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Lattice QCD calculation of the unpolarized transverse-momentum-dependent parton distributions

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Lattice Parton Collaboration

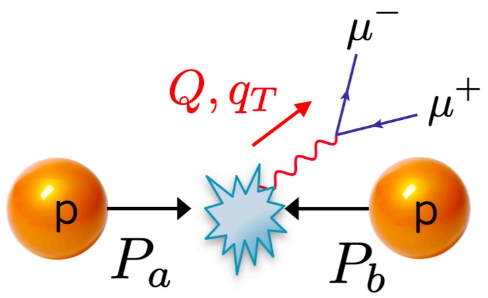
Based on *hep-lat/2211.02340*

OUTLOOK

- ➢ Motivation and recent progresses
- ➢ Lattice QCD calculation of unpolarized TMDPDF
  - Extract TMDPDF from LaMET
  - Quasi TMDPDF matrix elements and their renomalization
  - From Quasi TMDPDF to physical TMDPDF
  - Numerical results and discussion
- ➢ Summary and outlook

TMD processes:

Drell-Yan

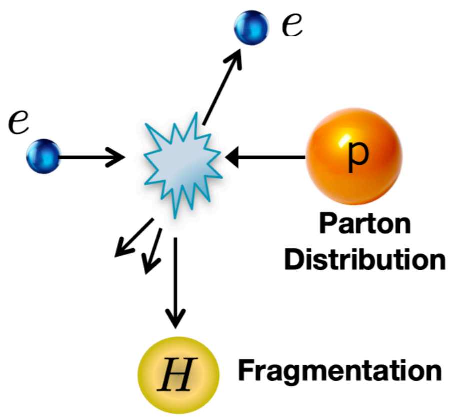


$q_T \ll Q$

LHC, FermiLab, RHIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

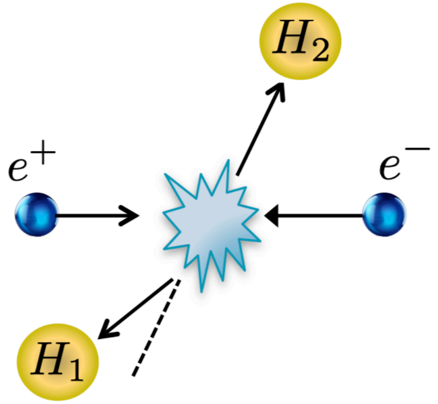
Semi-Inclusive DIS



HERMES, COMPASS, JLab, EIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

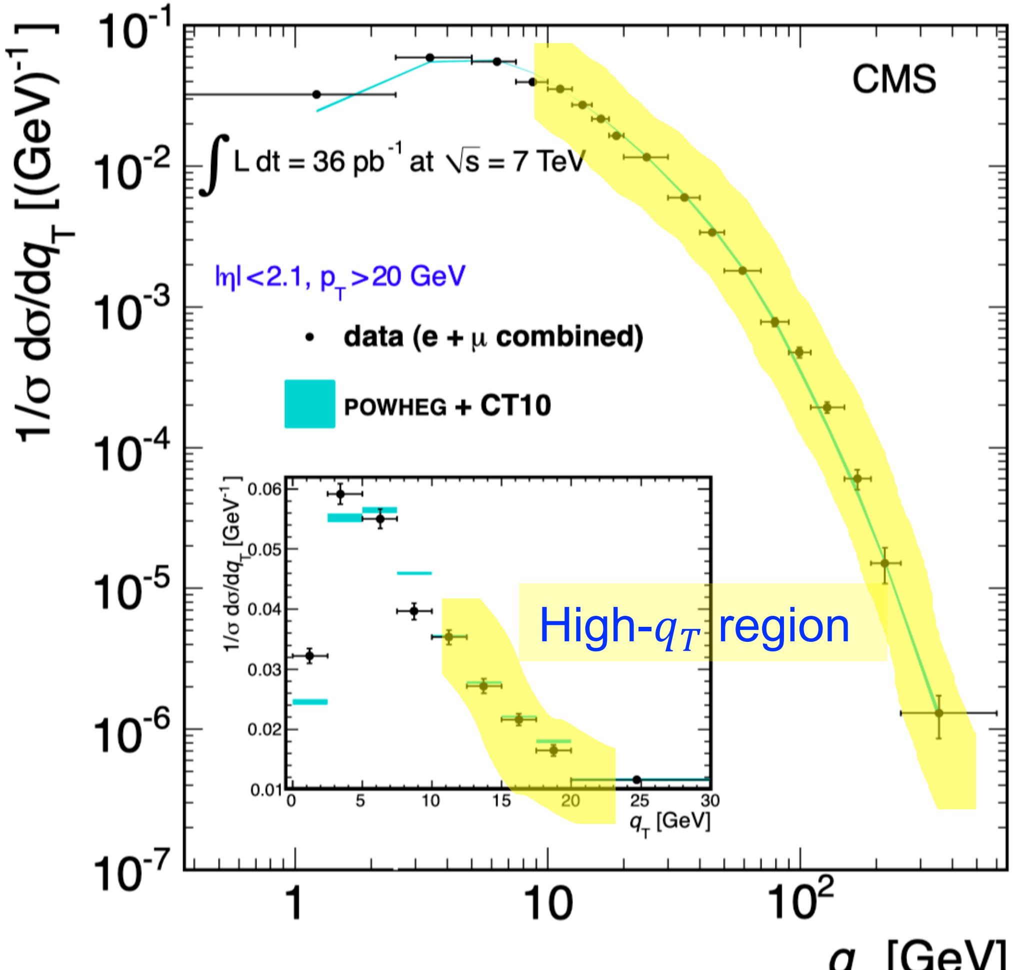
Dihadron in e^+e^-



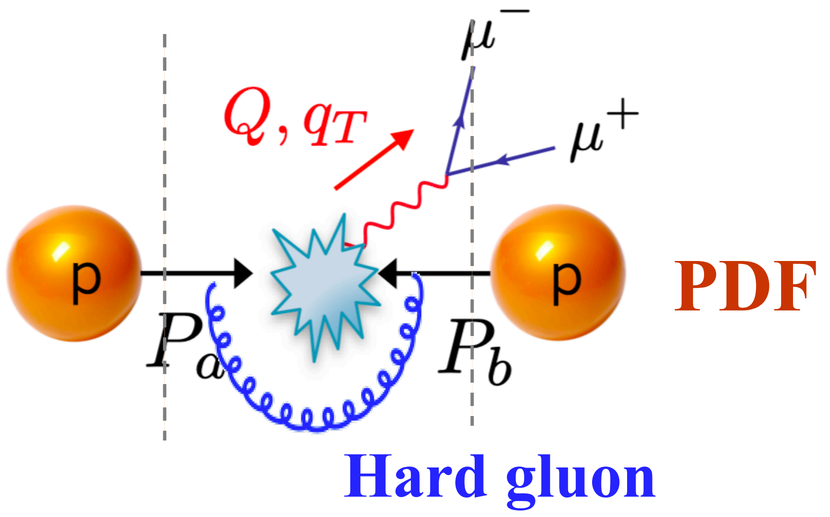
BESIII, Babar, Belle, ...

$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

Z-production q_T spectrum at LHC



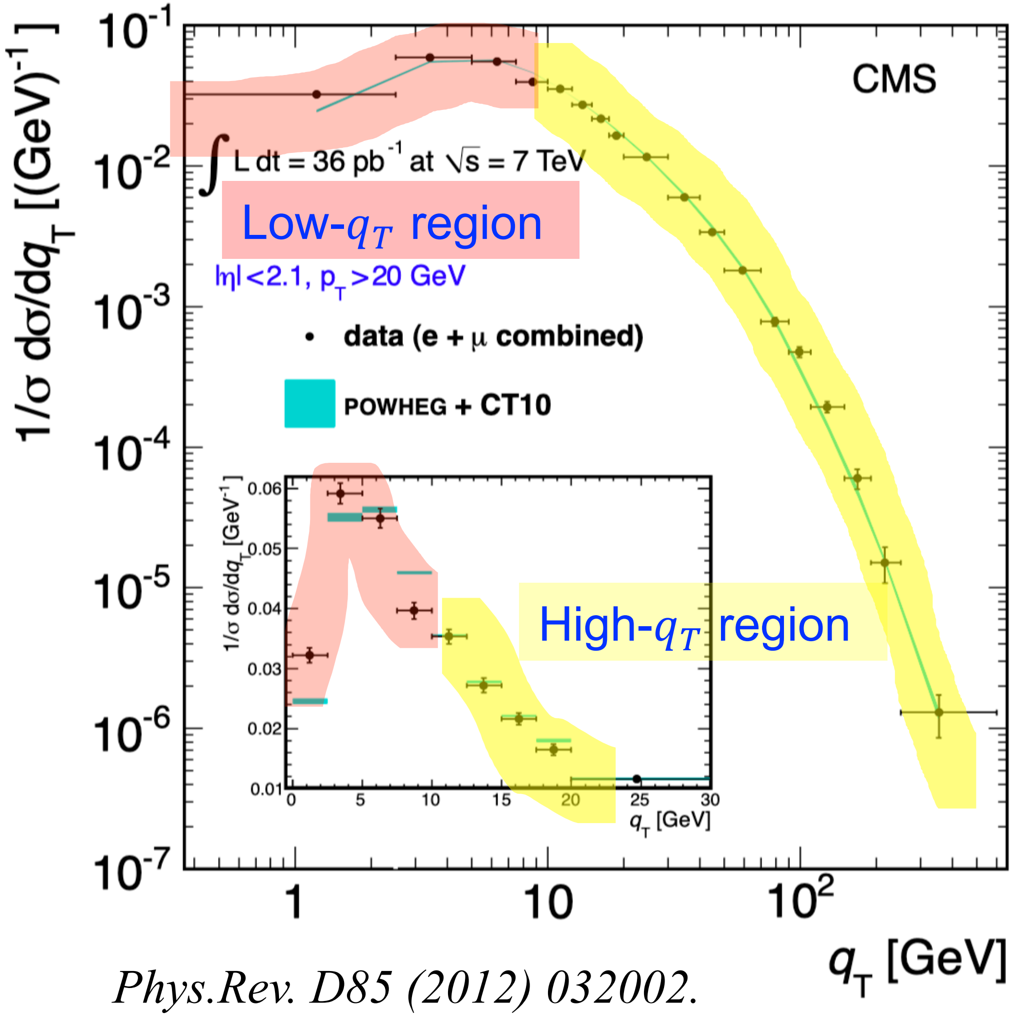
Phys.Rev. D85 (2012) 032002.



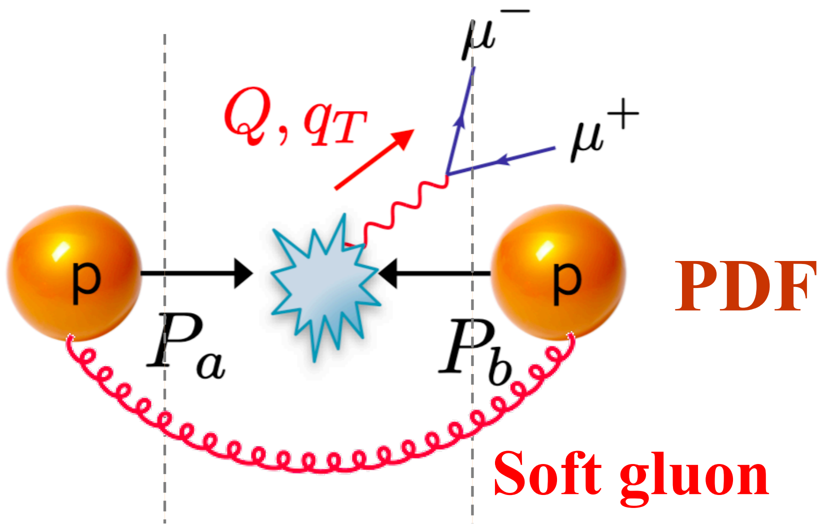
➢ **High-q_T region:**

Collinear factorization => PDF

Z-production q_T spectrum at LHC



Phys.Rev. D85 (2012) 032002.



