Connecting Euclidean to lightcone correlations: From forward to non-forward kinematics

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https://arxiv.org/abs/2212.14415

OUTLINE

Generalized parton observables (3D structure)

$$W(x, \mathbf{k_T}, \mathbf{b_T})$$

5D Wigner Distributions

$\int d^2\mathbf{b_T}$ ↙ ↘ $\int d^2\mathbf{k_T}$

$$f_{\mathrm{TMD}}(x, \mathbf{k_T})$$

Transverse Momentum Dependent (TMD) PDFs

$$f_{\mathrm{GPD}}(x, \mathbf{b_T})$$

Generalized Parton Distributions (GPDs)

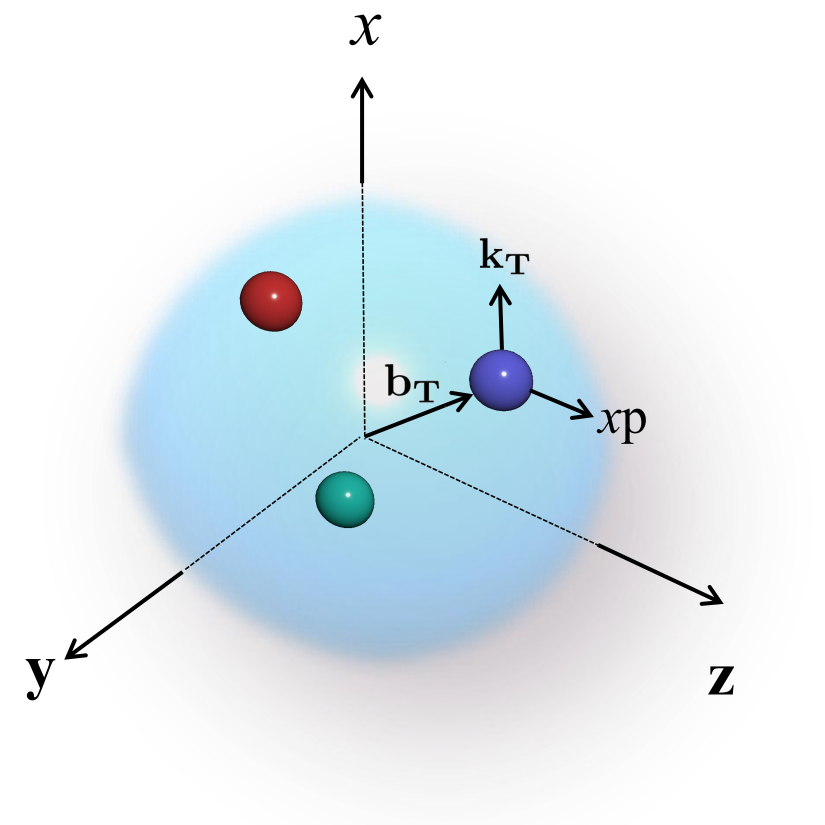
non-forward kinematics

$\int d^2\mathbf{k_T}$ ↘ ↙ $\int d^2\mathbf{b_T}$

$$f(x)$$

1D Conventional PDFs

forward kinematics



b_T denotes the impact parameter of the struck quark with respect to the center of momentum of the hadron

Theoretically, the unpolarized quark GPDs are defined as

$$F(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixp^+z^-} \left\langle p'' \middle| \bar{\psi}(-\frac{z}{2})\gamma^+ L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2}) \middle| p' \right\rangle_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2p^+}\left[H(x,\xi,t)\bar{u}(p'')\gamma^+ u(p') + E(x,\xi,t)\bar{u}(p'')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p')\right]$$

x	$\Delta^\mu = p''^\mu - p'^\mu$	$t = \Delta^2$	$\xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$
momentum fraction	momentum transfer	momentum transfer squared	skewness

$f_{\rm GPD}(x,\mathbf{b_T})$ → ($\mathbf{b_T} \leftrightarrow \Delta_{\rm T}$, FT) → $H(x,0,t)$, $E(x,0,t)$ ← ($\xi = 0$) ← $H(x,\xi,t)$, $E(x,\xi,t)$

Impact Parameter Distributions

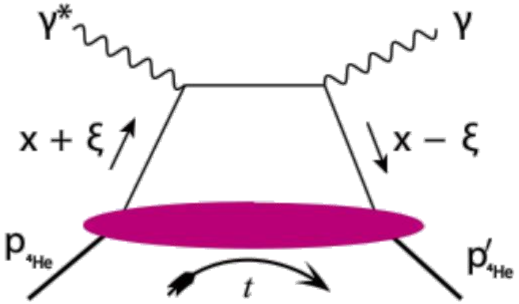
$H(x,0,t)$, $E(x,0,t)$ ↓ $\int dx$ → Form Factors (FFs)

$H(x,\xi,t)$, $E(x,\xi,t)$ ↓ $\int_{-1}^{1} dx x^{n-1}$ → Generalized form factors (GFFs)

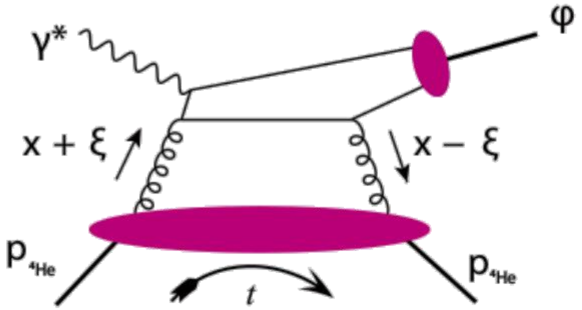
Mellin moments

Experimentally, GPDs can be accessed in exclusive processes

- deeply virtual Compton scattering (DVCS)
- deeply virtual meson production (DVMP) Armstrong et al, arxiv: 1708.00888



DVCS



DVMP

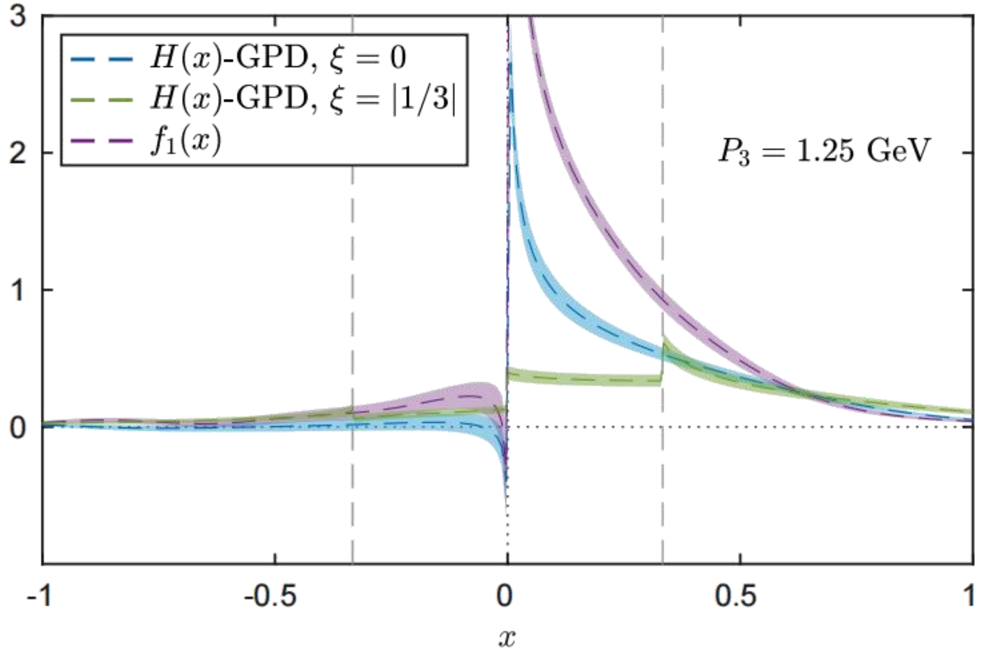
Limitations in global fit

- Only limited data, and they are indirectly related to GPD.
- Complicated kinematic dependence and no reliable framework (QCD models) for extracting 3D parton distributions.

