NuclearScience Computing Center at CCNU

Transversity PDFs of the proton from lattice QCD with physical quark masses

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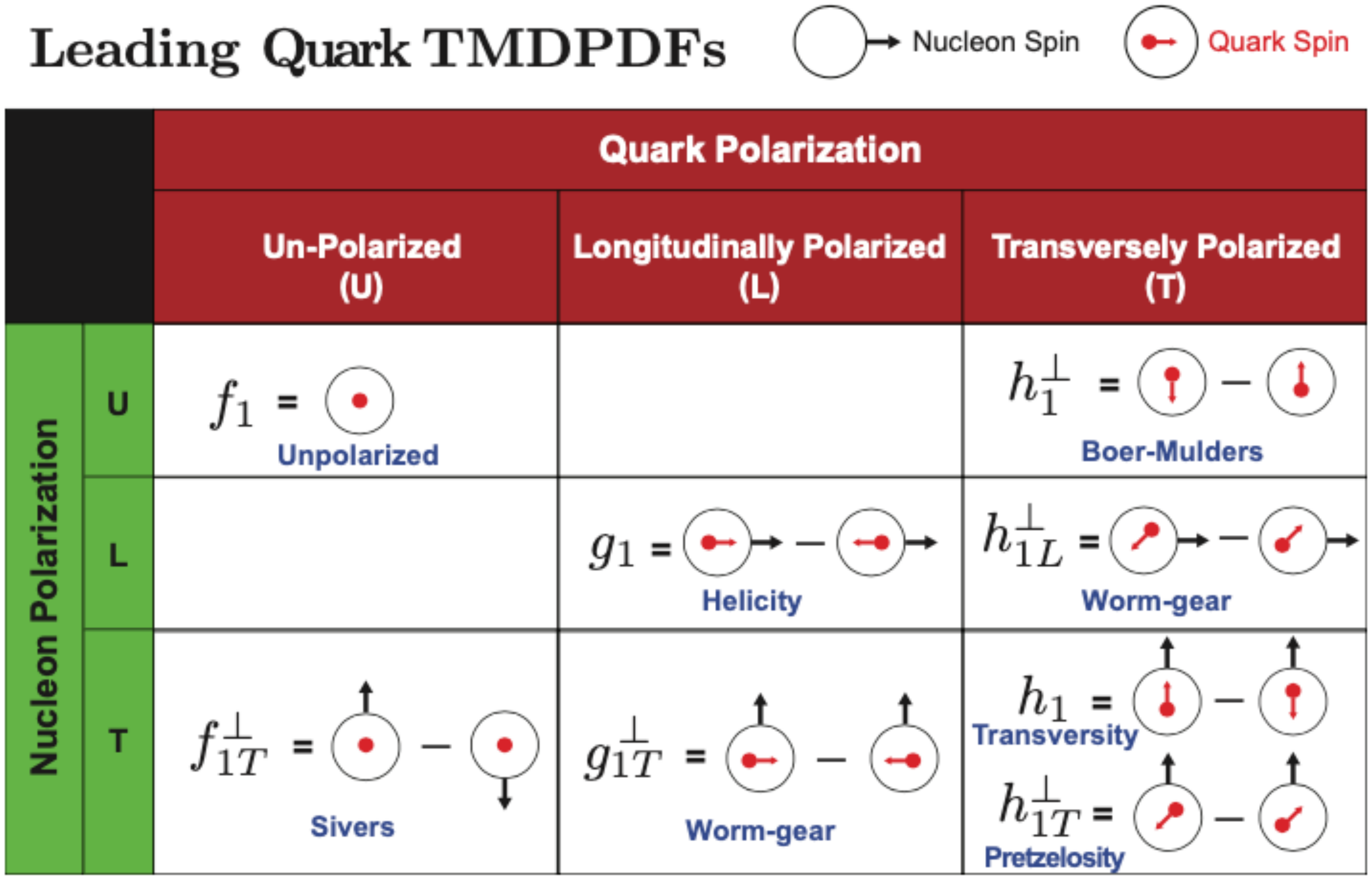
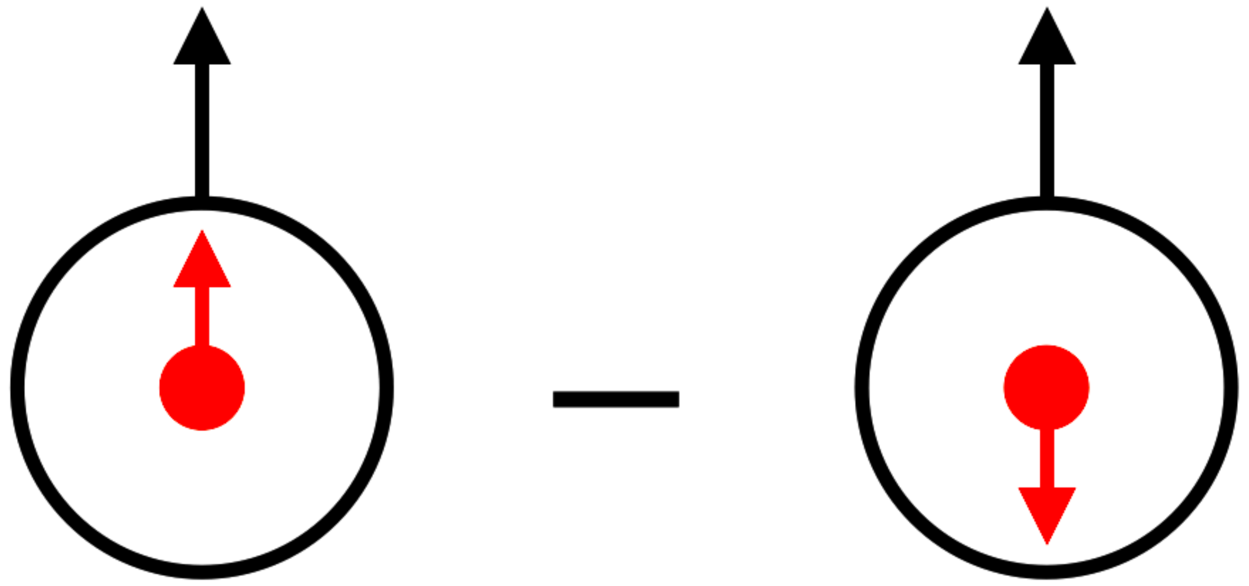
Outline