- **High-q_T region:**

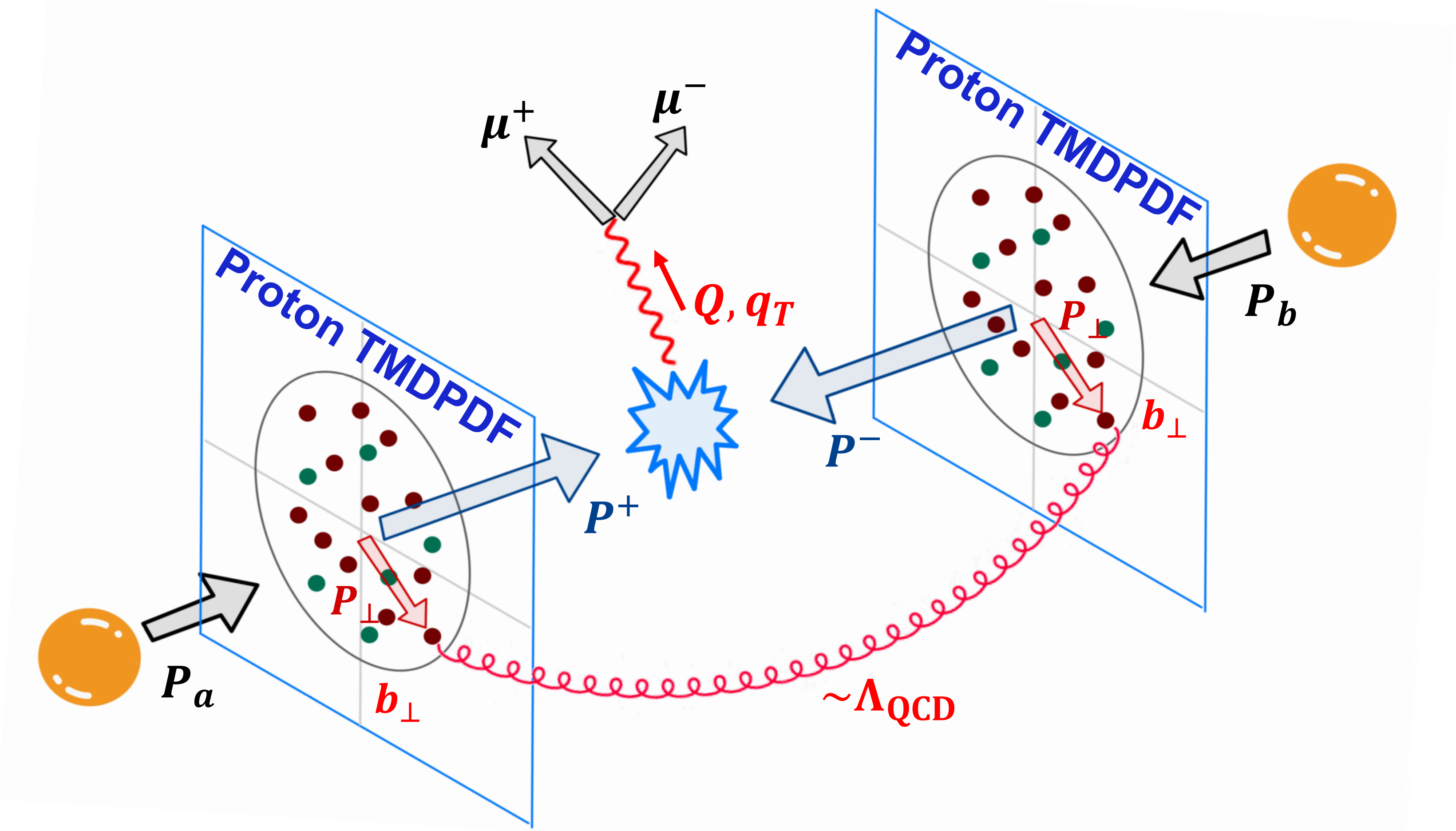
  Collinear factorization => PDF

- **Low-q_T region:**

  TMD factorization
  => Generalize to TMDPDFs

TMDPDFs: 3D tomography of the nucleon

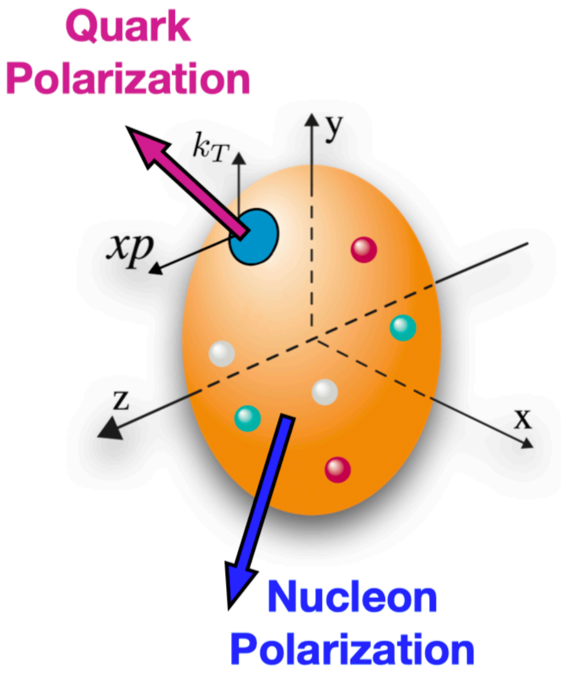
- **Low-q_T region of Drell-Yan Process:**



Revealing the confined motion of partons inside the nucleon

EPJA52, 268 (2016)

TMDPDFs: 3D tomography of the nucleon



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_{1L} = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

TMD Handbook, TMD Collaboration, 2304.03302

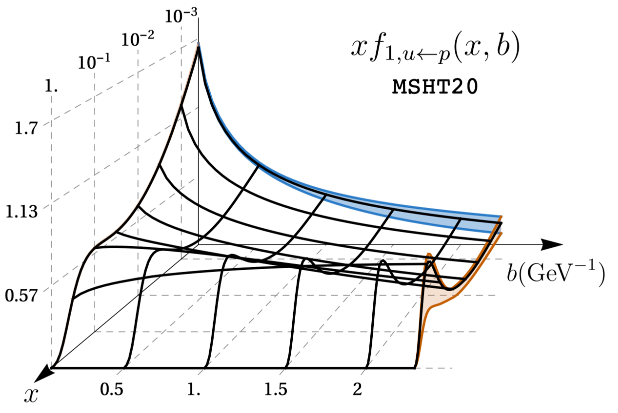
Progress in the study of TMDs

➢ **Theoretical analysis**

- **TMD factorization, evolution and resummation:**

  Boussarie et al., TMD handbook, 2304.03302;

  Collins, Foundations of perturbative QCD; ……



u-quark unpolarized TMDPDF, 2201.07114

➢ **Phenomenological parametrizations and extractions**

- **Unpolarized:**

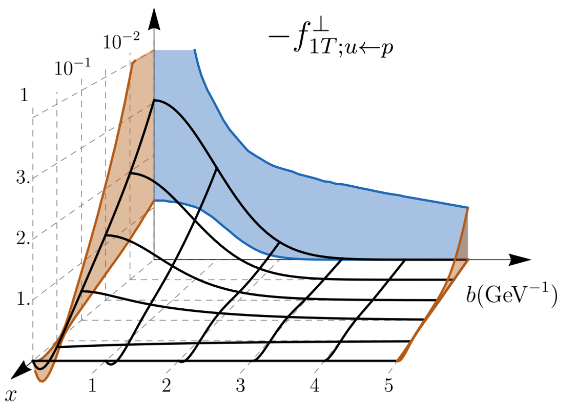
  Moos, 2305.07473; Bacchetta, 2206.07598; Bury, 2201.07114;

  Scimem, JHEP06 (2020); Bacchetta, JHEP06 (2017); ……

- **Sivers, Boer-Mulders:**

  Bury, PRL126 (2021), JHEP05 (2021) ; Cammarota, PRD102(2020);

  Zhang, PRD77 (2008), Lu, PRD81 (2010) ; ……



u-quark Sivers function, PRL126 (2021)

➢ Lattice calculations

- **Lorentz-invariant approach:** ratios of Mellin moments

  Hagler, EPL88(2009); Musch, PRD85(2012); Engelhardt, PRD93(2016); Yoon, 1601.05717, PRD96(2017); ……

- **LaMET formalism:**

  ✓ **I: theoretical analysis of matching kernel, soft function, Collins-Soper kernel, ……**

  Rio, 2304.14440; Ji, 2305.04416, RMP93(2021), NPB955(2020), PLB811(2020);

  Ebert, JHEP04(2022); Deng, JHEP09(2022)……

  ✓ **II: lattice calculation of intrinsic soft function, Collins-Soper kernel, beam function, ...**

  LPC, 2306.06488, PRL125(2020); Li, PRL128(2022); LPC, PRD106(2022); Shanahan,

  PRD104(2021); Schlemmer, JHEP08(2021); ……

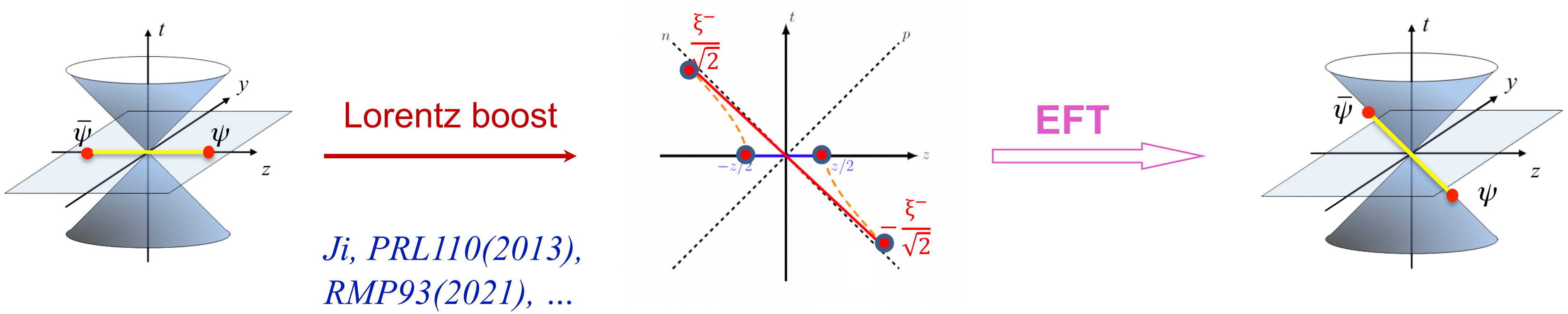
  ✓ **III: Nonperturbative renormalization, resummation, ……**

  Zhang, 2305.05212; Ji, 2305.04416; Su, NPB991(2023); LPC, PRL129(2022); 2209.01236……

  ➢ **IV: A real lattice calculation of TMD observable?**

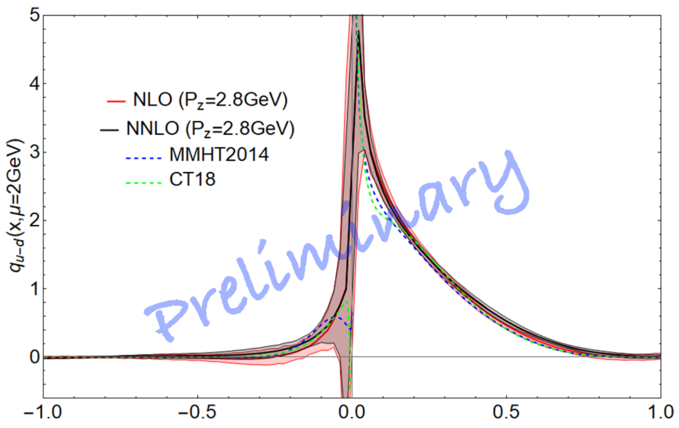
Extracting TMDs in LaMET formalism

- **Large-momentum effective theory: connecting Euclidean lattice and physical observables**

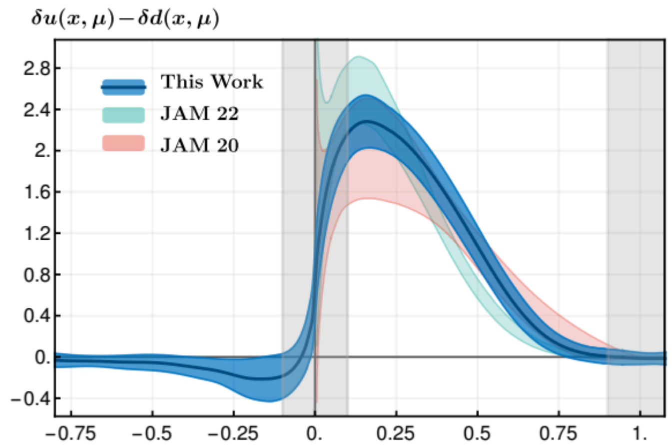


Ji, PRL110(2013), RMP93(2021), …

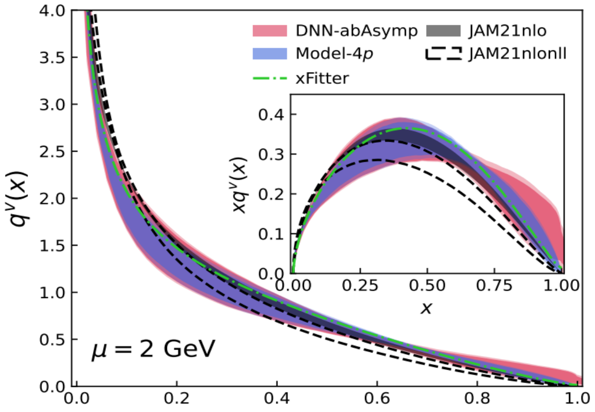
- **Achieved great success in the studies of PDF:**



