Progress in Lattice calculations for the Boer-Mulders function of the pion

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in collaboration with:

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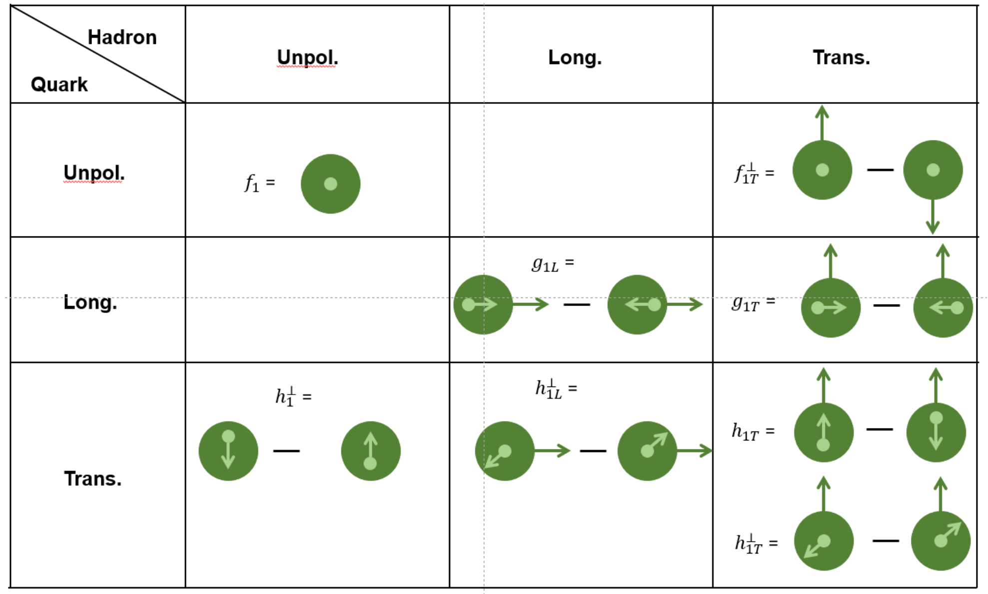
Motivation to calculate Boer-Mulders function of the pion

- Transverse-momentum-dependent parton distribution functions (TMDPDFs) provide description of the 3D internal structure of hadrons
- TMDPDFs and other TMD observables intensely studied in experiments as well as in Lattice calculations using for example ratios of Mellin moments [1] or LaMET [2-10]
- Good knowledge of TMDPDFs crucial for extracting them from experiments
- Eight leading-twist TMDPDFs, focus on T-odd Boer-Mulders function $h_1^\perp$ (describes net transverse polarization of quarks inside an unpolarized hadron)
- In the end: Want to calculate and compare Boer-Mulders function for pion and nucleon to test hypothesis that „all Boer-Mulders functions are alike“ [11]



1: M. Schlemmer at al., arXiv: 2103.16991
2: P. Shanahan et al., arXiv: 2107.11930
3: LPC, arXiv: 2211.02340
4: K. Zhang et al., arXiv: 2205.12809
5: M. A. Ebert et al., arXiv: 2201.08401
6: Z.-F. Deng et al., arXiv: 2207.07280
7: M.-H. Chu et al., arXiv: 2204.00200
8: M.-H. Chu et al., arXiv: 2306.06488
9: M.-H. Chu et al., arXiv: 2302.09961
10: X. Ji et al., 2305.04416
11: M. Burkhardt and B. Hannafious, arXiv: 0705.1573 [hep-ph]

Outline

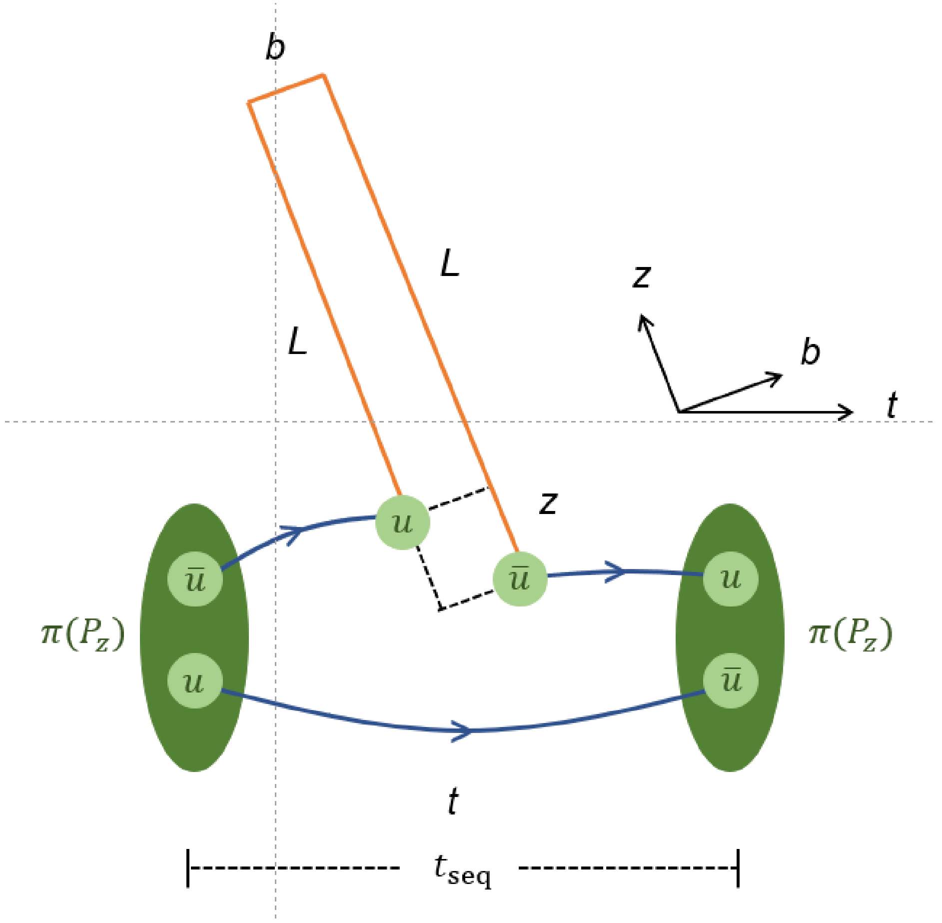
- Calculation of Boer-Mulders function in LaMET
- Lattice setup
- Dispersion relation
- Extraction of unsubtracted quasi-TMDPDF from fitting of correlation functions
- Square root of rectangular Wilson loop for renormalization
- Investigation of Wilson link length-dependence for limit $L \to \infty$
- Summary and next steps

Calculation in LaMET

- Start from unsubtracted quasi-TMDPDF:

$$\tilde{h}_{\chi,\Gamma}(b,z,L,P_z;1/a) = \langle \chi(P_z)|O_\Gamma(b,z,L)|\chi(P_z)\rangle$$

  - $O_\Gamma(b,z,L) \equiv \bar{\psi}(\vec{0}_\perp,0)\Gamma W(b,z,L)\psi(\vec{b}_\perp,z)$
  - $\chi(P_z)$: hadron (pion) state with momentum $P_\mu = (P_0,0,0,P_z)$
  - $\Gamma = \gamma^1\gamma^3$ for Boer-Mulders function (see parametrization of correlator $\tilde{\Phi}^{[i\sigma^{\mu\nu}\gamma^5]}_{\text{unsubtr.}}$ [12])
  - Staple-shaped Wilson link $W(b,z,L)$ with b and z separating the quark fields along transverse (longitudinal) direction
  - L needs to be large



12: B. U. Musch et al., Phys. Rev. D **85** (2012) 094510, arXiv: 1111.4249 [hep-lat]

Lattice setup on CLS [13] ensemble X650

- Lüscher-Weisz gauge action with tree-level coefficients
- $O(a)$-improved Wilson Dirac operator, $N_f = 2 + 1$

Ensemble	a (fm)	$L^3 \times T$	m_π (MeV)	$m_\pi L$	P_z (GeV)
X650	0.098	$48^3 \times 48$	338	8.1	0, 0.53, 0.79, 1.05, 1.32