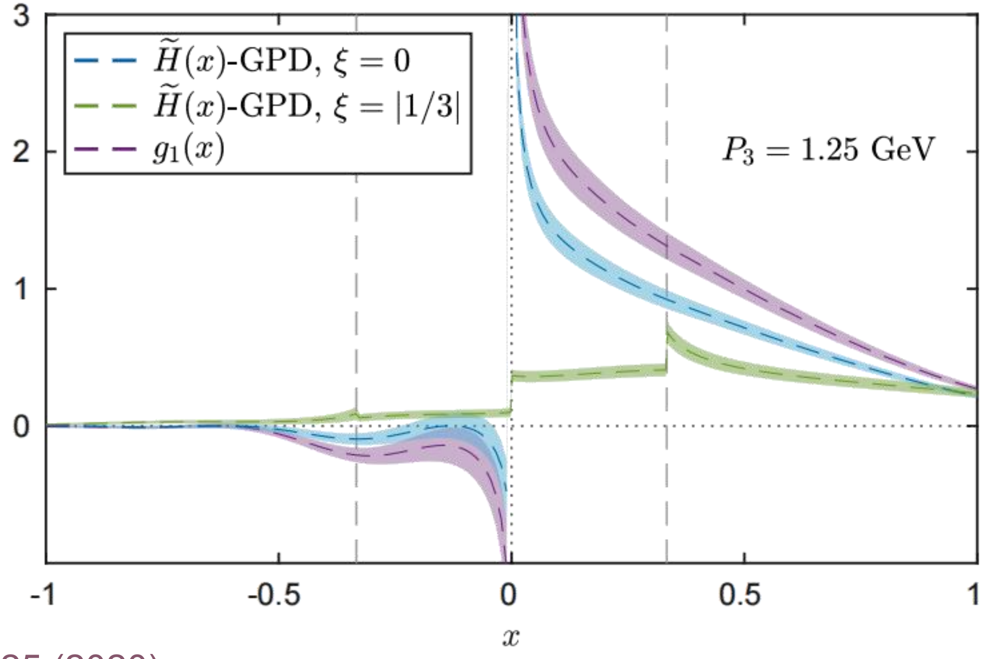
Extracting nucleon GPDs using lattice QCD

- Mellin moments of the GPDs (FFs and GFFs) Constantinou et al, PPNP 121 (2021)
- Large-momentum effective theory (LaMET)

Unpolarized GPDs H(x,ξ,t) and E(x,ξ,t)



Helicity GPDs $\widetilde{H}$(x,ξ,t) and $\widetilde{E}$(x,ξ,t)



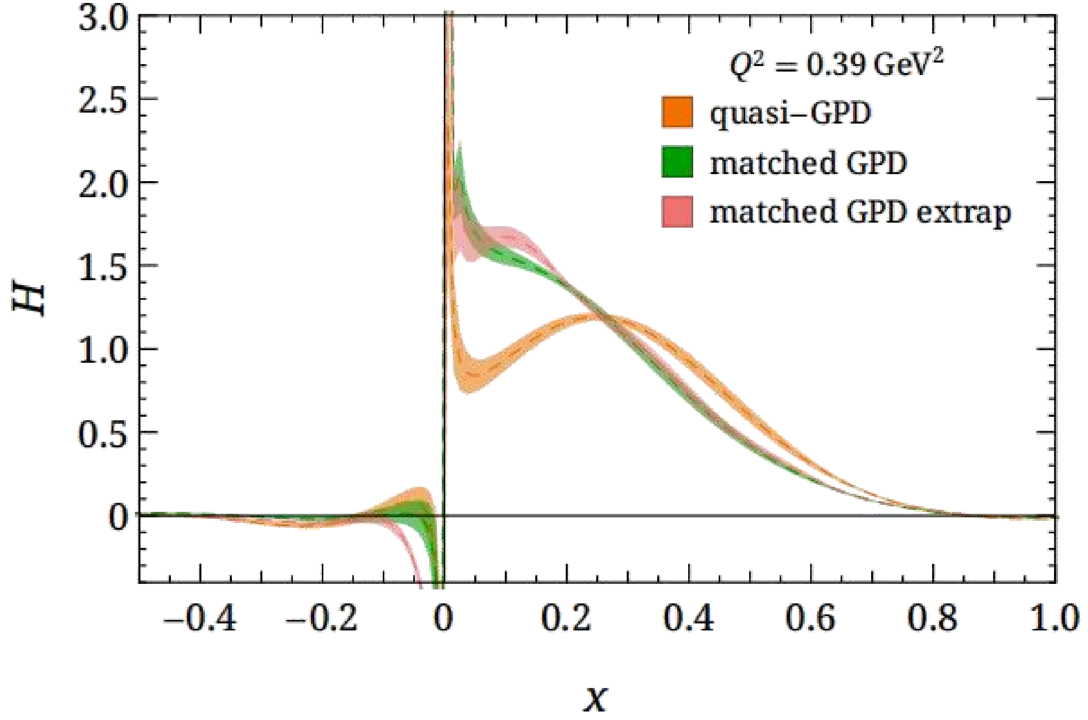
ETMC collaboration, PRL 125 (2020)

P_z = {0.83, 1.25, 1.67} GeV, $m_\pi \approx$ 260 MeV, Q^2 = 0.69 GeV², **RI/MOM scheme**.