Proton unpolarized PDF, in preparation

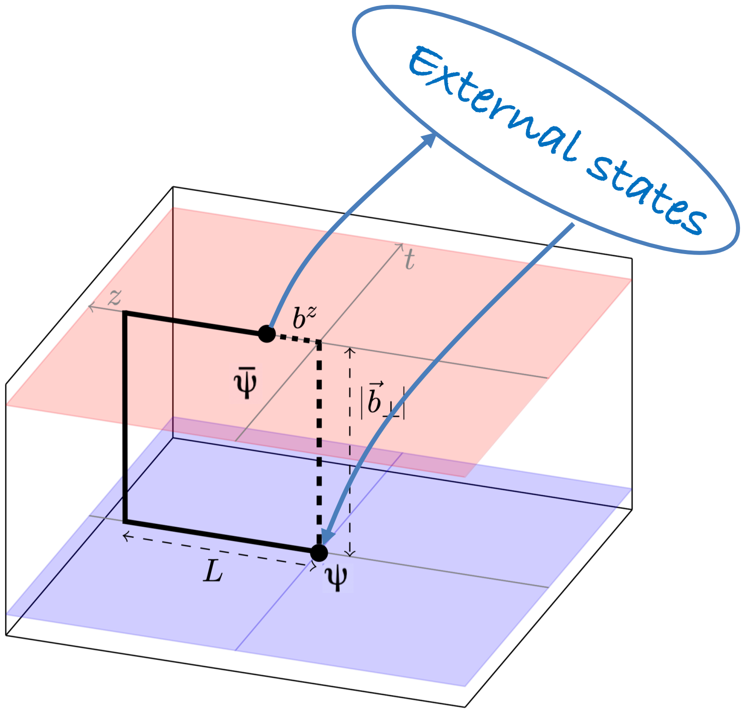


Proton transversity PDF, 2208.08008



Pion valance PDF, PRD106(2022)

- **Matching from quasi TMDs to TMDs**



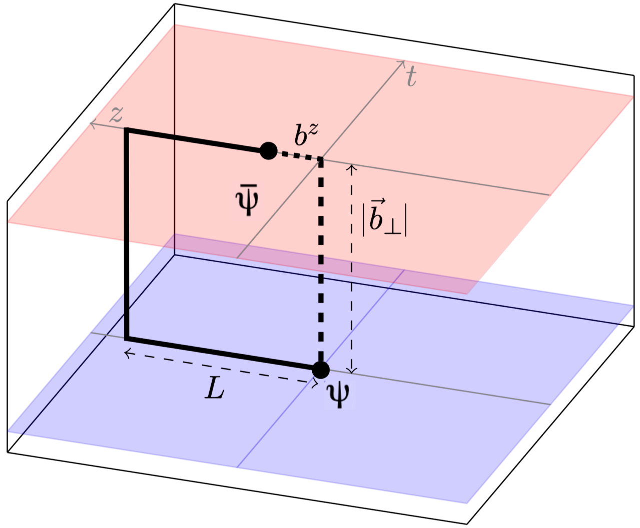
Equal-time correlators
with staple-shaped Wilson link,
directly calculable on lattice

- Hadronic matrix element reduced from equal-time correlators:

$$\tilde{h}^0_\Gamma(z, b_\perp, P^z) = \lim_{L\to\infty} \left\langle P^z \middle| \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma \right.$$
$$\times\, U_\sqsubset(b_\perp \hat{n}_\perp \leftarrow b_\perp \hat{n}_\perp + L\hat{n}_z; b_\perp \hat{n}_\perp + L\hat{n}_z \leftarrow L\hat{n}_z; L\hat{n}_z \leftarrow z\hat{n}_z)$$
$$\times\, \left. \psi(z\hat{n}_z) \middle| P^z \right\rangle$$

- Subtracted quasi TMDPDFs:

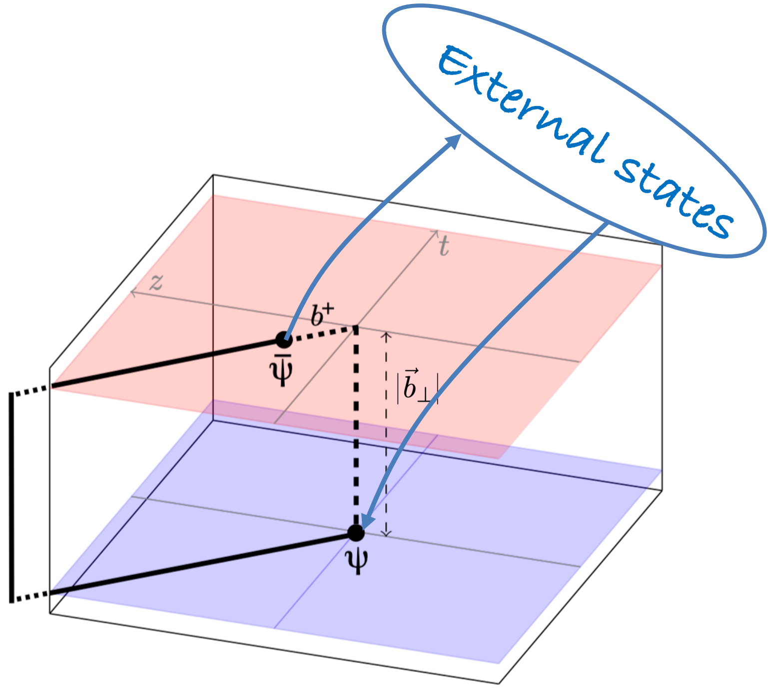
$$\tilde{f}_\Gamma(x, b_\perp, P^z, \mu) \equiv \lim_{\substack{a\to 0\\ L\to\infty}} \int \frac{dz}{2\pi} e^{-iz(xP^z)} \frac{\tilde{h}^0_\Gamma(z, b_\perp, P^z, a, L)}{\sqrt{Z_E(2L+z, b_\perp, a)}Z_O(1/a, \mu, \Gamma)}$$



Equal-time correlators,
directly calculable on lattice

Lorentz boost

$L \to \infty$



Space-like correlators,
NO effective method for directly calculation

Connected at large-momentum limit

Ji, PLB811(2020); Ebert, JHEP04(2022)

Matching kernel

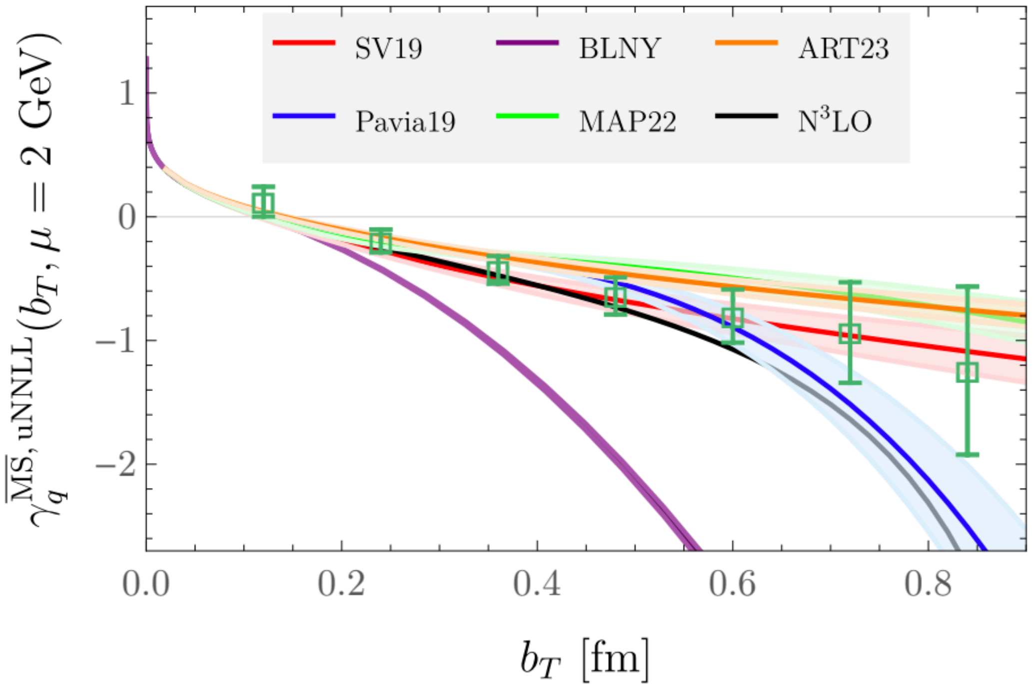
$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Quasi TMDPDF　Intrinsic soft function　Collins-Soper kernel　Light-cone TMDPDF

Collins-Soper kernel

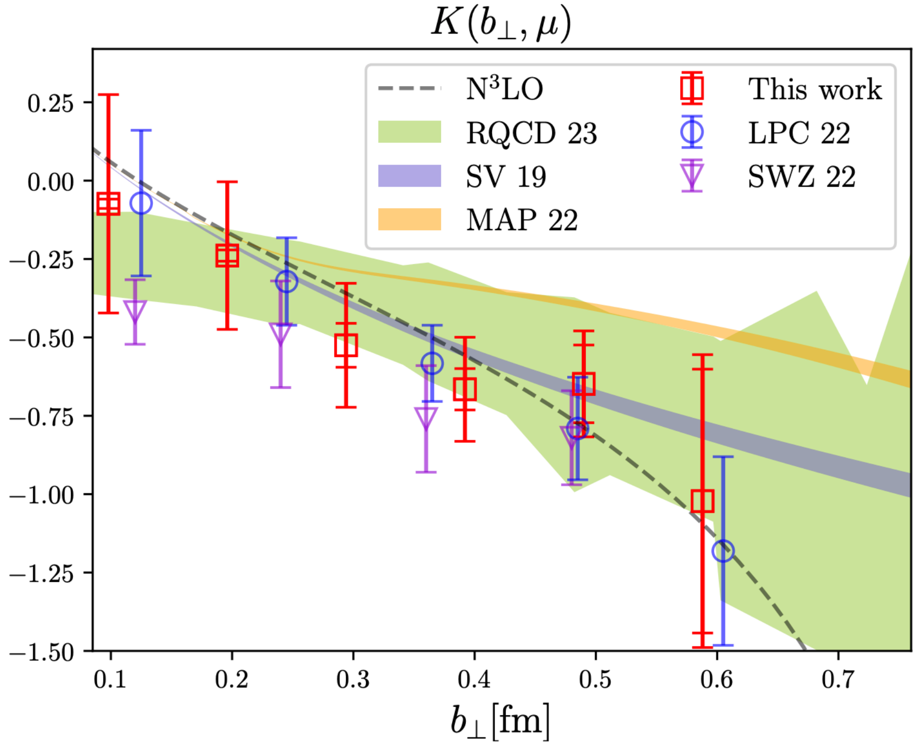
From quasi beam function:

Avkhadiev, 2307.12359; Shanahan, PRD104(2021), PRD102(2020);Schlemmer, JHEP08(2021); ……



From quasi TMDWF:

Chu, 2306.06488, PRD106(2022); Zhang, PRL125(2020); Li, PRL128(2022); ……



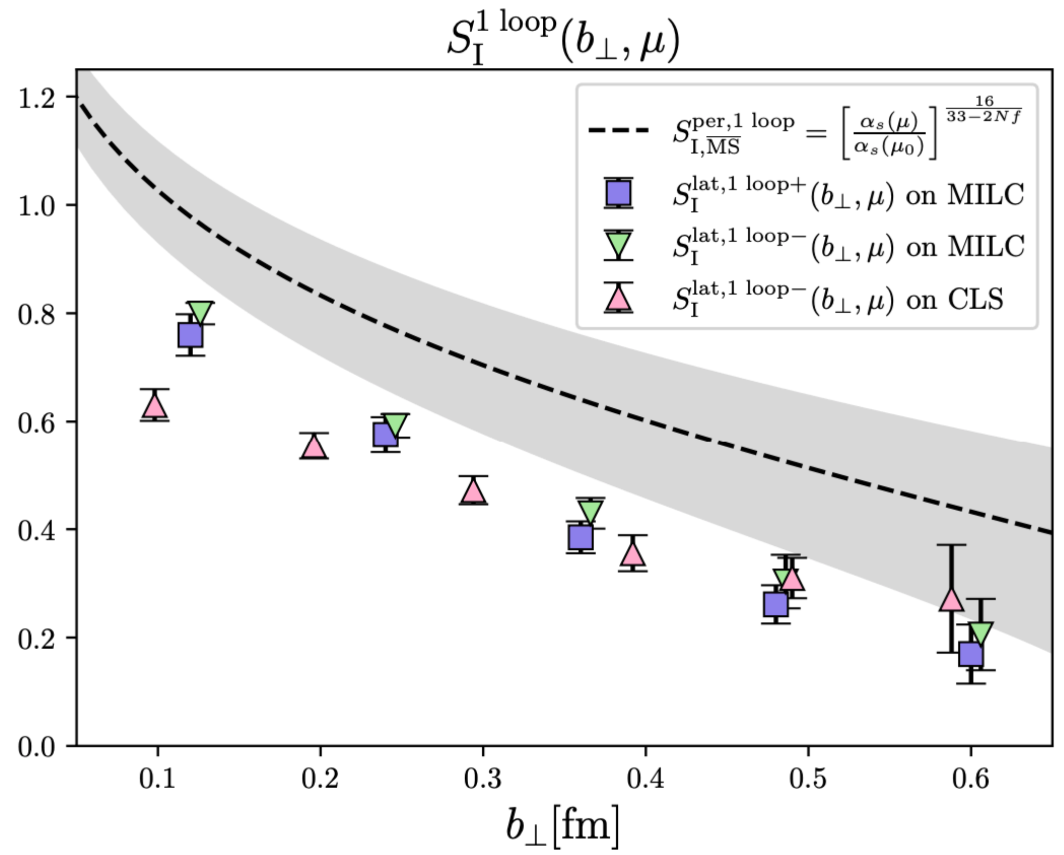
Intrinsic soft function

From quasi TMDWF + 4-quark matrix element:

Chu, 2302.09961; Ji, NPB955(2020);

Zhang, PRL125(2020); Li, PRL128(2022);

……

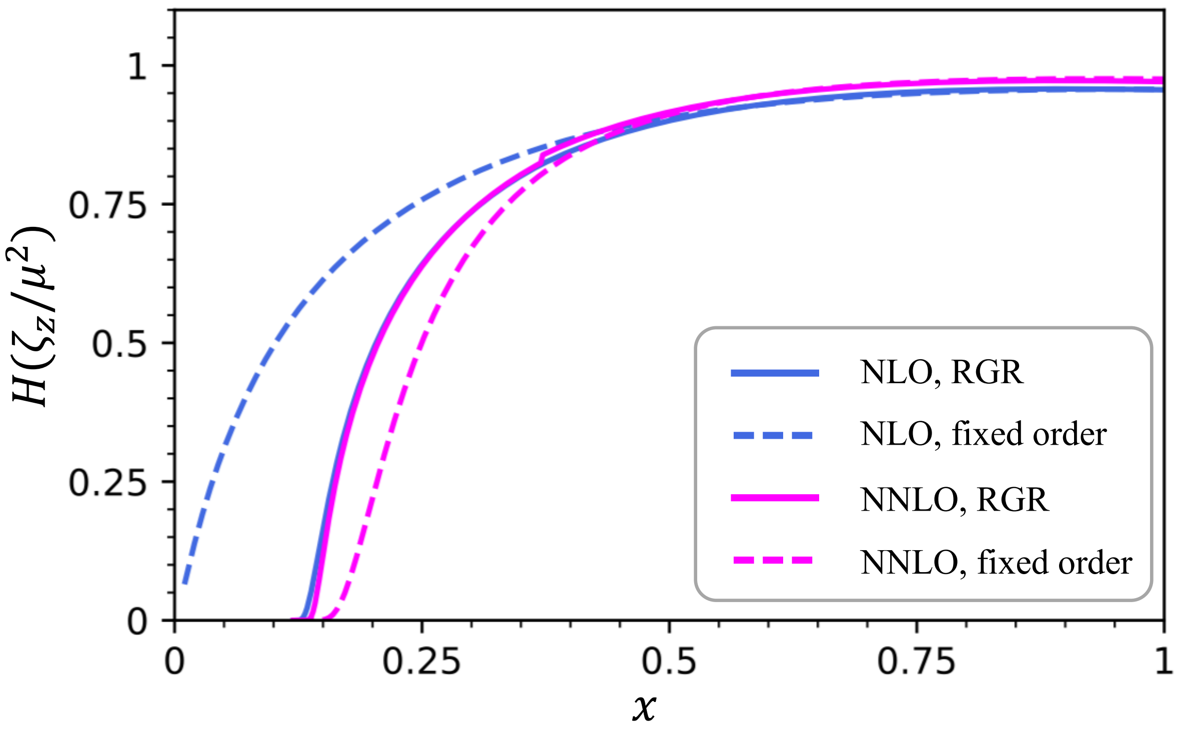


Perturbative matching kernel

- **NLO:** *Ji, PLB811(2020); RMP93(2021)*
- **NNLO:** *Río, 2304.14440; Ji, 2305.04416*

➢ **Fixed order: $\mu = 2\text{GeV}$;**

➢ **RGR: RG evolution from lattice scale $\sqrt{\zeta_z} = 2xP^z$ to $\overline{\text{MS}}$ scale $\mu = 2\text{GeV}$.**



Lattice calculation of physical TMDPDF?

Matching kernel ✓

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Quasi TMDPDF   Intrinsic soft function ✓   Collins-Soper kernel ✓

⇩

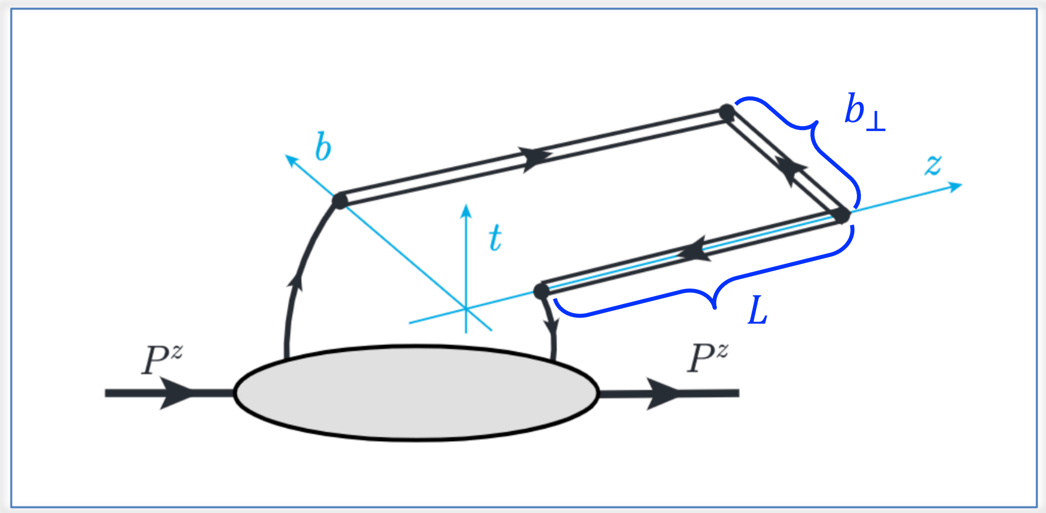
Simulating quasi TMDPDF on a Euclidean lattice:

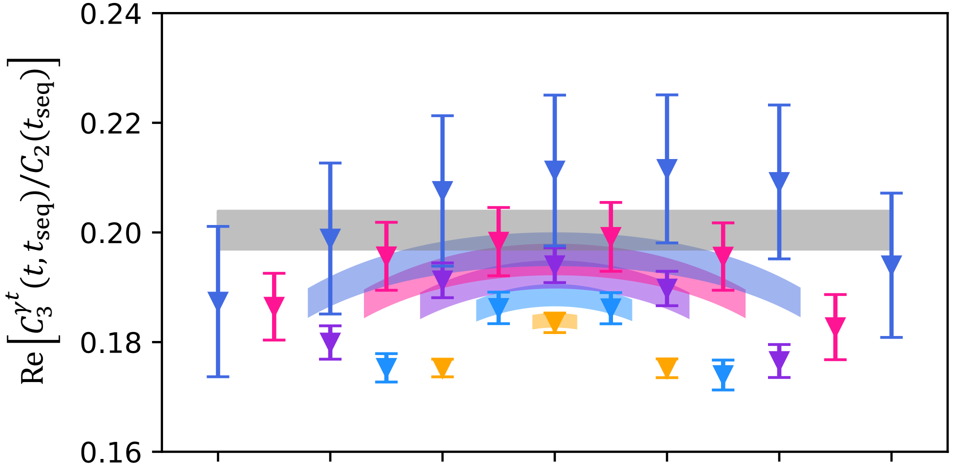
- MILC configuration: $48^3 \times 64$, $a = 0.12$fm;
- Pion mass: $m_\pi^{\text{sea}} = 130$MeV, $m_\pi^{\text{val}} = \{310, 220\}$MeV $\Rightarrow$ extrapolate to physical mass
- Large momentum: $P^z = \{1.72,\ 2.15,\ 2.58\}$GeV $\Rightarrow$ extrapolate to infinity
- Saturated length of Wilson link $L = 0.72$fm;
- $z_{\max} = 1.44$fm, $b_{\perp\max} = 0.6$fm $\Rightarrow$ momentum distribution.

Quasi TMDPDF matrix element

Bare quasi TMDPDF matrix element

$$\tilde{h}^0_\Gamma(z, b_\perp, P^z) = \lim_{L\to\infty} \left\langle P^z \middle| \bar{\psi}(b_\perp \hat{n}_\perp)\Gamma \; U_\sqsubset(b_\perp\hat{n}_\perp \leftarrow b_\perp\hat{n}_\perp + L\hat{n}_z; b_\perp\hat{n}_\perp + L\hat{n}_z \leftarrow L\hat{n}_z; L\hat{n}_z \leftarrow z\hat{n}_z) \; \psi(z\hat{n}_z) \middle| P^z \right\rangle$$

