Leading Power Accuracy in Lattice Calculation of Parton Distributions

Rui Zhang

(University of Maryland/Argonne National Laboratory)

LaMET2023, University of Regensburg, Germany
July 24th, 2023

UNIVERSITY OF MARYLAND 1856
DEPARTMENT OF PHYSICS

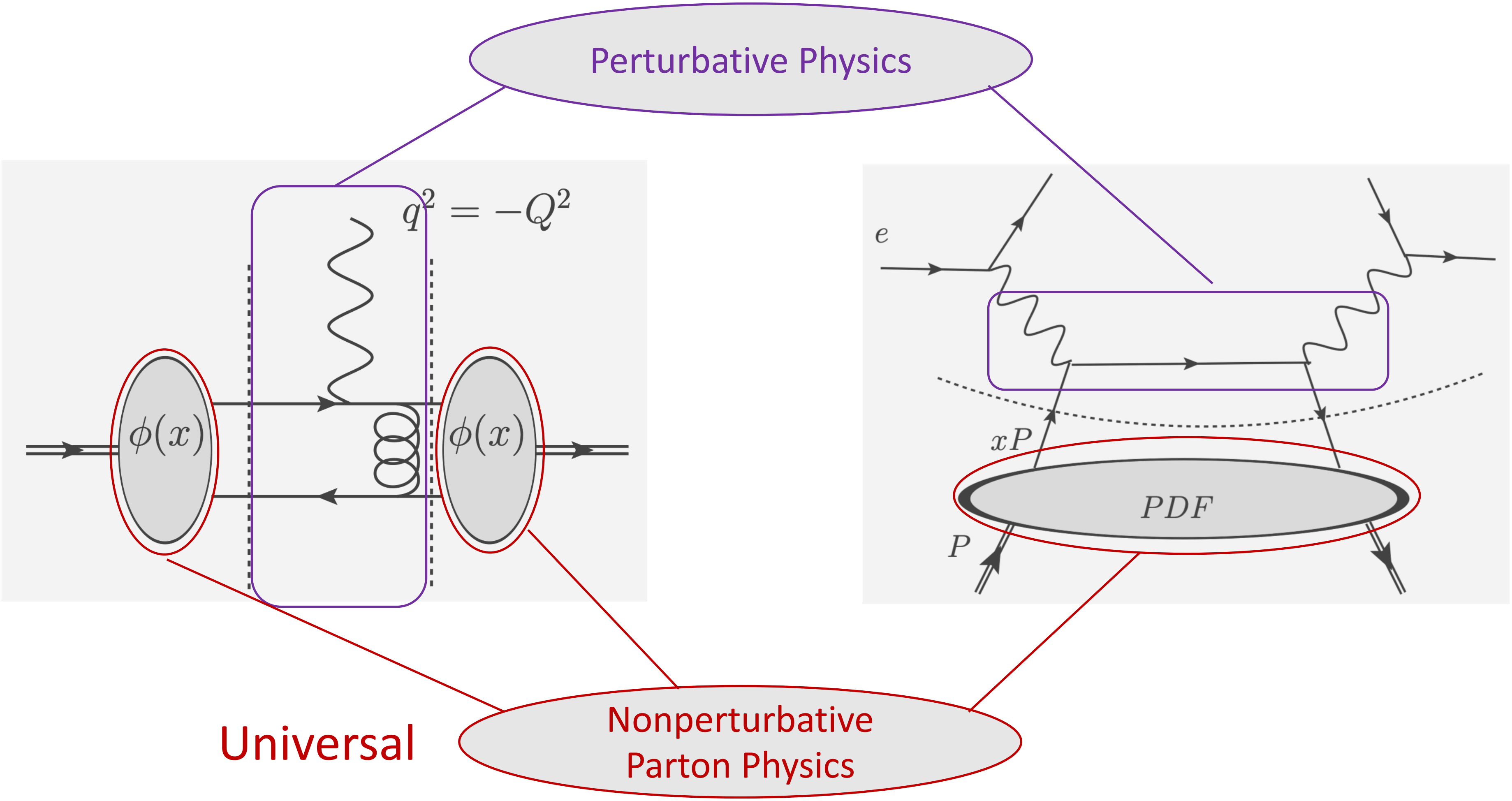
R. Zhang, J. Holligan, X. Ji, and Y. Su, PLB (2023)

Outline

- Introduction
- Leading Power Corrections in $1/|P_z|$Expansion
- Leading Power accuracy in $1/|P_z|$Expansion
- Conclusion and Outlook

Factorization of Hadronic Processes

Collins, et.al., ASDHEP (1989)



Important inputs to collider physics!

Factorization of Hadronic Processes

Collins, et.al., ASDHEP (1989)

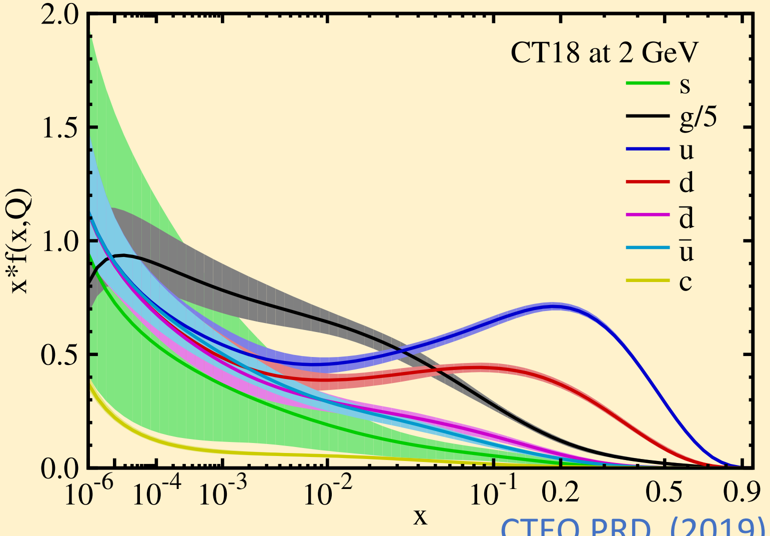
rbative Physics

Parton Distribution Function

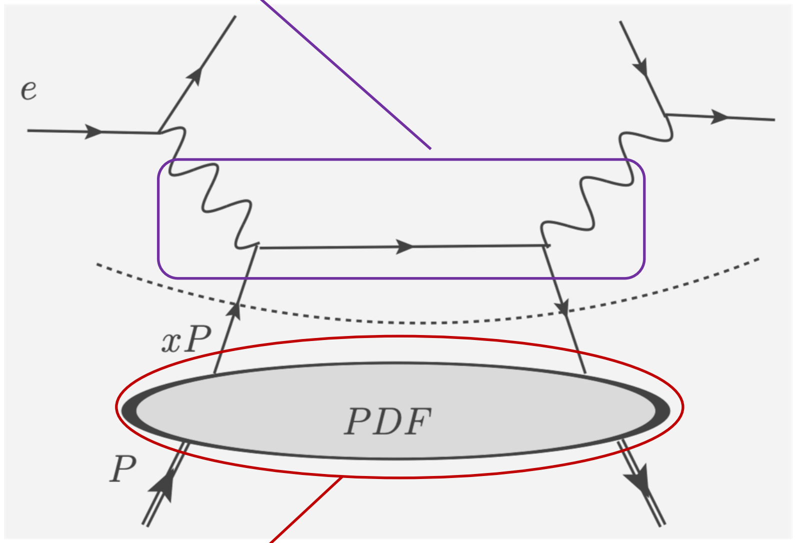
(Inclusive process)

probability density of finding a parton with momentum fraction x out of the hadron

Global fitting to experiments



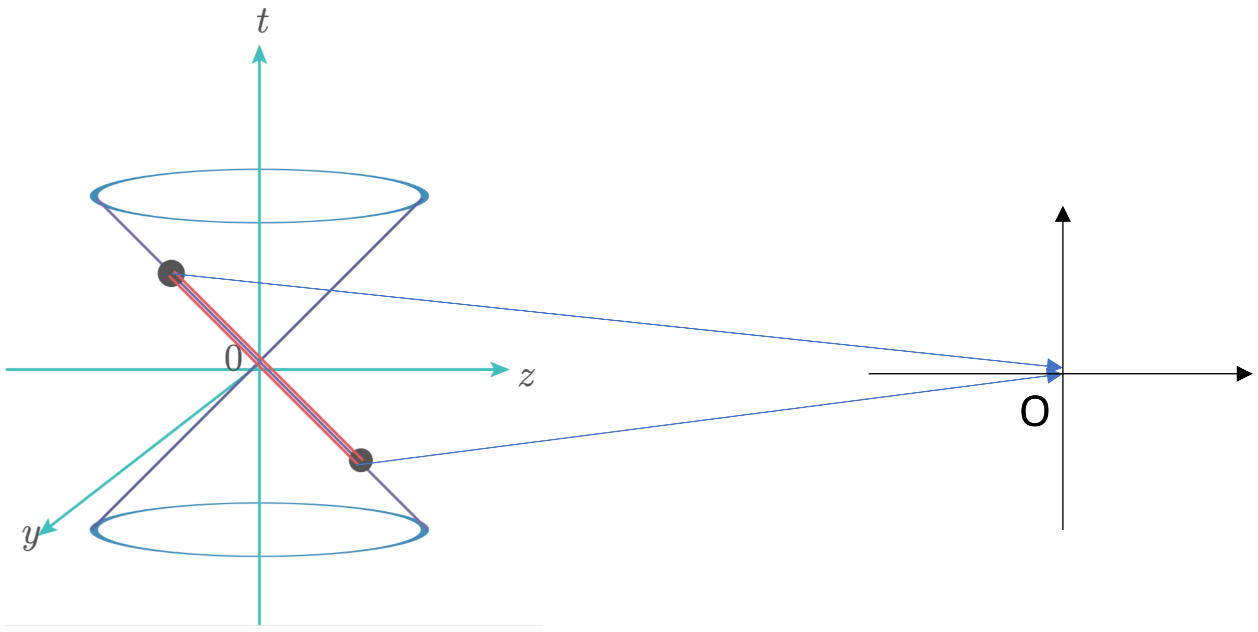
CTEQ PRD, (2019)



perturbative
rton Physics

Important inputs to collider physics!