- Motivation
- Lattice Setup
- Mellin Moments
- Transversity PDFs
- Summary

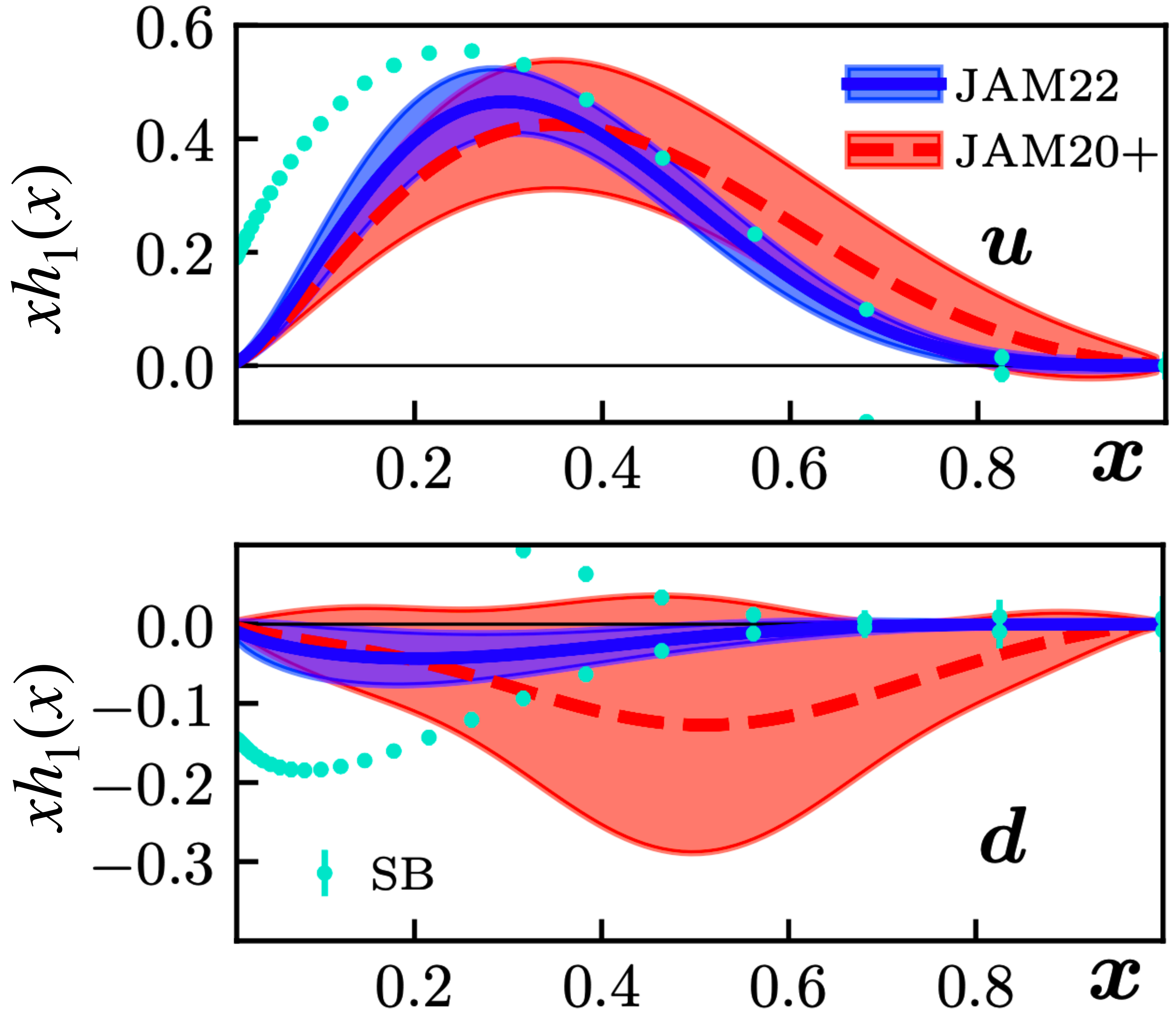
Collinear PDFs



Transversity PDFs

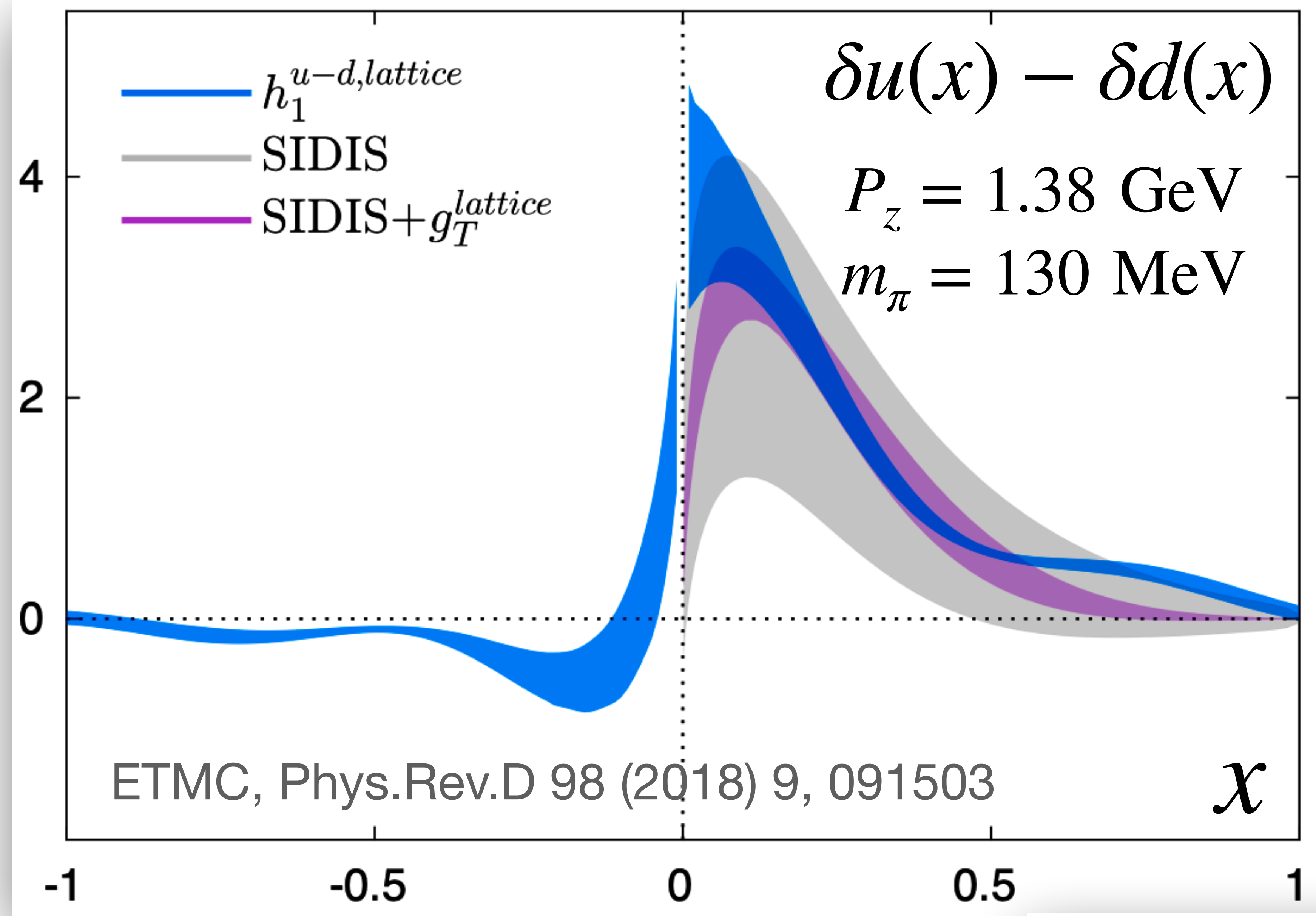


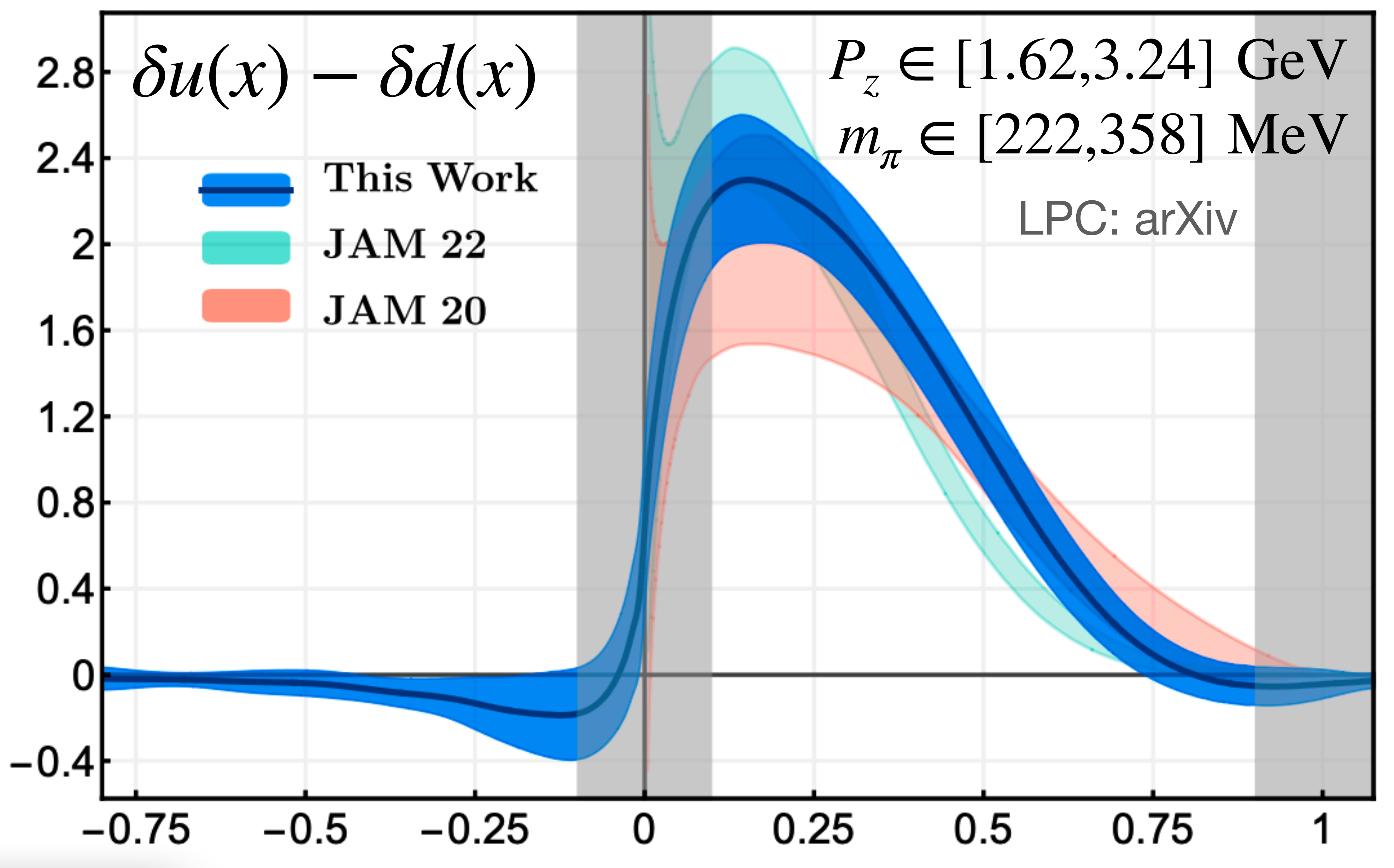
chiral-odd

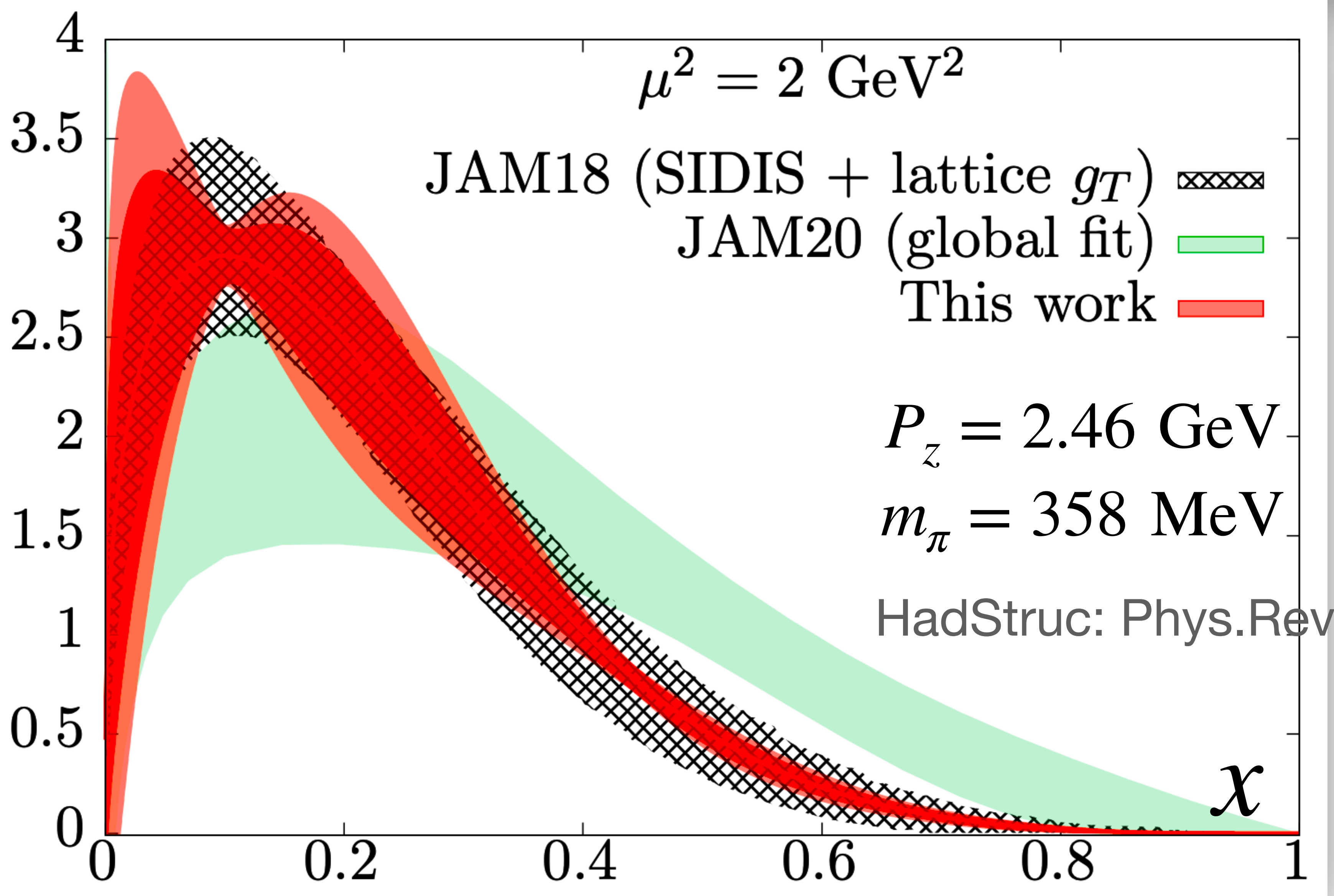


JAM: Phys.Rev.D 106 (2022) 3, 034014

Transversity distribution



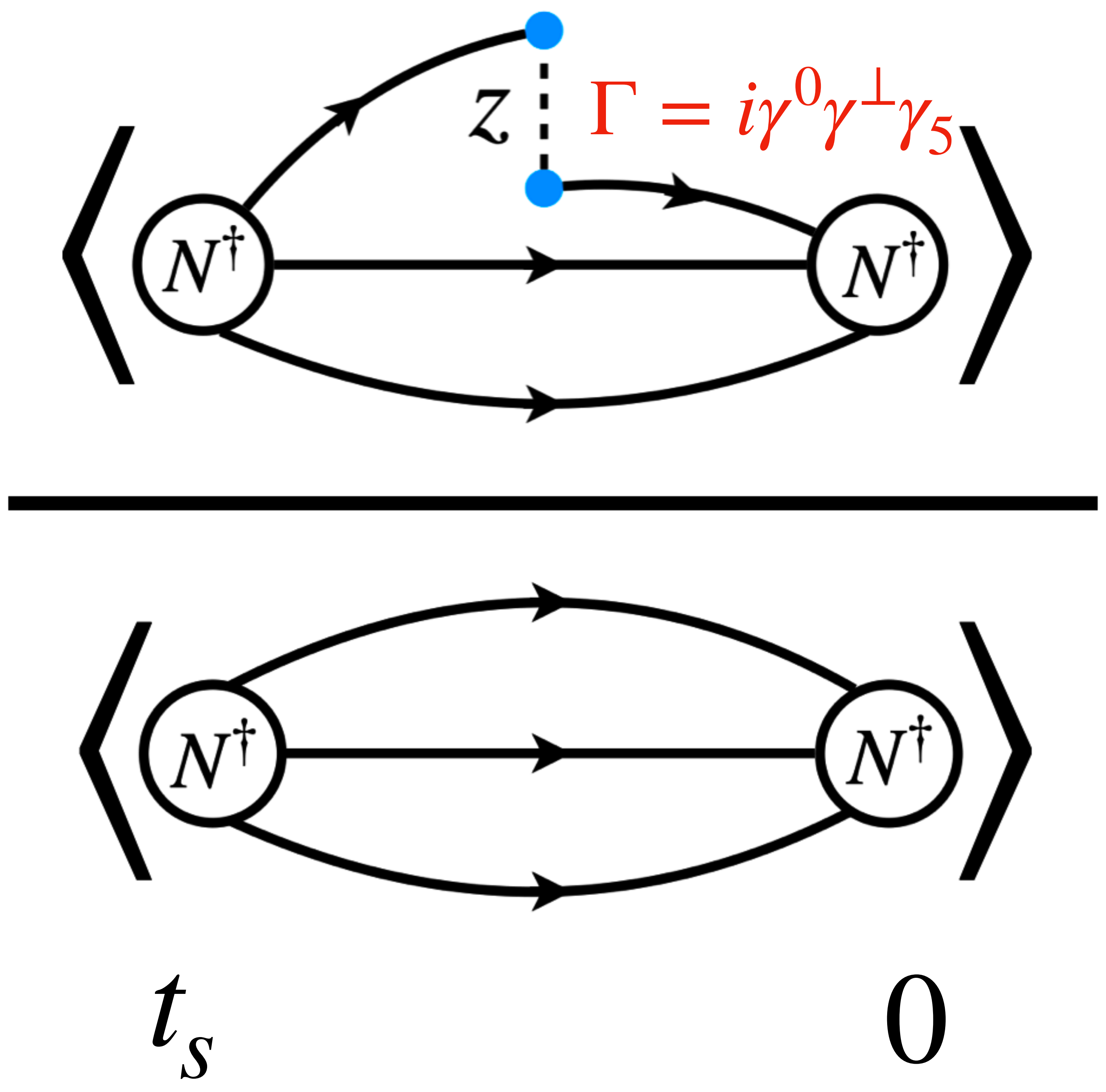




HadStruc: Phys.Rev.D 105 (2022) 3, 034507

Lattice setup

- $64^3 \times 64$, a = 0.076 fm
- 2+1f HISQ gauge ensembles
- Clover-fermion, $m_\pi = 140$ MeV
- 1-HYP smearing for Wilson line
- 4 momentum from 0 to 1.52 GeV using boosted smearing