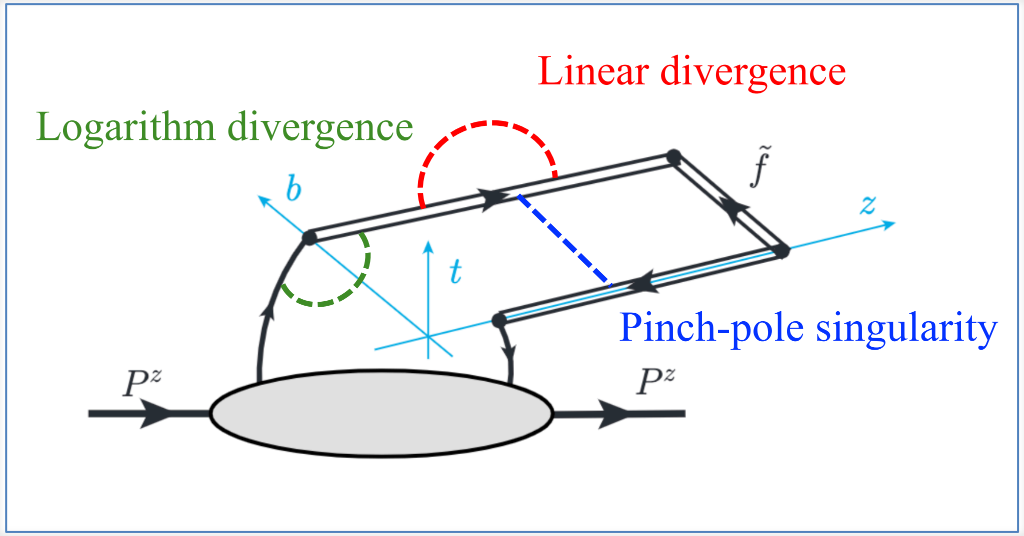




- **Extracted from 3- and 2-point functions**

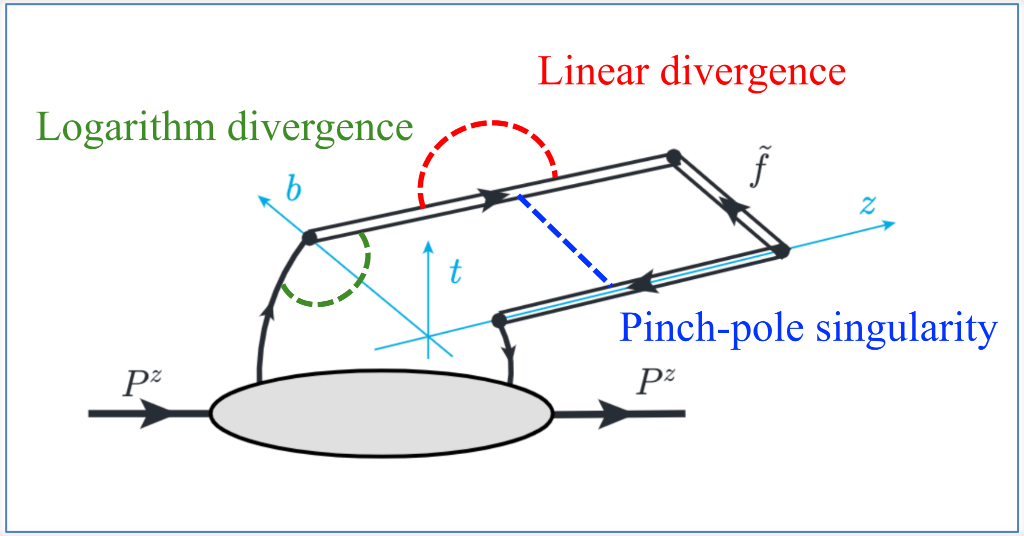
Quasi TMDPDF matrix element

1. Divergences in bare quasi TMDPDF

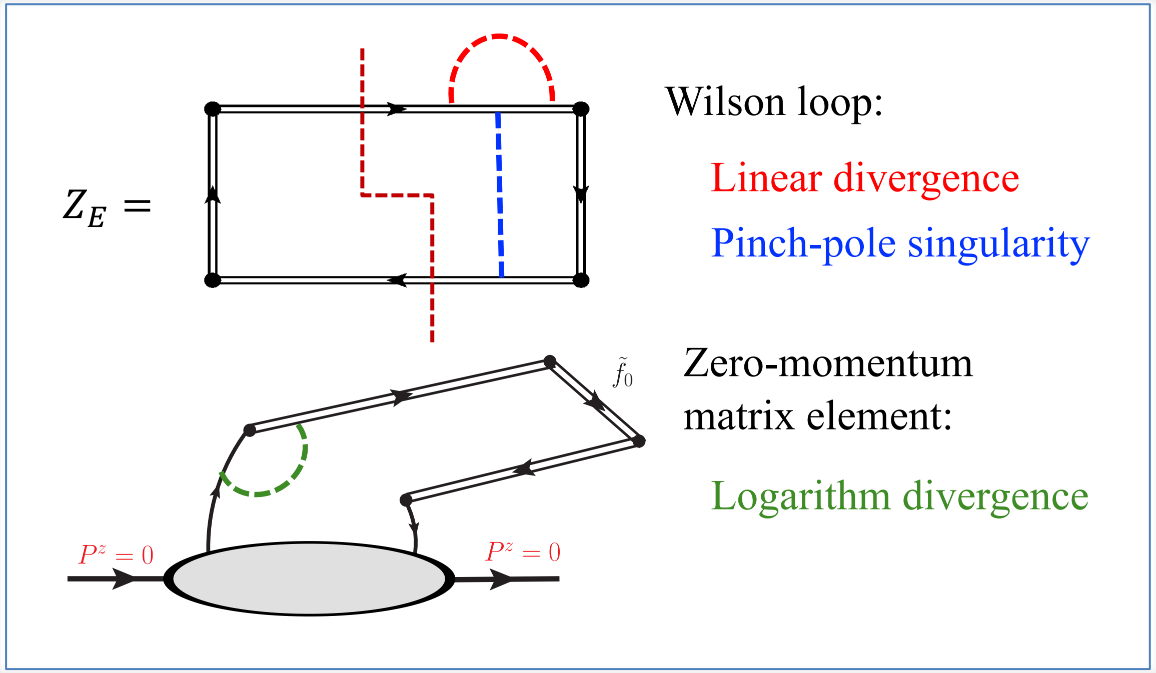


Quasi TMDPDF matrix element

1. Divergences in bare quasi TMDPDF



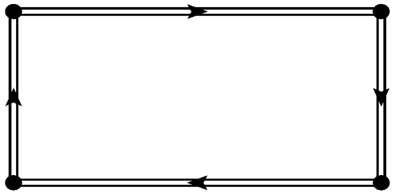
2. Renormalization

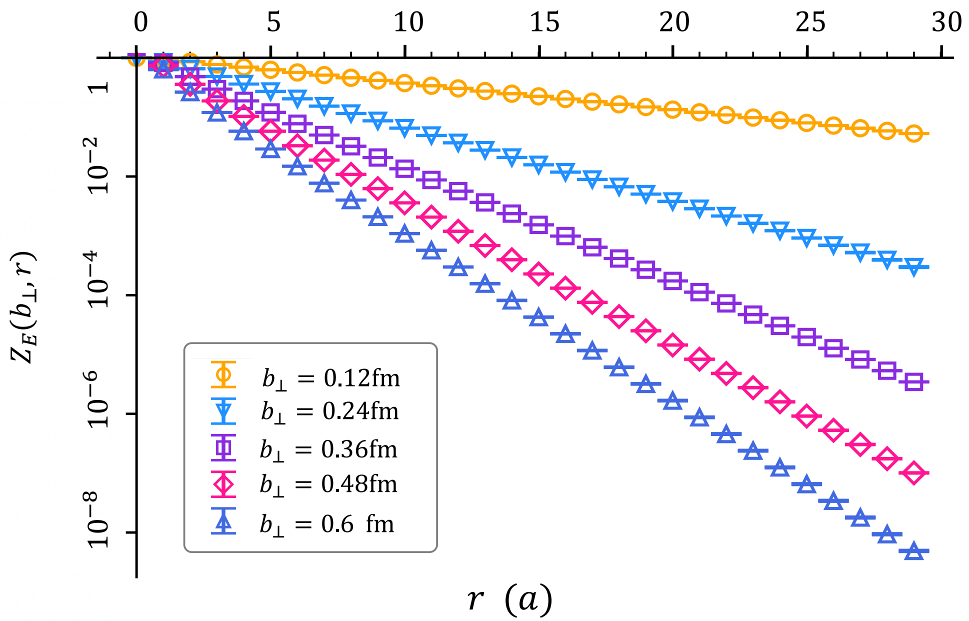


Ji, PRL120(2018), NPB964(2021), PLB257(1991); Zhang, PRD95(2017), NPB939(2019); Ishikawa, PRD96(2017); Green, PRL121(2018); Huo, NPB969(2021); Chen, NPB915(2017); Musch, PRD83(2011); ……

Quasi TMDPDF matrix element

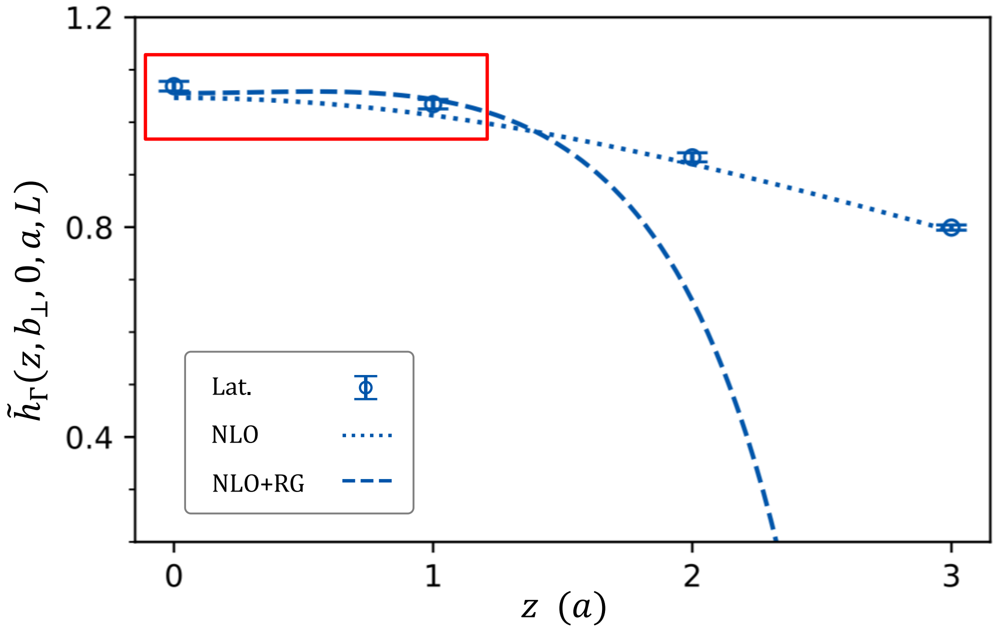
- **Wilson loop**

$Z_E =$ 



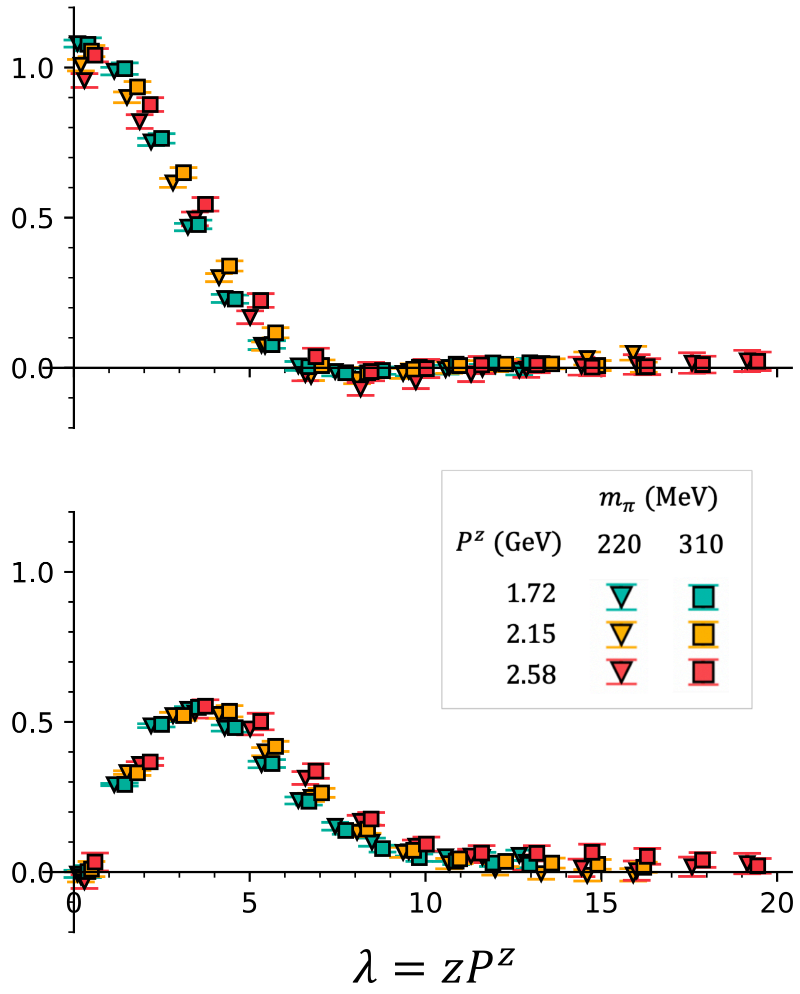
- **Logarithmic divergences factor**

$$Z_O(1/a, \mu, \Gamma) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma(z, b_\perp, 0, a, L)}{\sqrt{Z_E(2L+z, b_\perp, a)}\tilde{h}^{\overline{\rm MS}}_\Gamma(z, b_\perp, \mu)}$$