Parton Physics on Lattice?



Minkowski Spacetime

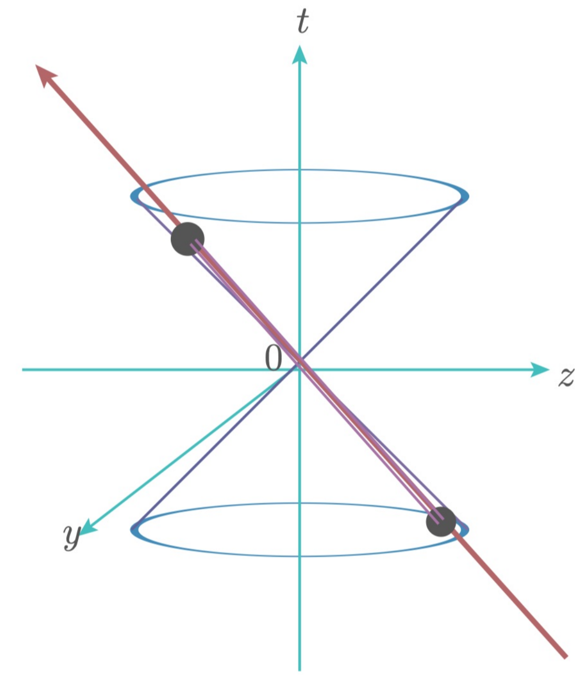
Euclidean Lattice

Not directly calculable?

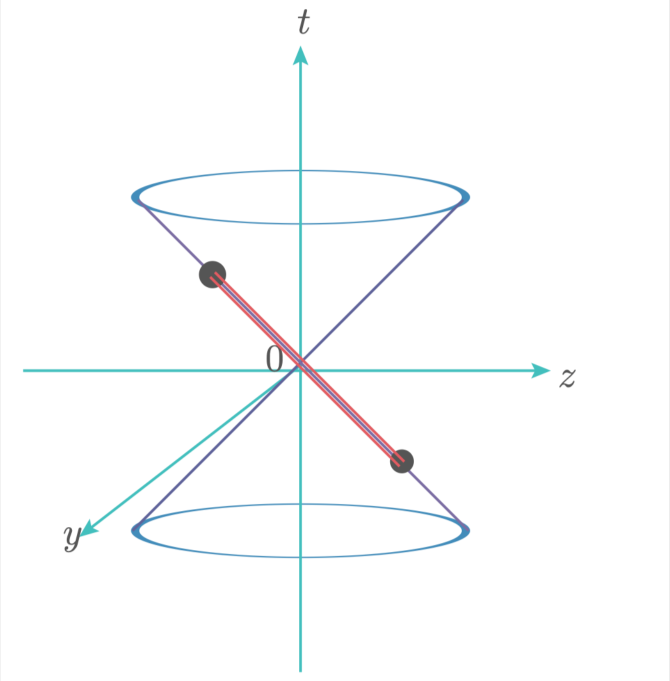
Large Momentum Effective Theory

Ji, PRL (2013)
Ji, SCPMA(2014)

Momentum distribution



Large P_z Expansion



$+\mathcal{O}\left(\frac{1}{|P_z|^n}\right)$

Quasi-PDF: $\tilde{f}(x, P_z) = \int \frac{dzP_z}{2\pi} e^{ixzP_z} \langle P|\bar{q}(0)\gamma_t U(0,z)q(z)|P\rangle$

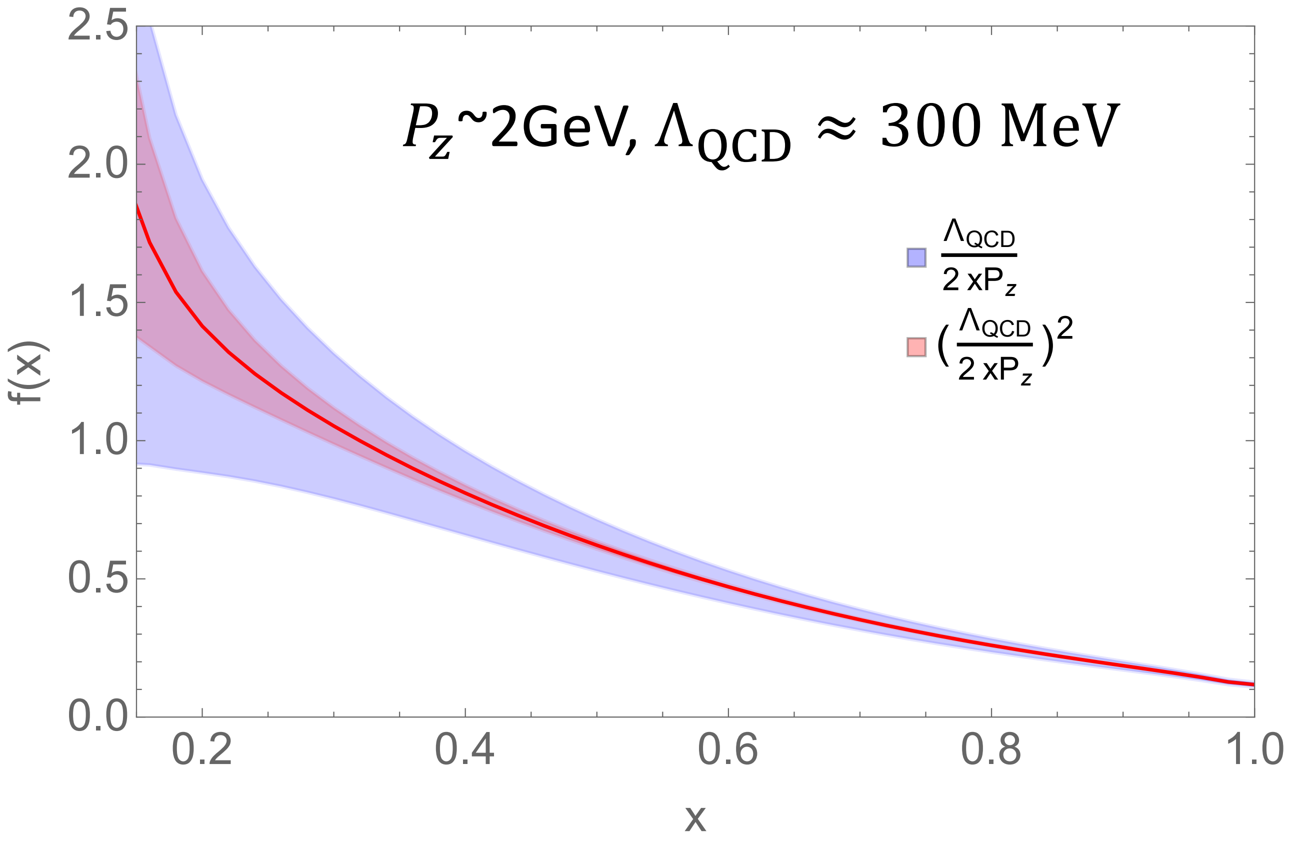
$C\ (x, y, \mu, P_z) \otimes f(y, \mu)$

$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$

Size of Power Correction

Ji, et.al, NPB (2021)

$$\tilde{f}(x, P_z) = C\,(x, y, \mu, P_z) \otimes f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{x\,|P_z|}\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

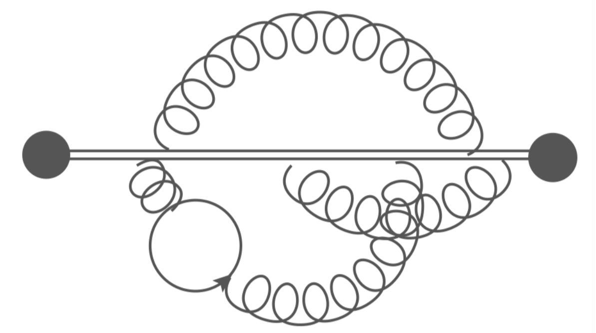


Linear power correction must be eliminated!

Not properly addressed in previous work

Why $1/P_z$ correction?

- Non-local operator: $\bar{q}(0)\Gamma U(0,z)q(z)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - A heavy quark propagating with "pole mass" $\delta m(a)$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a)\cdot|\mathrm{z}|}$

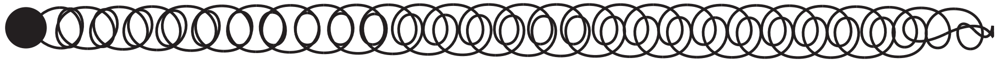


Ji, et.al, PRL (2017)

- What to subtract w/ linear divergence?

