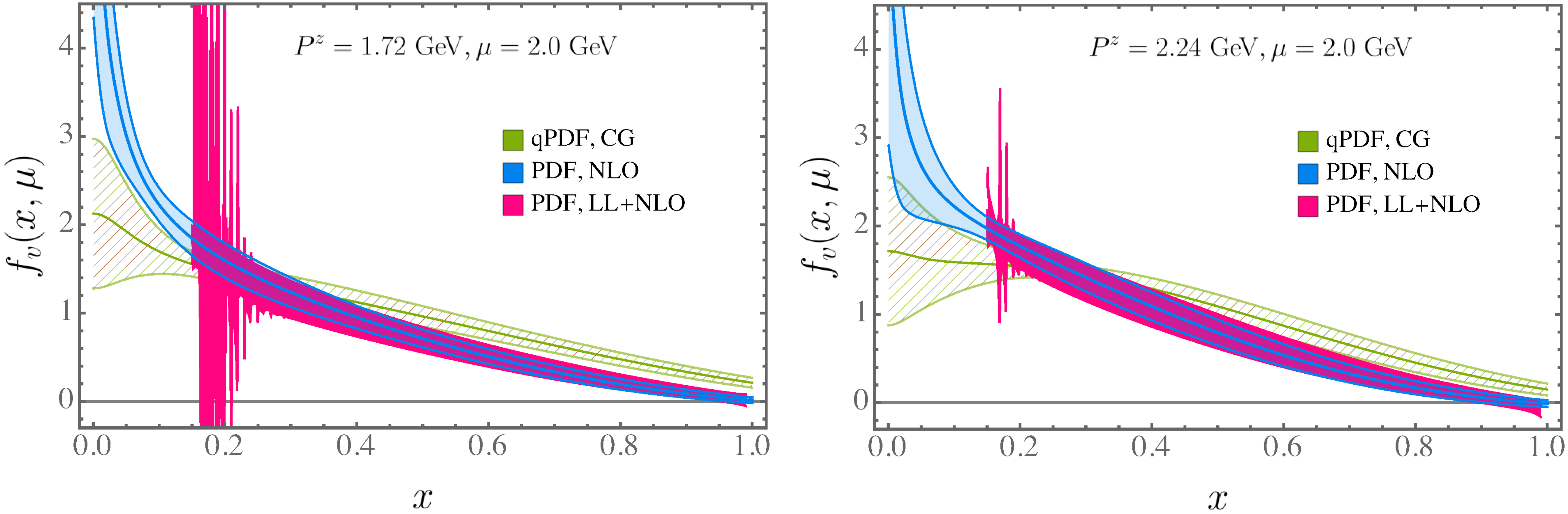



**While the quasi-PDFs are different by at least $1\sigma$, the matched results are consistent for $x \gtrsim 0.2$, demonstrating the universality in LaMET !**