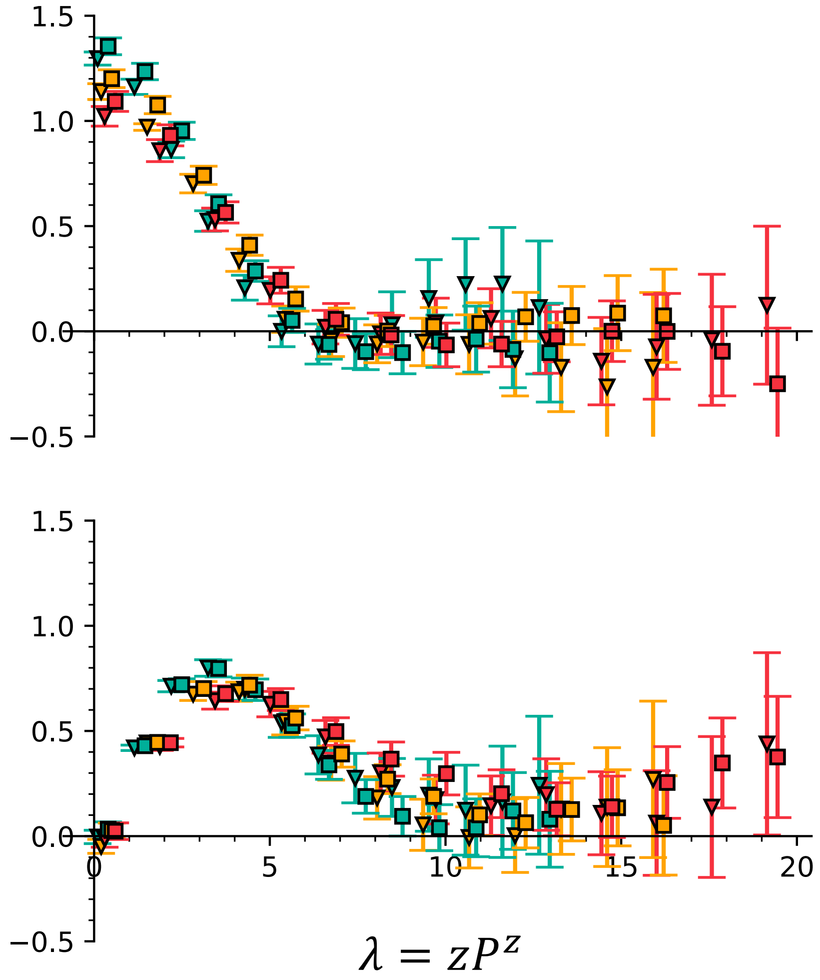


Quasi TMDPDF matrix element

Quasi TMDPDF matrix element at $b_\perp = 2a$



Quasi TMDPDF matrix element at $b_\perp = 5a$



Quasi TMDPDF matrix element

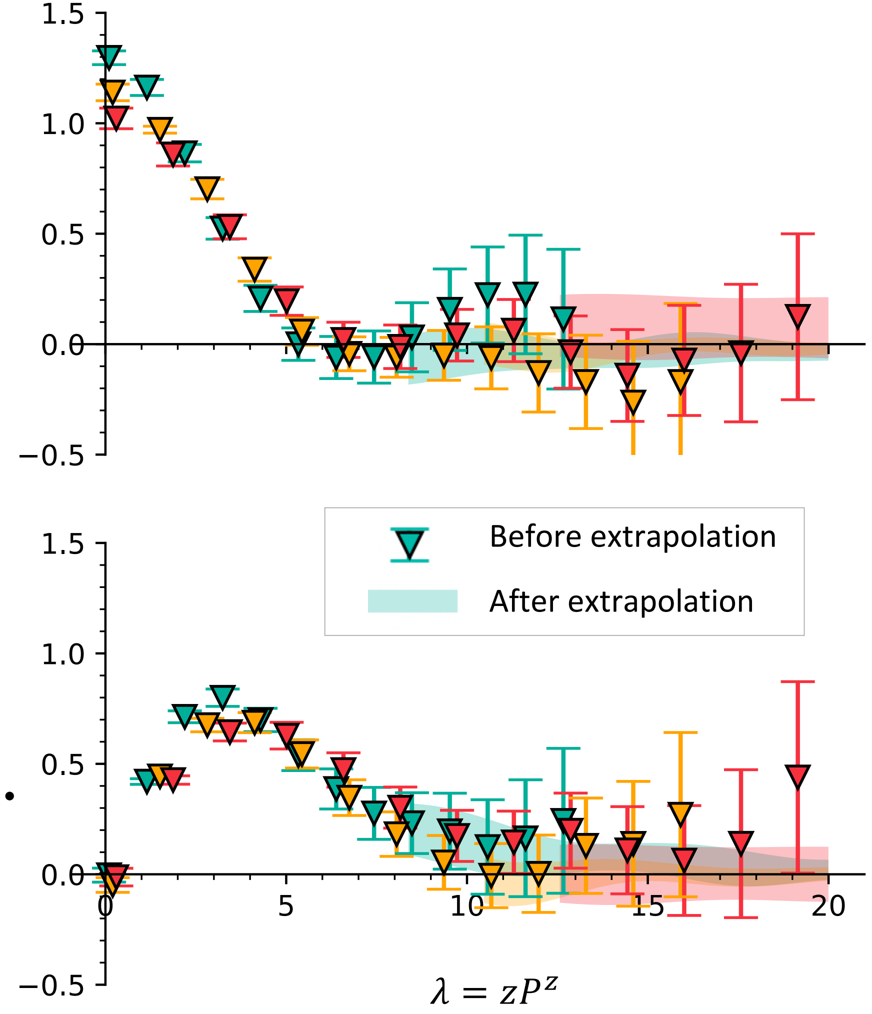
Physical based extrapolation:

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

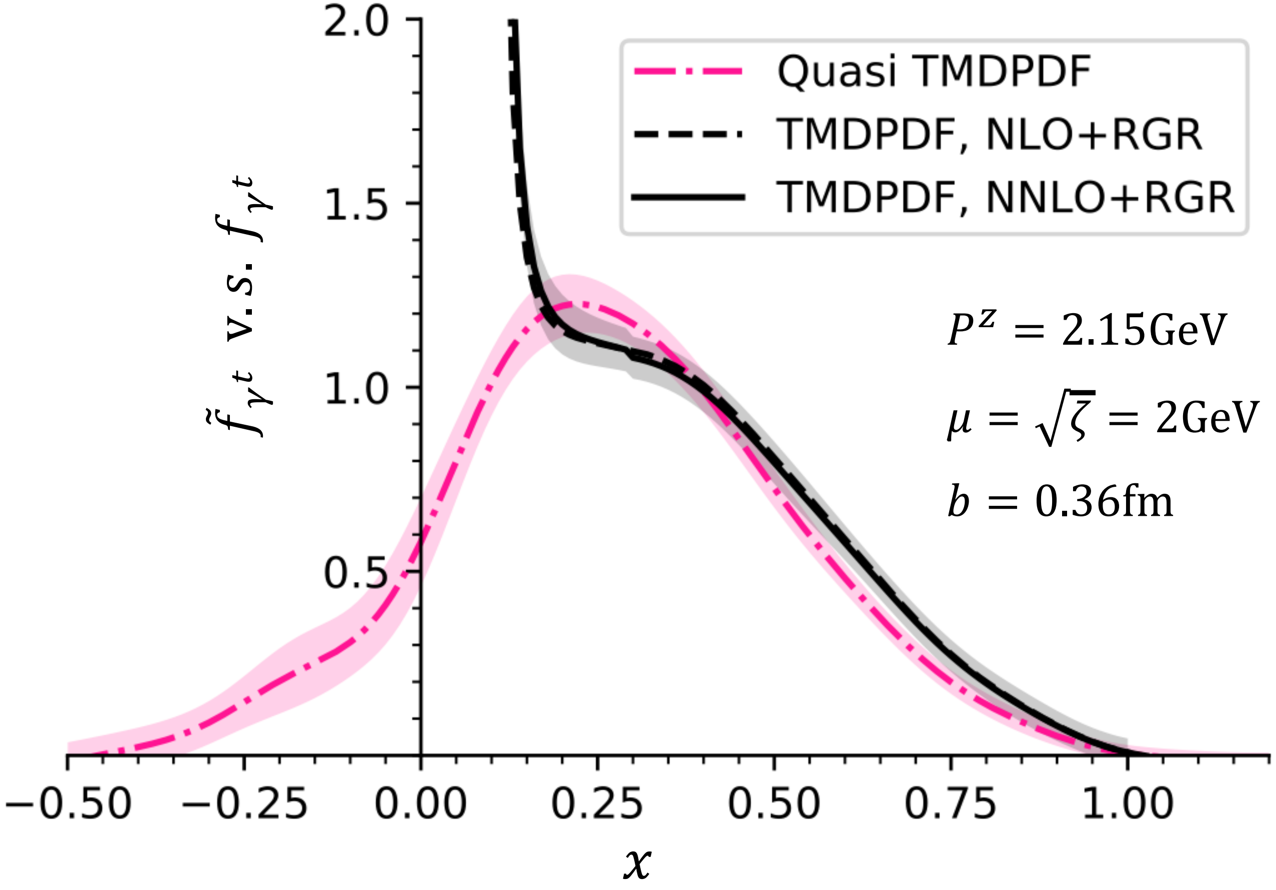
- end point power-law behavior $x^a(1-x)^b$;
- correlation function has a finite correlation length λ_0.

😢 **End point region in momentum space will be affected ……**

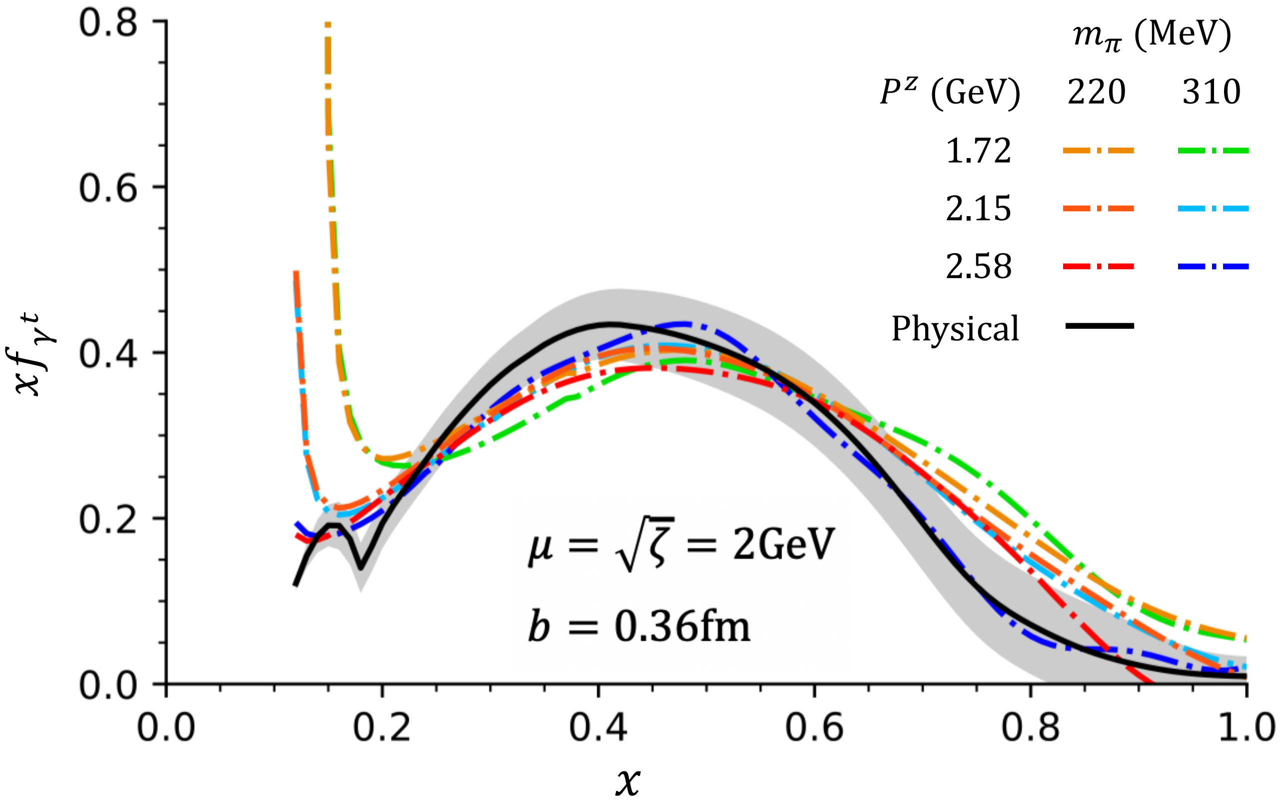
😁 **LaMET is not reliable in that region (power correction)**



From Quasi TMDPDF to TMDPDF



Physical TMDPDF



Physical extrapolation:

$$f^{\mathrm{lat}}(x, b_\perp, \mu, \zeta; m_\pi, P^z) = f^{\mathrm{phy}}(x, b_\perp, \mu, \zeta) * [1 + \mathrm{Corrections}]$$

$$d_0(m_\pi^2 - m_{\pi,\mathrm{phy}}^2) + \frac{d_1}{(P^z)^2}$$

Main form

$$d_0(m_\pi^2 - m_{\pi,\mathrm{phy}}^2)^2 + \frac{d_1}{(P^z)^2}$$

$$d_0(m_\pi^2 - m_{\pi,\mathrm{phy}}^2) + \frac{d_1}{(P^z)^2} + \frac{d_2}{P^z}$$

Used for estimating systematical errors

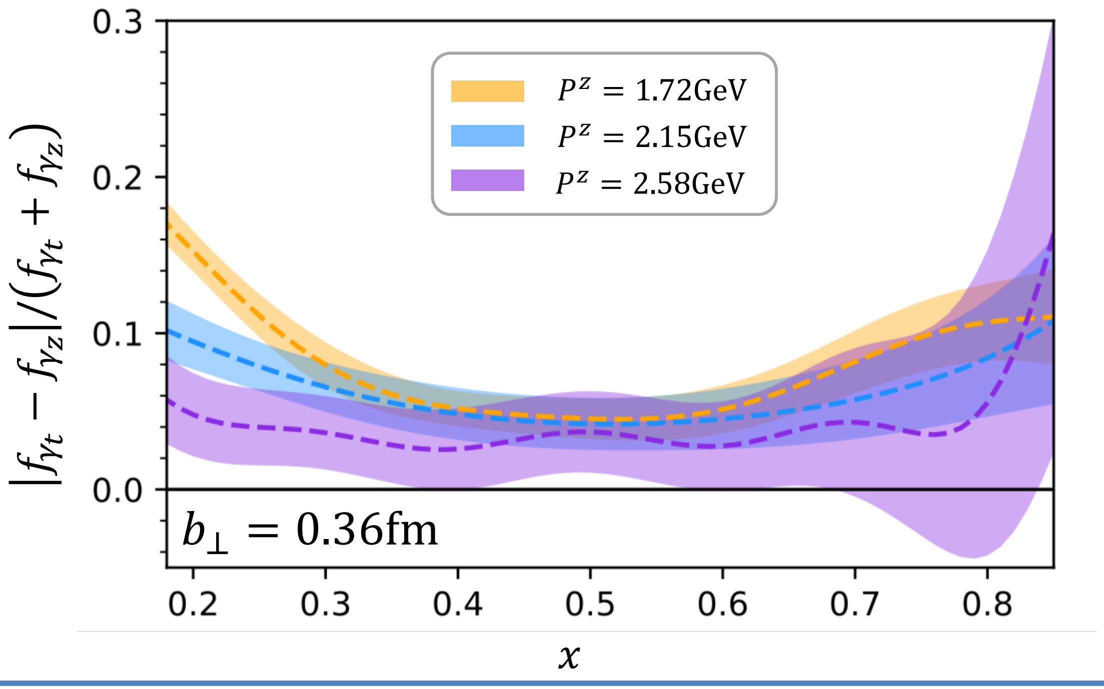
Error estimation

After Lorentz boost:

Leading power　　Higher power

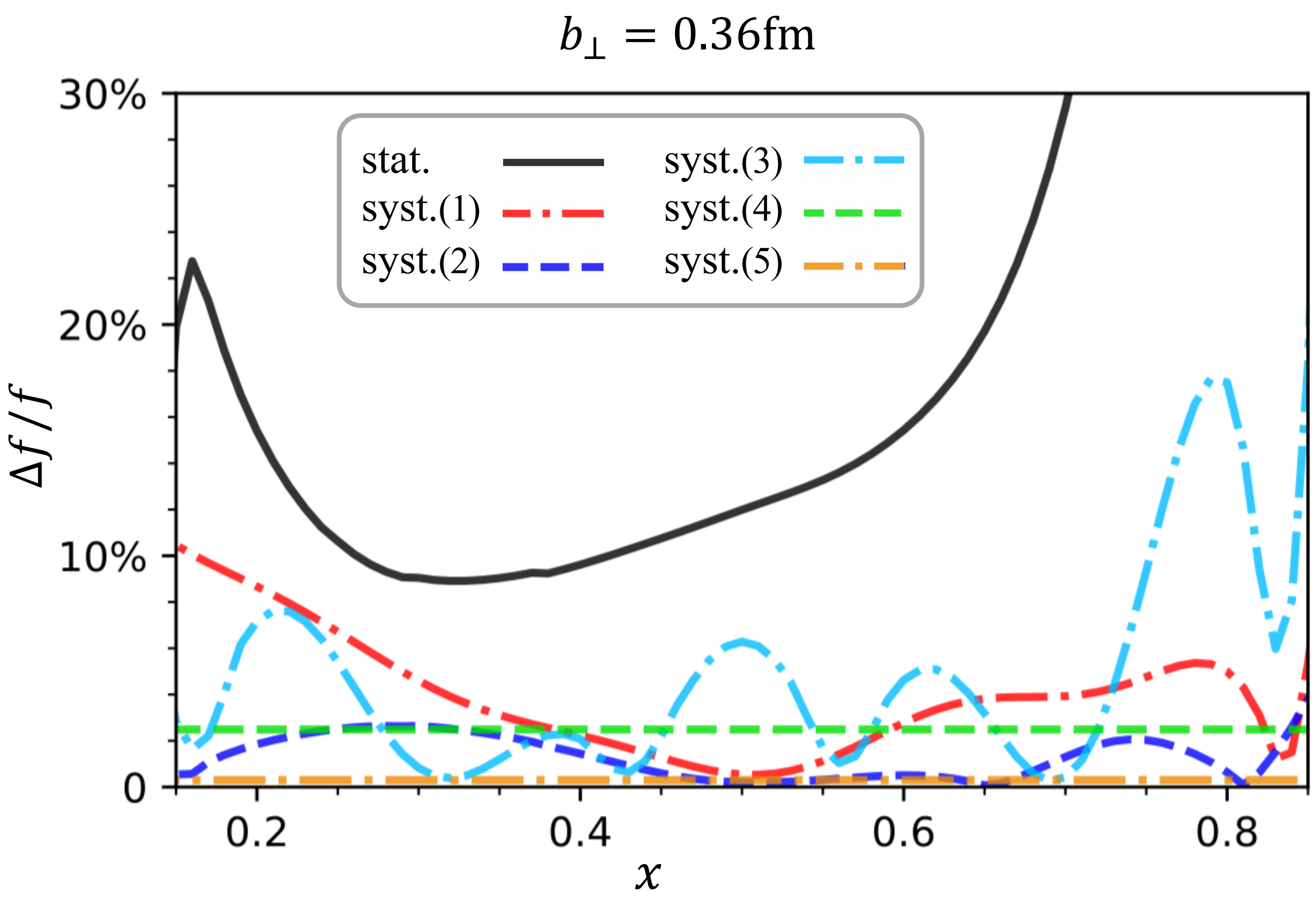
$$\bar{\psi}(z)\gamma^t\psi(0) = \frac{1}{2}\bar{\psi}(z)\gamma^+\psi(0) + \frac{1}{2}\bar{\psi}(z)\gamma^-\psi(0)$$

$$\bar{\psi}(z)\gamma^z\psi(0) = \frac{1}{2}\bar{\psi}(z)\gamma^+\psi(0) - \frac{1}{2}\bar{\psi}(z)\gamma^-\psi(0)$$



- **Ratios denote the deviations from light-like correlator with specific P^z;**
- **Ratio becomes smaller with P^z increasing.**

Error estimation



All errors:

- **Statistical error;**

(1) From power correction

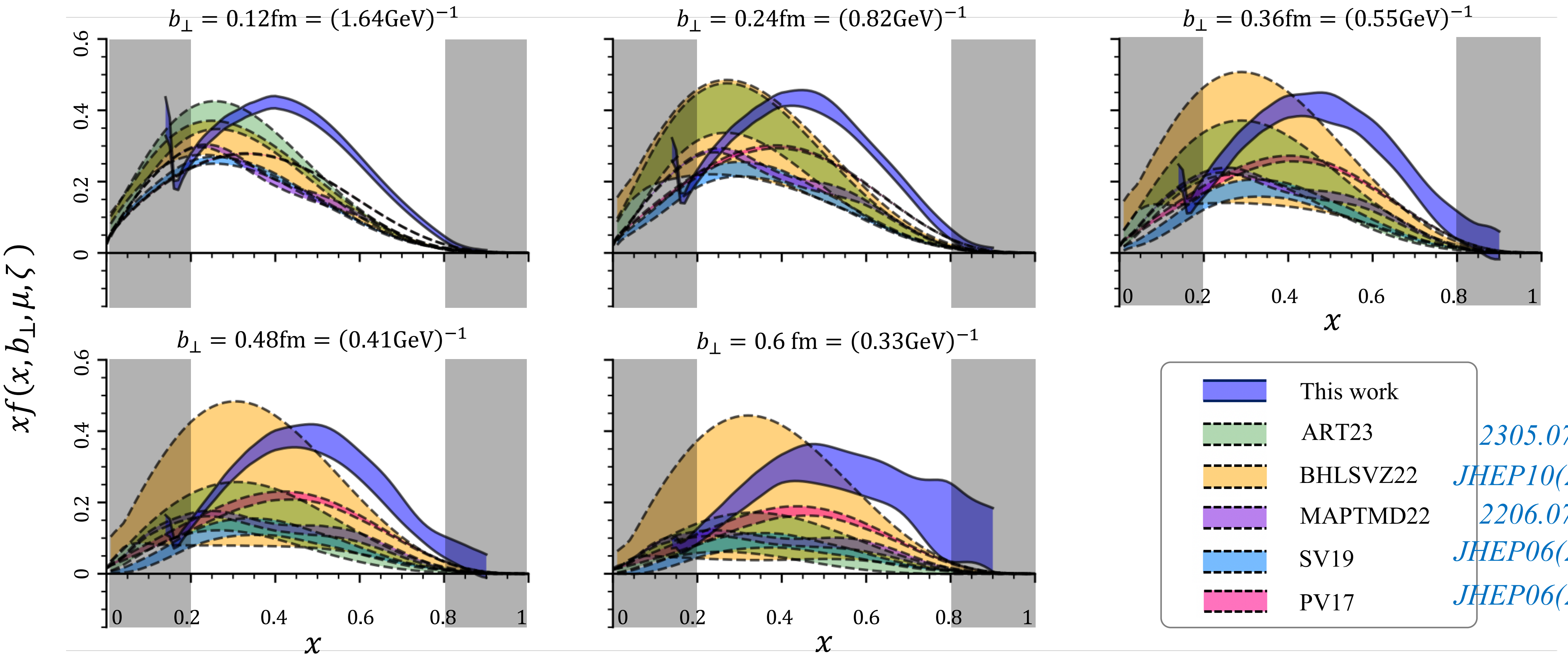
(2) From physical extrapolation

(3) From λ-extrapolation

(4) From soft function

(5) From Collins-Soper kernel

Final results and discussion



2305.07473

JHEP10(2022)

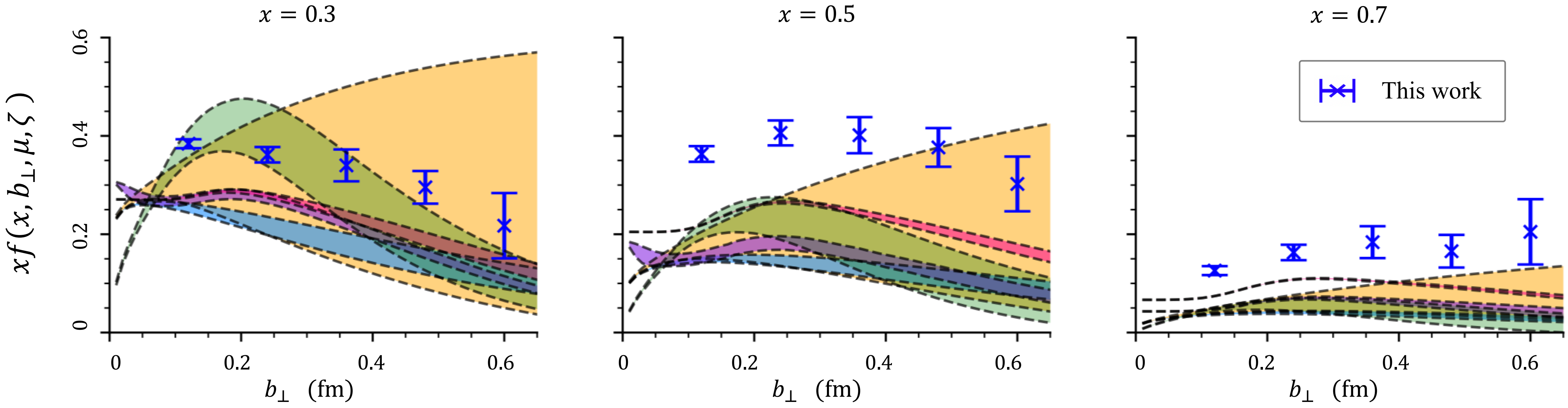
2206.07598

JHEP06(2020)

JHEP06(2019)

Final results and discussion

Compare the $b_\perp$-dependence of lattice and phenomenological results:



Final results and discussion

🤔 **Lattice discretization and finite-volume systematics are still absent in this preliminary work…**

- **It is a challenging work for calculating the TMDPDF at small lattice spacing**
- **From the previous experience of PDF (*Lin, 2011.14971*), we can roughly estimate that:**

  Finite-volume effect is less than 1%;

  Discretization effects overall within 2 standard deviations.

- **Furthermore, need more efforts to achieve the precise calculation on finer lattice.**

Summary and outlook

We present the lattice QCD calculation of TMDPDF at first attempt:

- ✓ **The state-of-the-art techniques in renormalization and extrapolation on the lattice;**
- ✓ **The latest perturbative kernel up to 2-loop with RG evolution;**
- ✓ **Physical extrapolation include chiral-continuum and infinity momentum;**
- ✓ **Comparable results with phenomenological global fits.**

Summary and outlook

<u>While there is still much room for further improvement</u>:

- 🤔 Better control of uncertainties: more statistic, first and most important;

- 🤔 Larger $b_\perp$ (up to nucleon radius?):

  More statistic, improved algorithm, better lattice extrapolation, ……

- 🤔 Continuum extrapolation: more lattice spacings;

- 🤔 Theoretical improvement: power correction (small-x region), operator mixing, ……

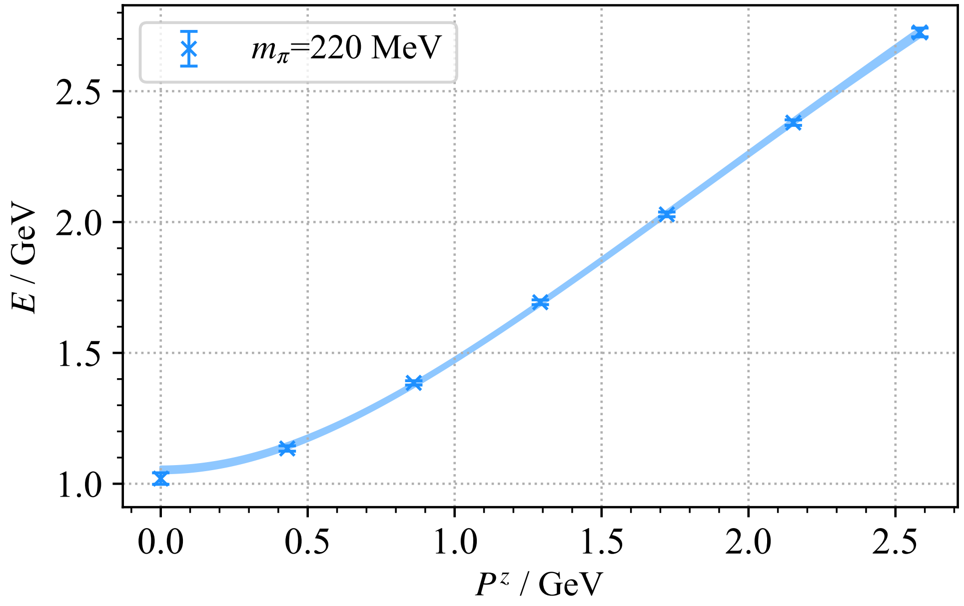
Thank you for your attention!