Freedom to choose the scheme

- Pole mass of a "free" quark?
  - Long range interactions contributing $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ ambiguously



Beneke, PLB (1995)

- $h^R(\mathrm{z}) \sim h^B(\mathrm{z})e^{\delta m\cdot|\mathrm{z}|+\mathcal{O}(|\mathrm{z}|\Lambda_{\mathrm{QCD}})}$

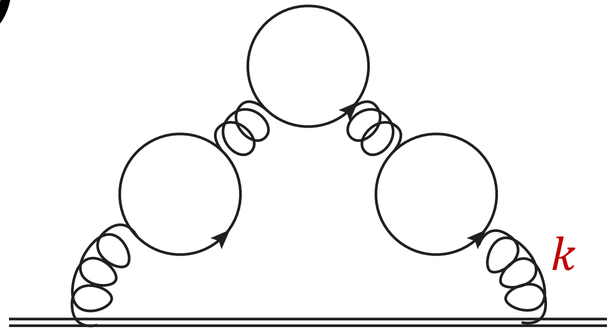
↓ Fourier Transform

$$\tilde{f}(x) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\mathrm{x}\,|P_z|}\right)$$

Ji, et.al, NPB (2021)

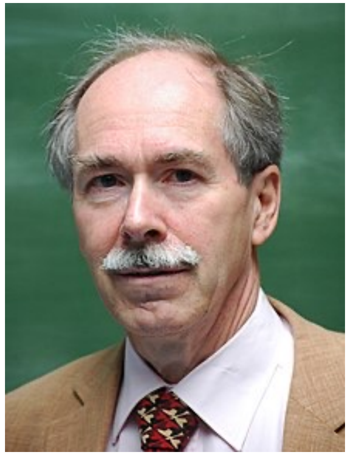
Perturbative determination of $\delta m(a)$

- In perturbation theory, $\delta m = \frac{1}{a}\sum \alpha_s^{n+1}(a) r_n$
  - $r_n \sim n!$ → Series is divergent for any α_s
- A lattice perturbative expansion of $\delta m(a)$ to **20th** order



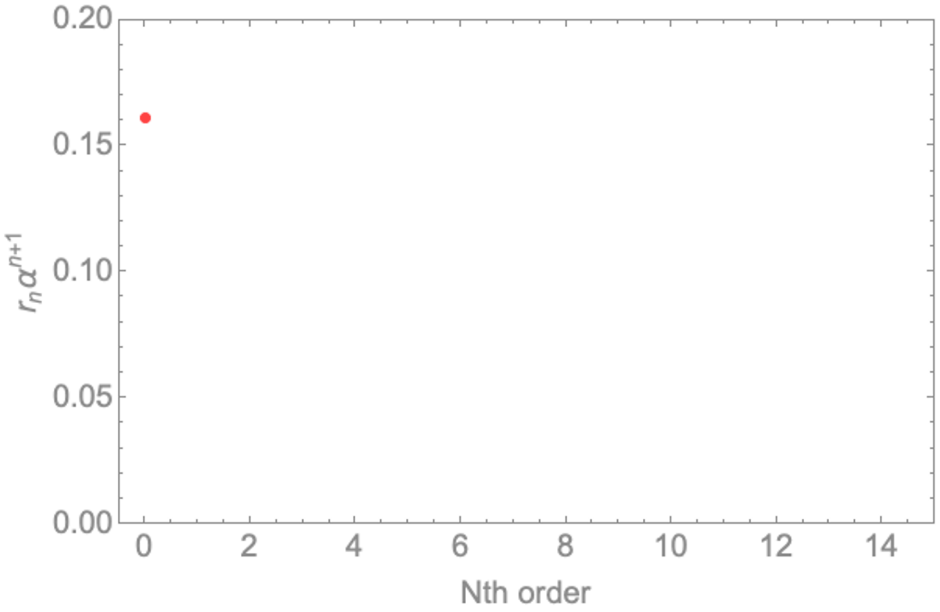
$\int dk^4 F(k)(\ln k^2)^n \sim n!$

Bali, et al., PRD (2013)



Gerard 't Hooft
1999 Nobel Prize

Renormalon Divergence



Infrared renormalon is partly related to the strong coupling $\alpha_s(k)$ becoming non-perturbative in the region $k \sim \Lambda_{QCD}$.

Beneke, RMP (1998)

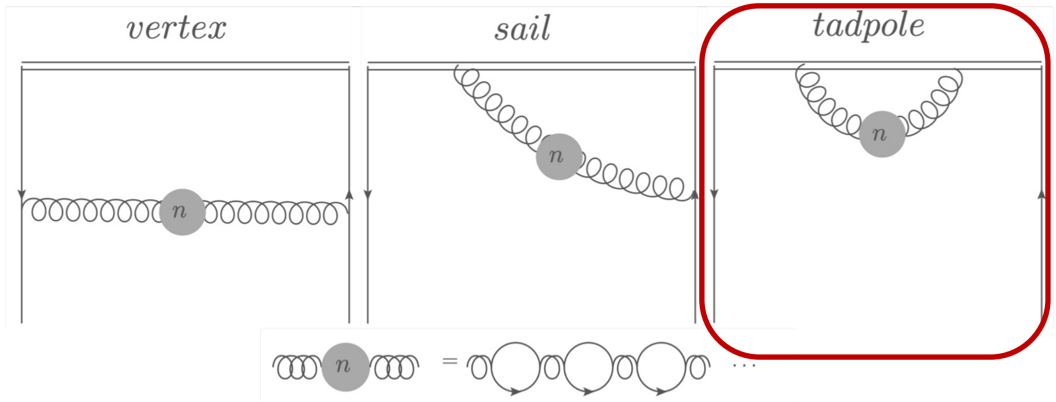
Renormalon in matching coefficients

$$\tilde{f}(x, P_z) = C(x, y, \mu, P_z) \otimes f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\text{x}\,|P_z|}\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{\text{x}^2 P_z^2}\right)$$

- $C(x, y, \mu, P_z)$ is obtained by perturbatively calculate the same operator $\bar{q}(0)\Gamma U(0, z)q(z)$, thus also has the same ambiguity:

$$C^{(n+1)} \sim n!$$

Braun, et.al., PRD(2018)



Linear Divergence

Remove IR ambiguity

- Regularizing infrared physics
  - Explicit IR cut off: $\int_0^{\Lambda_{\rm UV}} f(k)dk \to \int_{\Lambda_{\rm IR}}^{\Lambda_{\rm UV}} f(k)dk$ — Very difficult to calculate
  - Resumming the series to all orders with some prescription:

Ayala, PRD (2019)
Ayala, PRD (2020)

$$\sum_i^{\infty} \alpha_s^{i+1} r_i \to \int_C du\, e^{-u/\alpha_s} \sum_i^{\infty} \frac{r_i u^i}{i!}$$

Seems impossible to know high order terms?

But we know the divergent part

  - Neutralize color charge of the heavy quark
    - Non-perturbative determination of $\delta m(a)$
    - Depending on how to choose fitting parameters

Applicable to lattice data

  - Truncate at low order?

$$\tilde{h}^R(z, P_z) \to \tilde{h}^R(z, P_z, \tau)$$
$$C(x, y, \mu, P_z) \to C(x, y, \mu, P_z, \tau)$$
$$\mathcal{O}(|z|\Lambda_{\rm QCD}) \to m_0(\tau)|z|$$

Ambiguity is fixed to linear correction

Achieve Power Accuracy: Basic Idea

- OPE with twist-three accuracy

Matching Coefficients

$$h^R(z, P_z, \mu, \tau) = \Big(1 - m_0(\tau)z\Big) \sum_{k=0}^{\infty} C_k\left(\alpha_s(\mu), \mu^2 z^2\right) \lambda^k a_{k+1}(\mu) + \mathcal{O}(z^2)$$

$\sim\ e^{\delta m(a)\cdot z} h^B(z)$

PDF moments

$$= \sum_{k=0}^{\infty} \Big[C_k\left(\alpha_s(\mu), \mu^2 z^2\right) - z m_0(\tau)\Big] \lambda^k a_{k+1}(\mu) + \mathcal{O}(z\alpha_s, z^2),$$

- Twist-3 ambiguities regularized on both sides, h^R and C_k
- $m_0(\tau)$ matches schemes between renormalization of lattice data and regularization of the matching coefficients

Achieve Power Accuracy: Strategy

$$\tilde{f}(x, P_z) = C(x, y, \mu, P_z) \otimes f(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{x|P_z|}\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

