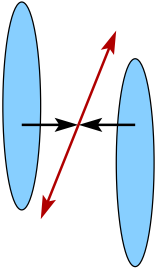
Double Parton Distributions off and on the lattice

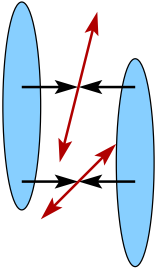
Christian Zimmermann, Daniel Reitinger, Markus Diehl,
Alexey Vladimirov, Andreas Schäfer et al.

- Why DPD physics needs lattice input.
- continuum QCD input - simple and over simplified models
- Lattice results for moments of DPDs
- Outlook and conclusion

DPIs: Simultaneous interaction of two parton pairs in one p+p, p+A or A+A collision. DPIs look unproblematic at first sight but they are not.



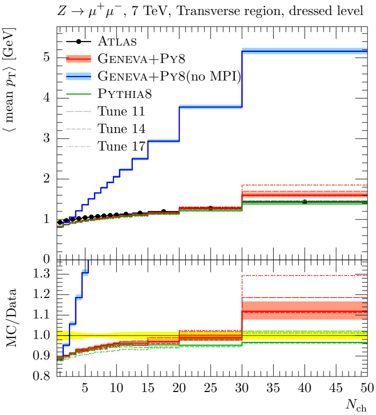
Single parton interaction



Double parton interaction

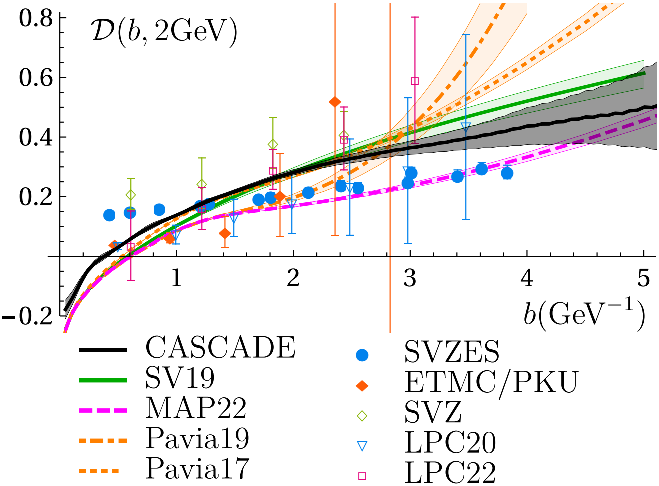
- A hadron is a strongly coupled highly entangled multi-particle state. Single parton distributions describe only very special aspects.
- high multiplicity $\Rightarrow$ enhanced MPI contributions; relevant for BSM searches

Phenomenological analysis of high energy data is largely based on event generators. Event generators mimic MPIs, but there is the danger of overfitting. Alioli, Bauer, Guns, Tackmann 1605.07192 mean charged particle p_T as function of N_{ch}.

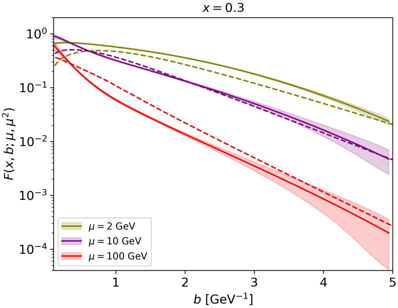


A similar problem is encountered for TMDs. One can use the predicted cross section from CASCADE to extract the CS kernel

Bermudez Martinez, Vladimirov 2206.01105



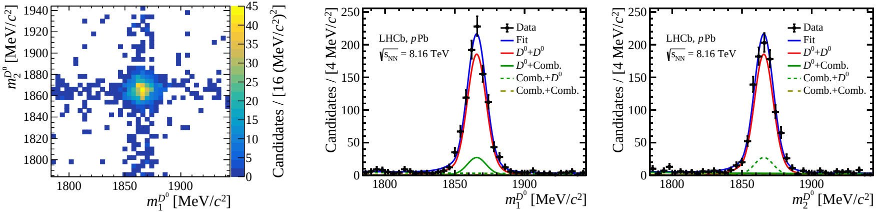
But the extracted TMDs differ between CASCADE and TMD factorization by about a factor two. In other words, each event generator defines a specific, largely unknown factorization scheme.



Empirically obtained matching between the two frames; solid lines: CASCADE, dashed lines: ART23

Another example: nuclear DPDs

R. Aaij *et al.* [LHCb] "Observation of Enhanced Double Parton Scattering in Proton-Lead Collisions at $\sqrt{s_{NN}}$ =8.16 TeV," PRL **125** (2020) 212001 arXiv:2007.06945.



To summarise, the production of LS and OS open charm hadron pairs as well as $J/\psi D^0$ pairs are studied in *p*Pb collisions at $\sqrt{s_{nn}} = 8.16$ TeV using fully reconstructed decays. The cross-section ratio between LS and OS pairs is found to be a factor of three higher than that in *pp* data. ... The effective cross-section and nuclear modification factor for $J/\psi D^0$ and $D^0 D^0$ are in general compatible with the expected enhancement factor of three for DPS over SPS production ratio from *pp* to *p*Pb collisions.

compatible with expectations: I. Helenius and H. Paukkunen, "Double D-meson production in proton-proton and proton-lead collisions at the LHC," arXiv:1906.06971.