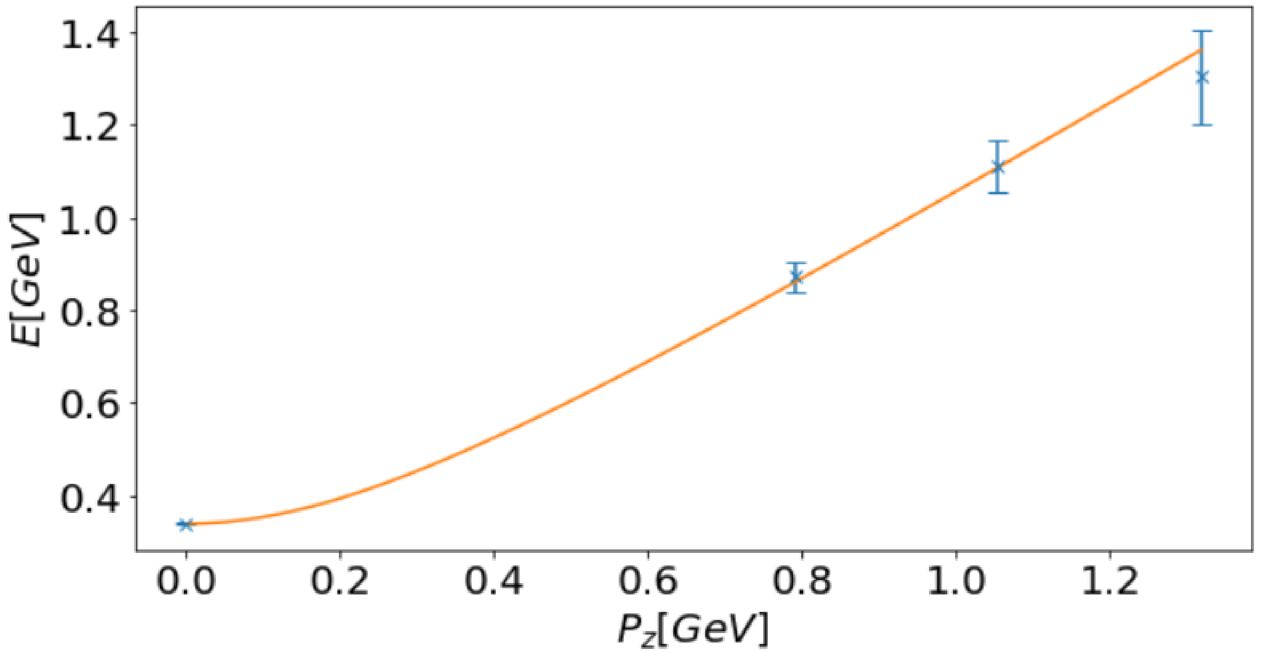
- Sources: 2 in *x*, *y*, *t*, 1 in *z* → 8 sources
- 930 configurations x 8 = 7440 measurements
- 1 step HYP-smearing on links, momentum smearing [14] of source and sink
- Source-sink separation t_{seq} : {6, 7, 8} a
- *z*: {0..18} a , *b*: {0..7} a
- *L*: {2, 4, 6, 8, 10, 12, (14, 16, 18)} a

13: M. Bruno et al., 1411.3982 [hep-lat]
14: G. S. Bali et al., Phys. Rev. D **93**, 094515 (2016), arXiv: 1602.05525 [hep-lat]

Correlation functions: Dispersion relation

- LQCD calculations performed using the Chroma software suite [15] and IDFLS solver [16]
- Calculate correlation functions $C^{2\mathrm{pt}}(P_z, t_{\mathrm{seq}})$ and $C_\Gamma^{3\mathrm{pt}}(P_z, t, t_{\mathrm{seq}}, b, z, L)$ on the lattice,

  t_{seq}: source-sink separation, *t*: operator insertion time
- Fit of $C^{2\mathrm{pt}}(P_z, t_{\mathrm{seq}})$ with parametrization form $c_4 e^{-E_0 t_{seq}}(1 + c_5\, e^{-\,\Delta E\, t_{seq}})$ gives pion mass



Comparison of fitted pion mass with relativistic dispersion relation $\mathrm{E} = \sqrt{m^2 + P_z^2}$

15: R. G. Edwards et al., arXiv: hep-lat/0409003
16: M. Lüscher, arXiv: 0710.5417

Correlation functions: Fit of $\tilde{h}_{\chi,\Gamma}$

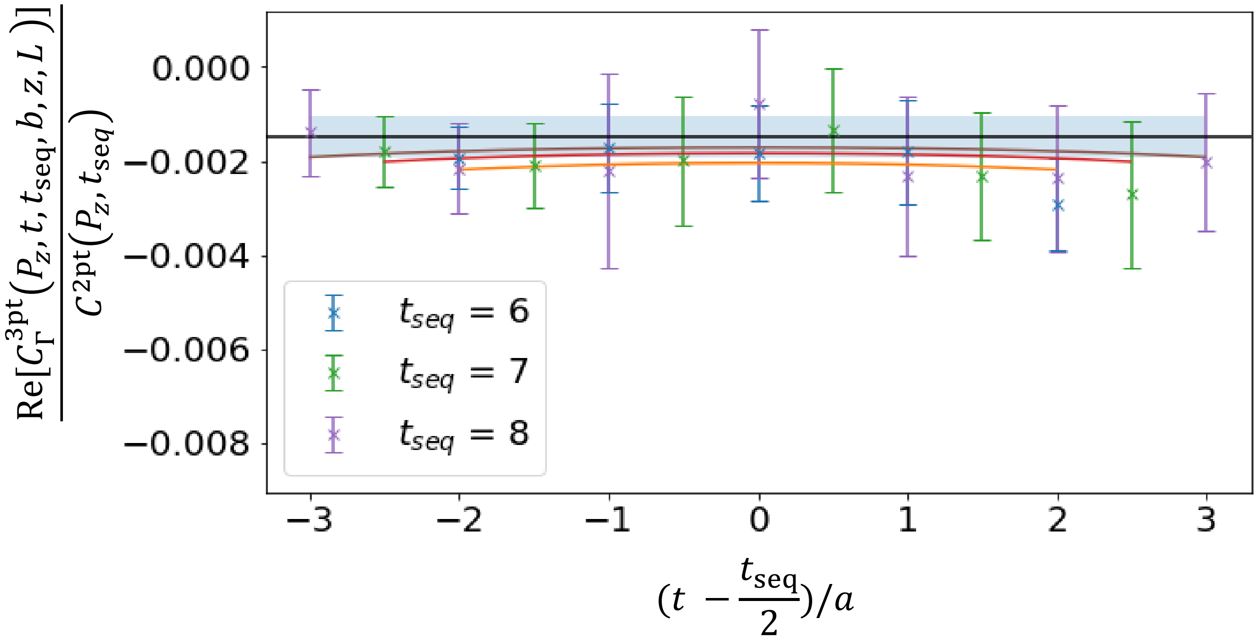
- Decomposition of correlation functions → unsubtracted quasi-TMDPDF can be extracted by fitting ratio

$$\frac{C_\Gamma^{3\mathrm{pt}}(P_z,t,t_{\mathrm{seq}},b,z,L)}{C^{2\mathrm{pt}}(P_z,t_{\mathrm{seq}})} \quad \text{with parametrization form} \quad \frac{\boldsymbol{c_0}+c_1(e^{-\Delta E\,(t_{\mathrm{seq}}-t)}+e^{-\Delta E\,t})+c_3(e^{-\Delta E\,t_{\mathrm{seq}}})}{1+c_5e^{-\Delta E t_{\mathrm{seq}}}}$$

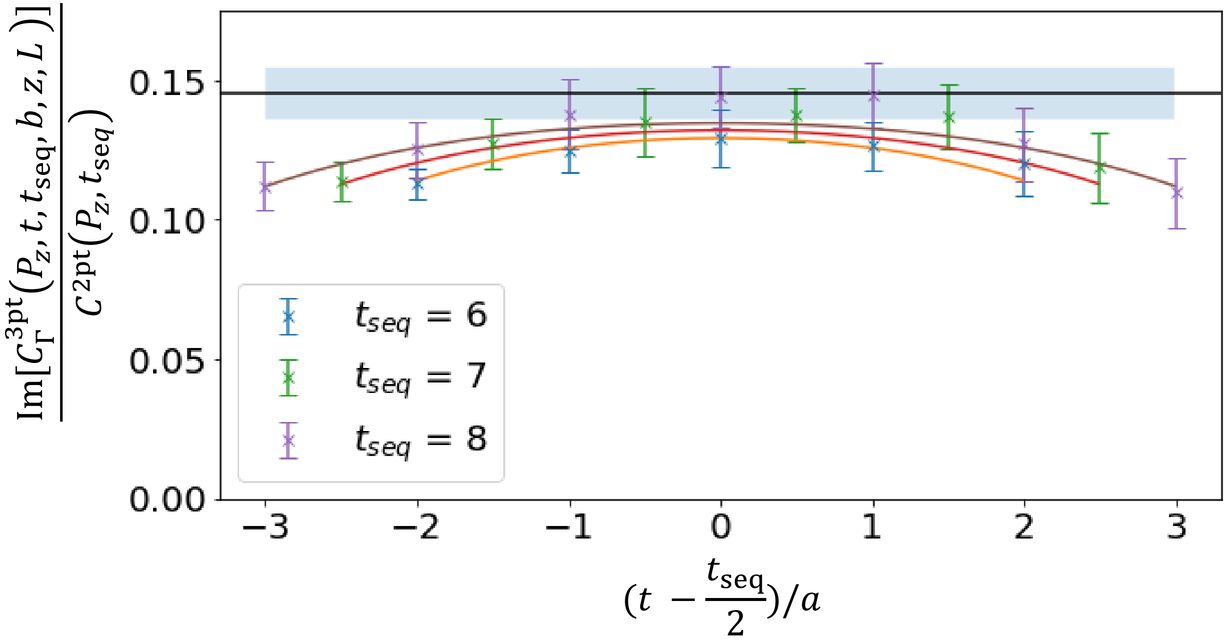
  - Combined fit of ratio and two-point function
  - $c_0, c_1, c_3, c_5, \Delta E$: fit parameters
  - ΔE: mass gap between ground state and first excited state
  - $c_0 = \tilde{h}_{\chi,\Gamma}(b,z,L,P_z;1/a)$: unsubtracted quasi-TMDPDF
  - Correlated fit
  - Fitted using Python package lsqfit