Renormalize with scheme-dependent non-perturbative parameter $m_0(\tau)$

$Z(a, \tau) \propto e^{(\delta m(a) + m_0(\tau)) \cdot |z|}$

+

Define τ scheme: Leading renormalon resummation

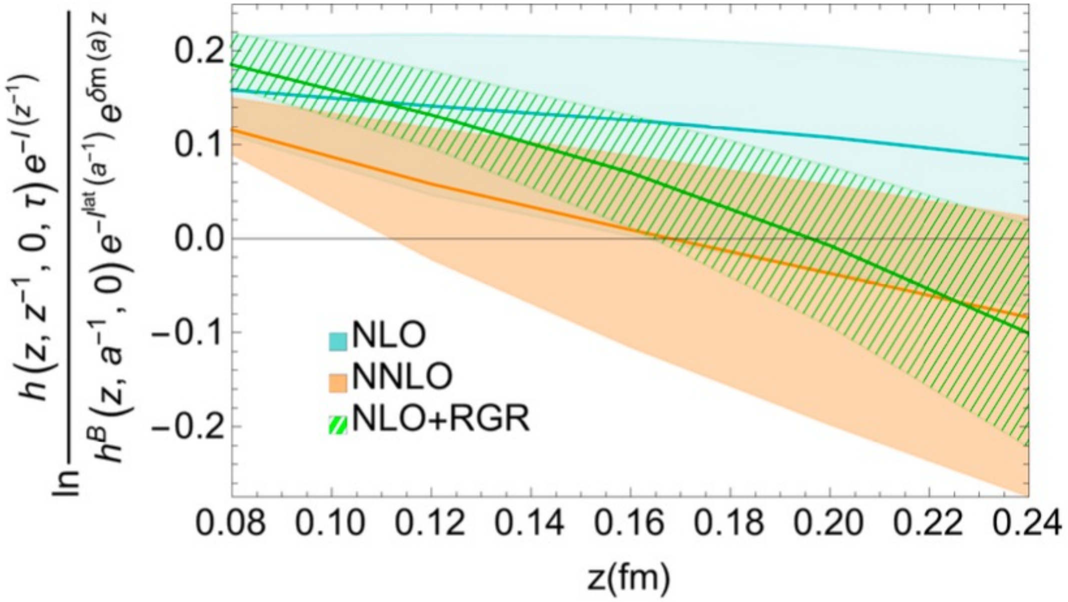
Matching Condition

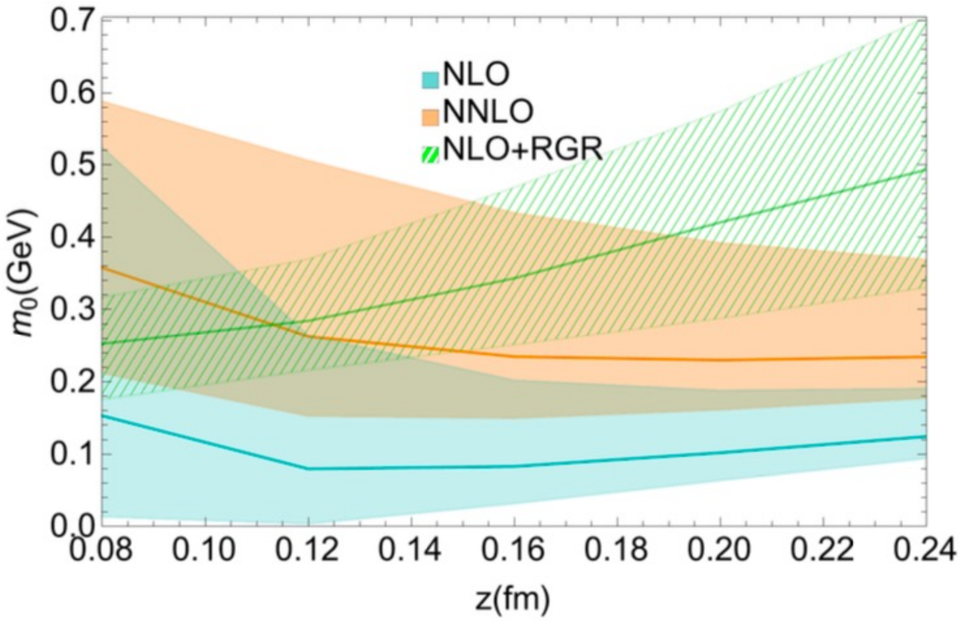
Extract $m_0(\tau)$: Fitting to $P_z = 0$ Lattce data

Extract $m_0(\tau)$ from fixed-order pert theory

$$\ln\left(\frac{h^R(z, P_z = 0, \mu)}{C_0(z, \mu^2 z^2)}\right) = c + m_0(\tau) z$$

Too large uncertainty!



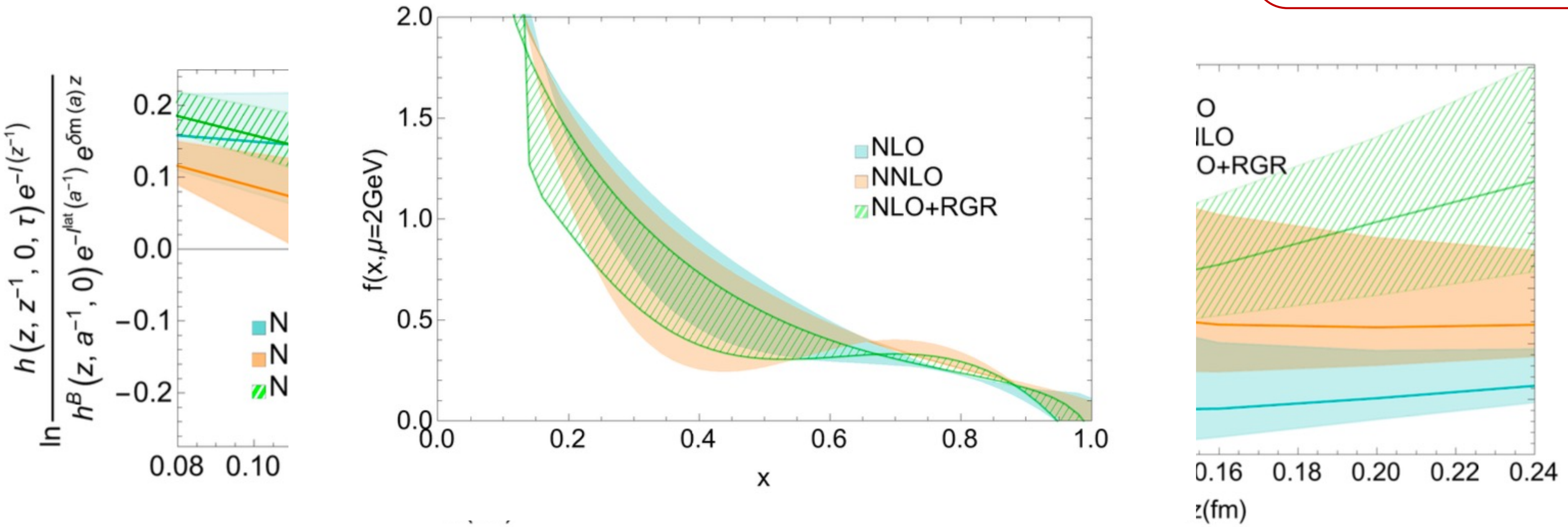


Fixed-order truncation is not a good regularization method!

Extract $m_0(\tau)$ from fixed-order pert theory

$$\ln\left(\frac{h^R(z, P_z = 0, \mu)}{C_0(z, \mu^2 z^2)}\right) = c + m_0(\tau) z$$

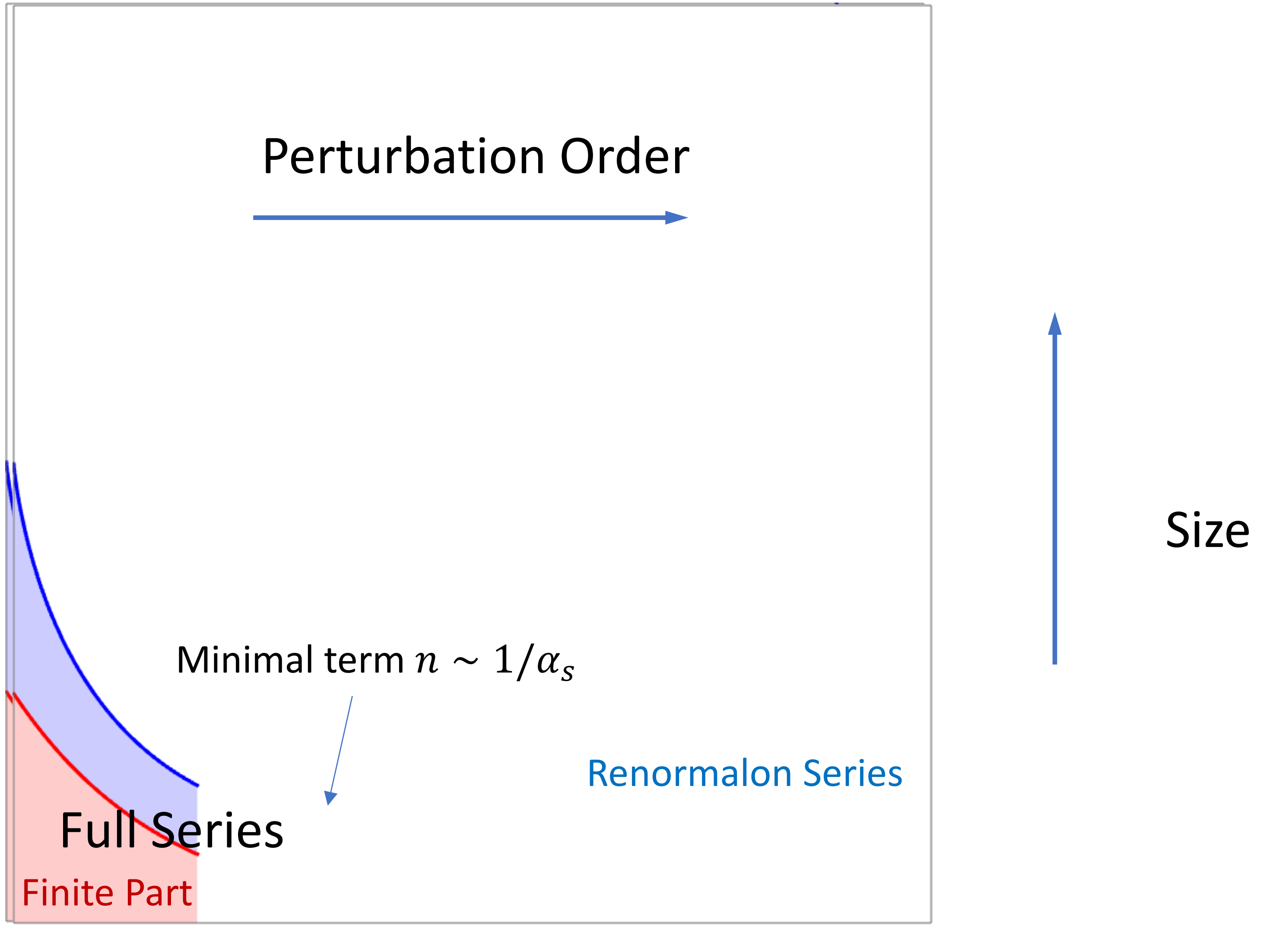
Too large uncertainty!