# Perturbative matching

NLO V.S. Leading-logarithmic (LL) small-*x* resummation

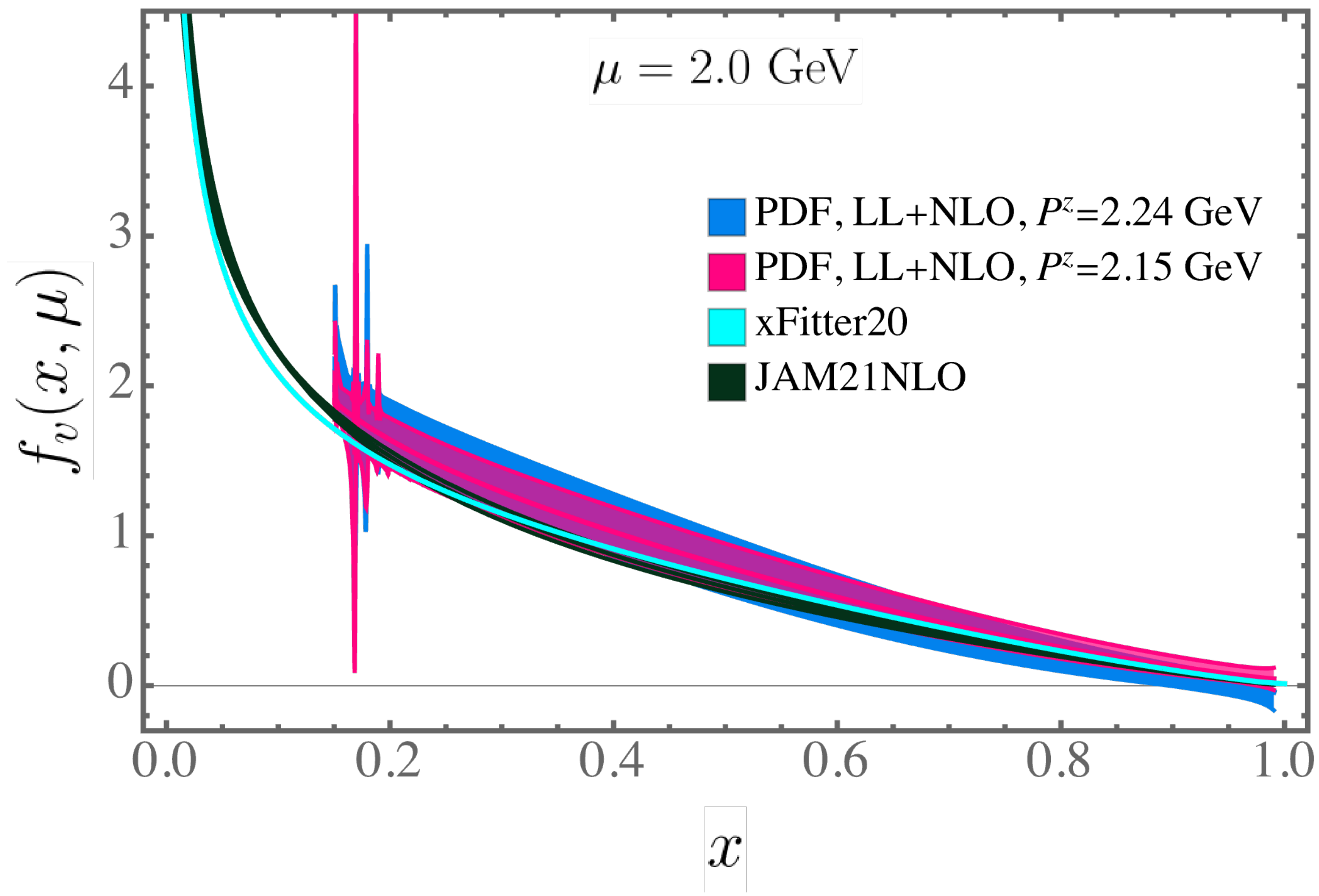
Inverse matching at $\mu = \kappa \cdot 2xP^z$ → DGLAP evolution from $\kappa \cdot 2xP^z$ to $\mu = 2$ GeV → Vary $\kappa \in [1/\sqrt{2}, \sqrt{2}]$ to estimate scale uncertainty




**Small-*x* resummation makes almost no difference for $x \gtrsim 0.4$, but becomes important at smaller $x$ and is out of control at $2xP^z \sim 0.8$ GeV where $\alpha_s \sim 1$.**

# Final result

## Comparison with global fits




**Global fits at NLO**

- **JAM21NLO**, PRL **127** (2021);
- **xFitter (2020)**, PRD **102** (2020).

- **Agreement with global fits for $x \gtrsim 0.2$ within the (large) error;**
- **Precision can be considerably improved with larger statistics.**

# Summary

- We verify the factorization of CG quasi-PDF to the PDF at NLO;
- We demonstrate the universality in LaMET through the equivalence of CG and GI quasi-PDF methods;
- The CG correlations have the advantages of access to larger off-axis momenta (at a lower computational cost), absence of linear divergence, and enhanced long-range precision;
- It is almost free to compute the GI and CG matrix elements at the same time.

# Outlook

- **Open questions:**
  - Effects of Gribov copies seem negligible, but should be further studied;
  - Threshold resummation is necessary and similar to the quasi-PDF;
  - OPE and mixings complicated by breaking of Lorentz symmetry.
- **Wider applications:**
  - GPDs. Straightforward extension from the PDF.
  - TMDs. Staple-shaped Wilson lines with infinite extension.
    - Absence of Wilson line provides much convenience in computation and renormalization;
    - Factorization should be provable as boosted quarks in a physical gauge capture the right collinear degrees of freedom.