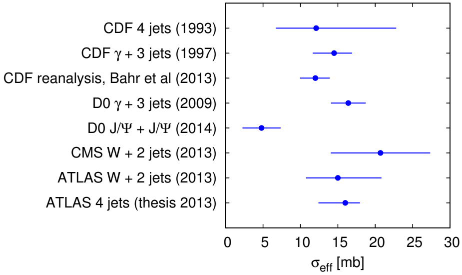
We, i.e. P. Zurita (Madrid) and Peter Plößl (DESY) et al. try to improve on this. The uncertainties for DPIs in nuclei seem to be huge.

Naive expectations suggest roughly a factor 9 enhancement for Pb-Pb.

The value of phenomenology is very limited for DPDs because it is usually based on strong assumptions.

$$d\sigma_{DPS} = \frac{d\sigma_{SPS} d\sigma_{SPS}}{2\sigma_{eff}}$$

which might or might not be good approximations



We came a long way

- M. Diehl, D. Ostermeier and A. Schäfer, "Elements of a theory for multiparton interactions in QCD," JHEP **1203** (2012) 089, arXiv:1111.0910
- M. Diehl, J. R. Gaunt, D. Ostermeier, P. Plößl and A. Schäfer, "Cancellation of Glauber gluon exchange in the double Drell-Yan process," JHEP **1601** (2016) 076, arXiv:1510.08696
- M. Diehl, J. R. Gaunt, P. Plößl and A. Schäfer "Two-loop splitting in double parton distributions," SciPost Phys. **7** (2019) 017, arXiv:1902.08019
- M. Diehl, P. Plößl and A. Schäfer, "Proof of sum rules for double parton distributions in QCD," Eur. Phys. J. C **79** (2019) 253, arXiv:1811.00289

- $\Rightarrow$ constraints for DPDs
  M. Diehl, J. R. Gaunt, D. M. Lang, P. Plößl and A. Schäfer, "Sum rule improved double parton distributions in position space," Eur. Phys. J. C **80** (2020) 468, arXiv:2001.10428
- lattice calculations for pion and nucleon
  G. S. Bali, L. Castagnini, M. Diehl, J. R. Gaunt, B. Gläßle, A. Schäfer and C. Zimmermann, "Double parton distributions in the pion from lattice QCD," JHEP **02** (2021), 067, arXiv:2006.14826
  G. S. Bali, M. Diehl, B. Gläßle, A. Schäfer and C. Zimmermann, "Double parton distributions in the nucleon from lattice QCD," JHEP **09** (2021), 106, arXiv:2106.03451.
- The upshot: no justification for simple models

Definition of DPDs:

$$F_{a_1 a_2}(x_1, x_2, \mathbf{y}) = 2p^+ \int dy^- \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle h(p)|\mathcal{O}_{a_1}(y, z_1)\mathcal{O}_{a_2}(0, z_2)|h(p)\rangle$$

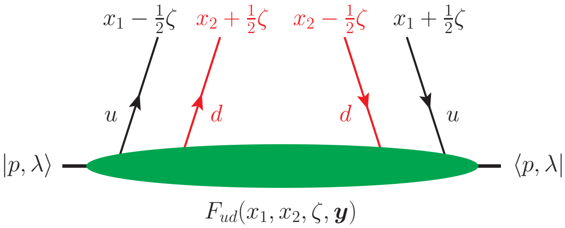


Illustration for the case where all fractions $x_i \pm \zeta/2$ are positive.

Cross sections can be written in terms of two DPDs, integrated over the transverse parton distance:

$$\int d^2\mathbf{y}\ F_{a_1 a_2}(x_1, x_2, \mathbf{y}) F_{b_1 b_2}(x_1', x_2', \mathbf{y})$$

Evolution equation for a quark DPD $F_{a_1 a_2}(x_1, x_2, \vec{y}_\perp; \mu)$

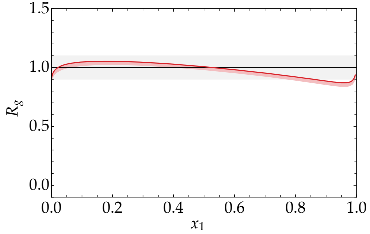
$$\begin{aligned} \frac{dF_{a_1 a_2}(x_1, x_2, \vec{y}_\perp; \mu)}{d\log\mu^2} &= \frac{\alpha_s(\mu)}{2\pi} \sum_{b_1} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} P_{a_1 b_1}\left(\frac{x_1}{z_1}\right) F_{b_1 a_2}(z_1, x_2, \vec{y}_\perp; \mu) \\ &+ \frac{\alpha_s(\mu)}{2\pi} \sum_{b_2} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} P_{a_2 b_2}\left(\frac{x_2}{z_2}\right) F_{a_1 b_2}(x_1, z_2, \vec{y}_\perp; \mu) \end{aligned}$$