Fixed-order truncation is not a good regularization method!

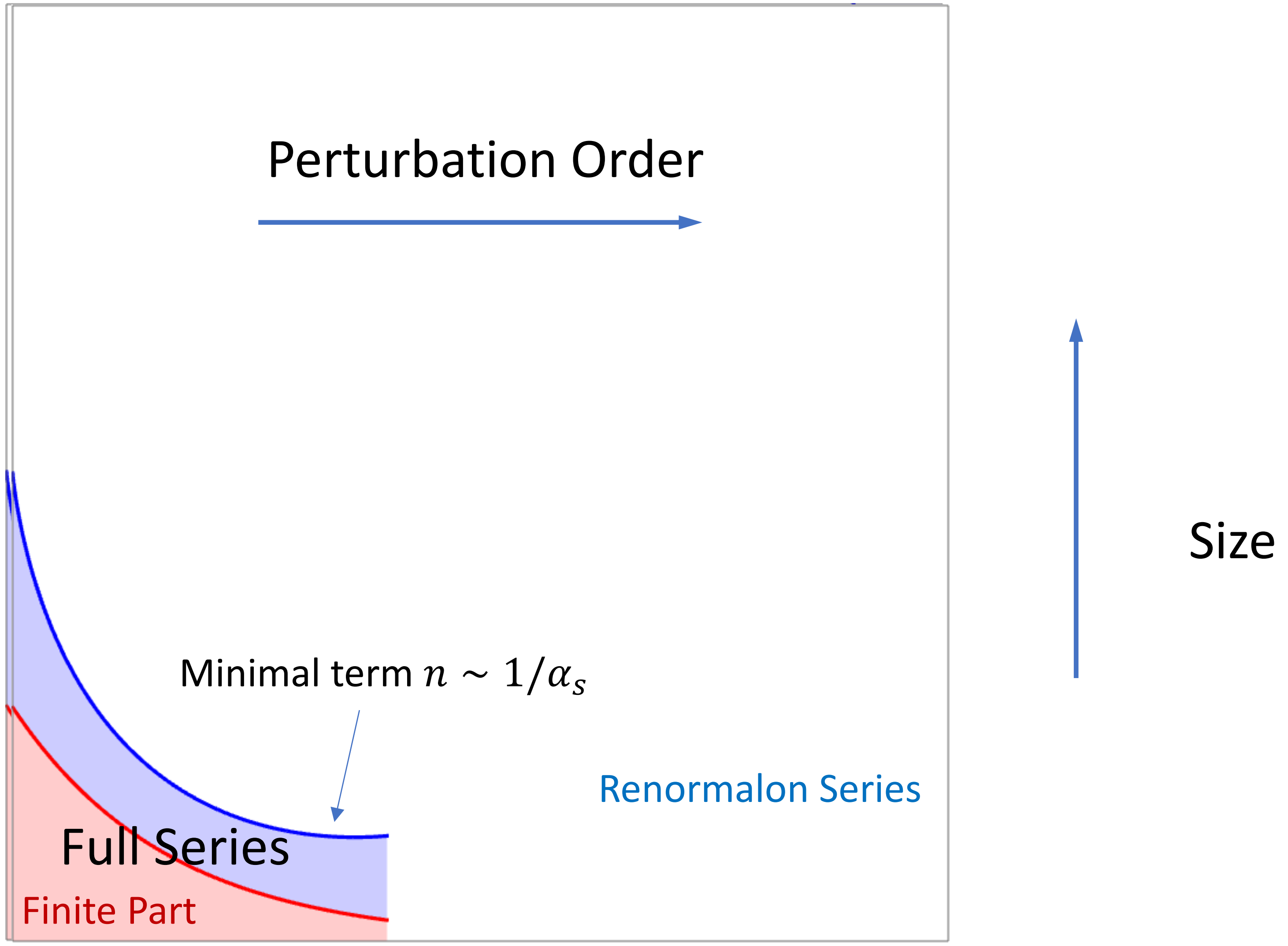
Fixed-order Truncation

Results changes dramatically when including higher order terms



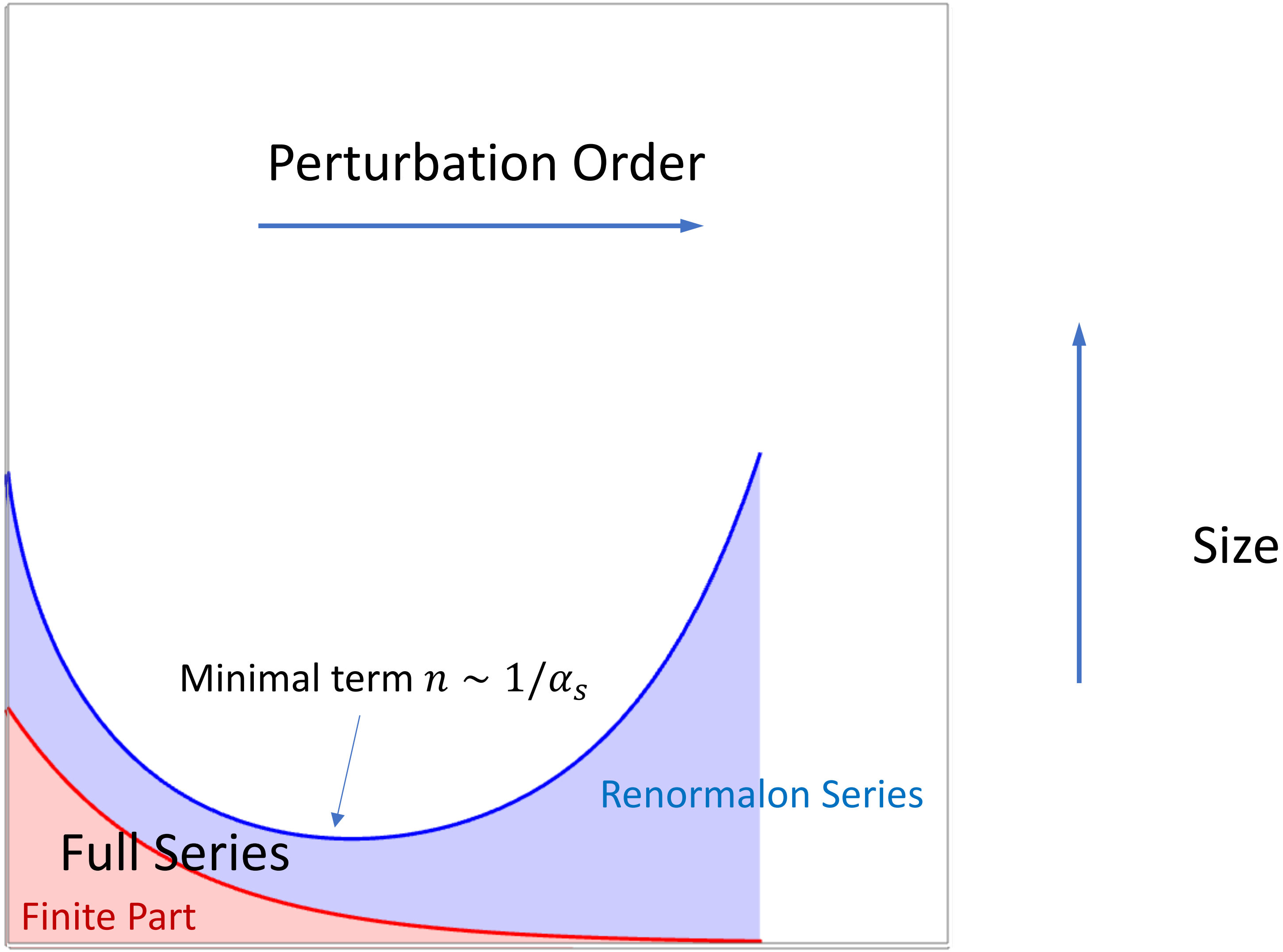
Fixed-order Truncation

Results changes dramatically when including higher order terms



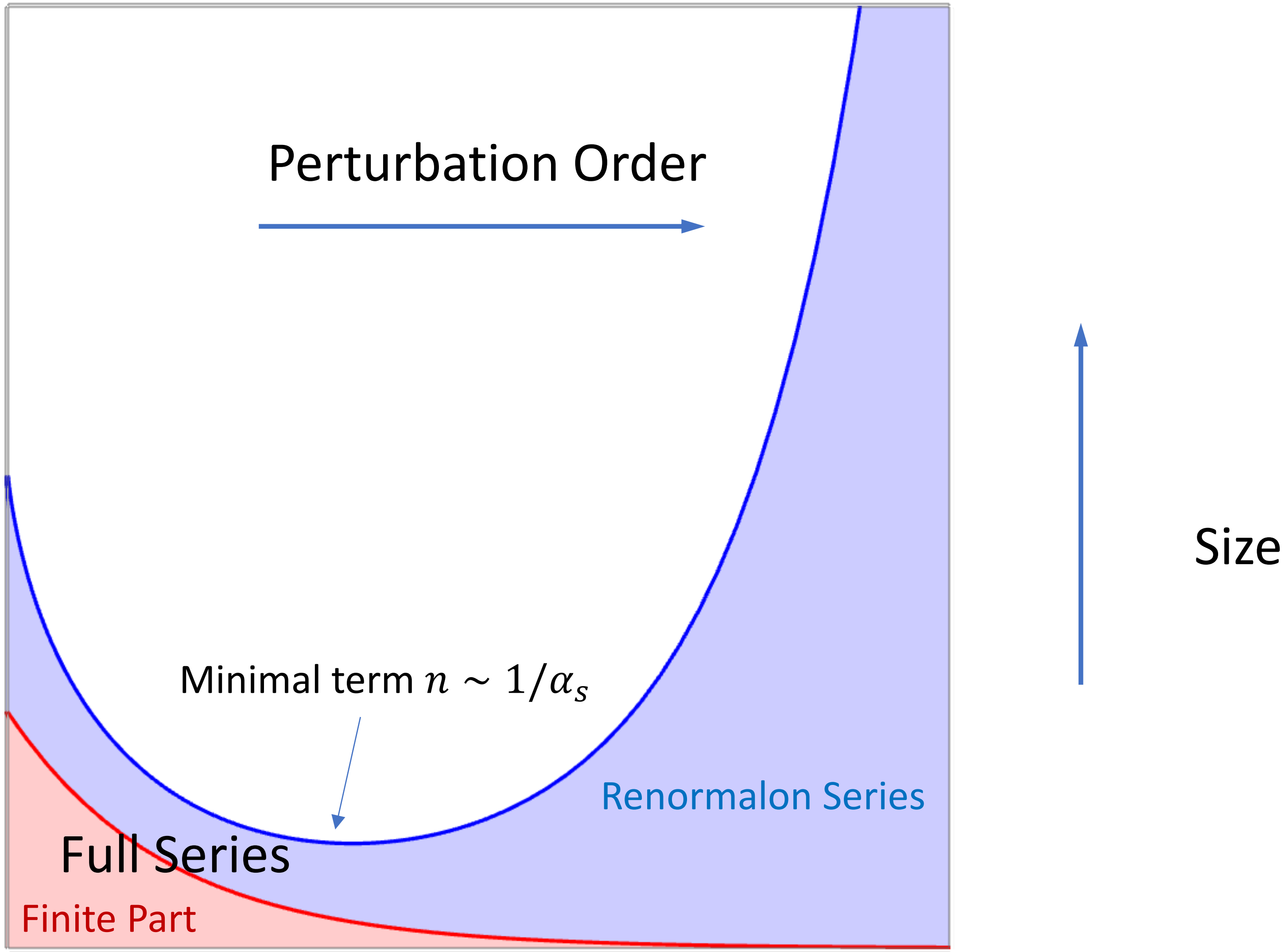