Correlation functions: Fit of $\tilde{h}_{\chi,\Gamma}$, real part

real part



imaginary part

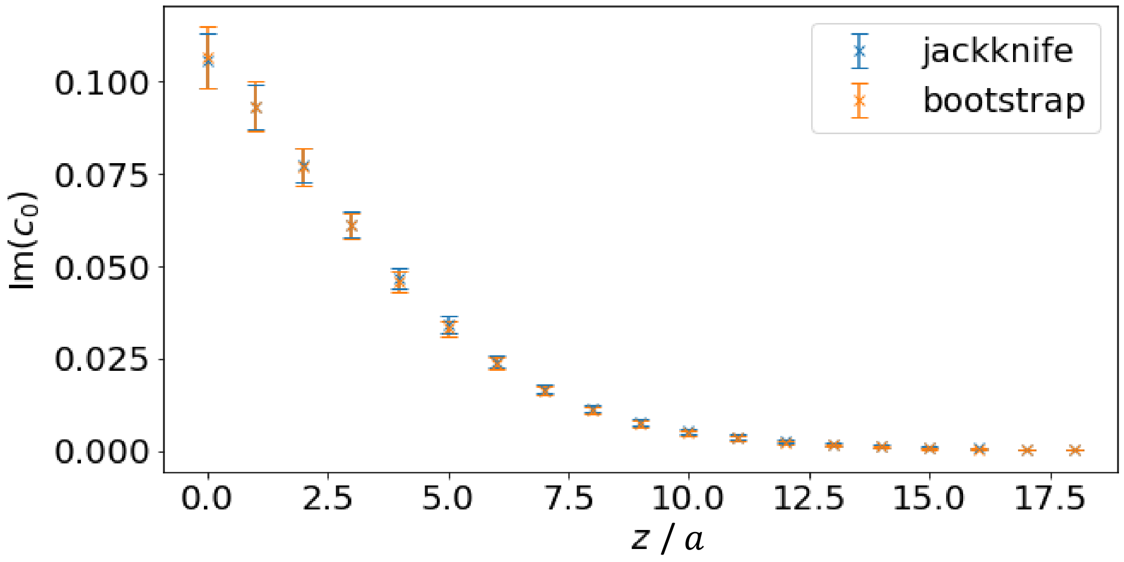


- $P_z = 3 * \frac{2\pi}{n_S a} = 0.79$ GeV, $L = 8a \approx 0.8$ fm, $b = a \approx 0.1$ fm, $z = a \approx 0.1$ fm
- Fitted c_0 **almost** zero within errors for real part (1 % of imaginary part)
- Show only fit of imaginary part in the next slides

Comparison of jackknife and bootstrap resampling

- Before fitting:
  - Twisted-mass reweighting to compensate for additional term added to the action [17]
  - Binning with bin size 10
  - Resampling (jackknife / bootstrap)

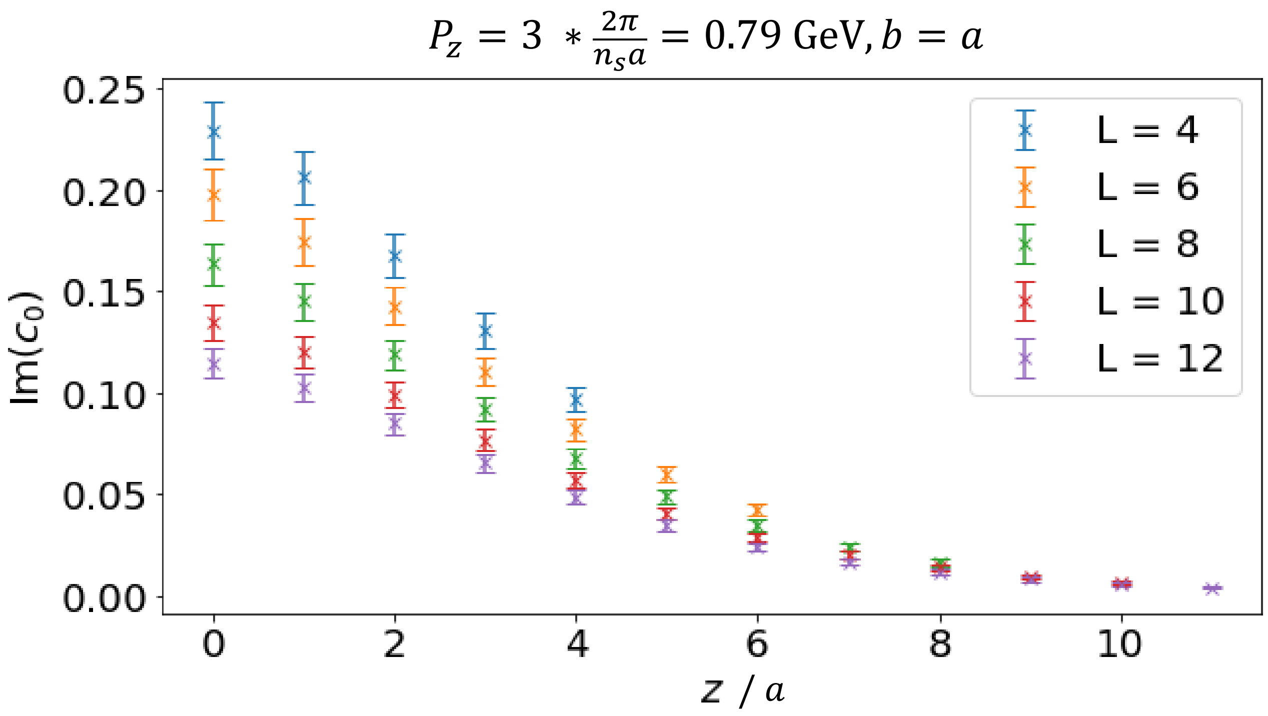
$P_z = 4 * \frac{2\pi}{n_s a} = 1.05 \text{ GeV}, L = 8\text{a}, b = a$



Good agreement between fitting results with jackknife and bootstrap (500 samples) resampled data

17: M. Lüscher and F. Palombi, PoS LATTICE 2008 (2008), arXiv: 0810.0946

Fitting results for $\tilde{h}_{\chi,\Gamma} : P_{\text{lat}} = 3 \ (P_z = 0.79 \text{ GeV})$



Fit results for imaginary part of unsubtracted quasi-TMDPDF $\tilde{h}_{\chi,\Gamma}$ for $P_z = 3 * \frac{2\pi}{n_s a} =$ 0.79 GeV , b = *a*, L = (4, 6, 8, 10, 12) *a*

Renormalization as in [4]