Sum rules for quark number and momentum

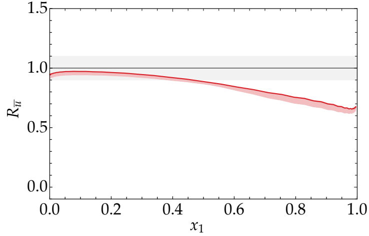
Gaunt and Stirling 0910.4347

$$\begin{aligned} \int_0^{1-x_1} dx_2\, F_{a_1 q_v}(x_1, x_2, \vec{\Delta}_\perp = \vec{0}_\perp; \mu) &= (N_{q_v} + \delta_{a_1, \bar{q}} - \delta_{a_1, q}) f_{a_1}(x_1; \mu) \\ \sum_{a_2} \int_0^{1-x_1} dx_2\, x_2\, F_{a_1 a_2}(x_1, x_2, \vec{\Delta}_\perp = \vec{0}_\perp; \mu) &= (1 - x_1) f_{a_1}(x_1; \mu) \end{aligned}$$

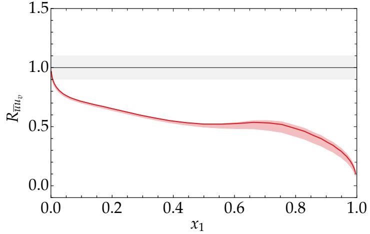
A simple but not naive model: $\mu = 2.25$ GeV; ν between $\mu/2$ and 2μ



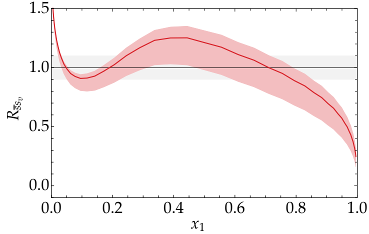
(a) g momentum sum rule



(b) $\bar{u}$ momentum sum rule

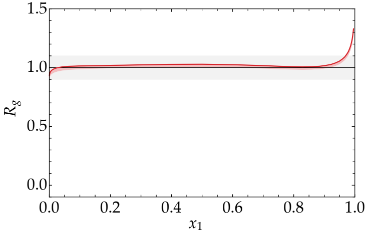


(c) $\bar{u}u_v$ number sum rule

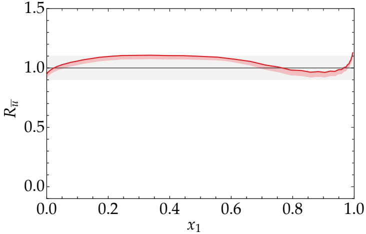


(d) $\bar{s}s_v$ number sum rule

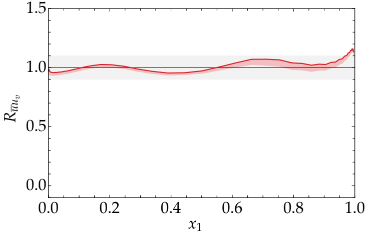
The improved model: $\mu = 2.25$ GeV; ν between $\mu/2$ and 2μ



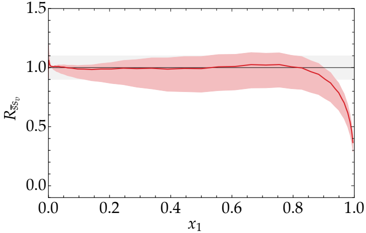
(e) g momentum sum rule



(f) $\bar{u}$ momentum sum rule



(g) $\bar{u}u_v$ number sum rule



(h) $\bar{s}s_v$ number sum rule

The evolution for perturbatively small $\vec{y}_\perp$ is treated separately, e.g. for the "simple model" at $\mu_0 = 1$ GeV.

$$
\begin{aligned}
R_{a_1 q_v} &= \frac{\int dx_2 F_{a_1 q_v}(x_1, x_2; \mu)}{N_{q_v} + \delta_{a_1,\bar{q}} - \delta_{a_1,q}) f_{a_1}(x_1; \mu)} \\
R_{a_1} &= \frac{\sum_{a_2} \int dx_2 x_2 F_{a_1 a_2}(x_1, x_2; \mu)}{(1 - x_1) f_{a_1}(x_1; \mu)} \\
F_{a_1 a_2}(x_1, x_2, \vec{y}_\perp; \mu) &= F_{a_1 a_2, int}(x_1, x_2, \vec{y}_\perp; \mu) + F_{a_1 a_2, spl}(x_1, x_2, \vec{y}_\perp; \mu) \\
F_{a_1 a_2, int}(x_1, x_2, \vec{y}_\perp; \mu) &= f_{a_1}(x_1; \mu) f_{a_2}(x_2; \mu) \frac{e^{-\vec{y}_\perp^2 / 4 h_{a_1 a_2}}}{4\pi h_{a_1 a_2}} \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 (1 - x_2)^2} \\
F_{a_1 a_2, spl}(x_1, x_2, \vec{y}_\perp; \mu) &= \frac{f_{a_0}(x_1 + x_2; \mu_y)}{\pi \vec{y}_\perp^2 (x_1 + x_2)} \frac{\alpha_s(\mu_y)}{2\pi} P_{a_1 a_0}\left(\frac{x_1}{x_1 + x_2}\right) \\
\mu_y &= \frac{b_0 \sqrt{1 + \vec{y}_\perp^2 / y_{max}^2}}{\sqrt{\vec{y}_\perp^2}}
\end{aligned}
$$

Take home message: Sum rule preservation under evolution severely constraints DPD models.