Fixed-order Truncation

Results changes dramatically when including higher order terms



Fixed-order Truncation

Results changes dramatically when including higher order terms

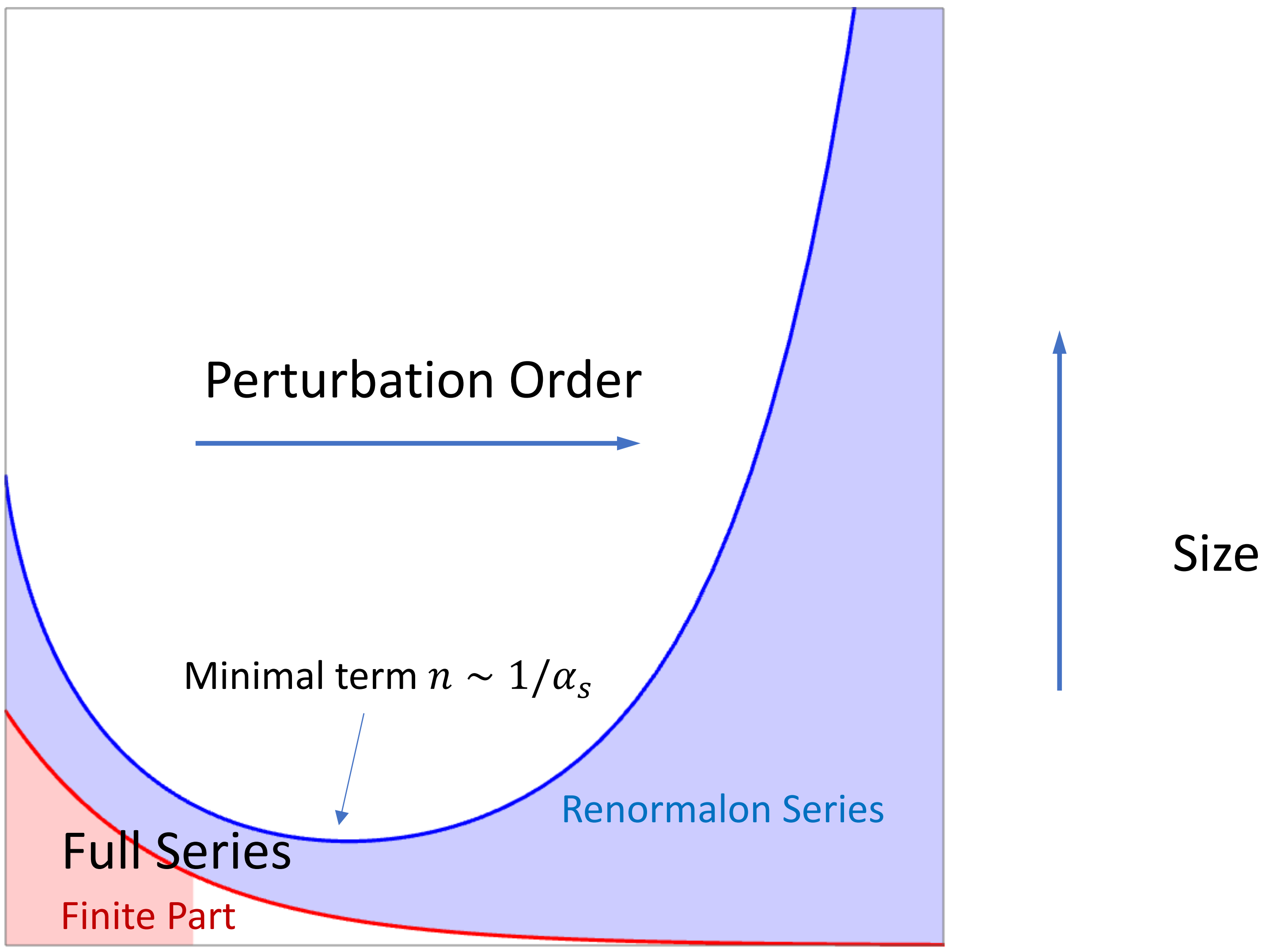


Leading Renormalon Resummation

LRR resums the factorially growing part.

The remaining part is convergent.

The scheme choice is invariant under scale variation.