Extracting nucleon GPDs using lattice QCD (LaMET)

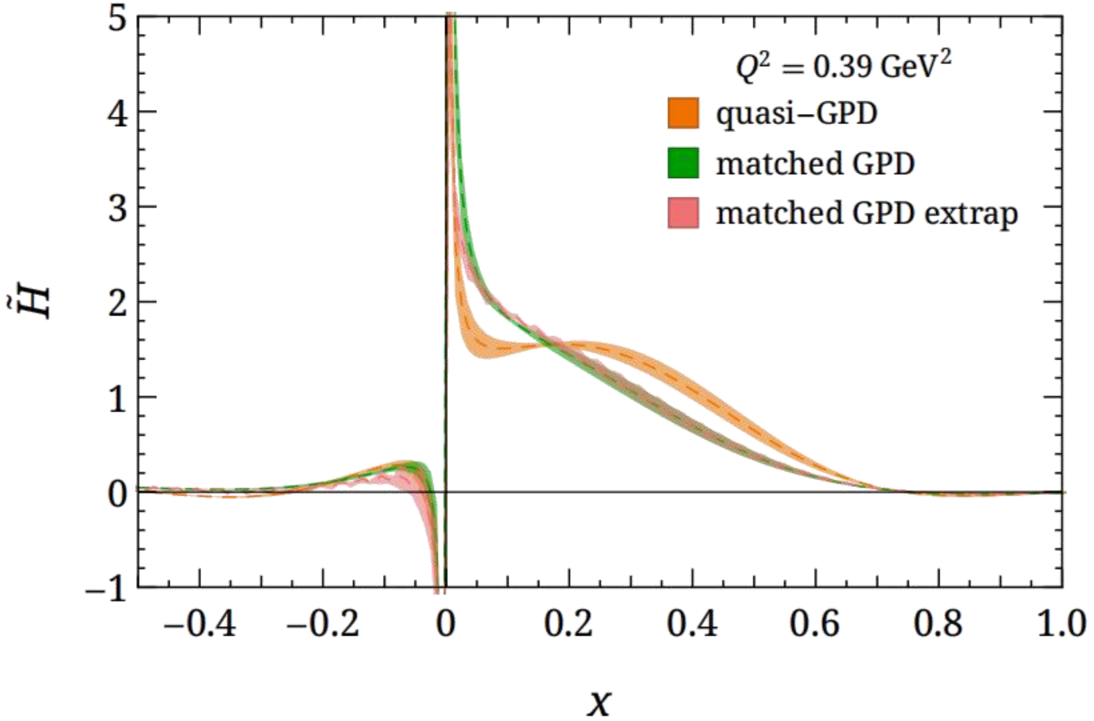
Lin, PRL 127 (2021)

Unpolarized GPDs H(x,ξ,t) and E(x,ξ,t)



Lin, PLB 824 (2022)

Helicity GPD $\tilde{H}(x,\xi,t)$



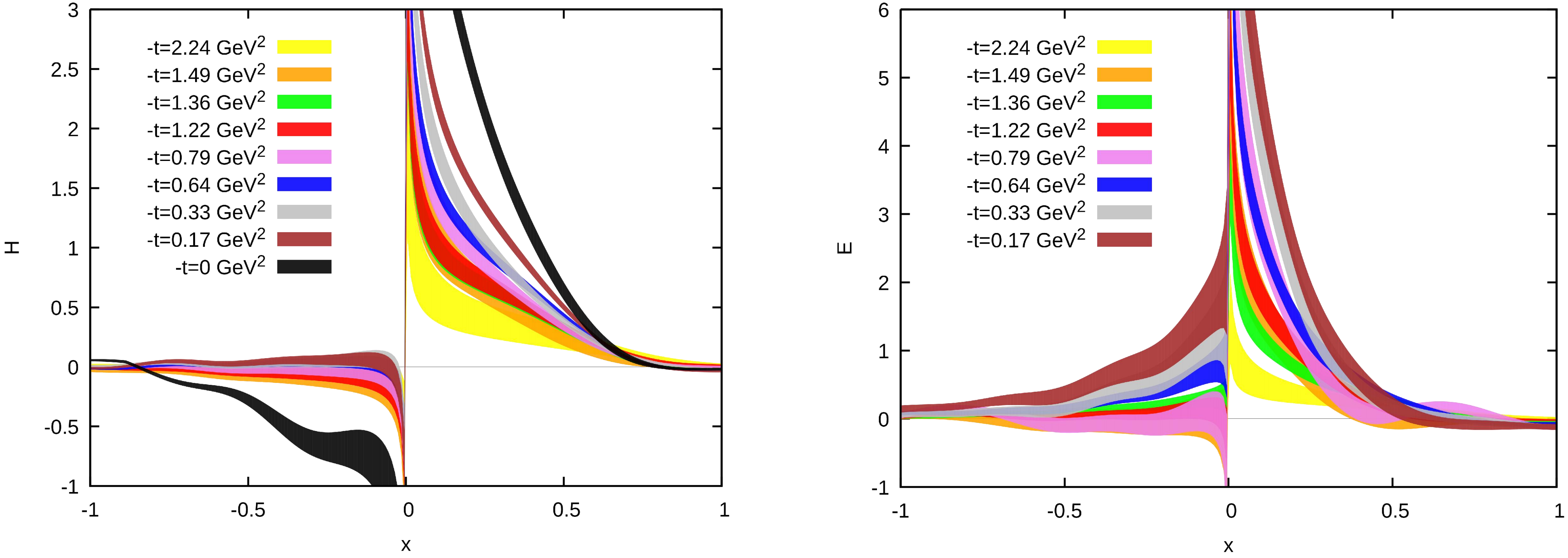
ξ=0, $P_z \approx 2.2$ GeV, m_π =135 MeV, Q^2 = {0, 0.19, 0.39, 0.77, 0.97} GeV², **RI/MOM scheme.**

For helicity operator $\Gamma = \gamma_z\gamma_5$, $\tilde{E}(x,\xi,t)$ couples with $q_z = (P_i - P_f)_z$.

Extracting nucleon GPDs using lattice QCD

- Studying proton GPDs **in asymmetric frames.** Bhattacharya et al, PRD 106 (2022)

Unpolarized GPDs $H(x,\xi,t)$ and $E(x,\xi,t)$ Cichy et al, arXiv:2304.14970, See talks on the 26th



$\xi=0$, $P_z = \{0.83, 1.25, 1.67\}$ GeV, $m_\pi \approx 260$ MeV, **RI/MOM scheme**.

The lattice quasi-observables connect the light-cone observables via a **perturbative matching coefficient.**

Motivation and Goal

- Only quark GPDs in flavor non-singlet case without mixing (isovector).
- Renormalization and matching using RI/MOM scheme.
- The flavor-singlet quark GPDs and gluon GPDs have been much less studied.

- To have a unified framework for perturbative matching including flavor non-singlet and singlet case, both in coordinate and momentum space.
- In a state-of-the-art scheme.
- Provide a manual for extracting all leading-twist GPDs, PDFs and DAs from lattice QCD.

Spatial nonlocal operator

upper sign: non-singlet (ns)

lower sign: singlet (s)

quark operator	
unpolarized	$O_{q,u}(z_1,z_2)=\frac{1}{2}\left[\bar\psi(z_1)\gamma^t[z_1,z_2]\psi(z_2)\pm(z_1\leftrightarrow z_2)\right]$
helicity	$O_{q,h}(z_1,z_2)=\frac{1}{2}\left[\bar\psi(z_1)\gamma^z\gamma_5[z_1,z_2]\psi(z_2)\mp(z_1\leftrightarrow z_2)\right]$
transversity	$O_{q,t}(z_1,z_2)=\frac{1}{2}\left[\bar\psi(z_1)\gamma^t\gamma^\perp\gamma_5[z_1,z_2]\psi(z_2)+(z_1\leftrightarrow z_2)\right]$
gluon operator	
unpolarized	$O_{g,u}(z_1,z_2)=g_\perp^{\mu\nu}\mathbf{F}_{\mu\nu}$
helicity	$O_{g,h}(z_1,z_2)=i\epsilon_\perp^{\mu\nu}\mathbf{F}_{\mu\nu}$
transversity	$O_{g,t}(z_1,z_2)=\frac{1}{2}\left[\mathbf{F}_{\mu\nu}+\mathbf{F}_{\nu\mu}\right]-\frac{1}{d-2}g_\perp^{\mu\nu}\mathbf{F}_\alpha{}^\alpha$

$$\mathbf{F}_{\mu\nu}\equiv \mathrm{F}_{z_{12}\mu}(z_1)[z_1,z_2]\,\mathrm{F}_{\nu z_{12}}(z_2) \qquad \{\mu,\nu,\alpha=1,2\}$$

➡ ns: $q(x)-\bar q(x)$

s: $q(x)+\bar q(x)$

Belitsky and Radyushkin, Phys.Rept. 418 (2005), (3.38)-(3.43).