$$R(t_s, \tau) = \frac{\langle N(t_s)\mathcal{O}_\Gamma^f(\vec{z}, \tau)\overline{N}(0)\rangle}{\langle N(t_s)\overline{N}(0)\rangle} \xrightarrow{t_s \to \infty} h^B(z,\ P_z)$$

Parton distribution functions

X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

- Quasi-PDF, **L**arge-**M**omentum **E**ffective **T**heory

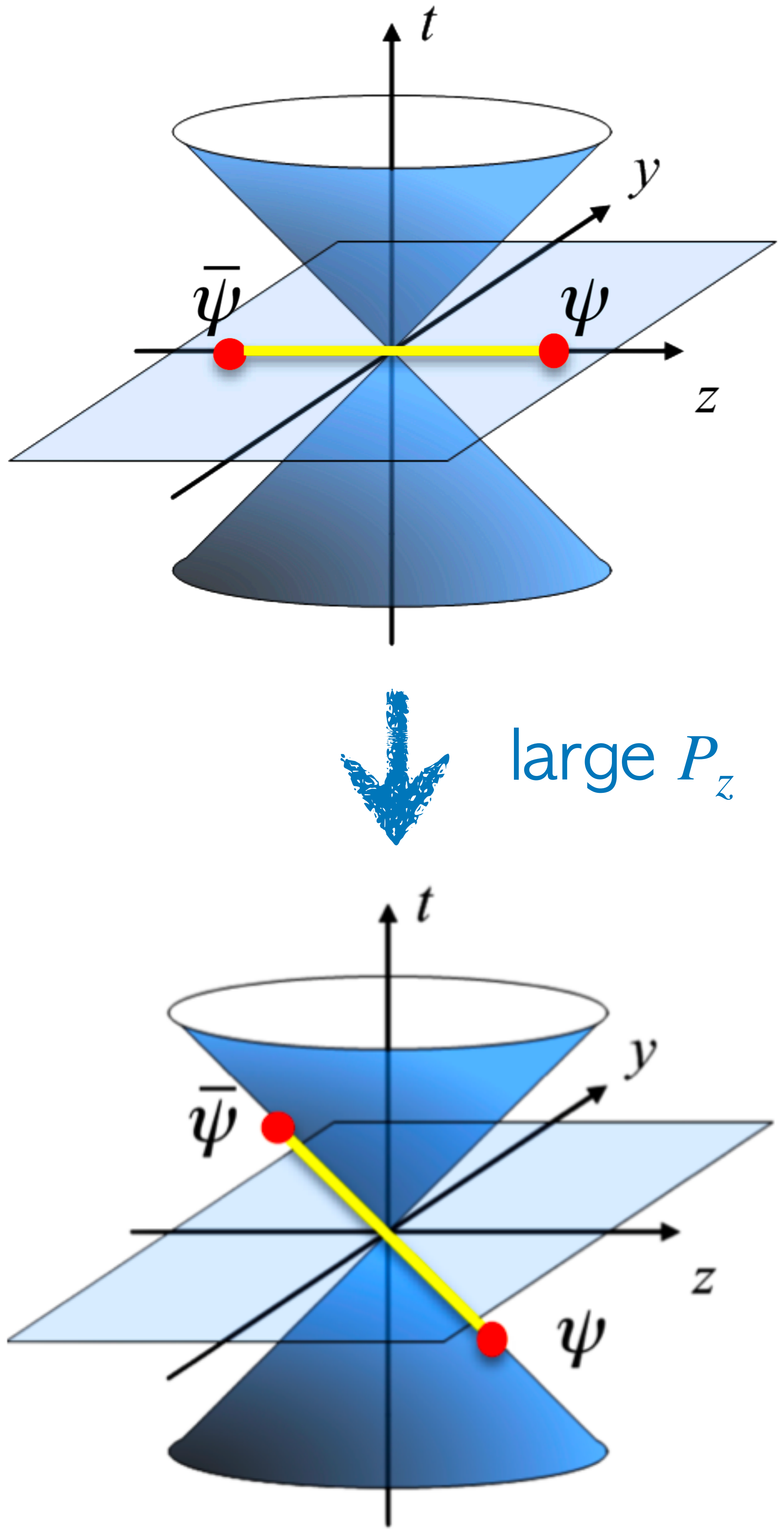
$$\tilde{q}(x, P_z, \mu) = \int \frac{\mathrm{d}\lambda}{2\pi} e^{-ix\lambda} h^R(x, z, P_z, \mu)$$

$$q(x, \mu) = \int \frac{\mathrm{d}y}{y} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) \tilde{q}(y, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{x^2 P_z^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-x)^2 P_z^2}\right)$$

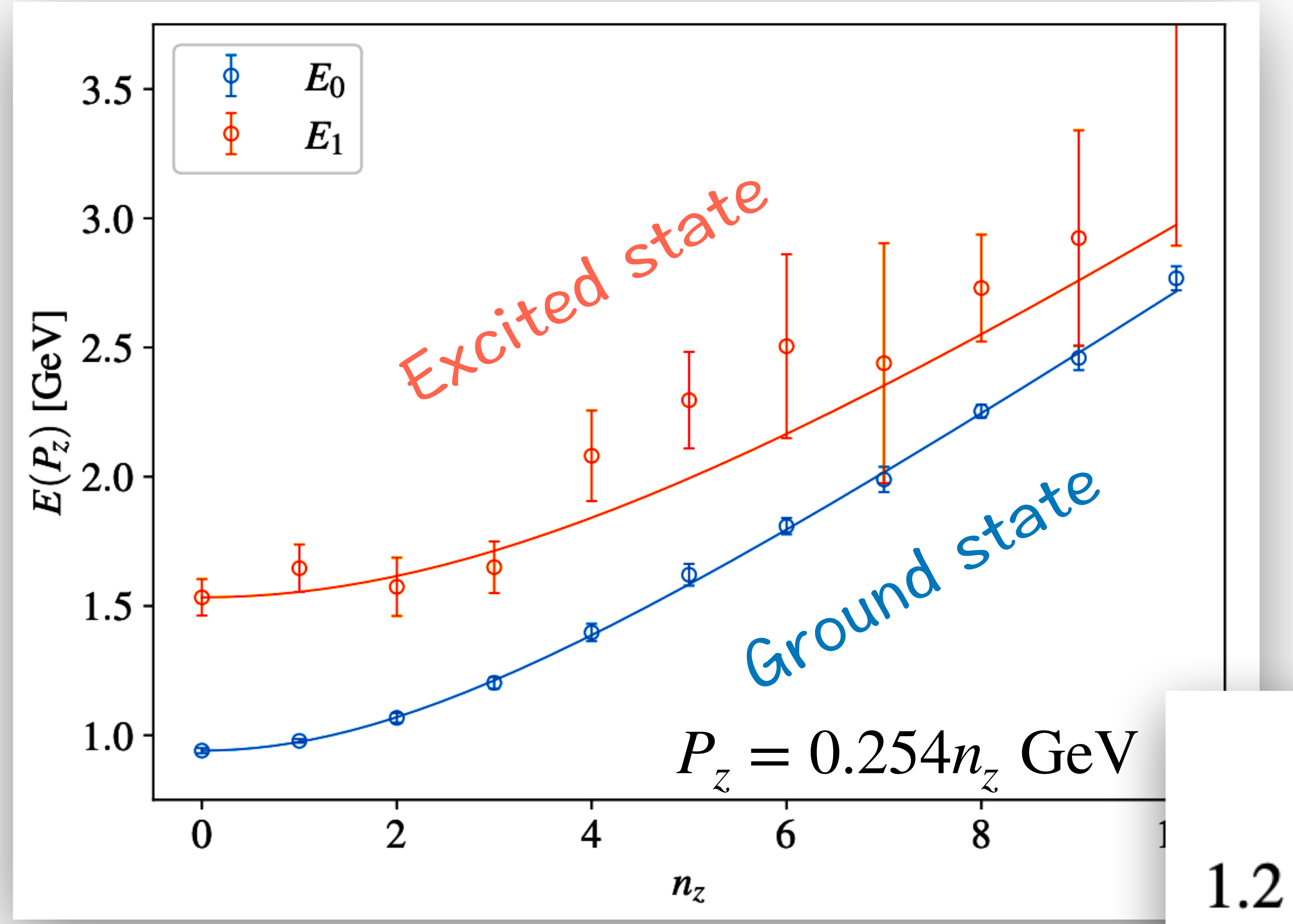
- Pseudo-PDF, **S**hort **D**istance **F**actorization