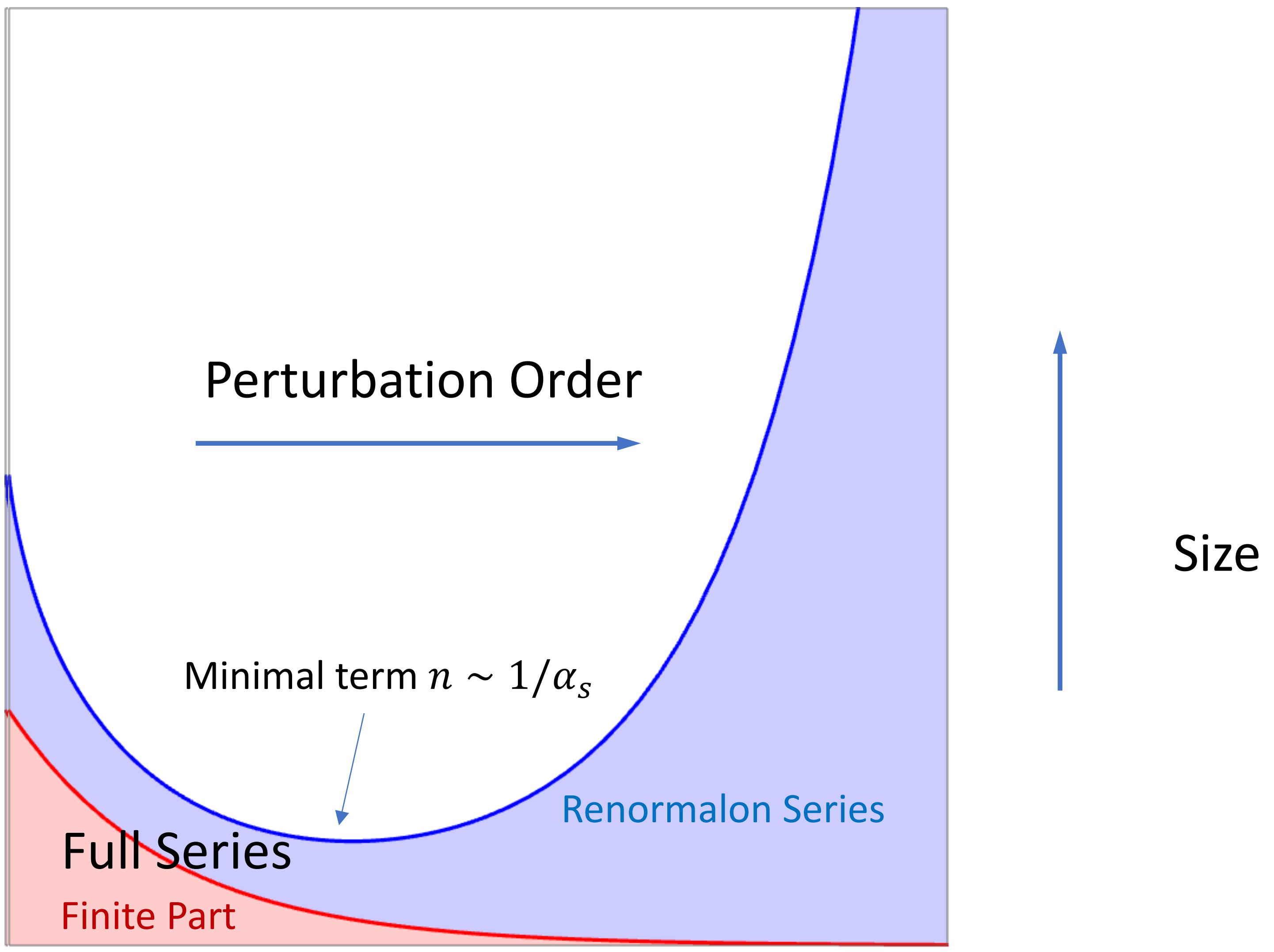


Leading Renormalon Resummation

LRR resums the factorially growing part.

The remaining part is convergent.

The scheme choice is invariant under scale variation.



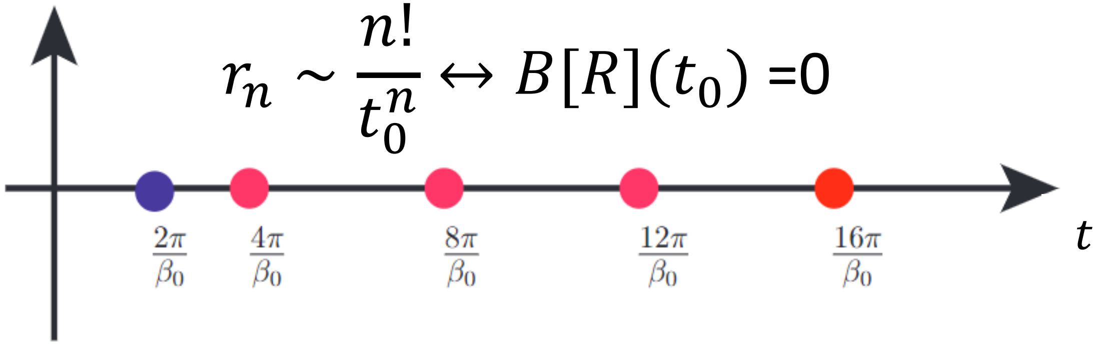
How to resum the renormalon series?

- Borel transformation: $$R = \sum_{n=0} r_n \, \alpha_s^{n+1}$$

$$B[R](t) = \sum_n \frac{r_n}{n!} t^n , \qquad \tilde{R} = \int_0^{+\infty} dt \, e^{-\frac{t}{\alpha_s}} B[R](t)$$

Divergent Series ↔ Poles in the Borel Plane

$$r_n \sim \frac{n!}{t_0^n} \leftrightarrow B[R](t_0) = 0$$



Beneke, RMP (1998)

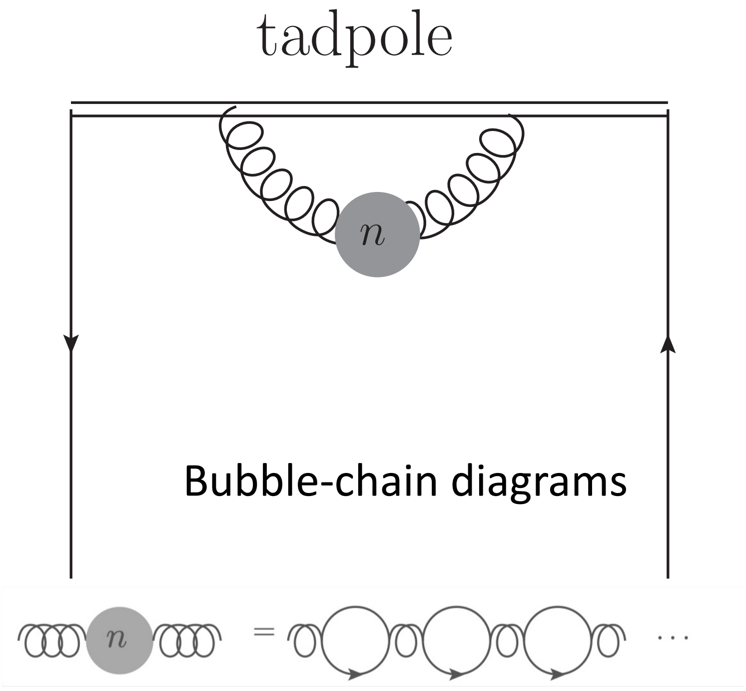
Braun, et.al., PRD(2018)

$\tilde{R}$ depends on the integral path (regularization schemes)

Large β_0 approximation

- The only calculable diagrams to infinite order

$$C_{\rm tp}(\alpha_s(\mu), z^2\mu^2)|_{\rm PV} = \int_0^\infty du e^{-4\pi u/\alpha(\mu)\beta_0} \frac{2C_F}{\beta_0 u}\left(\frac{\Gamma(1-u)e^{\frac{5}{3}u}(z^2\mu^2/4)^u}{(1-2u)\Gamma(1+u)} - 1\right)\Bigg|_{\rm PV}$$



- Not including all leading renormalon effects

- Introducing higher renormalons (higher power corrections)

Beyond β_0 approximation

- Leading renormalon series follow certain properties:
  - The divergent rate is determined by the pole in Borel plane $r_n \sim \left(\frac{\beta_0}{2\pi}\right)^n n!$
  - The IR renormalon series is independent of UV renormalization
- We can infer the asymptotic form:
  - When $n \to \infty$, invariant under the change of renormalization scheme/scale $\alpha_s \to \alpha_s', \beta \to \beta'$

Beneke, PLB (1995)

$$r_n = N_m \mu \left(\frac{\beta_0}{2\pi}\right)^n \frac{\Gamma[n+1+b]}{\Gamma[1+b]}\left(1 + \frac{b}{n+b} c_1 + \frac{b(b-1)}{(n+b)(n+b-1)} c_2 + \cdots\right)$$

$b = \frac{\beta_1}{2\beta_0^2}, c_1 = \frac{1}{4 b \beta_0^3}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right)$ all from β functions

Determined from known series with the same renormalon at high orders

Pineda, JHEP (2001)
Bali, et.al., PRD (2013)

LRR Beyond β_0 approximation

- Resumming the asymptotic series:

$$C_k(\alpha_s(1/z),1)_{\rm PV} = N_m \frac{4\pi}{\beta_0}\int_{0,{\rm PV}}^{\infty} du e^{-\frac{4\pi u}{\alpha_s(1/z)\beta_0}} \frac{1}{(1-2u)^{1+b_0}}(1+c_1(1-2u)+...),$$

- Correcting the perturbative results:

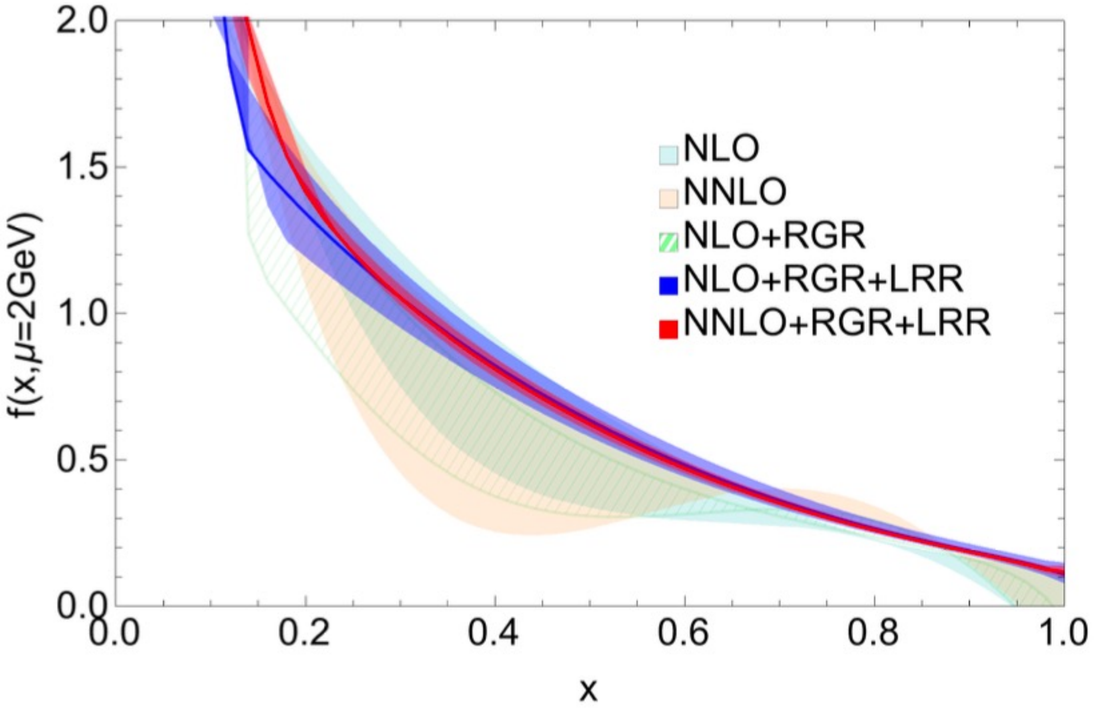
$$C^{\rm LRR}(\alpha_s(z^{-1}),1) = C_k(\alpha_s(z^{-1}),1) + \left[C_k(\alpha_s(z^{-1}),1)_{\rm PV} - \sum_i \alpha_s^{i+1}(z^{-1}) r_i\right].$$