The quasi-GPDs v.s light-cone GPDs (unpolarized quark operator)

Quasi-LF correlation

$$\langle P_1 S_1 | O_{q,u} | P_2 S_2 \rangle = \int_{-1}^{1} dx\, e^{i(x+\xi)P\cdot z_1 - i(x-\xi)P\cdot z_2} \bar{u}(P_1 S_1)\left[\mathbb{H}(x,\xi,t)\gamma^t + \mathbb{E}(x,\xi,t)\frac{i\sigma^{t\mu}\Delta_\mu}{2M}\right] u(P_2 S_2) \quad i = 1,2$$

LF correlation

$$\langle P_1 S_1 | O^{l.t.}_{q,u} | P_2 S_2 \rangle = \int_{-1}^{1} dx\, e^{i(x+\xi)P^+ z_1^- - i(x-\xi)P^+ z_2^-} \bar{u}(P_1 S_1)\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right] u(P_2 S_2)$$

where the **light-cone** quark operator is

$$O^{l.t.}_{q,u} = \frac{1}{2}\left[\bar{\psi}(z_1 n_-)\gamma^+[z_1, z_2]\psi(z_2 n_-) \pm (z_1 \leftrightarrow z_2)\right]$$

- They can be related by a factorization formula. The matching coefficients for the light-cone unpolarized quark GPDs H(x,ξ,t) and E(x,ξ,t) should be same. Liu et al, PRD 100 (2019), Ji et al, PRD 92 (2015)

l.t. stands for the leading-twist projection which acts as the generating function of leading-twist local operators.

Factorization formula (non-singlet):

$$O_q^{ns}(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, \boxed{C_{qq}^{ns}(\alpha, \beta, \mu^2 z_{12}^2)} O_q^{l.t.,ns}(z_{12}^{\alpha}, z_{21}^{\beta})$$

Factorization formula (singlet): Quark and gluon quasi-distributions can mix with each other,

$$\begin{pmatrix} O_q \\ O_g \end{pmatrix} = \begin{pmatrix} C_{qq} & C_{qg} \\ C_{gq} & C_{gg} \end{pmatrix} \otimes \begin{pmatrix} O_q^{l.t.} \\ O_g^{l.t.} \end{pmatrix}$$

$$O_q(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left[\boxed{C_{qq}(\alpha, \beta, \mu^2 z_{12}^2)} O_q^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) + C_{qg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) \right]$$

$$O_g(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left[C_{gq}(\alpha, \beta, \mu^2 z_{12}^2) O_q^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) + C_{gg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) \right]$$

l.t. stands for the leading-twist projection which acts as the generating function of leading-twist local operators.

Sandwiched between quark or gluon external states (GPDs)

	$O_q/O_q^{l.t.}$	$O_g/O_g^{l.t.}$
$\|q\rangle$	**Quark in quark (Non-singlet case)** $C_{qq}^{(1)} = \frac{\langle q\|O_q\|q'\rangle^{(1)} - \langle q\|O_q^{l.t.}\|q'\rangle^{(1)}}{\langle q\|O_q^{l.t.}\|q'\rangle^{(0)}}$	**Gluon in quark** $C_{gq}^{(1)} = \frac{\langle q\|O_g\|q'\rangle^{(1)} - \langle q\|O_g^{l.t.}\|q'\rangle^{(1)}}{\langle q\|O_q^{l.t.}\|q'\rangle^{(0)}}$
$\|g\rangle$	**Quark in gluon** $C_{qg}^{(1)} = \frac{\langle g\|O_q\|g'\rangle^{(1)} - \langle g\|O_q^{l.t.}\|g'\rangle^{(1)}}{\langle g\|O_g^{l.t.}\|g'\rangle^{(0)}}$	**Gluon in gluon** $C_{gg}^{(1)} = \frac{\langle g\|O_g\|g'\rangle^{(1)} - \langle g\|O_g^{l.t.}\|g'\rangle^{(1)}}{\langle g\|O_g^{l.t.}\|g'\rangle^{(0)}}$

Quark in gluon: gluon matrix element of the quark quasi-GPD operator

- PDFs (forward) $\langle q|O_q|q\rangle$
- DAs $\langle q\bar{q}'|O_q|0\rangle$ or $\langle 0|O_q|q\bar{q}'\rangle$

Fourier transformation

$$\mathscr{P}\left(\tau,\,\xi,\,\mu^2 z_{12}^2\right) = N \int \frac{dz_1}{2\pi} \int \frac{dz_2}{2\pi}\, e^{-i(\xi+\tau)P\cdot z_1 - i(\xi-\tau)P\cdot z_2}\, \widetilde{\mathscr{H}}\left(z_i, P_i, \mu^2 z_{12}^2\right)$$

$$\mathbb{H}\left(x,\xi,\frac{\mu}{P_z}\right) = P_z^2 \int_{-1}^{1} d\tau_1 \int_{-1}^{1} d\tau_2 \int \frac{dz_1}{2\pi} \int \frac{dz_2}{2\pi}\, e^{iP_z[(x_1-\tau_1)z_1+(x_2-\tau_2)z_2]}\, \mathscr{P}\left(\tau_1,\tau_2,\frac{\mu^2\zeta^2}{P_z^2}\right)$$

Quasi-LF correlations $\widetilde{\mathscr{H}}(z_i, P_i, \mu^2 z_{12}^2)$

FT

Radyushkin, PRD 100 (2019)

Pseudo-GPDs $\mathscr{P}\left(\tau,\,\xi,\,\mu^2 z_{12}^2\right)$

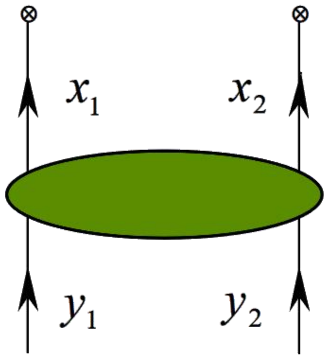
double FT

Quasi-GPDs $\mathbb{H}\left(x,\xi,\frac{\mu}{P_z}\right)$

$$C\left(\alpha,\beta;\mu^2 z_{12}^2\right) = \begin{pmatrix} C_{qq} & C_{qg} \\ C_{gq} & C_{gg} \end{pmatrix}$$

$$\mathcal{C}\left(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2\right) = \begin{pmatrix} \mathcal{C}_{qq} & \mathcal{C}_{qg} \\ \mathcal{C}_{gq} & \mathcal{C}_{gg} \end{pmatrix}$$