A. Radyushkin, PRD 100 (2019)
A. Radyushkin, Int.J.Mod.Phys.A 2020

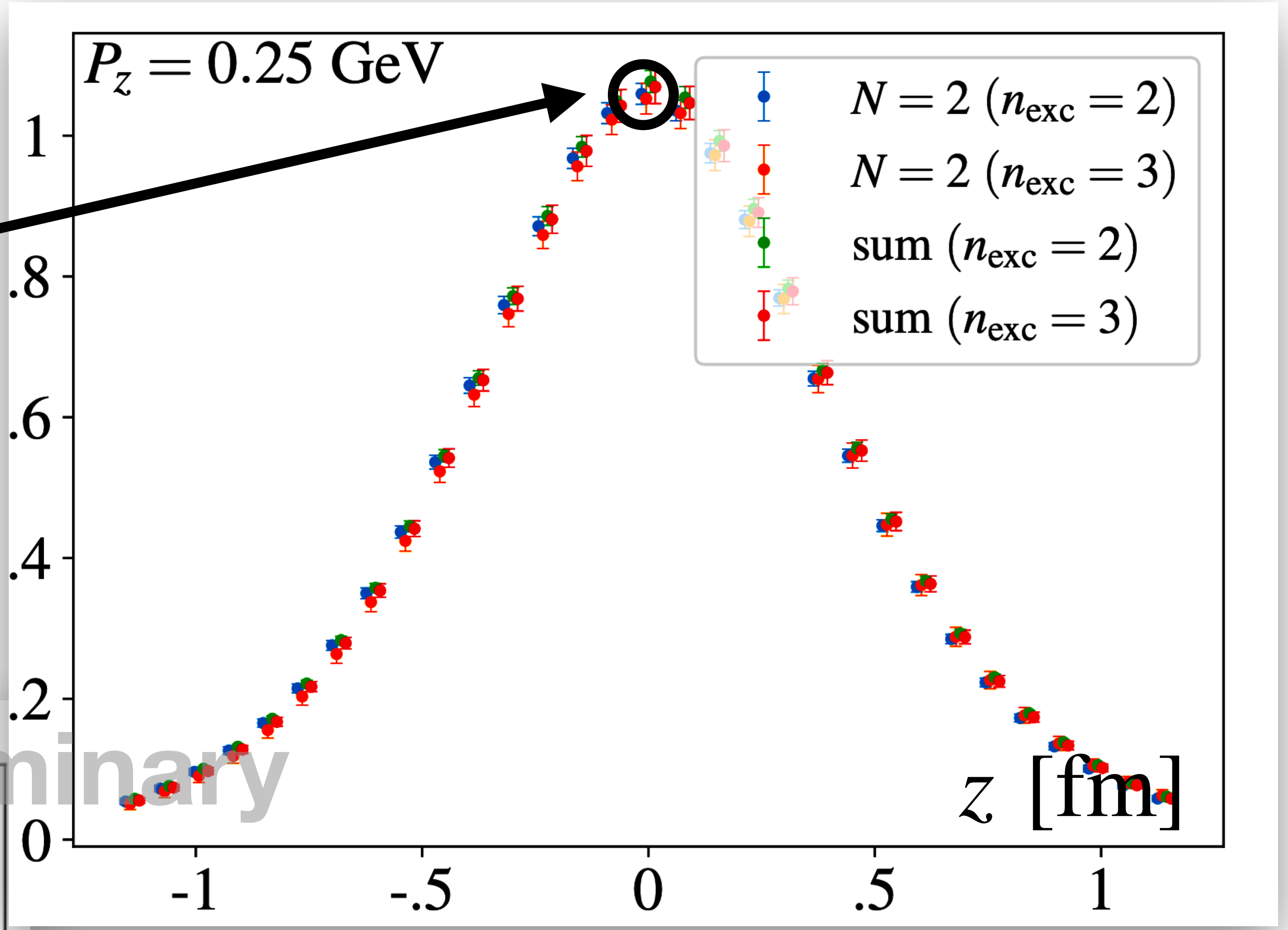
- …



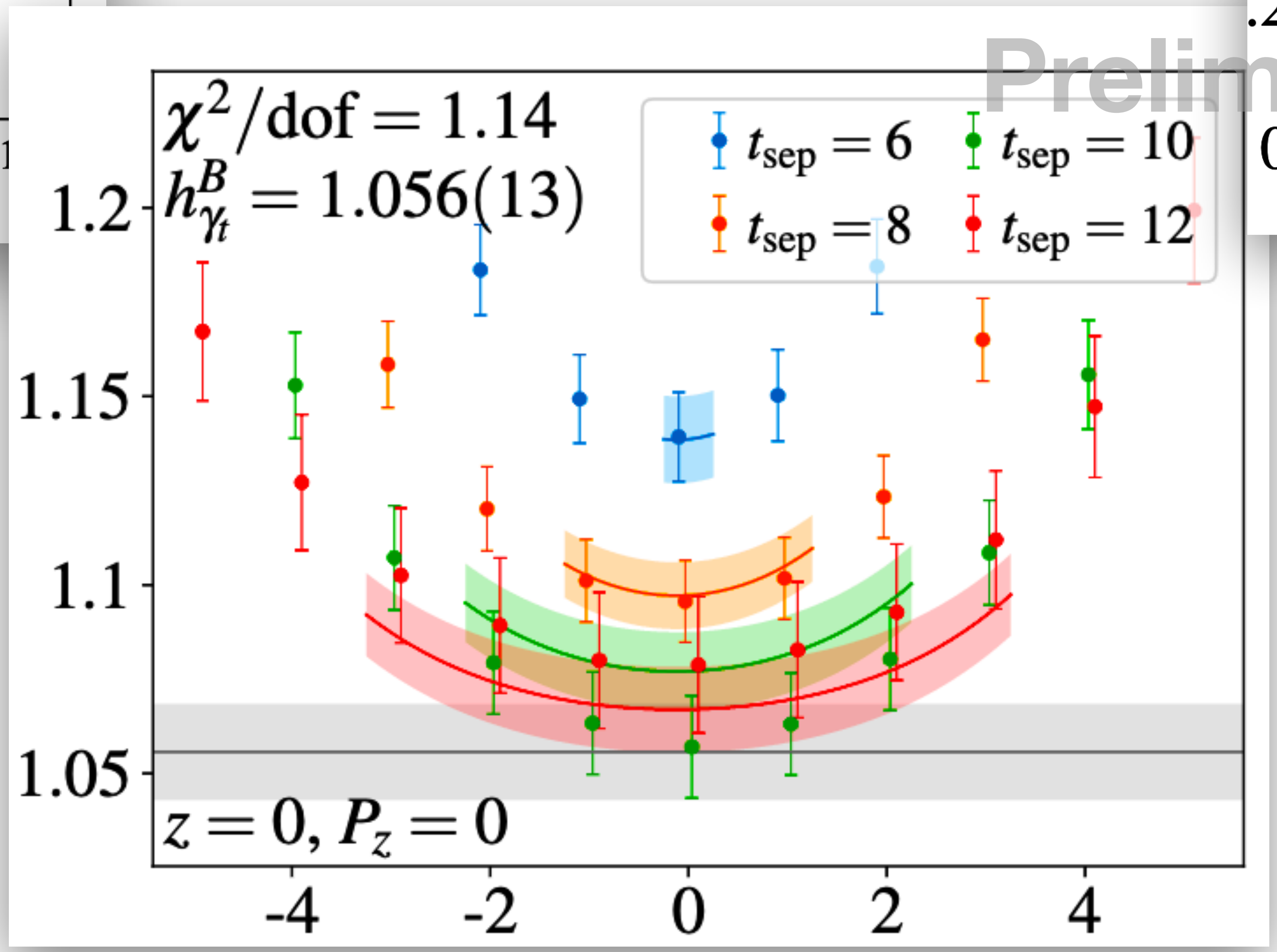
8 Bare matrix elements



$$h^B(z=0,\, P_z) = g_T/Z_T$$



$$h^B(z,\, P_z)$$



$$R(t_s, \tau) \xrightarrow{t_s \to \infty} h^B(z,\, P_z)$$

Ratio-scheme renormalization

X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
J. Green, K. Jansen and F. Steffens, PRL121 (2018)
T. Ishikawa, et al, PRD 96 (2017)

$$h^B = e^{-\delta m z} Z(a) h^R$$

- Ratio scheme renormalization

A. V. Radyushkin, PRD 2017
K. Orginos, et al, PRD 96, 2017
B. Joó, et al, PRL125, 2020
X. Gao, et al, PRD 102, 2020
Z. Fan, et al, PRD 102, 2020

$$\mathscr{M}(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

- RG invariant double ratio.

$$\mathscr{M}(z^2, P_z, P_z^0) \equiv \left(\frac{h^B(z, P_z)}{h^B(z, P_z^0)}\right)\left(\frac{h^B(0, P_z^0)}{h^B(0, P_z)}\right) = \left(\frac{h^R(z, P_z)}{h^R(z, P_z^0)}\right)\left(\frac{h^R(0, P_z^0)}{h^R(0, P_z)}\right)$$