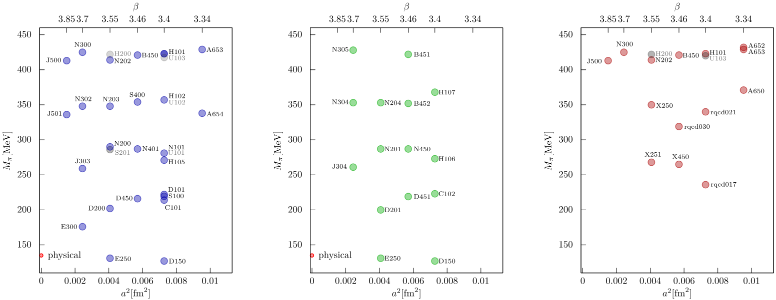
“Simple Models” are mostly wrong.

Sum rules are insufficient to pin DPDs down. Additional input is needed from Lattice QCD.

So far we have only calculated Mellin moments.

Recently Jianhui Zhang (arXiv:2304.12481; next talk) and Jaarsma et al. (arXiv:2305.09716) have analysed the possibility of quasiDPD calculations.

We use CLS ensembles



Available CLS ensembles; Left: $m_u + m_d + m_s = (m_u + m_d + m_s)$(physical); Middle: $m_s = m_s$(physical); Right: $m_u = m_d = m_s$.

For the nucleon we started with H102. Presently we analyse S400.

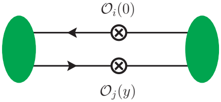
Pion

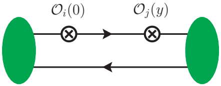
The main question: Is the product form

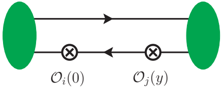
$$
\begin{aligned}
C^{ij}_{\Gamma\Gamma'}(\vec{y}) &= \langle \pi^+(p')|\bar{q}(0,\vec{y})\Gamma q(0,\vec{y})\ \bar{q}'(0,\vec{0})\Gamma' q'(0,\vec{0})|\pi^+(p)\rangle \\
&= \sum_X \langle \pi^+(p')|\bar{q}(t,\vec{y})\Gamma q(t,\vec{y})|X\rangle\ \langle X|\bar{q}'(t,\vec{0})\Gamma' q'(t,\vec{0})|\pi^+(p)\rangle \\
&\overset{?}{\approx} \int \frac{d^3k}{(2\pi)^3 2k^0}\ \langle \pi^+(p')|\bar{q}(t,\vec{y})\Gamma q(t,\vec{y})|\pi^+(k)\rangle \\
&\quad \langle \pi^+(k)|\bar{q}'(t,\vec{0})\Gamma' q'(t,\vec{0})|\pi^+(p)\rangle
\end{aligned}
$$

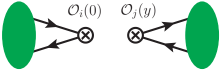
a good approximation or not ?