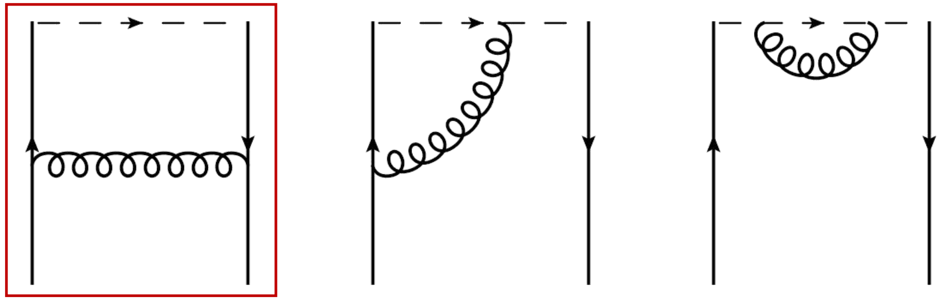
$$\mathbb{C}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) = \begin{pmatrix} \mathbb{C}_{qq} & \mathbb{C}_{qg} \\ \mathbb{C}_{gq} & \mathbb{C}_{gg} \end{pmatrix}$$



$$X_1 = \xi + X, \qquad X_2 = \xi - X \qquad (X = \tau, x, y)$$

Belitsky and Radyushkin, Phys.Rept. 418 (2005)

Quark in quark Cqq (In coordinate space)



$$C_{qq}^{\overline{\mathrm{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = \delta(\alpha)\delta(\beta) + 2a_s C_F \Big\{ \Big(A_2 + [\bar{\alpha}/\alpha]_+ \delta(\beta) + [\bar{\beta}/\beta]_+ \delta(\alpha) \Big) (\mathrm{L_z} - 1) + \mathrm{A}_3 - 2\,[\ln(\alpha)/\alpha]_+ \delta(\beta) - 2\,[\ln(\beta)/\beta]_+ \delta(\alpha) \Big\} + 2a_s C_F(-2\mathrm{L_z} + 2)\delta(\alpha)\delta(\beta)\,,$$

$$\mathrm{L_z} = \ln \frac{4\mathrm{e}^{-2\gamma_\mathrm{E}}}{-\mu^2 \mathrm{z}_{12}^2}$$

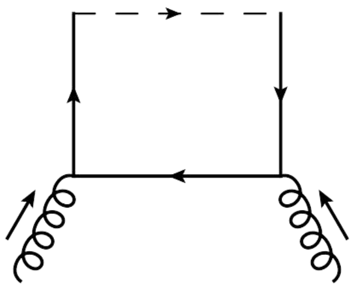
$$A_{2,u} = 1\,, \qquad A_{2,h} = 1\,, \qquad A_{2,t} = 0$$

$$A_{3,u} = 2\,, \qquad A_{3,h} = 4\,, \qquad A_{3,t} = 0$$

The matching coefficients are consistent with Ref. [Radyushkin, PRD 100 (2019)].

- Applicable both to quark flavor non-singlet case and to DA case (the difference lies in the phase structure).

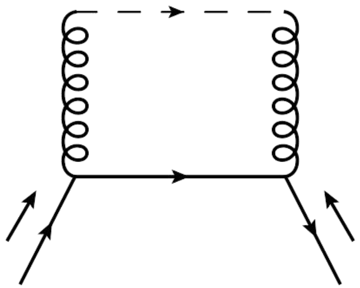
Quark in gluon Cqg (In coordinate space)



$$C_{qg}^{\overline{\mathrm{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = 4ia_s T_F N_f \, \mathbf{z}_{12} \, B_3 \, \mathrm{L_z}$$

$$B_{3,u} = \bar{\alpha}\bar{\beta} + 3\alpha\beta \qquad B_{3,h} = \bar{\alpha}\bar{\beta} - \alpha\beta$$

Gluon in quark Cgq (In coordinate space)

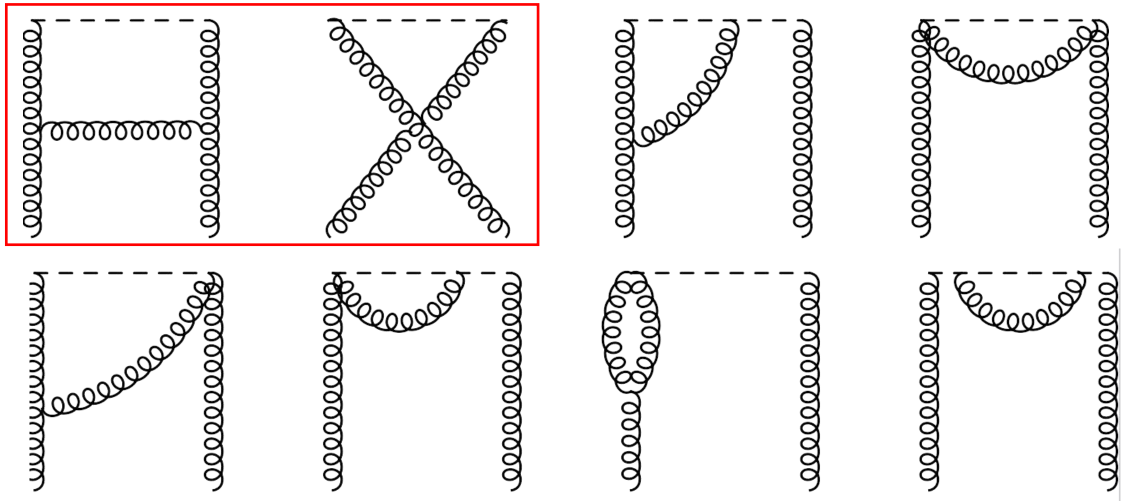


$$C_{gq}^{\overline{\mathrm{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) = \frac{-2ia_s C_F}{\mathbf{z}_{12}} \Big\{ \Big(\delta(\alpha)\delta(\beta) + D_3 \Big) (\mathrm{L_z} + 1) + D_4 - 2\Big(\delta(\alpha) + \delta(\beta) \Big) \Big\}$$

$$D_{3,u} = 2\,, \qquad D_{3,h} = -2\,, \qquad D_{4,u} = 6\,, \qquad D_{4,h} = 4\,.$$

- Note that the operator defining the quark transversity is chiral-odd and thus does not mix with gluons.
- Mixing terms should ensure dimension consistency.

Gluon in gluon Cgg (In coordinate space)



$$C_{gg}^{\overline{\rm MS}}(\alpha,\beta,\mu^2 z_{12}^2) =\delta(\alpha)\delta(\beta) + 2a_s C_A\Big\{ \Big(E_1 + \left[\bar{\alpha}^2/\alpha\right]_+ \delta(\beta) + \left[\bar{\beta}^2/\beta\right]_+ \delta(\alpha)\Big) (\mathrm{L_z}-1) + E_2 - 2\left[\ln(\alpha)/\alpha\right]_+ \delta(\beta) - 2\left[\ln(\beta)/\beta\right]_+ \delta(\alpha)\Big\} + 2a_s C_A\,(-3\,\mathrm{L_z}+2)\,\delta(\alpha)\delta(\beta)\,,$$

$$E_{1,u} =4(1-\alpha-\beta+3\alpha\beta)\,, \qquad E_{1,h} = 4(1-\alpha-\beta)\,, \qquad E_{1,t} = 0,$$

$$E_{2,u} =\frac{5}{2}\,E_{1,u} + 6\alpha\beta\,, \qquad E_{2,h} = \frac{3}{2}\,E_{1,h}\,, \qquad E_{2,t} = 2(1+\alpha+\beta-2\alpha\beta).$$

The evolution kernels are all consistent with Ref. [Belitsky and Radyushkin, Phys.Rept. 418 (2005)].