- Definition of unsubtracted quasi-TMDPDF $\tilde{h}_{\chi,\Gamma}(b, z, L, P_z; 1/a)$

  pinch pole singularity (from interaction between the two Wilson lines in *z* direction),

  linear divergence from Wilson link self-energy

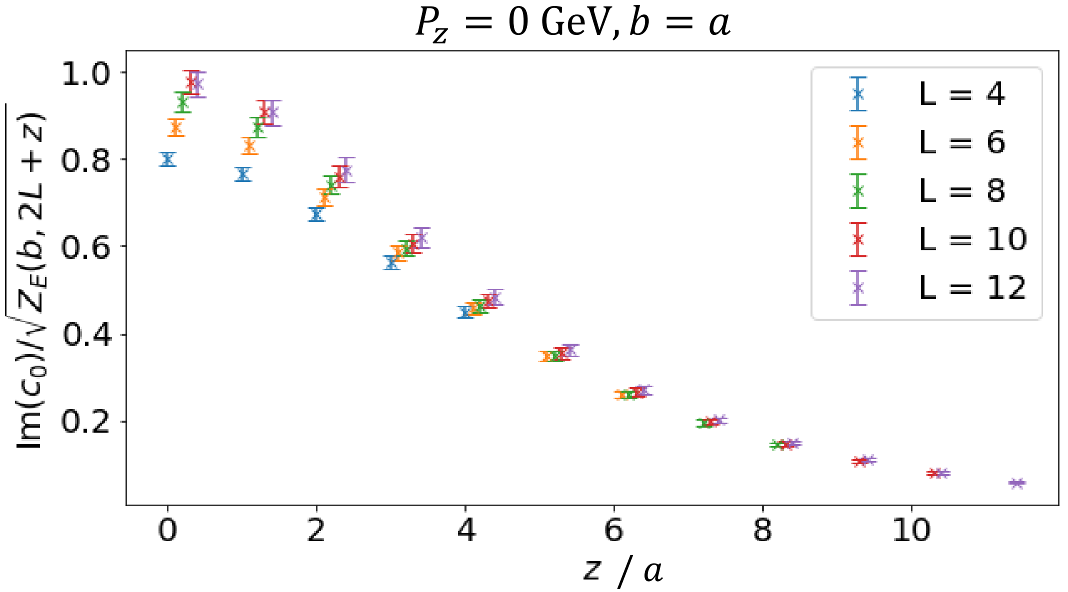
- Subtracted quasi-TMDPDF to cancel those divergences:

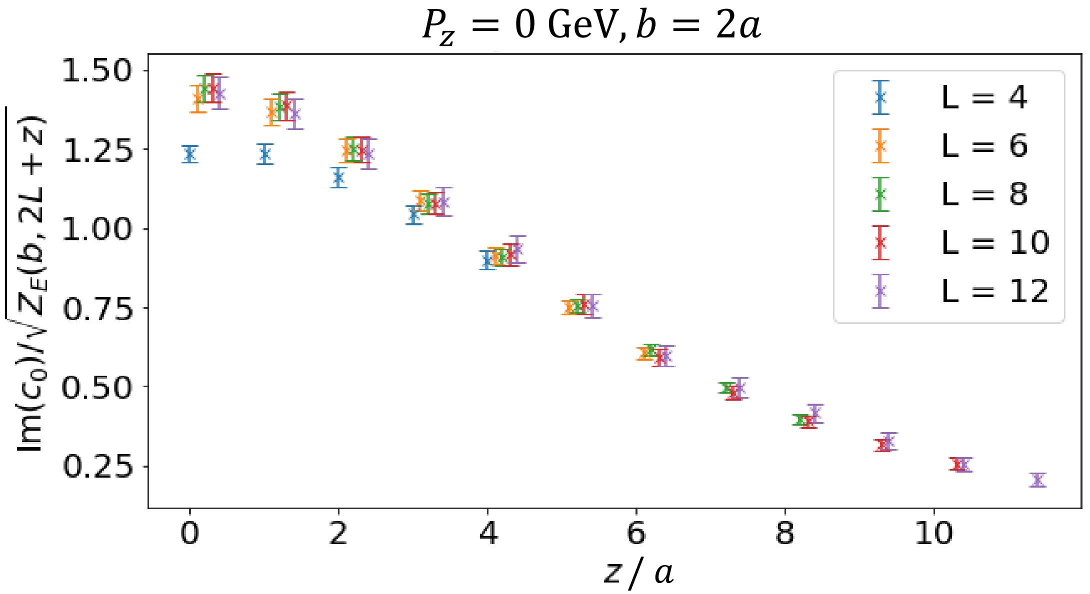
$$h_{\chi,\Gamma}(b, z, P_z; 1/a) = \lim_{L\to\infty} \frac{\tilde{h}_{\chi,\Gamma}(b, z, L, P_z; 1/a)}{\sqrt{Z_E(b, 2L+z; 1/a)}}$$

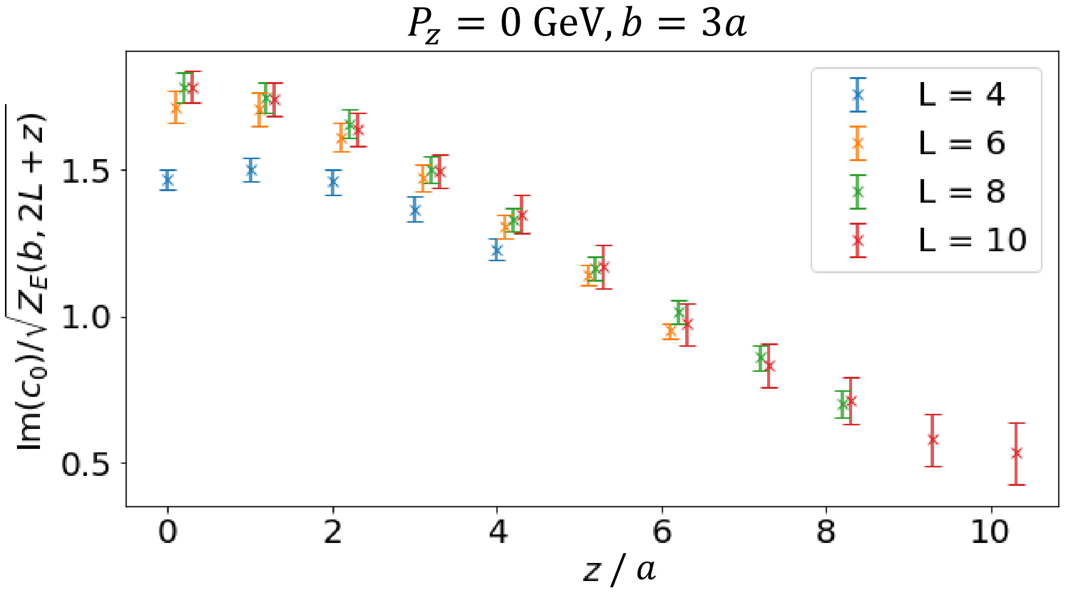
$Z_E(b, 2L+z; 1/a)$: rectangular Wilson loop with side lengths *b* and *2L+z*

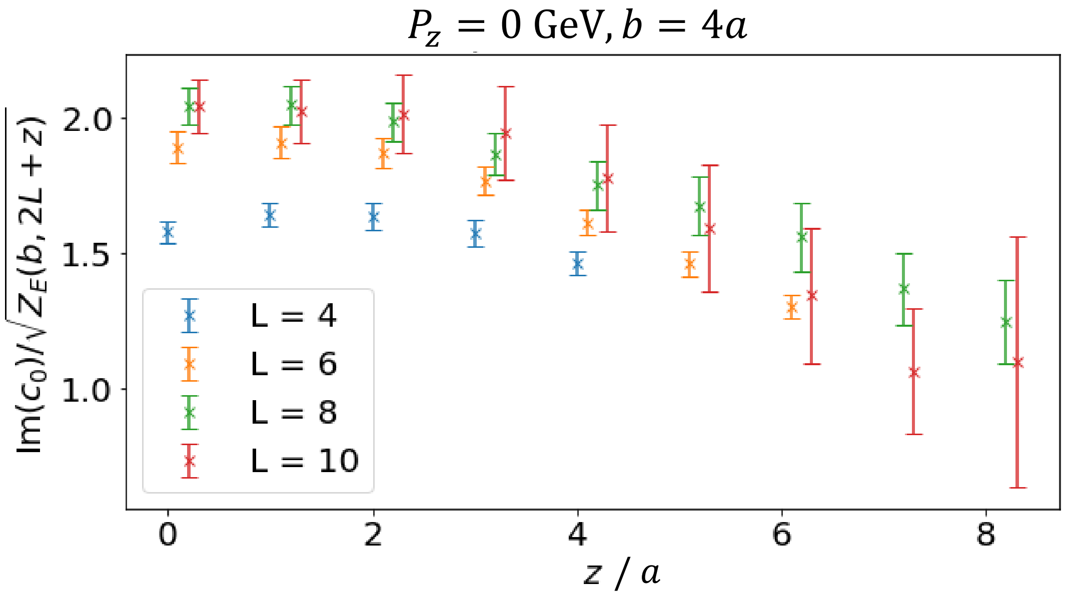
4: K. Zhang et al., arXiv: 2205.13402 [hep-lat]