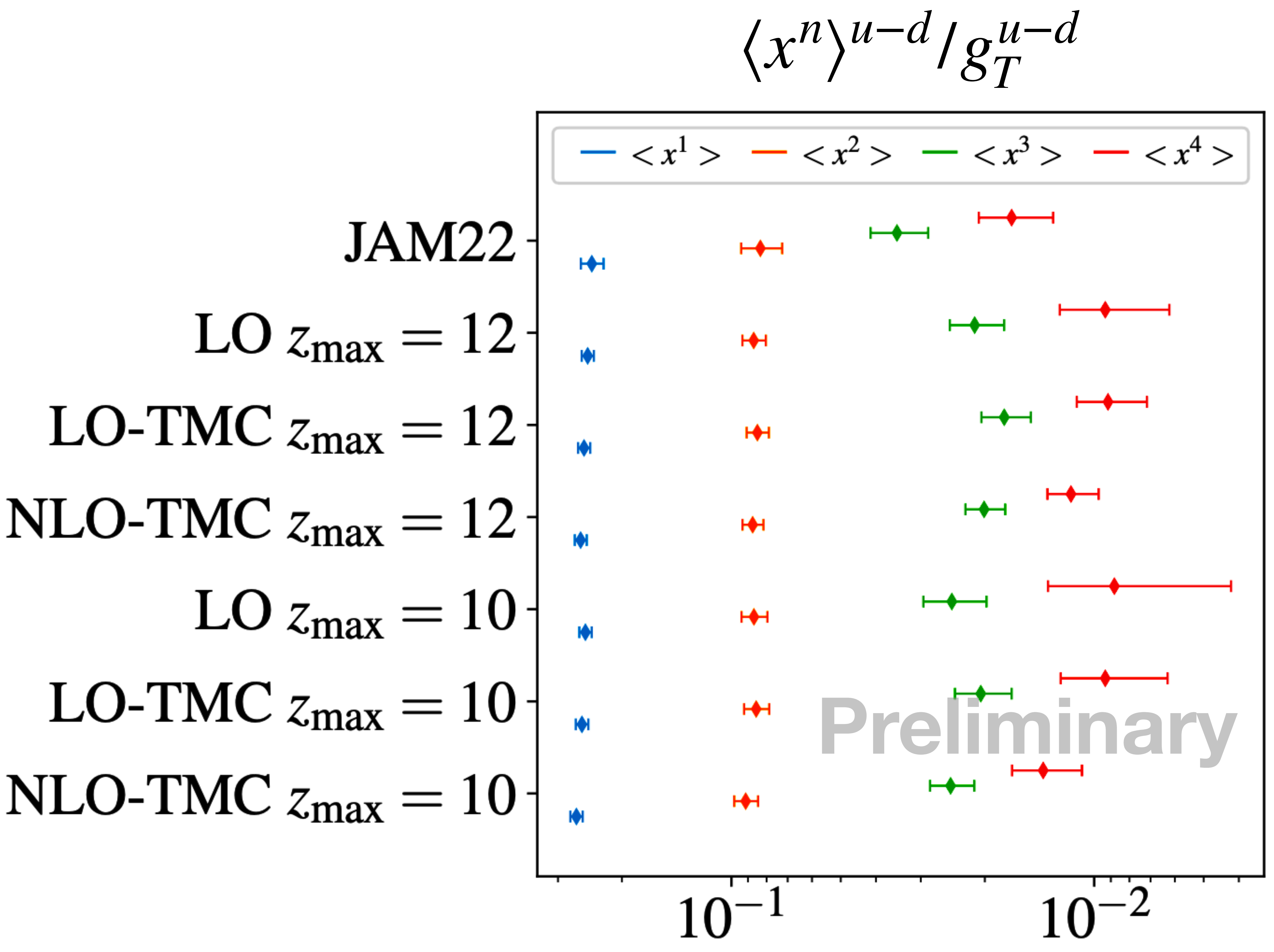
- The twist-2 OPE formula.

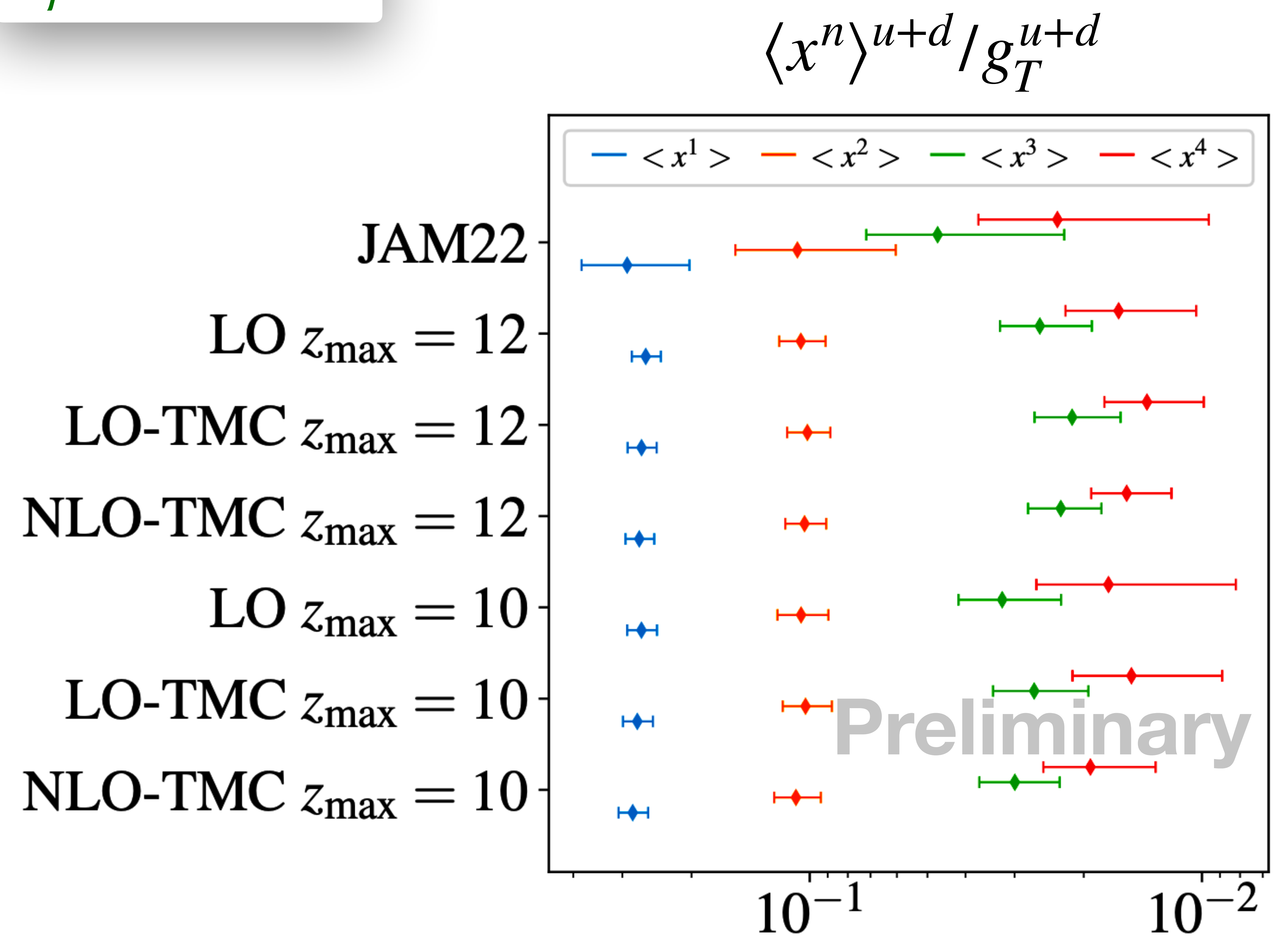
$$\mathscr{M}(z^2, P_z, 0) = \sum_{n=0}^{\infty} \frac{(-izP_z)^n}{n!} \frac{C_n(z^2\mu^2)}{C_0(z^2\mu^2)} \frac{\langle x^n \rangle}{g_T}$$

- Real part: even moments.
- Imaginary part: odd moments.