- Fourier transform to pseudo space

$$\mathcal{C}(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2) = \int_0^1 d\alpha \int_0^1 d\beta\, C(\alpha,\beta,\mu^2 z_{12}^2)\,\delta(\tau_1 - \bar{\alpha} y_1 - \bar{\alpha}\beta y_2)$$

Ji and Belitsky, NPB 894 (2015)

- Fourier transform to momentum space

$$\mathbb{C}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) = P_z^2 \int_{-1}^1 d\tau_1 \int_{-1}^1 d\tau_2 \int \frac{dz_1}{2\pi}\int \frac{dz_2}{2\pi}\, e^{iP_z[(x_1-\tau_1)z_1+(x_2-\tau_2)z_2]}\,\mathcal{C}\left(\tau_1,\tau_2,y_1,y_2;\frac{\mu^2\zeta^2}{P_z^2}\right)$$

e.g. the matching coefficient of quark GPDs

$$\mathbb{C}_{qq}^{\overline{\text{MS}}}(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}) = \delta(x_1-y_1) + a_s C_F\, \mathbb{C}_{qq}^{(1)}(x_1,x_2,y_1,y_2;\frac{\mu}{P_z})$$

with

$$\mathbb{C}_{qq,u}^{(1)} = \left\{\left(\frac{|x_1|}{y_1(y_1+y_2)}(\text{Ł}_x - 1) + (x_1\leftrightarrow x_2, y_1\leftrightarrow y_2)\right) - \frac{|x_1-y_1|}{y_1 y_2}(\text{Ł}_{xy}-1)\right\} + \mathbb{C}_{qq,t}^{(1)}\,,$$

$$\mathbb{C}_{qq,h}^{(1)} = \mathbb{C}_{qq,u}^{(1)} + 2\left\{\frac{|x_1|}{y_1(y_1+y_2)} + \frac{|x_2|}{y_2(y_1+y_2)} - \frac{|x_1-y_1|}{y_1 y_2}\right\},$$

$$\mathbb{C}_{qq,t}^{(1)} = \left\{\left(\frac{|x_1|}{y_1(y_1-x_1)}(\text{Ł}_x - 1) + (x_1\leftrightarrow x_2, y_1\leftrightarrow y_2)\right) + \left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\frac{1}{|x_1-y_1|}(\text{Ł}_{xy}-1)\right\}.$$

$$\text{Ł}_x = \ln\frac{4P_z^2 x_1^2}{\mu^2}$$

$$\text{Ł}_{xy} = \ln\frac{4P_z^2(x_1-y_1)^2}{\mu^2}$$

Having same results in Ref. [Ma et al, JHEP 08 (2022)], they calculate directly in momentum space.

- Fourier transform to pseudo space

$$\mathcal{C}(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2) = \int_0^1 d\alpha \int_0^1 d\beta\, C(\alpha,\beta,\mu^2 z_{12}^2)\,\delta(\tau_1-\bar{\alpha}y_1-\bar{\alpha}\beta y_2)$$

Ji and Belitsky, NPB 894 (2015)

- Fourier transform to momentum space

$$\mathbb{C}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) = P_z^2\int_{-1}^{1} d\tau_1 \int_{-1}^{1} d\tau_2 \int \frac{dz_1}{2\pi}\int \frac{dz_2}{2\pi}\, e^{iP_z[(x_1-\tau_1)z_1+(x_2-\tau_2)z_2]}\,\mathcal{C}\left(\tau_1,\tau_2,y_1,y_2;\frac{\mu^2\zeta^2}{P_z^2}\right)$$

e.g. the matching coefficient of quark GPDs

$$\begin{aligned}\mathbb{C}_{qq}^{(1)}(x,y,\xi) = \frac{1}{y}\Big[&G_1(x,y,\xi)\theta(x<-\xi)\theta(x<y) + G_2(x,y,\xi)\theta(-\xi<x<\xi)\theta(x<y)\\ &+ G_3(-x,-y,\xi)\theta(-\xi<x<\xi)\theta(x>y) + G_3(x,y,\xi)\theta(x>\xi)\theta(x<y)\\ &- G_1(x,y,\xi)\theta(x>\xi)\theta(x>y)\Big]\end{aligned}$$

where the region $\theta(-\xi < x < \xi)\,\theta(x > y)$ is missing in the kinematic setup in Refs. [Ji et al, PRD 92 (2015)], [Xiong et al, PRD 92 (2015)], [Liu et al, PRD 100 (2019)] (switching to the notation of these references).

Radyushkin, PRD 98 (2018) — Ji et al, NPB 964 (2021)

	MSbar scheme	Ratio scheme	Ratio-hybrid scheme
Coordinate space	$C\left(\alpha,\beta;\mu^2 z_{12}^2\right)$	$C^{\mathrm{ratio}}(\alpha,\beta,\mu^2 z_{12}^2) = C^{\overline{\mathrm{MS}}}(\alpha,\beta,\mu^2 z_{12}^2) - Z^{(1)}(\mu^2 z_{12}^2)\delta(\alpha)\delta(\beta)$ Z represents the same operator matrix element at zero momentum.	$C^{\mathrm{hybrid}}\left(\alpha,\beta,\mu^2 z_{12}^2,\frac{z_{12}^2}{z_s^2}\right) = C^{\mathrm{ratio}}(\alpha,\beta,\mu^2 z_{12}^2) - T_L \ln\frac{z_{12}^2}{z_s^2}\delta(\alpha)\delta(\beta)\,\theta\left(\lvert z_{12}\rvert - z_s\right)$ where z_s denotes a truncation point and T_L is anomalous dimension.
Pseduo space	$\mathcal{C}\left(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2\right)$	$\mathcal{C}^{\mathrm{ratio}}(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2) = \mathcal{C}^{\overline{\mathrm{MS}}}(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2) - Z^{(1)}\left(\mu^2 z_{12}^2\right)\,\delta(\tau_1-y_1)$	
Momentum space	$\mathbb{C}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right)$	$\mathbb{C}^{\mathrm{ratio}}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) = \mathbb{C}^{\overline{\mathrm{MS}}}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) - T_L\frac{1}{\lvert x_1-y_1\rvert}$	$\mathbb{C}^{\mathrm{hybrid}}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) = \mathbb{C}^{\mathrm{ratio}}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right) - T_L\left[-\frac{1}{\lvert x_1-y_1\rvert} + \frac{2\mathrm{Si}((x_1-y_1)\lambda_s)}{\pi(x_1-y_1)}\right]_+$

Chou and Chen, PRD 106 (2022), See Chen's talk

- Ratio scheme introduce undesired IR effects at large distances in LaMET.