Results for subtracted quasi-TMDPDF $h_{\chi,\Gamma}$: $P_z = 0$ GeV



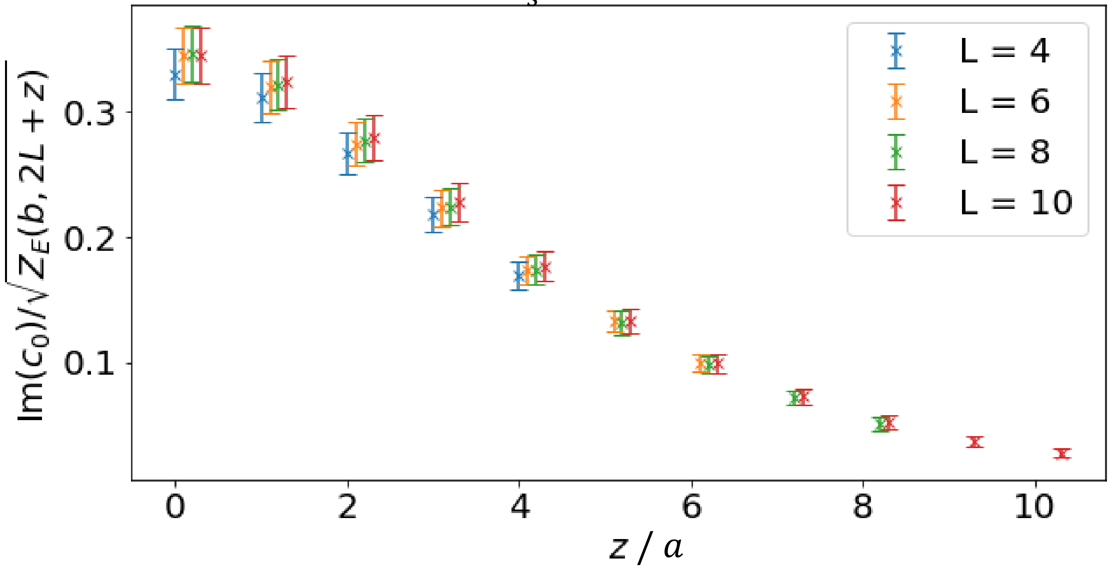




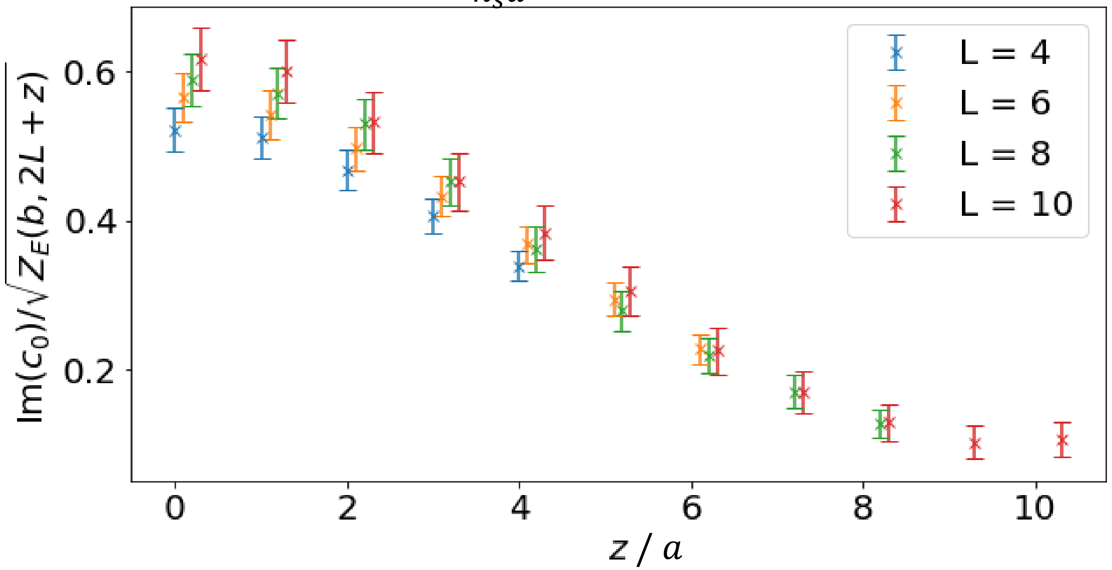


Results for subtracted quasi-TMDPDF $h_{\chi,\Gamma}$: $P_{\text{lat}} = 3$ ($P_z = 0.79$ GeV)

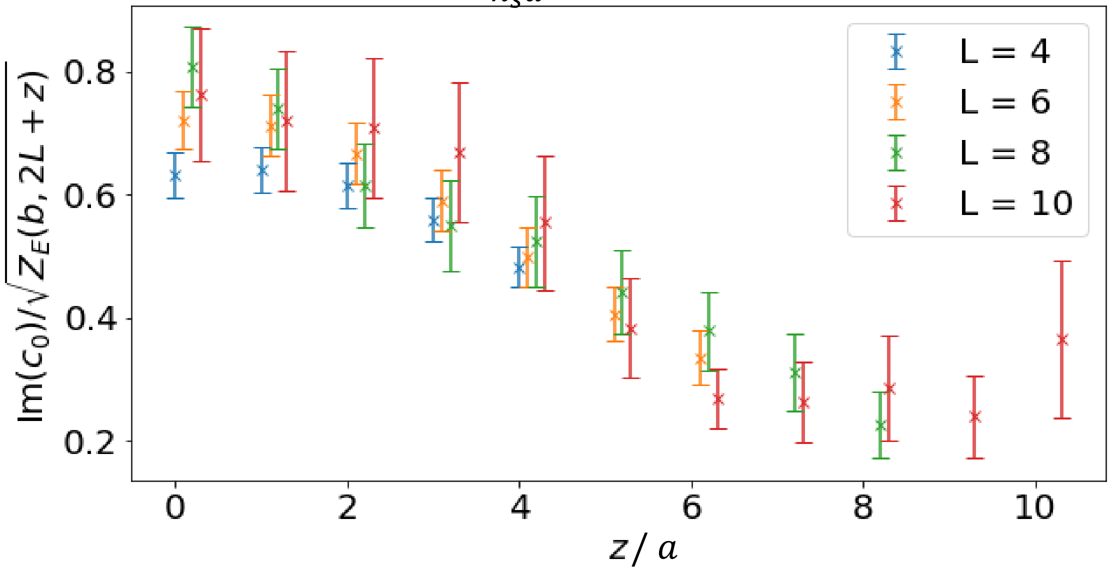
$P_z = 3 * \frac{2\pi}{n_s a} = 0.79\ \text{GeV}, b = a$