1. Mellin moments

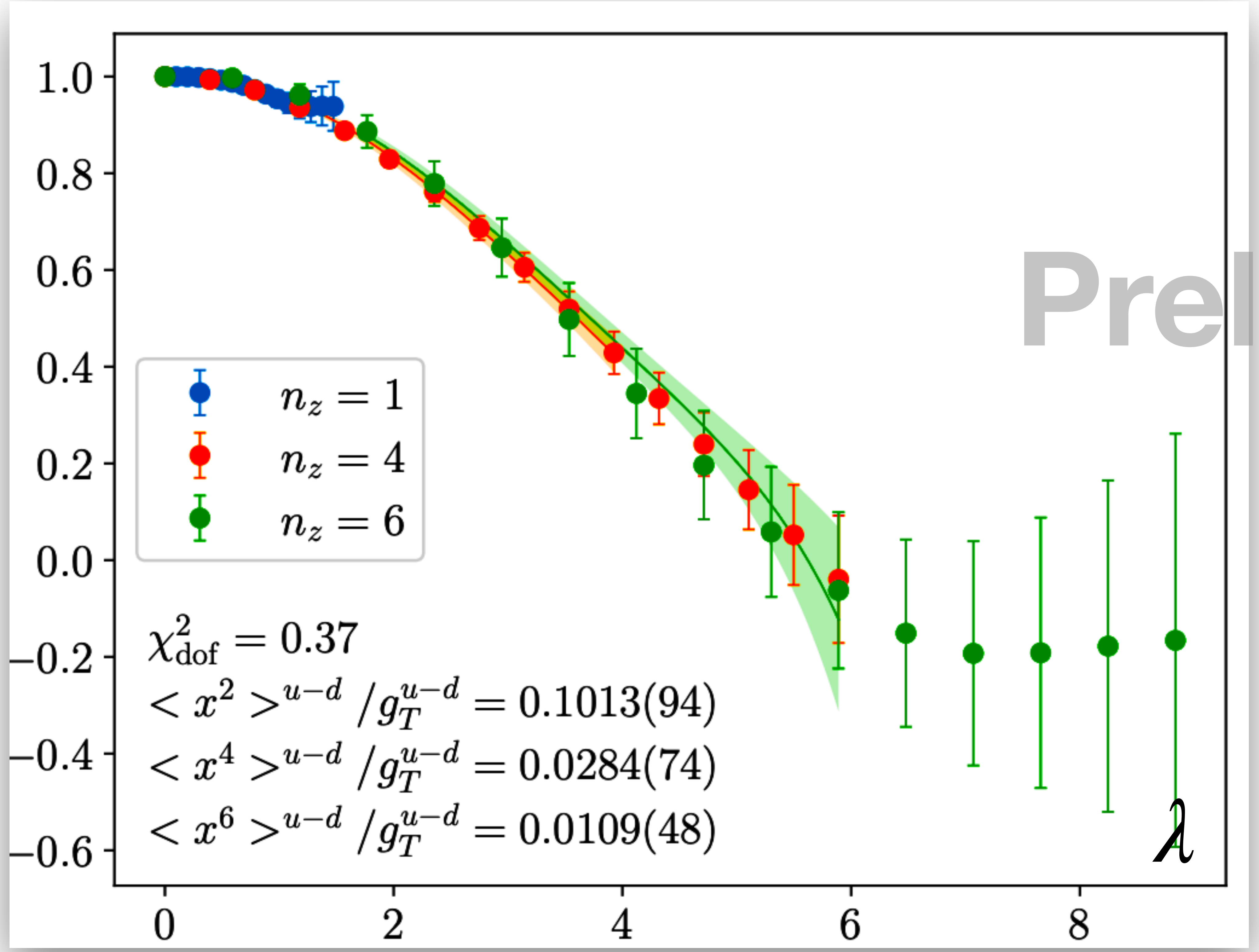
$\mu = 2$ GeV



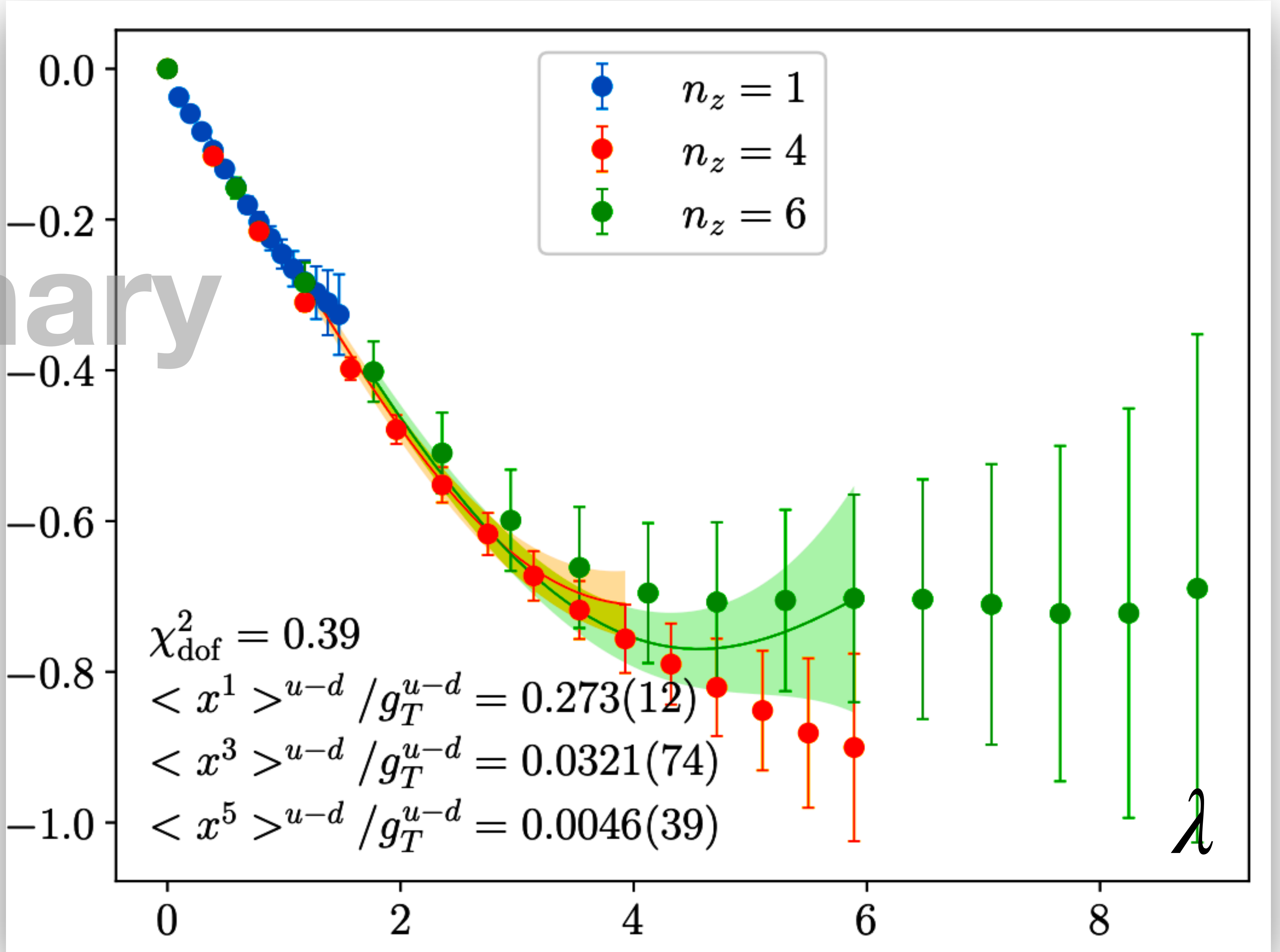


1. Mellin moments

$\mathrm{Re}[\mathscr{M}(\lambda, z^2, P_z, 0)]$



$\mathrm{Im}[\mathscr{M}(\lambda, z^2, P_z, 0)]$



Preliminary

2. Transversity PDFs from model reconstruction

Leading twist factorizaiton:

$$h^R(z, P_z, \mu) = \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

Model

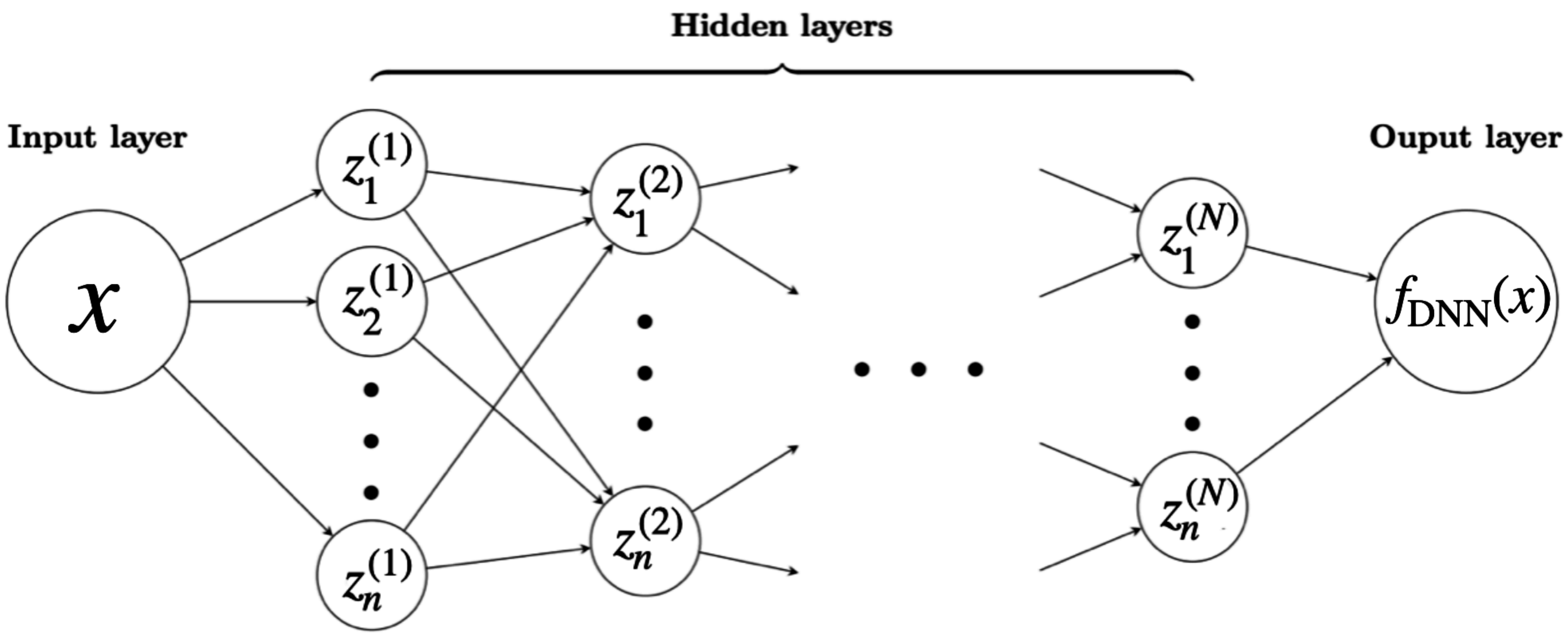
$$q(x) = Ax^\alpha (1-x)^\beta (1 + \text{subleading terms})$$

Deep neural network (DNN)

$$q(x; \alpha, \beta, \boldsymbol{\theta}) \equiv Ax^\alpha (1-x)^\beta \{1 + \epsilon(x) \cdot \sin[f_{\text{DNN}}(x; \boldsymbol{\theta})]\}$$

BNL-ANL, Phys.Rev.D 107 (2023) 7, 074509

2. Transversity PDF from model reconstruction



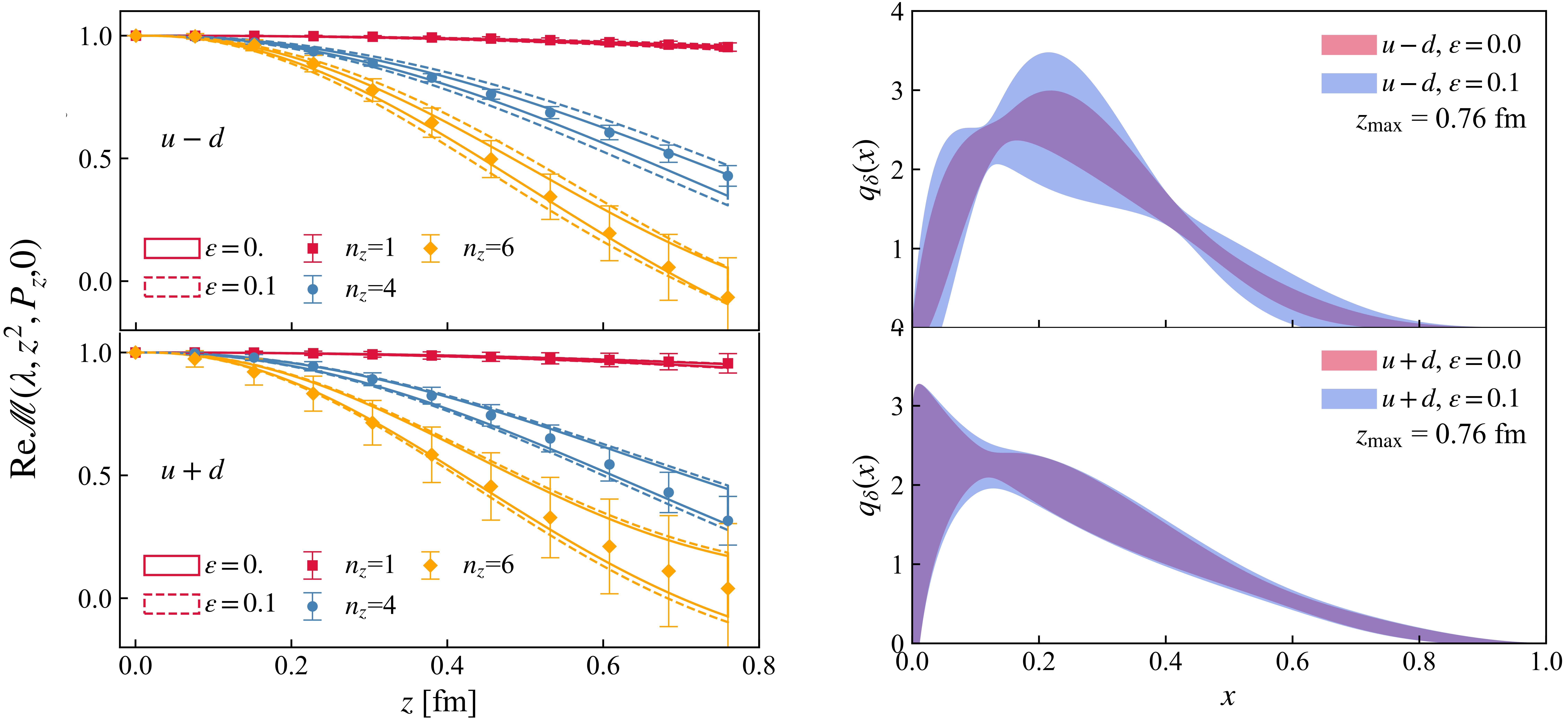
Hidden layer:

Linear transformation: input $z_i^{(l)} = b_i^{(l)} + \sum_j W_{ij}^{(l)} a_j^{(l-1)}$

Activation: output $a_i^{(l)} = \sigma_{\mathrm{elu}}^{(l)}(z_i^{(l)}) = \theta(-z)(e^z - 1) + \theta(z)z$

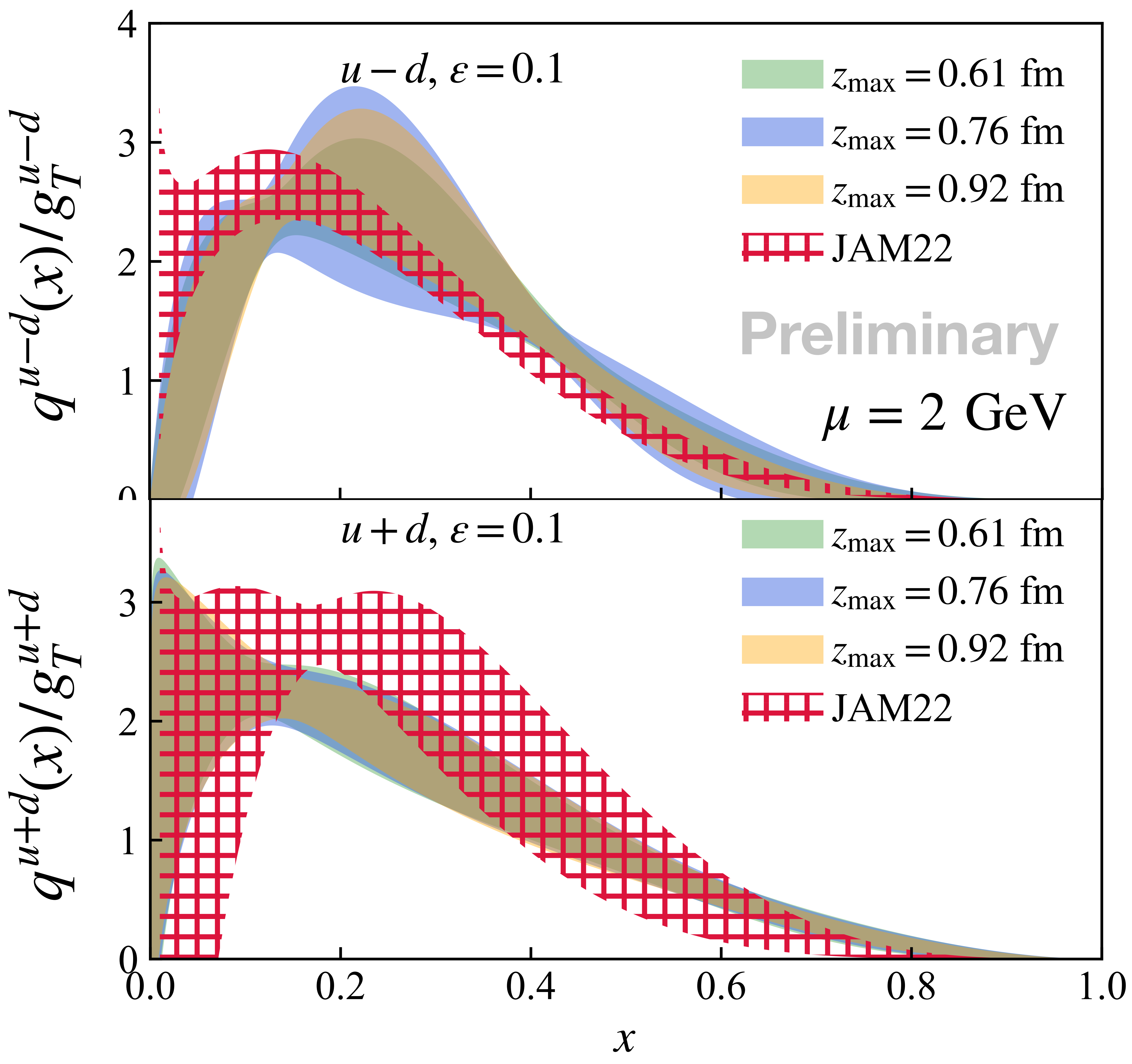
Loss function: $J \equiv \frac{\eta}{2}\mathcal{O}(b, W) \cdot \mathcal{O} + \frac{1}{2}\chi^2(\mathcal{O}, \alpha, \beta)$

2. Transversity PDF from model reconstruction



Anti-quark distributions are negligible.

2. Transversity PDF from model reconstruction



Hybrid renormalization

$$h^B = e^{-\delta m z} Z(a) h^R$$

Hybrid scheme renormalization

- Short distance $z \in [0, z_s]$, $z_s \ll \Lambda_{\mathrm{QCD}}$:

$$h^R = \frac{h^B(z, P_z, a)}{h^B(z, 0, a)}$$

- Long distance $z \in [z_s, +\infty]$:

$$h^R = e^{\delta m |z - z_s|} \frac{h^B(z, P_z, a)}{h^B(z_s, 0, a)}$$

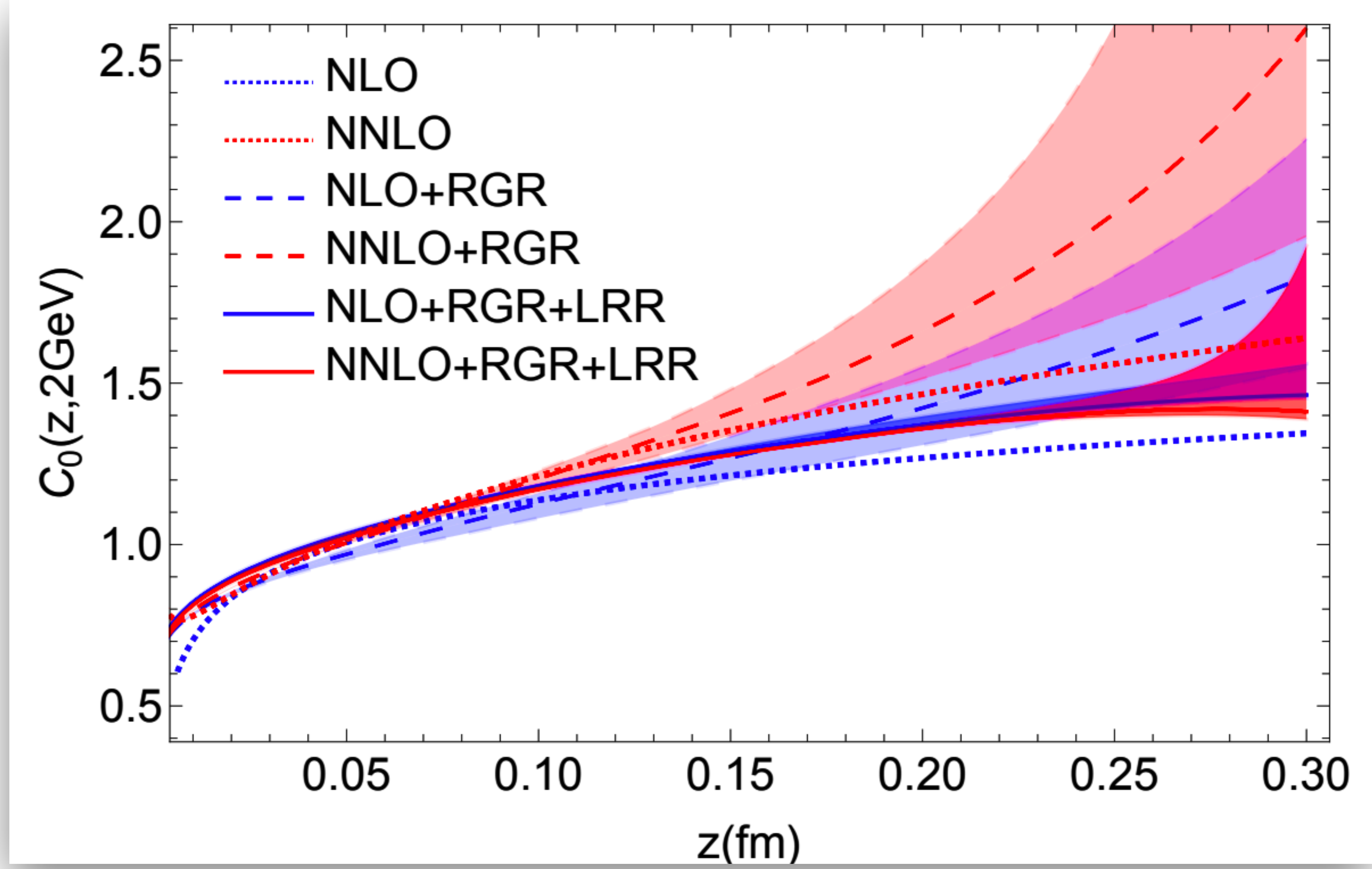
$$\delta m' = \delta m + \bar{m}_0$$

Renormalon ambiguity

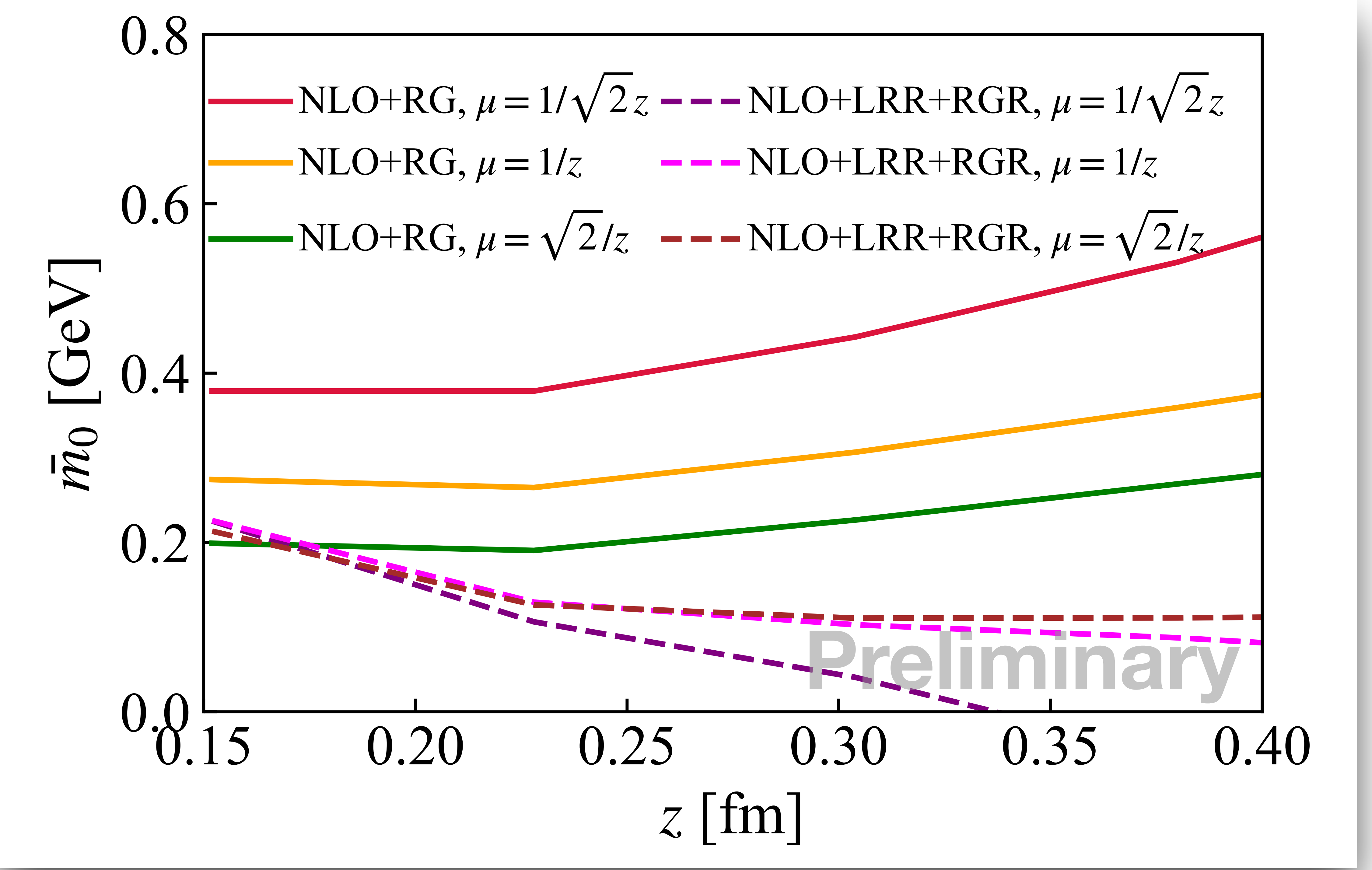
$$h^B(z, p_z = 0) = Z(a) e^{-\delta m z} e^{-\bar{m}_0 z} C_0(z^2 \mu^2),$$