Radyushkin, PRD 98 (2018), Balitsky et al, PLB 808 (2020)

	GPDs	Reduction to PDFs (forward limit)	Reduction to DAs
Coordinate space	$C\left(\alpha,\beta;\mu^2 z_{12}^2\right)$	Factorization formula: $\tilde{h}(z_{12},p_z,\mu)=\int_0^1 d\alpha\, \boldsymbol{C}(\alpha,\mu^2 z_{12}^2)h^{l.t.}(\bar{\alpha},\mu)$ Integrating one of Feynman parameters, $\boldsymbol{C}(\alpha',\mu^2 z_{12}^2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, \delta(\alpha'-\alpha-\beta)C(\alpha,\beta,\mu^2 z_{12}^2)$	Factorization formula: $\widetilde{M}(z_i,p_i,\mu) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} C(\alpha,\beta,\mu^2 z_{12}^2)M^{l.t.}(z_i,p_i,\mu)$ The matching coefficient is same as the GPDs.
Momentum space	$\mathbb{C}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P_z}\right)$	Factorization formula: $\tilde{q}\left(x,P_z\right)=\int_{-1}^1 dy\, F\left(x,y;\frac{\mu}{P_z}\right)q(y,\mu)$ Taking the zero-skewness limit ξ → 0, $F\left(x,y;\frac{\mu}{P_z}\right)=\mathbb{C}\left(x,-x,\,y,-y;\frac{\mu}{P_z}\right)$	Factorization formula: $\tilde{\phi}\left(x,P_z\right)=\int_0^1 dy\, V\left(x,y;\frac{\mu}{P_z}\right)\phi(y,\mu)$ Taking limit, $V\left(x,y;\frac{\mu}{P_z}\right)=\mathbb{C}\left(x,1-x,y,1-y;\frac{\mu}{P_z}\right)$

Complete matching for PDFs in MSbar scheme [Wang et al, EPJC 78 (2018)] and RI/MOM scheme [Liu et al, 100 (2019)].
Matching for non-singlet the meson DAs in Ref. [Liu et al, PRD 99 (2019)].

The discrepancy in **gluon PDFs** (unpolarized)

$$x\tilde{g}(x,P_z)=\int_{-1}^{1}\frac{dy}{|y|}F_{gq}\left(\frac{x}{y},\frac{\mu}{P_z}\right)q(y,\mu)+\int_{-1}^{1}\frac{dy}{|y|}F_{gg}\left(\frac{x}{y},\frac{\mu}{P_z}\right)yg(y,\mu)$$

Wang et al, EPJC 78 (2018)

$$F_{gg}^{(1)}\left(t,\frac{P_z}{\mu}\right)=2a_sC_A\begin{cases}\frac{2(t^2-t+1)^2}{t-1}\ln\frac{t-1}{t}+2t^2-t+\boxed{\frac{8}{3}+1} & t>1\\ \frac{2(t^2-t+1)^2}{t-1}\ln\frac{\mu^2}{4t(1-t)P_z^2}+\frac{10t^4-16t^3+21t^2-15t+6}{3(t-1)}+\frac{4}{3}+1 & 0<t<1\\ -\frac{2(t^2-t+1)^2}{t-1}\ln\frac{t-1}{t}-2t^2+t-\frac{8}{3}-1 & t<0\end{cases}$$

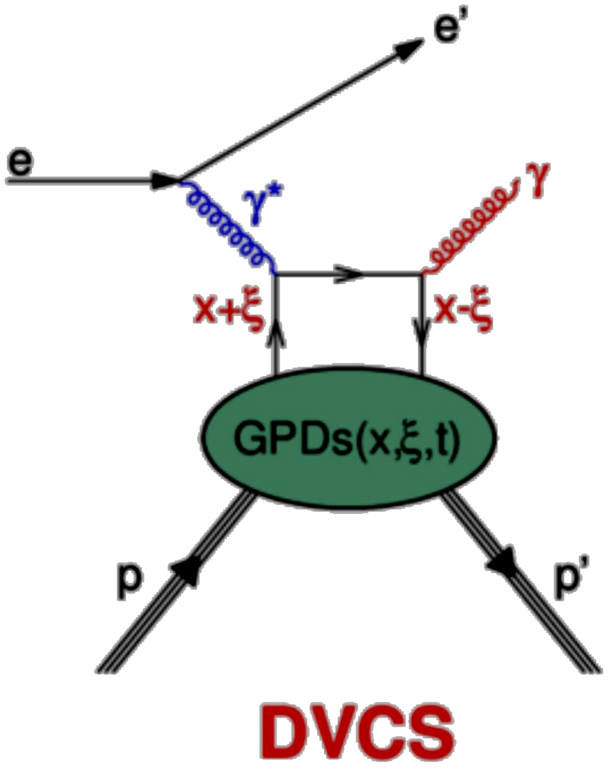
- The discrepancy in F_{gq} affect **the matching process**, while that of F_{gg} does not.
- The contributions **in unphysical region** are completely determined by the evolution kernel (ratio scheme),

$$\mathcal{K}_{gg}(\alpha)=2\left[\frac{(1-\alpha\bar{\alpha})^2}{1-\alpha}\right]_+,\quad \int_0^1 d\alpha\frac{\mathcal{K}_{gg}(\alpha)}{t-\alpha}=2\frac{(1-t\bar{t})^2}{t-1}\ln\frac{t-1}{t}+\frac{11}{6}\frac{1}{t-1}+t(2t-1)+\boxed{\frac{11}{3}}$$

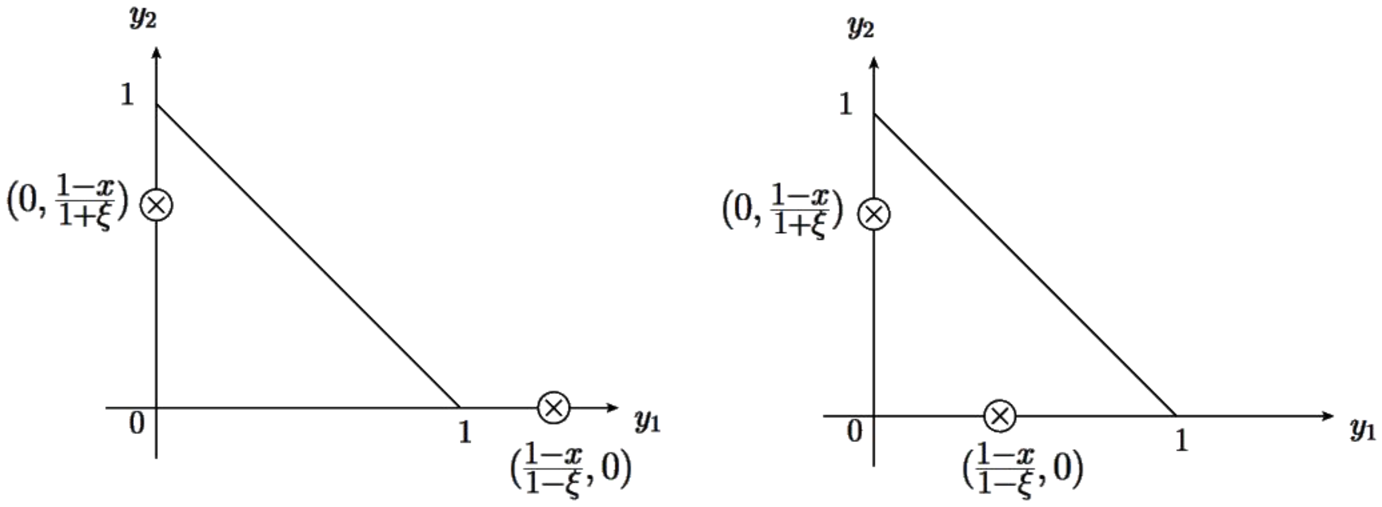
There is similar discrepancy between Ref. [Balitsky et al, PLB 808 (2020)] and Ref. [Wang et al, PRD 100 (2019)], **while our results in the ratio scheme are completely consistent with the former**.