$P_z = 3 * \frac{2\pi}{n_s a} = 0.79\ \text{GeV}, b = 2a$

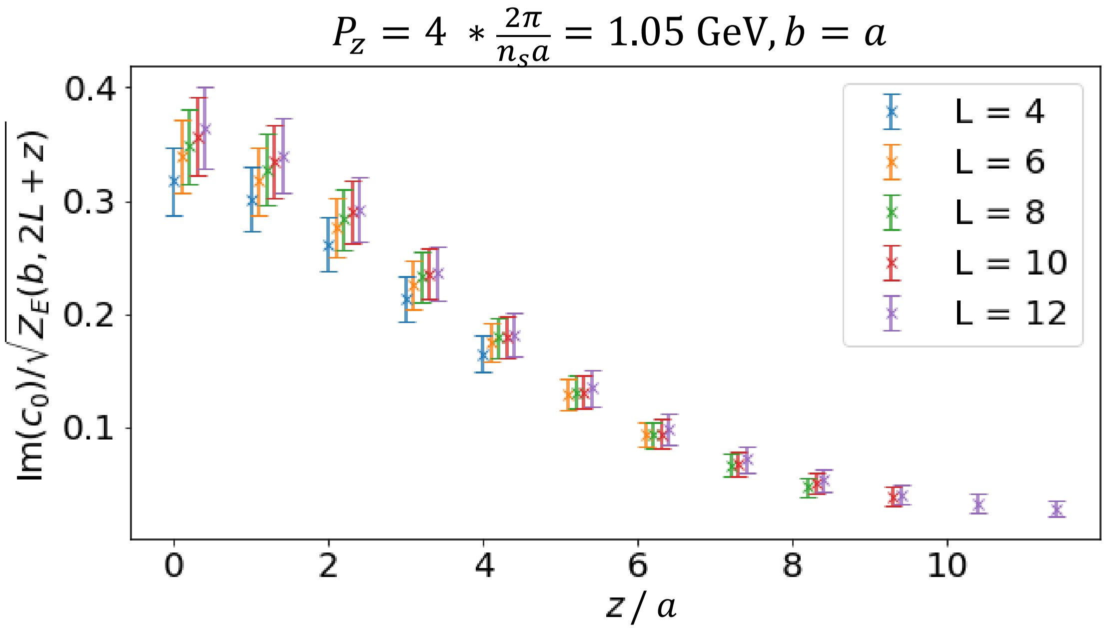


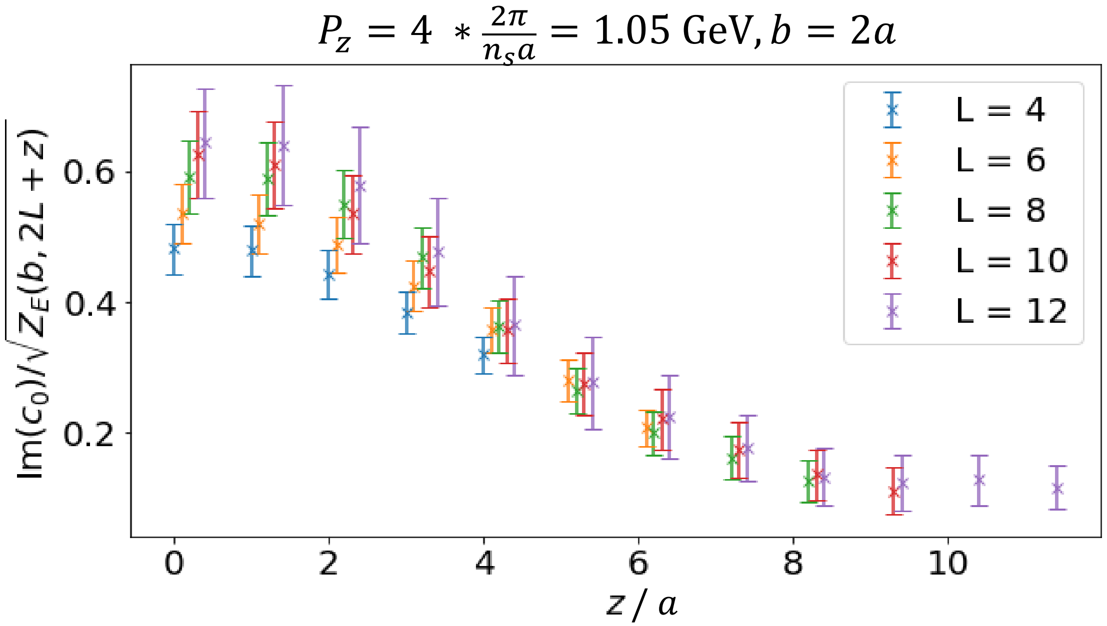
$P_z = 3 * \frac{2\pi}{n_s a} = 0.79\ \text{GeV}, b = 3a$

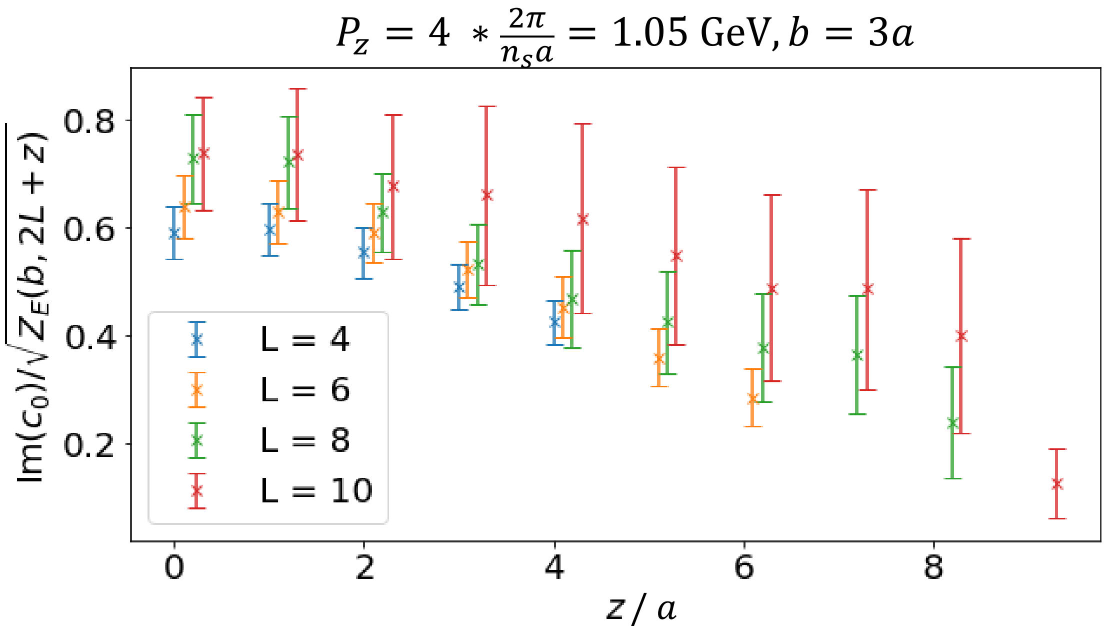


signal worse for increasing *b*

Results for subtracted quasi-TMDPDF $h_{\chi,\Gamma}$: $P_{\text{lat}} = 4$ ($P_z = 1.05$ GeV)