$$\frac{h^B(z, p_z = 0)}{h^B(z - a, p_z = 0)} = e^{-\delta m a} e^{-\bar{m}_0 a} \frac{C_0(z^2 \mu^2)}{C_0((z - a)^2 \mu^2)}$$

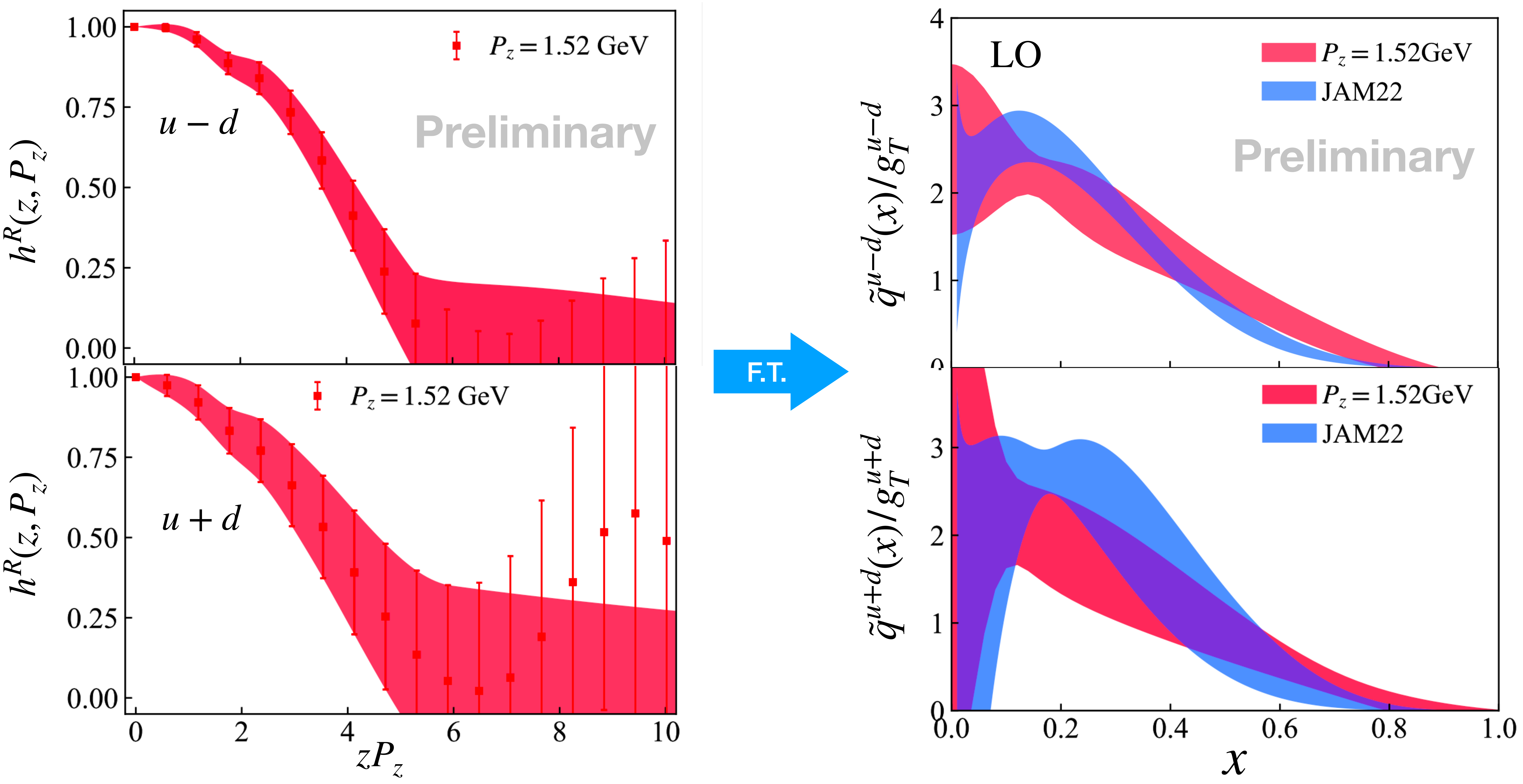
Hybrid renormalization



R. Zhang, et al, PLB, 844, 2023



3. Transversity quasi-PDF



Summary

Bare transversity matrix elements of iso-vector and iso-scalar.

- The first few Mellin moments, ratio-scheme.
- Reconstruct the PDF using DNN, ratio-scheme.
- Quasi-PDF, hybrid-scheme.
  The NLO+LRR+RG matching in on going.

Thanks for your attention!

Backup

Transversity PDF from model reconstruction

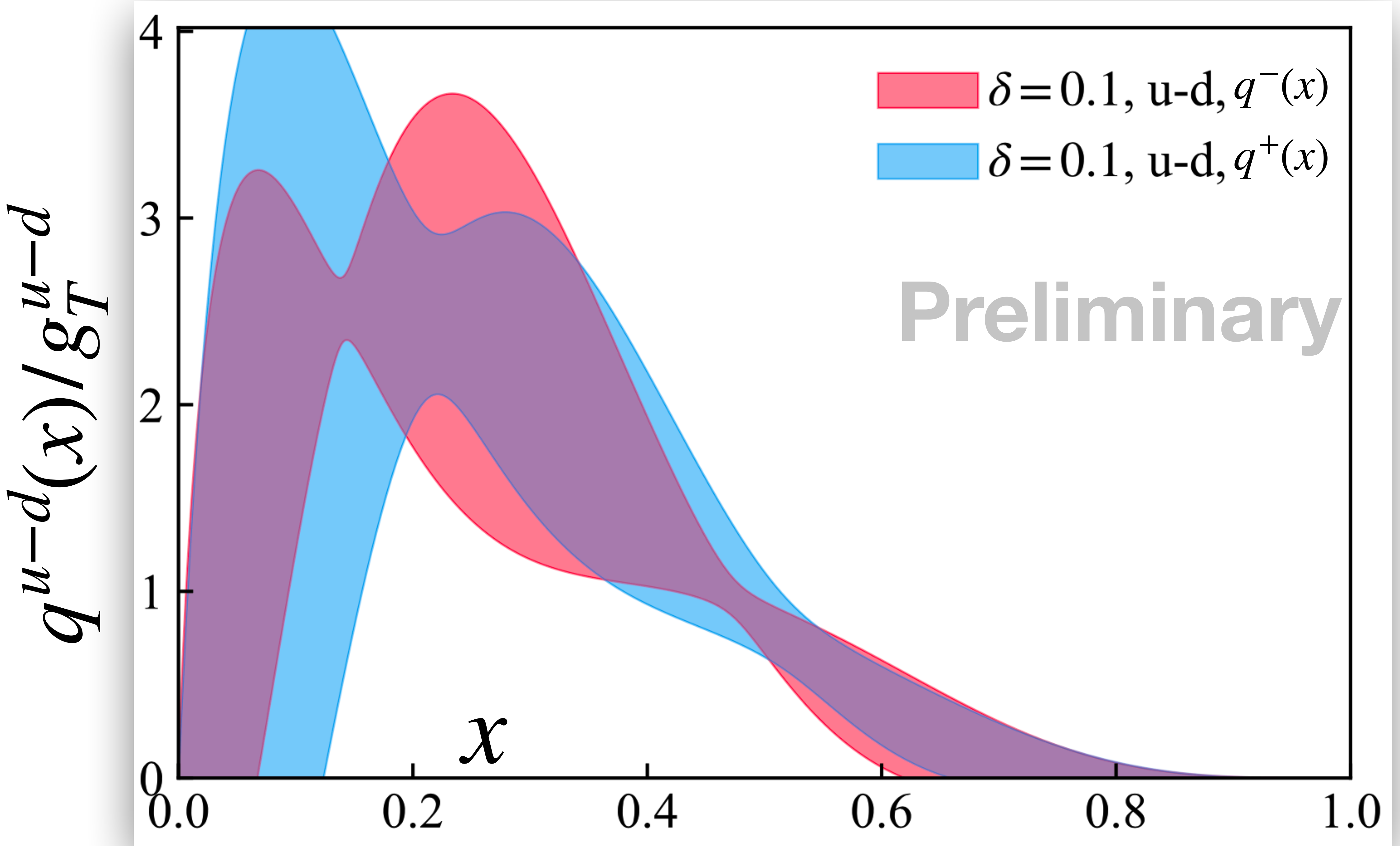
$$\mathscr{M}(\lambda, z^2, P_z, 0) = \sum_{n=0}^{\infty} \frac{(-izP_z)^n}{n!} \frac{C_n(z^2\mu^2)}{C_0(z^2\mu^2)} \frac{\langle x^n \rangle}{g_T}$$

Real part:

$$q^-(x) \equiv q^q(x) - q^{\bar{q}}(x)$$

Imaginary part:

$$q^+(x) \equiv q^q(x) + q^{\bar{q}}(x)$$



The difference:

$$q^+(x) - q^-(x) \equiv 2q^{\bar{q}}(x)$$

- Anti-quark distribution negligible.

LaMET factorization and hybrid renormalization

$$\tilde{q}(x, P_z, \mu) = \int \mathrm{d}z e^{ixP_z z} h^R(x, z, P_z, \mu)$$

$$q(x, \mu) = \int \frac{\mathrm{d}y}{y} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) \tilde{q}(y, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$