Lattice correlations we measure

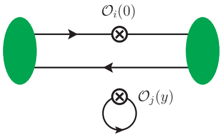
$C_1^{ij}(y) =$ 

$C_2^{ij}(y) =$ 

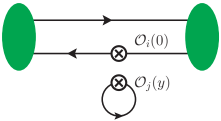
$= \eta_C^{ij} \times$ 

$A^{ij}(y) =$ 

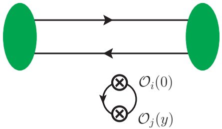
$S_1^{ij}(y) =$



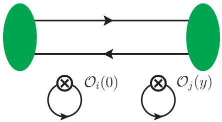
$= \eta_C^{ij} \times$



$S_2^{ij}(y) =$



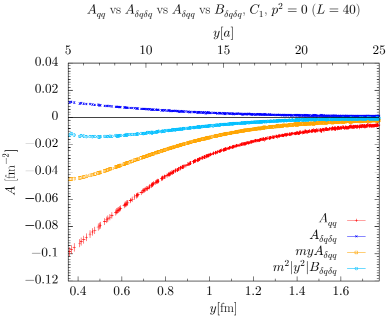
$D^{ij}(y) =$

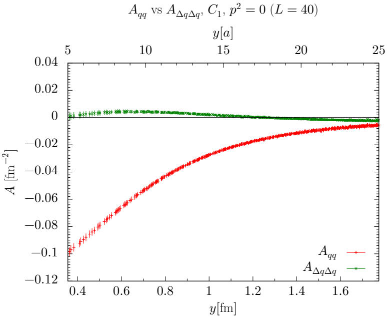


pion matrix elements for $N_f = 2$

$$
\begin{aligned}
\langle\pi^+|\mathcal{O}_i^{uu}(0)\mathcal{O}_j^{dd}(y)|\pi^+\rangle &= C_1^{ij}(y) + \left[S_1^{ij}(y) + S_1^{ji}(-y)\right] + D^{ij}(y) \\
\langle\pi^+|\mathcal{O}_i^{uu}(0)\mathcal{O}_j^{uu}(y)|\pi^+\rangle &= \left[C_2^{ij}(y) + C_2^{ji}(-y)\right] + \left[S_1^{ij}(y) + S_1^{ji}(-y)\right] \\
&+ D^{ij}(y) + S_2^{ij}(y) \\
\langle\pi^0|\mathcal{O}_i^{uu}(0)\mathcal{O}_j^{dd}(y)|\pi^0\rangle &= \left[S_1^{ij}(y) + S_1^{ji}(-y)\right] + D^{ij}(y) \\
&- \tfrac{1}{2}\left[A^{ij}(y) + A^{ji}(-y)\right] \\
\langle\pi^0|\mathcal{O}_i^{uu}(0)\mathcal{O}_j^{uu}(y)|\pi^0\rangle &= C_1^{ij}(y) + \left[S_1^{ij}(y) + S_1^{ji}(-y)\right] + D^{ij}(y) \\
&+ \left[C_2^{ij}(y) + C_2^{ji}(-y)\right] + \tfrac{1}{2}\left[A^{ij}(y) + A^{ji}(-y)\right] \\
&+ S_2^{ij}(y) \\
\langle\pi^+|\mathcal{O}_i^{du}(0)\mathcal{O}_j^{ud}(y)|\pi^+\rangle &= 2C_2^{ij}(y) + A^{ij}(y) + S_2^{ij}(y) \\
\langle\pi^0|\mathcal{O}_i^{du}(0)\mathcal{O}_j^{ud}(y)|\pi^0\rangle &= -C_1^{ij}(y) + \left[C_2^{ij}(y) + C_2^{ji}(-y)\right] + S_2^{ij}(y) \\
\langle\pi^-|\mathcal{O}_i^{du}(0)\mathcal{O}_j^{du}(y)|\pi^+\rangle &= 2C_1^{ij}(y) + \left[A^{ij}(y) + A^{ji}(-y)\right] \\
\sqrt{2}\,\langle\pi^0|\mathcal{O}_i^{du}(0)\mathcal{O}_j^{uu}(y)|\pi^+\rangle &= C_1^{ij}(y) + \left[C_2^{ij}(y) - C_2^{ji}(-y)\right] + A^{ij}(y)
\end{aligned}
$$

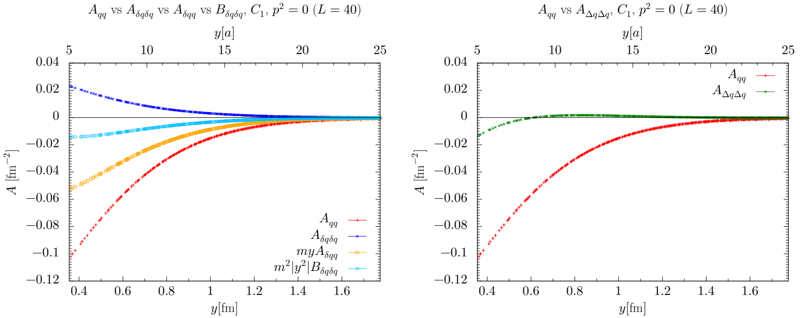
light quarks





Effects of transverse (left) and longitudinal (right) polarisation for graph C_1.

strange quarks



Effects of transverse (left) and longitudinal (right) polarisation for graph C_1.

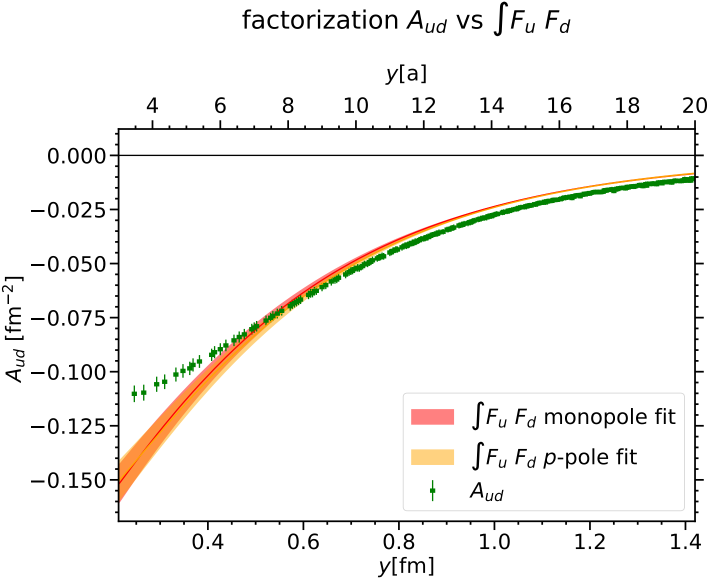
The RQCD ensembles used; $N_f = 2$, Clover-Wilson fermions, down to nearly physical mass ($m_\pi = 150$ MeV). N number of configurations, $N_{\rm sm}$ number of Wuppertal smearing iterations, $t_{\rm f}$ sink-source time differences. The error of the pion mass combines statistical and systematic errors.