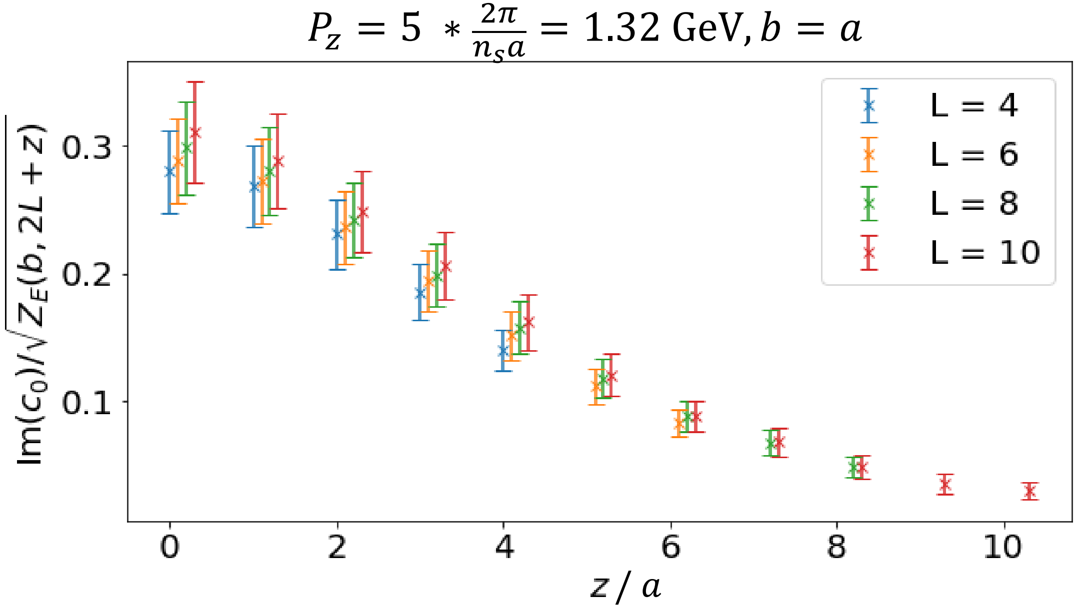


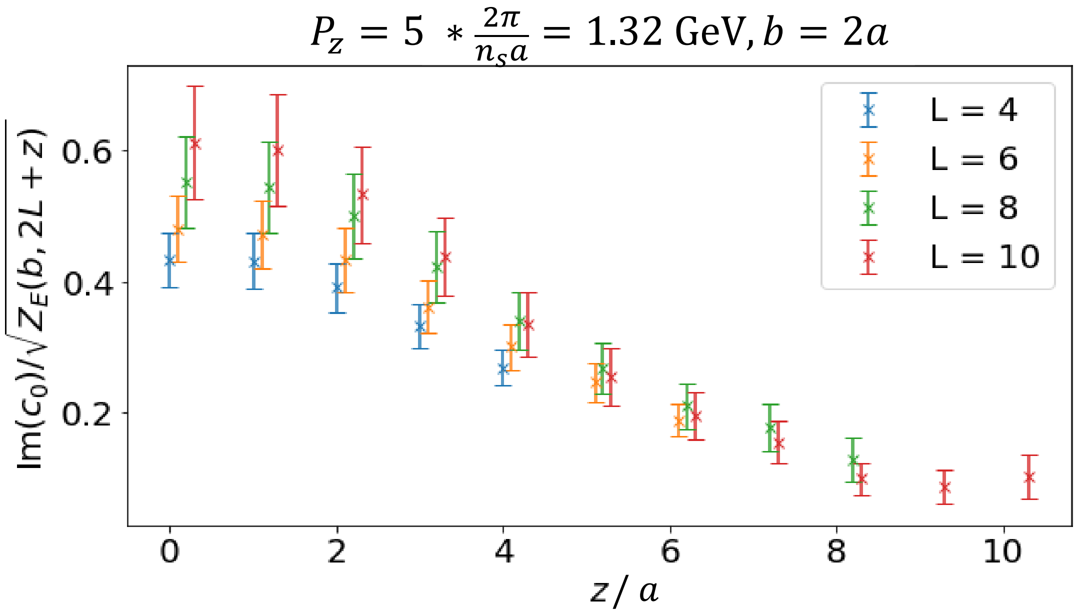


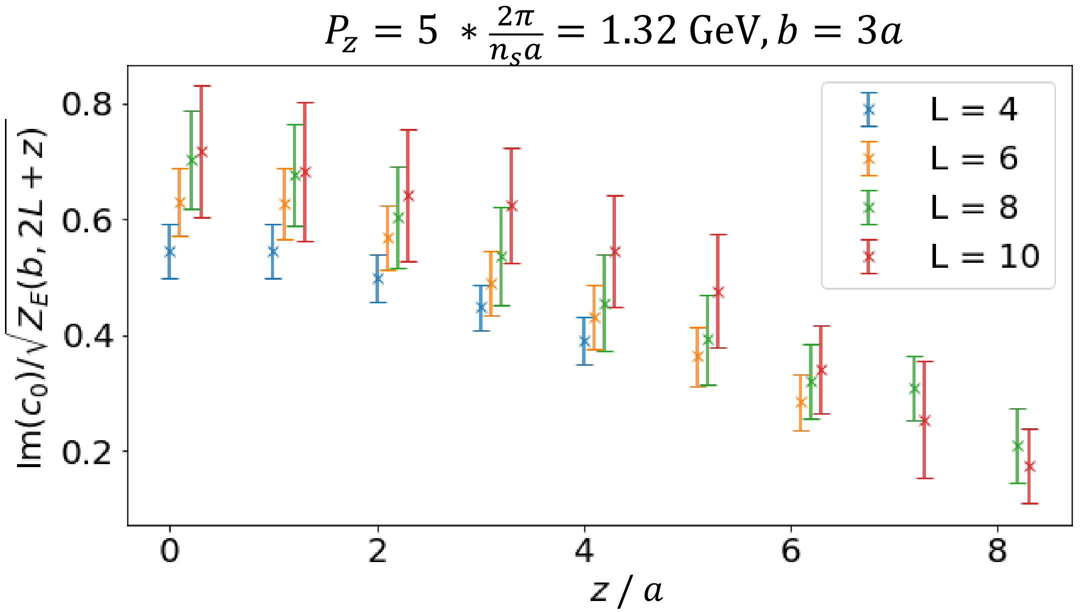


signal worse for increasing *b*

Results for subtracted quasi-TMDPDF $h_{\chi,\Gamma}$: $P_{\mathrm{lat}} = 5$ ($P_z = 1.32$ GeV)



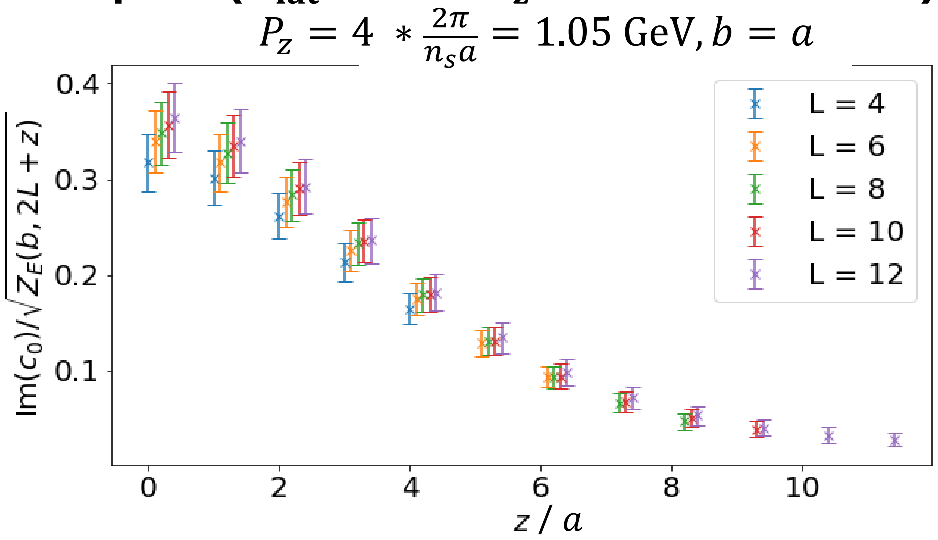


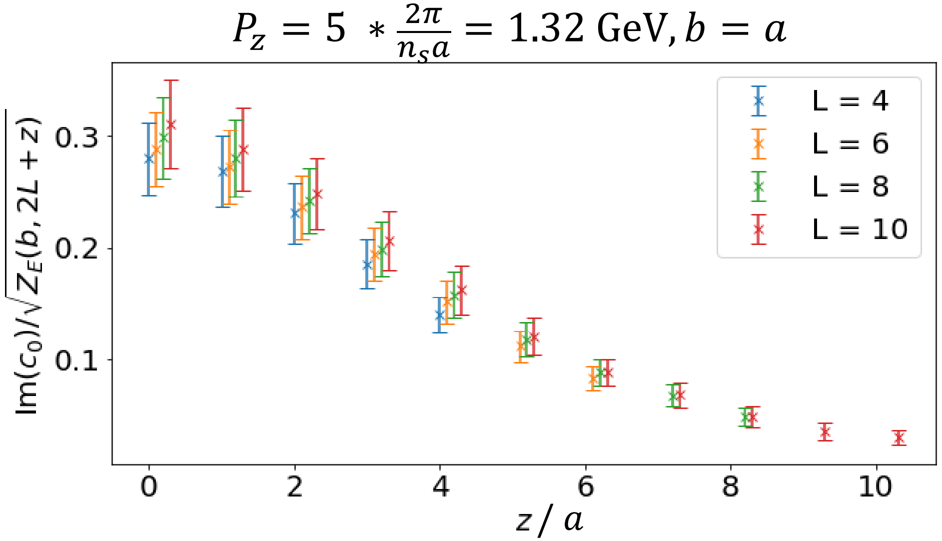


signal worse for increasing *b*

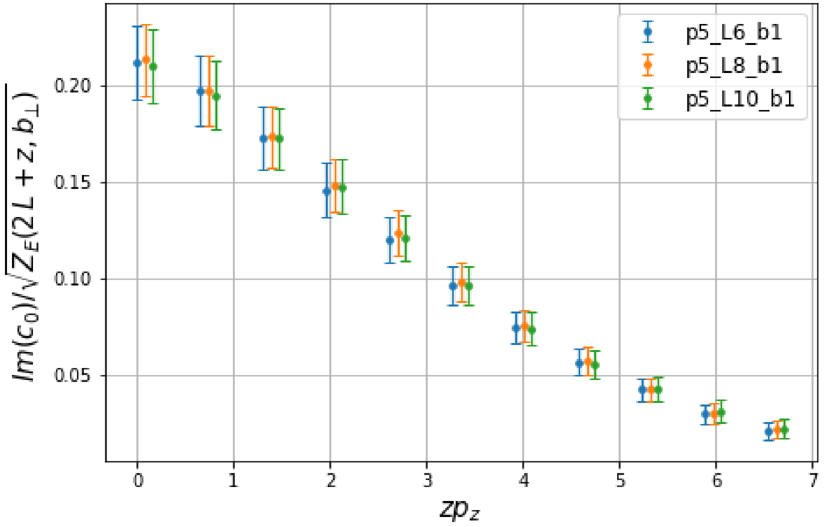
Results for subtracted quasi-TMDPDF $h_{\chi,\Gamma}$: π, n ($b = a$)

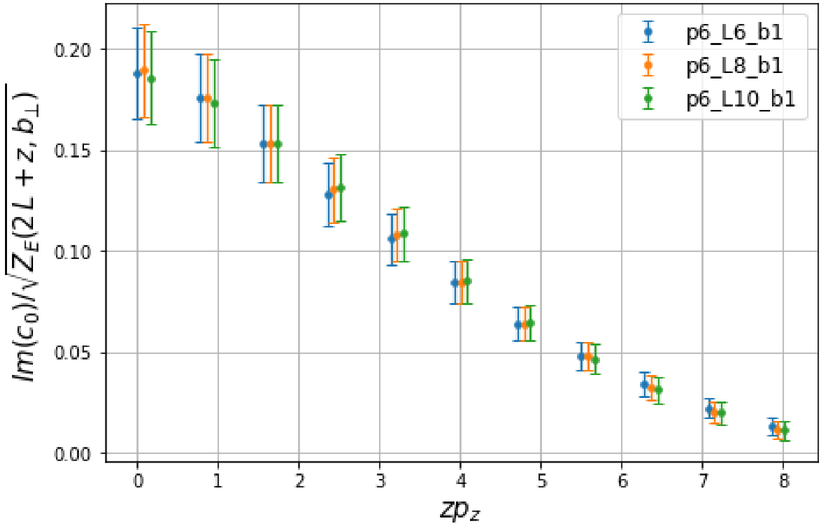
pion ($P_{\text{lat}} = 4, 5$; $P_z = 1.05, 1.32$ GeV)