- GPDs plays an important role in the detailed understanding of the inner 3D structure of nucleon.
- Lattice QCD calculations can provide great help to extract GPDs.
- We provide a unified framework for perturbative matching connecting Euclidean to lightcone correlations.
  - Both for non-singlet and singlet (GPDs, PDFs, DAs).
  - In coordinate and momentum space.
  - In a state-of-the-art scheme (ratio and hybrid scheme).
- Follow-up:
  - Studying the discrepancy.
  - Two-loop level.

Thank you for listening !



$eA \to e' A\gamma$



Integration in ERBL (left) and DGLAP (right) regions: The singularities are denoted by cross

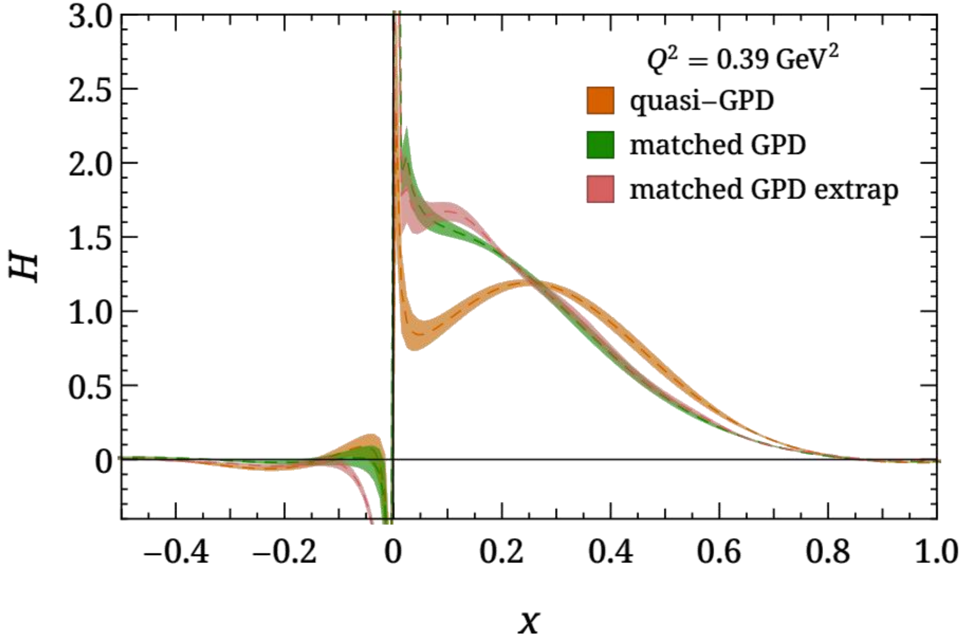
Liu et al — PRD 100 (2019)

Extracting nucleon GPDs using lattice QCD (LaMET)

Lin, PRL 127 (2021) 18, 182001

Unpolarized nucleon GPD

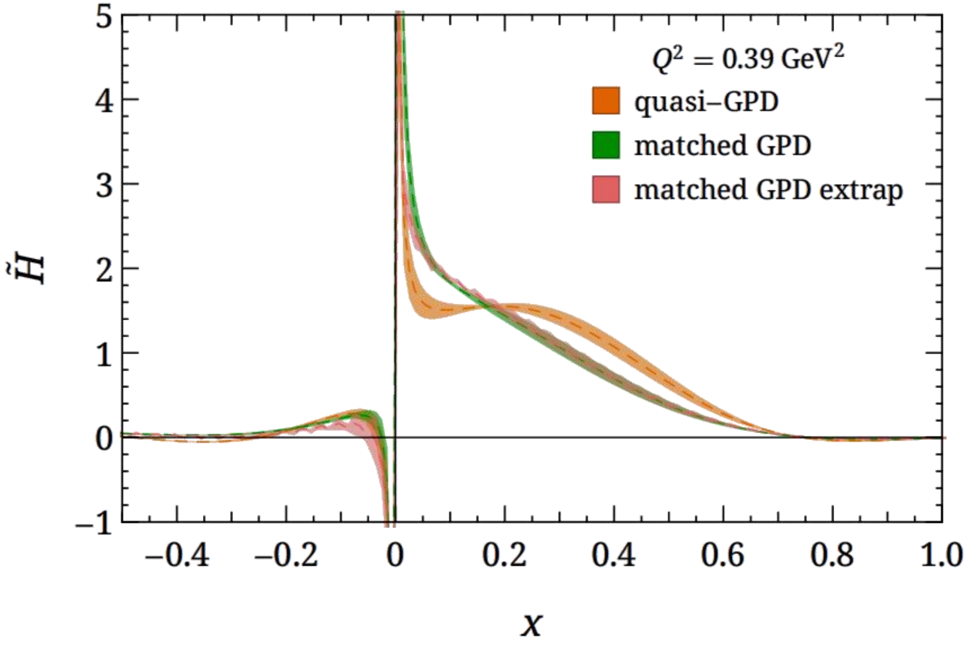
$$F(x,\xi,t)=\int\frac{dz^-}{4\pi}e^{-ixp^+z^-}\left\langle p''\left|\bar\psi(-\frac{z}{2})\gamma^+L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})\right|p'\right\rangle_{z^+=0,\vec z_\perp=0}$$
$$=\frac{1}{2p^+}\left[H(x,\xi,t)\bar u(p'')\gamma^+u(p')+E(x,\xi,t)\bar u(p'')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p')\right]$$



Lin, PLB 824 (2022) 136821

Helicity nucleon GPD

$$\tilde F(x,\xi,t)=\int\frac{dz^-}{4\pi}e^{-ixp^+z^-}\left\langle p''\left|\bar\psi(-\frac{z}{2})\gamma^+\gamma_5L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})\right|p'\right\rangle_{z^+=0,\vec z_\perp=0}$$
$$=\frac{1}{2p^+}\left[\tilde H(x,\xi,t)\bar u(p'')\gamma^+\gamma_5u(p')+\tilde E(x,\xi,t)\bar u(p'')\frac{\gamma_5\Delta^+}{2M}u(p')\right]$$



$\xi=0$, $P_z\approx 2.2$ GeV at the physical pion mass, $Q^2\in\{0, 0.19, 0.39, 0.77, 0.97\}$ GeV^2.