Ensemble	β	a [fm]	κ	V	m_π [GeV]	Lm_π	N	$N_{\rm sm}$	$t_{\rm f}/a$
IV	5.29	0.071	0.13632	$32^3 \times 64$	0.2946(14)	3.42	2023	400	15
V	5.29	0.071	0.13632	$40^3 \times 64$	0.2888(11)	4.19	2025	400	15

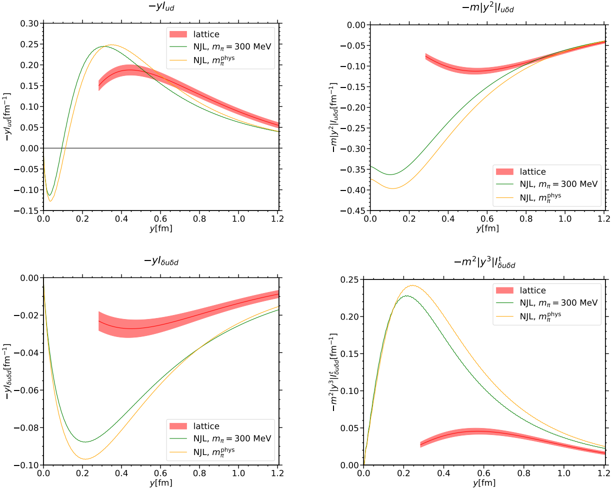
Direct Tests of naive factorization: Integrals based on

$$\int 4pt - correlator \stackrel{?}{=} \int (formfactor)^2$$

Direct tests of naive factorization VV case



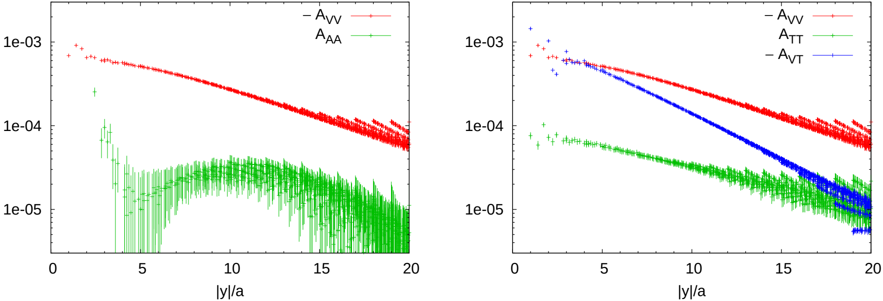
Comparison with a NJL model



Courtoy et al. arXiv:1909.09530

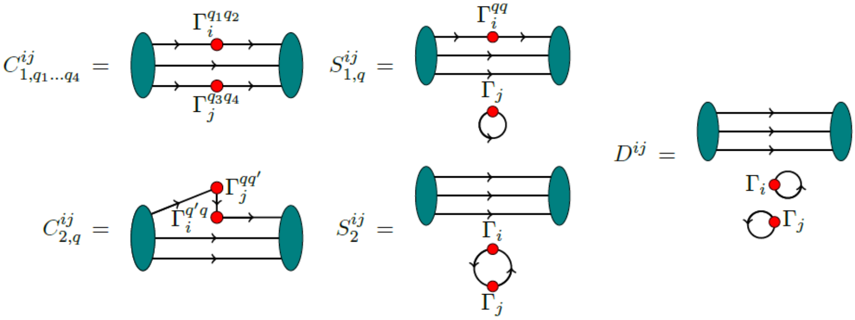
Spin correlations

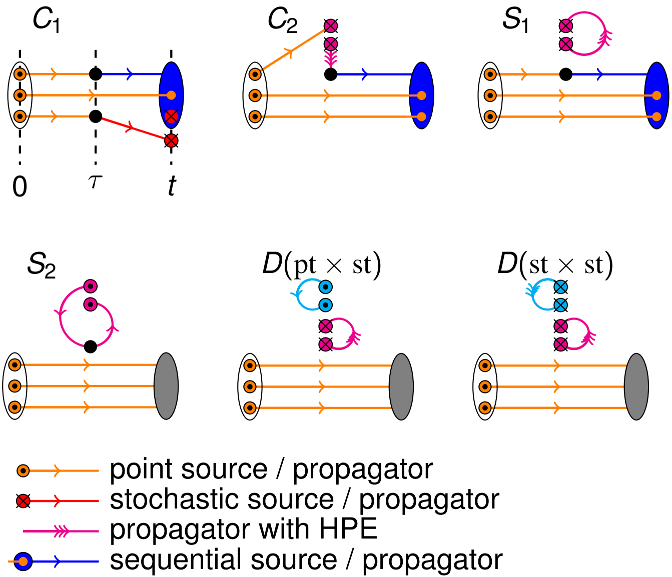
AA: longitudinal spin correlation $u^{\uparrow}\bar{d}^{\uparrow} + u^{\downarrow}\bar{d}^{\downarrow} - u^{\uparrow}\bar{d}^{\downarrow} - u^{\downarrow}\bar{d}^{\uparrow}$
TT: transverse spin correlation $\vec{s}_u \cdot \vec{s}_d$
VT: $\vec{y} \cdot \vec{s}_{\bar{d}}$



This work equally well for the nucleon.