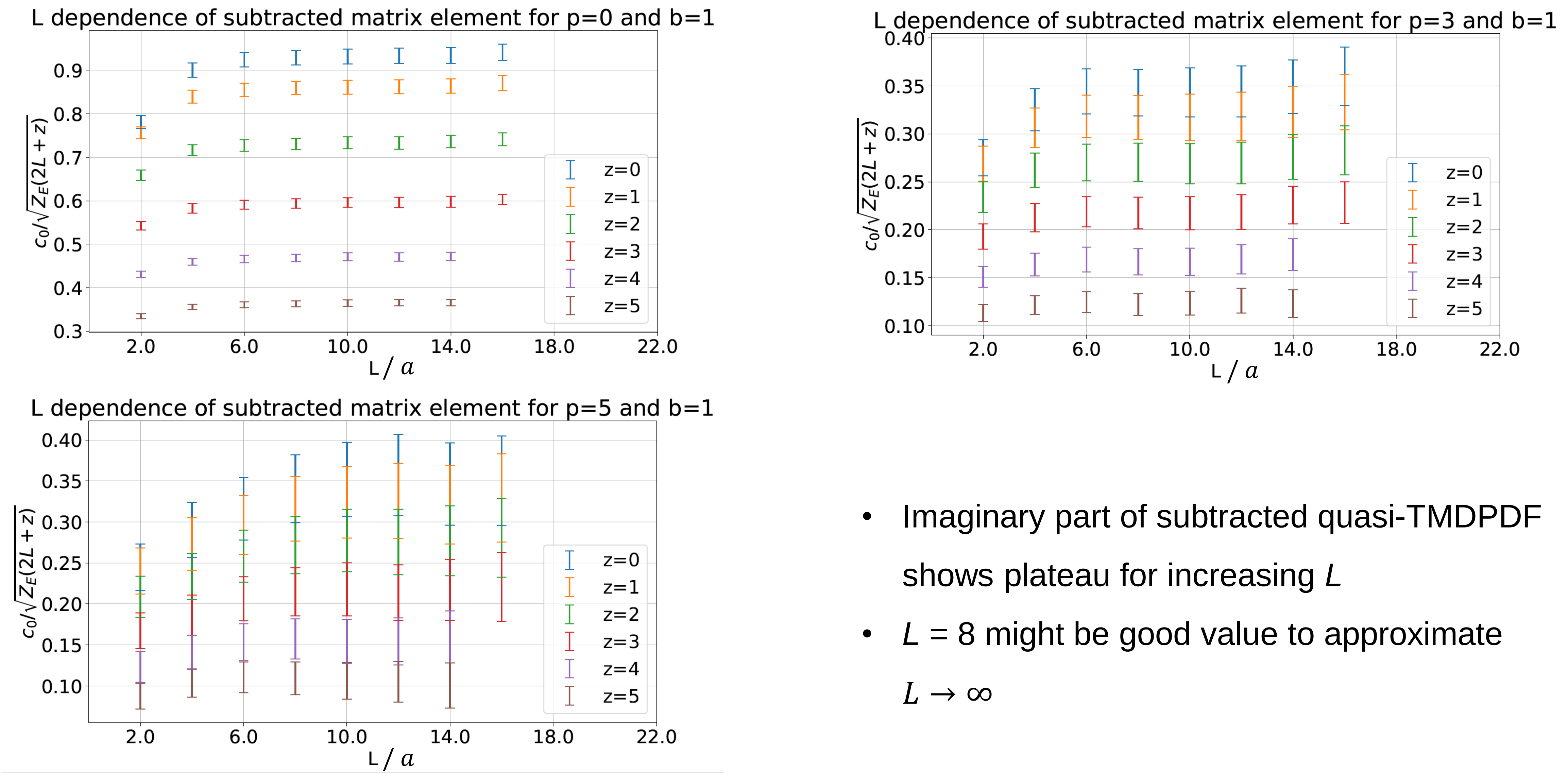
nucleon ($P_{\text{lat}} = 5, 6$; $P_z = 1.32, 1.58$ GeV)





Nucleon plots provided by Lingquan Ma

Limit $L \to \infty$: investigation of L-dependence of $h_{\chi,\Gamma}$



- Imaginary part of subtracted quasi-TMDPDF shows plateau for increasing *L*
- *L* = 8 might be good value to approximate $L \to \infty$

To do: Cancellation of remaining divergences

- Remaining logarithmic divergences from the endpoint of the Wilson links
- UV divergence multiplicative
- As in [3] / Qi-An Zhang‘s talk at 13:30:

  Extract logaritmic divergence factor Z_O from bare matrix elements at p = 0 and short distances z, $b_\perp$

$$Z_O(1/a, \mu, \Gamma) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma(z, b_\perp, 0, a, L)}{\sqrt{Z_E(2L + z, b_\perp, a)}\, \tilde{h}^{\overline{\mathrm{MS}}}_\Gamma(z, b_\perp, \mu)}$$

  - $z \ll \Lambda^{-1}_{\mathrm{QCD}}$, $b_\perp$ small→ Perturbation theory works well
  - $\tilde{h}^{\overline{\mathrm{MS}}}_\Gamma(z, b_\perp, \mu)$: RG evolved perturbation results

- Divide subtracted quasi-TMDPDF $h_{\chi,\Gamma}$ by logarithmic divergence factor Z_O

3: J.-C. He at al., arXiv: 2211.02340 [hep-lat]

Question about momentum P_z necessary for pion and nucleon

- Lorentz boost factor:

$$\gamma = \sqrt{1 + \left(\frac{p}{m_o c}\right)^2}$$

- Pion: m_π = 339 MeV

P_{lat} = 3	P_z = 0.79 GeV	γ = 2.54
P_{lat} = 4	P_z = 1.05 GeV	γ = 3.25
P_{lat} = 5	P_z = 1.32 GeV	γ = 4.03

- Nucleon: m_N = 1 106 MeV (from Lingquan Ma's 2pt fit)

P_{lat} = 7	P_z = 1.85 GeV	γ = 1.94
P_{lat} = 8	P_z = 2.11 GeV	γ = 2.15
P_{lat} = 10	P_z = 2.63 GeV	γ = 2.58

→ Need smaller momenta for pion as for nucleon

– Matching formula between quasi-TMDPDF and TMDPDF has power-corrections of order $(M^2/P_z)^2$ [18] (amongst others)

→ Is $P_z = 5 * \frac{2\pi}{n_s a} = 1.32\ \text{GeV}$ large enough? (need to check convergence with increasing P_z)

18: X. Ji at al., Rev. Mod. Phys. 93, 35005 (2021), arXiv: 2004.03543 [hep-ph]

Summary

- Unsubtracted quasi-Boer-Mulders function $\tilde{h}_{\chi,\Gamma}(b, z, L, P_z; 1/a\ )$ for pion extracted from fits of correlation functions calculated on CLS ensemble X650 (for multiple values of b, L, P_z)
- Comparison of jackknife and bootstrap resampling shows good agreement
- Calculation of rectangular Wilson loop $Z_E(b, 2L + z; 1/a)$
- Subtracted quasi-Boer-Mulders function shows good convergence with increasing *L*
- $L = 8$ might be good choice for approximation of $L \to \infty$

Next steps

- Calculation of more source-sink separations (t_{seq} = 9*a*, 10*a* currently running on Athene)
- Increasing the statistics (currently ½ of X650 configurations is used)

- Cancellation of logarithmic divergences with Z_O from matrix elements at $P_z = 0$ and short distances z, $b_\perp$
- Extrapolation to large $\lambda = zP_z$ [19] with

$$\tilde{h}_\Gamma(\lambda) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\lambda/\lambda_0}$$

- Fourier transformation of renormalized quasi-TMDPDF to momentum space
- Extract physical TMDPDF by using instrinsic soft function and CS kernel (calculated for X650 in [8]) and extrapolating $P_z \to \infty$

- Repeat calculation for other CLS ensembles with smaller lattice spacings

19: X. Ji at al., Nucl.Phys. B964 (2021) 115311, arXiv: 2008.03886 [hep-ph]
8: M.-H. Chu et al., arXiv: 2306.06488 [hep-lat]