Backup slides

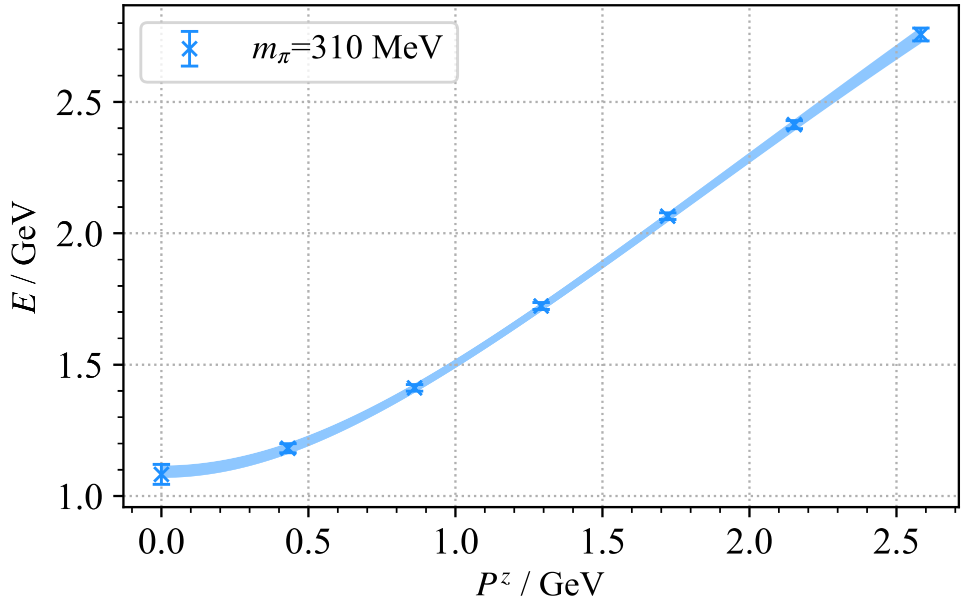
Dispersion relation

$$E = \sqrt{m^2 + c_1(P^z)^2 + c_2(P^z)^4 a^2}$$

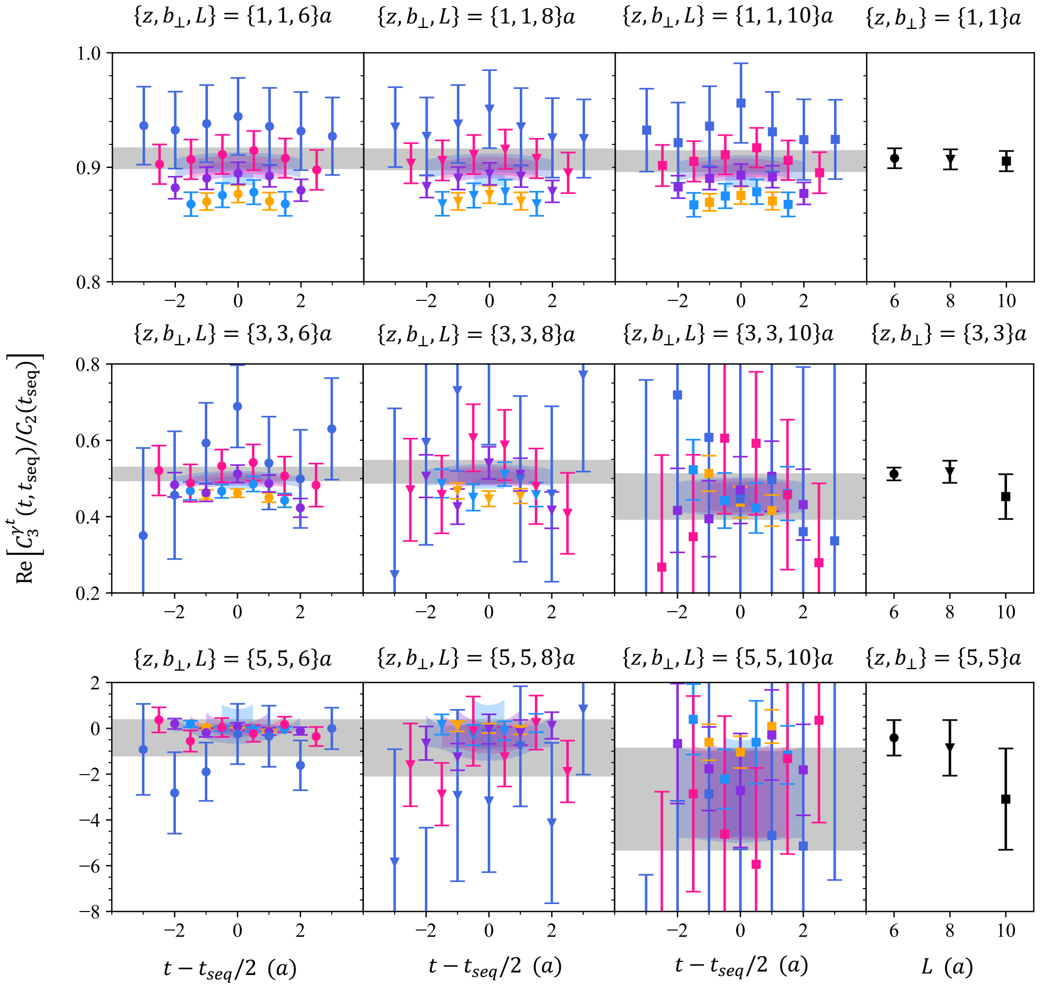
$c_1 = 1.081(20),\ c_2 = -0.0548(96)$

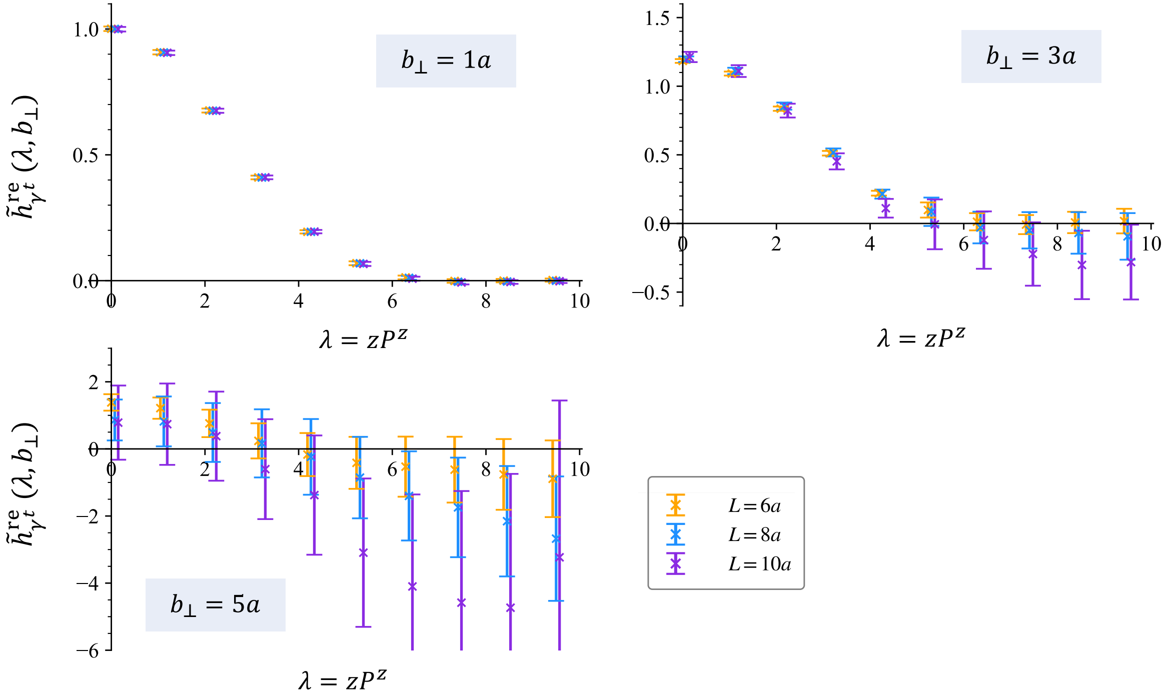


$c_1 = 1.087(28),\ c_2 = -0.053(13)$



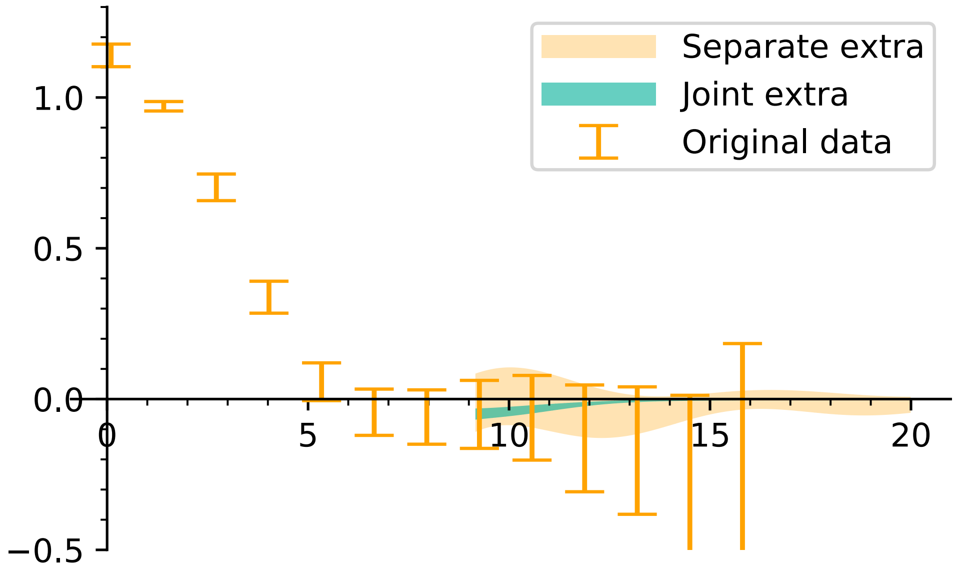
L-dependence





λ-extrapolation

$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$



The power-law behavior and correlation length for each $b_\perp$ should be similar,

but the joint fit will give a strict limit for large-$b_\perp$ cases:

	m_1	m_2	n_1	n_2	λ_0	χ^2/d.o.f
$b = 1a$	$-8.8(3.7)$	$0.25(50)$	$0.909(39)$	$1.13(74)$	$2.63(38)$	1.0
$b = 2a$	$-6.3(2.9)$	$-3.9(5.5)$	$0.943(61)$	$2.37(68)$	$4.1(1.1)$	1.1
$b = 3a$	$-66(60)$	$-78(76)$	$0.89(12)$	$1.71(31)$	$2.42(85)$	1.4
$b = 4a$	$-8.0(4.4)$	$-3.3(2.9)$	$0.801(78)$	$1.55(38)$	$4.3(1.6)$	0.75
$b = 5a$	$-8.5(10)$	$-3.8(5.3)$	$0.84(16)$	$1.22(44)$	$4.4(2.8)$	0.57
Joint fit	–	–	$0.887(28)$	$1.65(12)$	$2.53(28)$	1.2

Perturbative matching kernel and RG resummation

- **Fixed-order perturbative results up to the 2-loop level:**

$$h^{(1)}\left(\frac{\zeta_z}{\mu^2}\right) = \frac{\alpha_s C_F}{2\pi}\left(-2+\frac{\pi^2}{12}+\ln\frac{\zeta_z}{\mu^2}-\frac{1}{2}\ln^2\frac{\zeta_z}{\mu^2}\right),$$

$$h^{(2)}\left(\frac{\zeta_z}{\mu^2}\right) = \alpha_s^2\left[c_2-\frac{1}{2}\left(\gamma_C^{(2)}-\beta_0 c_1\right)\ln\frac{\zeta_z}{\mu^2}-\frac{1}{4}\left(\Gamma_{\text{cusp}}^{(2)}-\frac{\beta_0 C_F}{2\pi}\right)\ln^2\frac{\zeta_z}{\mu^2}-\frac{\beta_0 C_F}{24\pi}\ln^3\frac{\zeta_z}{\mu^2}\right]$$

- **RG equation of the matching kernel:**

$$\mu^2\frac{d}{d\mu^2}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \frac{1}{2}\Gamma_{\text{cusp}}\left(\alpha_s\right)\ln\frac{\zeta_z}{\mu^2}+\frac{\gamma_C\left(\alpha_s\right)}{2},$$

and its solution:

$$H\left(\zeta_z/\mu^2\right) = H\left(\zeta_z/\mu_0^2\right)\exp\left[\int_{\mu_0}^{\mu}\frac{d\mu}{\mu}\left(\Gamma_{\text{cusp}}^{(1)}\ln\frac{\zeta_z}{\mu^2}\alpha_s(\mu)+\gamma_C^{(1)}\alpha_s(\mu)+\Gamma_{\text{cusp}}^{(2)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^2(\mu)\right.\right.$$
$$\left.\left.+\gamma_C^{(2)}\alpha_s^2(\mu)+\Gamma_{\text{cusp}}^{(3)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^3(\mu)+\gamma_C^{(3)}\alpha_s^3(\mu)+\Gamma_{\text{cusp}}^{(4)}\ln\frac{\zeta_z}{\mu^2}\alpha_s^4(\mu)\right)\right].$$