- Correcting the matching kernel:

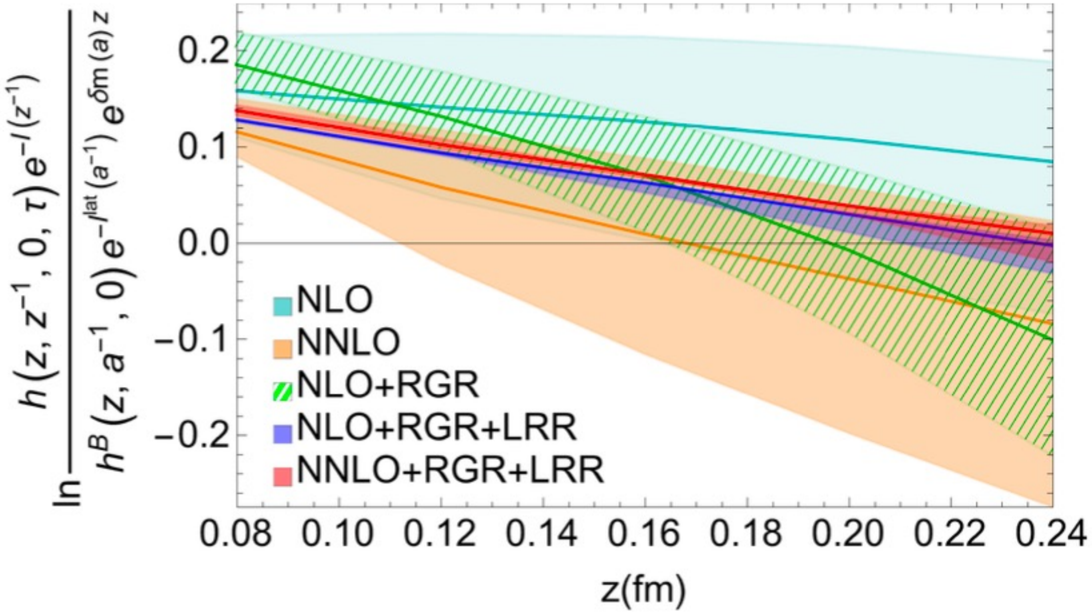
$$\Delta C^{\rm LRR}(x,y,\mu,P_z) = \int \frac{p_z dz}{2\pi} e^{i(x-y)zP_z}\left[\mu z C_k(\alpha_s(\mu), z^2\mu^2)_{\rm PV} - \sum_i \mu z \alpha_s^{i+1} r_i(\mu)\right].$$

LRR improved perturbation theory

$C_0(z, \mu^2 z^2)$:



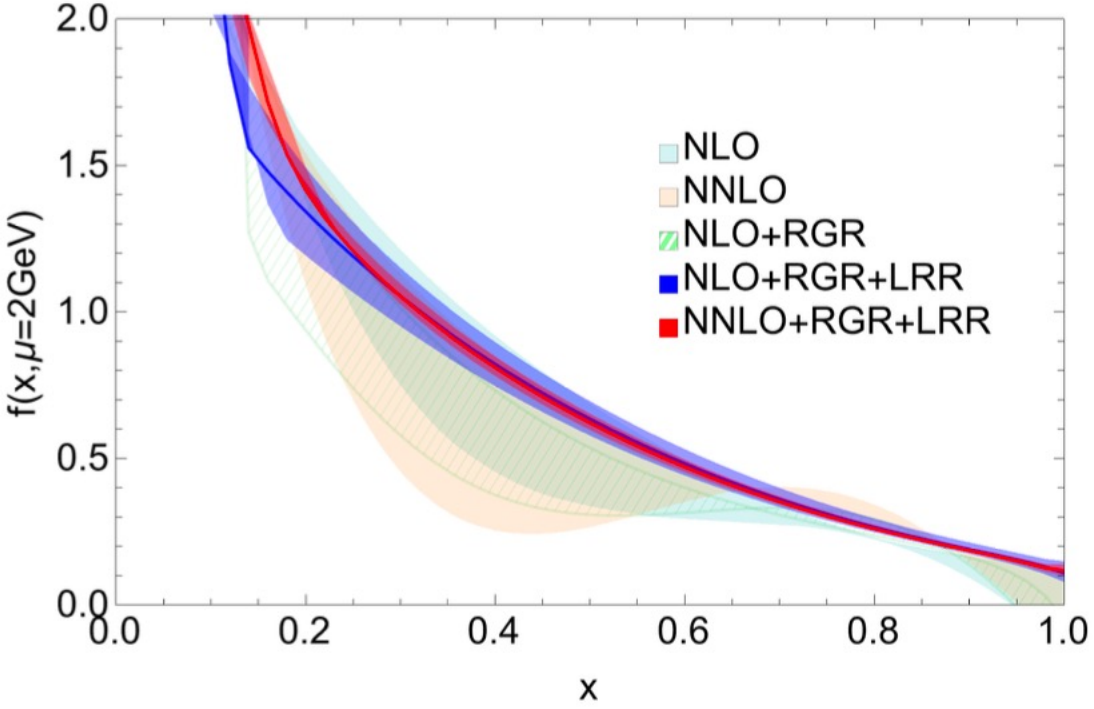
$\ln\left(\frac{h^R(z, P_z=0, z^{-1})}{C_0(z,1)}\right)$:



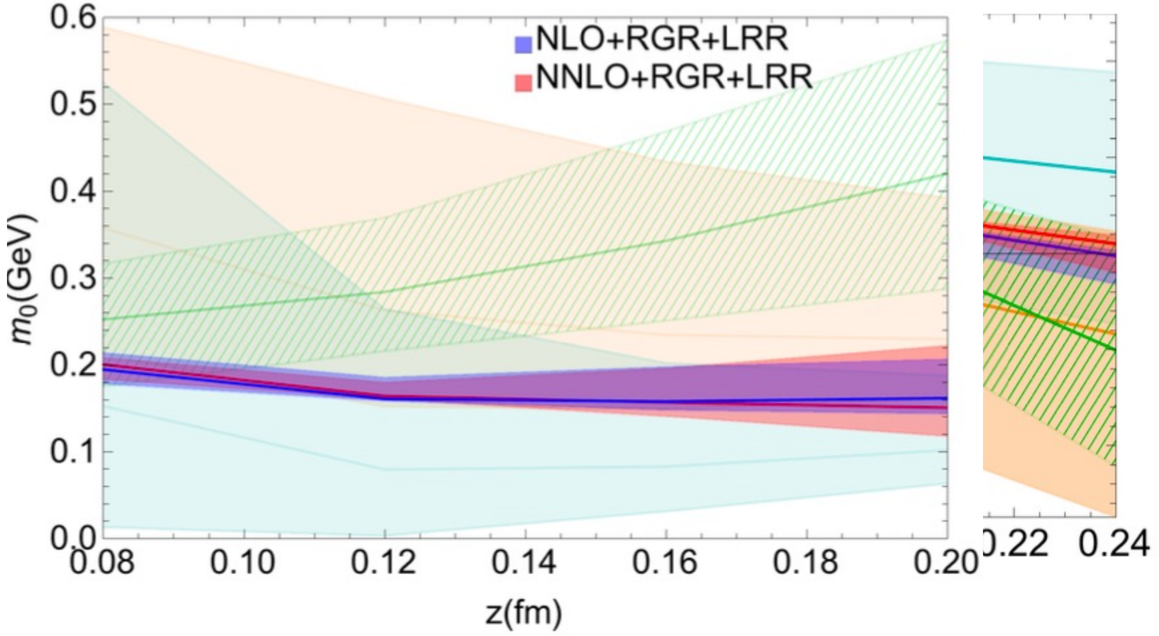
- Reduce the uncertainty 3~5 times from scale variation
- Improve the convergence when going to higher order

LRR improved perturbation theory

$C_0(z, \mu^2 z^2)$:



$\ln\left(\frac{h^R(z, P_z=0, z^{-1})}{C_0(z,1)}\right)$:



- Reduce the uncertainty 3~5 times from scale variation
- Improve the convergence when going to higher order

Summary

- Parton physics can be calculated from lattice QCD through large momentum expansion precisely
- Power correction is an important source of systematic uncertainty
- We propose the first systematic approach to achieve $1/P_z$ accuracy
- The leading renormalon resummation reduces the scale variation and the convergence of perturbation theory.

Outlook

- More solid determination of the renormalon contribution
- Generalization to more complicated parton observables (polarized, GPD)
- Including more systematic uncertainties

Thank you!