Proton


$C^{ij}_{1,q_1 \ldots q_4} =$
$\Gamma_i^{q_1 q_2}$
$\Gamma_j^{q_3 q_4}$
$S^{ij}_{1,q} =$
Γ_i^{qq}
Γ_j
$D^{ij} =$
Γ_i
Γ_j
$C^{ij}_{2,q} =$
$\Gamma_j^{qq'}$
$\Gamma_i^{q'q}$
$S^{ij}_2 =$
Γ_i
Γ_j


C_1
C_2
S_1
0
τ
t
S_2
$D(\mathrm{pt} \times \mathrm{st})$
$D(\mathrm{st} \times \mathrm{st})$
point source / propagator
stochastic source / propagator
propagator with HPE
sequential source / propagator

$$M^{\mu_1\cdots\mu_2\cdots}_{q_1q_2,i_1i_2}(p,y) \;=\; \sum_\lambda \langle p,\lambda|J^{\mu_1\cdots}_{q_1,i_1}(y)\,J^{\mu_2\cdots}_{q_2,i_2}(0)|p,\lambda\rangle$$

We calculated only with $J^\mu_{q,V}(y)=\bar{q}(y)\gamma^\mu q(y)$ and $J^\mu_{q,A}(y)=\bar{q}(y)\gamma^\mu\gamma_5 q(y)$ and $J^{\mu\nu}_{q,T}(y)=\bar{q}(y)\sigma^{\mu\nu}q(y)$

Expansion in Lorentz-invariant functions

$$\begin{aligned}
M^{\{\mu\nu\}}_{q_1q_2,VV}-\frac{1}{4}g^{\mu\nu}g_{\alpha\beta}M^{\alpha\beta}_{q_1q_2,VV} &= u^{\mu\nu}_{VV,A}A_{q_1q_2}+u^{\mu\nu}_{VV,B}m^2B_{q_1q_2}+u^{\mu\nu}_{VV,C}m^4C_{q_1q_2}\\
M^{\mu\nu\rho}_{q_1q_2,TV}+\frac{2}{3}g^{\rho[\mu}M^{\nu]\alpha\beta}_{q_1q_2,TV}g_{\alpha\beta} &= u^{\mu\nu\rho}_{TV,A}mA_{\delta q_1q_2}+u^{\mu\nu\rho}_{TV,B}m^3B_{\delta q_1q_2}\\
\frac{1}{2}\left[M^{\mu\nu\rho\sigma}_{q_1q_2,TT}+M^{\rho\sigma\mu\nu}_{q_1q_2,TT}\right] &= \tilde{u}^{\mu\nu\rho\sigma}_{TT,A}A_{\delta q_1\delta q_2}+\tilde{u}^{\mu\nu\rho\sigma}_{TT,B}m^2B_{\delta q_1\delta q_2}+\tilde{u}^{\mu\nu\rho\sigma}_{TT,C}m^2C_{\delta q_1\delta q_2}\\
&+ \tilde{u}^{\mu\nu\rho\sigma}_{TT,D}m^4D_{\delta q_1\delta q_2}+u^{\mu\nu\rho\sigma}_{TT,E}m^2\widetilde{E}_{\delta q_1\delta q_2}
\end{aligned}$$

The DPD Mellin moments

$$\int_{-\infty}^{\infty} dy^- \, e^{-i\zeta y^- p^+} M^{++}_{q_1 q_2, VV}(p,y)\Big|_{y^+=0,\, \vec{p}_\perp=\vec{0}_\perp} = 2p^+ I_{q_1 q_2}(\zeta, y^2)$$

$$\int_{-\infty}^{\infty} dy^- \, e^{-i\zeta y^- p^+} M^{++}_{q_1 q_2, AA}(p,y)\Big|_{y^+=0, \vec{p}_\perp=\vec{0}_\perp} = 2p^+ I_{\Delta q_1 \Delta q_2}(\zeta, y^2)$$

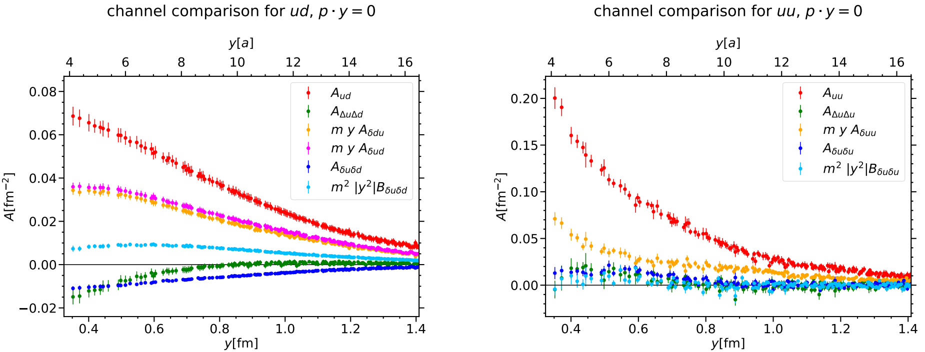
$$\int_{-\infty}^{\infty} dy^- \, e^{-i\zeta y^- p^+} M^{k_1 ++}_{q_1 q_2, TV}(p,y)\Big|_{y^+=0, \vec{p}_\perp=\vec{0}_\perp} = 2p^+ \vec{y}^{k_1}_\perp m I_{\delta q_1 q_2}(\zeta, y^2)$$

$$\int_{-\infty}^{\infty} dy^- \, e^{-i\zeta y^- p^+} M^{+k_2+}_{q_1 q_2, VT}(p,y)\Big|_{y^+=0, \vec{p}_\perp=\vec{0}_\perp} = 2p^+ \vec{y}^{k_2}_\perp m I_{q_1 \delta q_2}(\zeta, y^2)$$

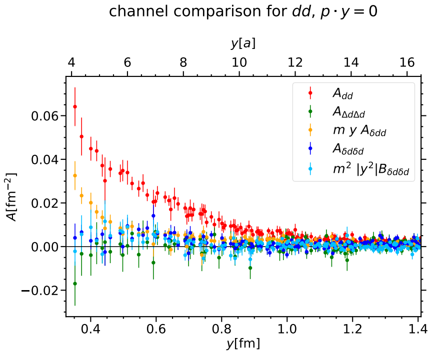
$$\int_{-\infty}^{\infty} dy^- \, e^{-i\zeta y^- p^+} M^{k_1 + k_2 +}_{q_1 q_2, TT}(p,y)\Big|_{y^+=0, \vec{p}_\perp=\vec{0}_\perp} = 2p^+ \big[\delta^{k_1 k_2} I_{\delta q_1 \delta q_2}(\zeta, y^2) - (2\vec{y}^{k_1}_\perp \vec{y}^{k_2}_\perp - \delta^{k_1 k_2} \vec{y}^2_\perp) m^2 I^t_{\delta q_1 \delta q_2}(\zeta, y^2)\big]$$

and finally:

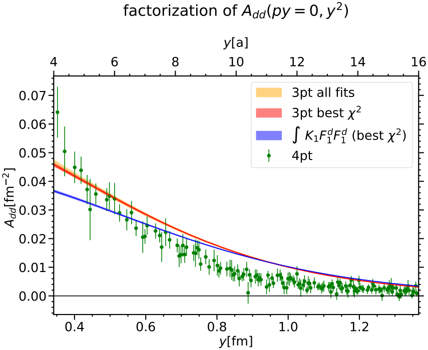
$$A_{a_1 a_2}(py = 0, y^2) = \frac{1}{\pi} \int_0^1 d\zeta \, I_{a_1 a_2}(\zeta, y^2)$$

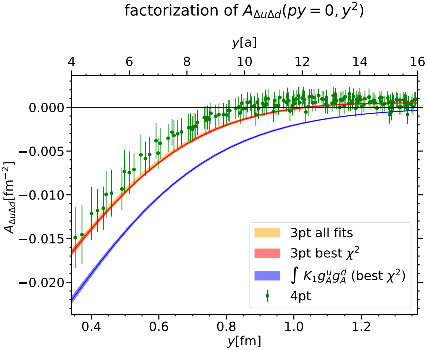


Comparison of the results for different quark polarizations for the flavor combinations Left: ud, Right: uu at $p \cdot y = 0$

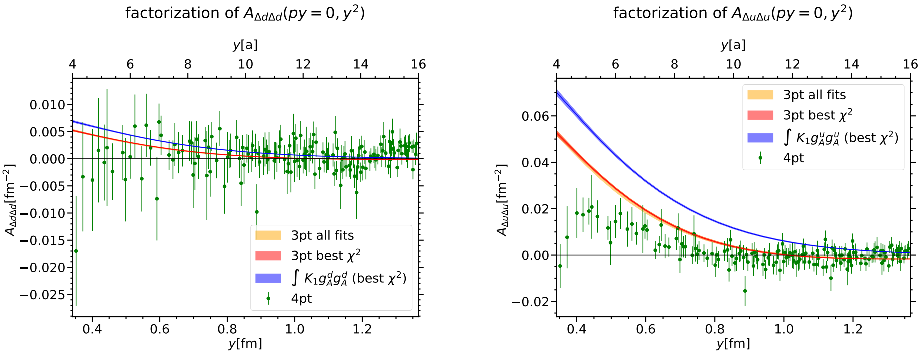


Comparison of the results for different quark polarizations for the flavor combination dd at $p \cdot y = 0$





The twist-two functions Left: A_{dd}, Right: $A_{\Delta u \Delta d}$ compared to the corresponding form factor integral.



The twist-two functions Left: $A_{\Delta d \Delta d}$, Right: $A_{\Delta u \Delta u}$ compared to the corresponding form factor integral.

Conclusions

- DPDs parameterize parton correlations in hadrons.
- Therefore, DPIs are not only a troublesome background but DPDs encode most relevant hadron physics.
- Event generators and phenomenological models give only a very approximate estimate for DPIs. High energy does not really help.
- Lattice input is crucial for a reliable description of DPDs.
- DPDs are large enough to give clear results, also for interference DPDs.
- quasi DPDs are an exciting perspective.