Moments of GPDs from the OPE of nonlocal quark bilinears

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In collaboration with: S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, and Y. Zhao

LaMET 2023

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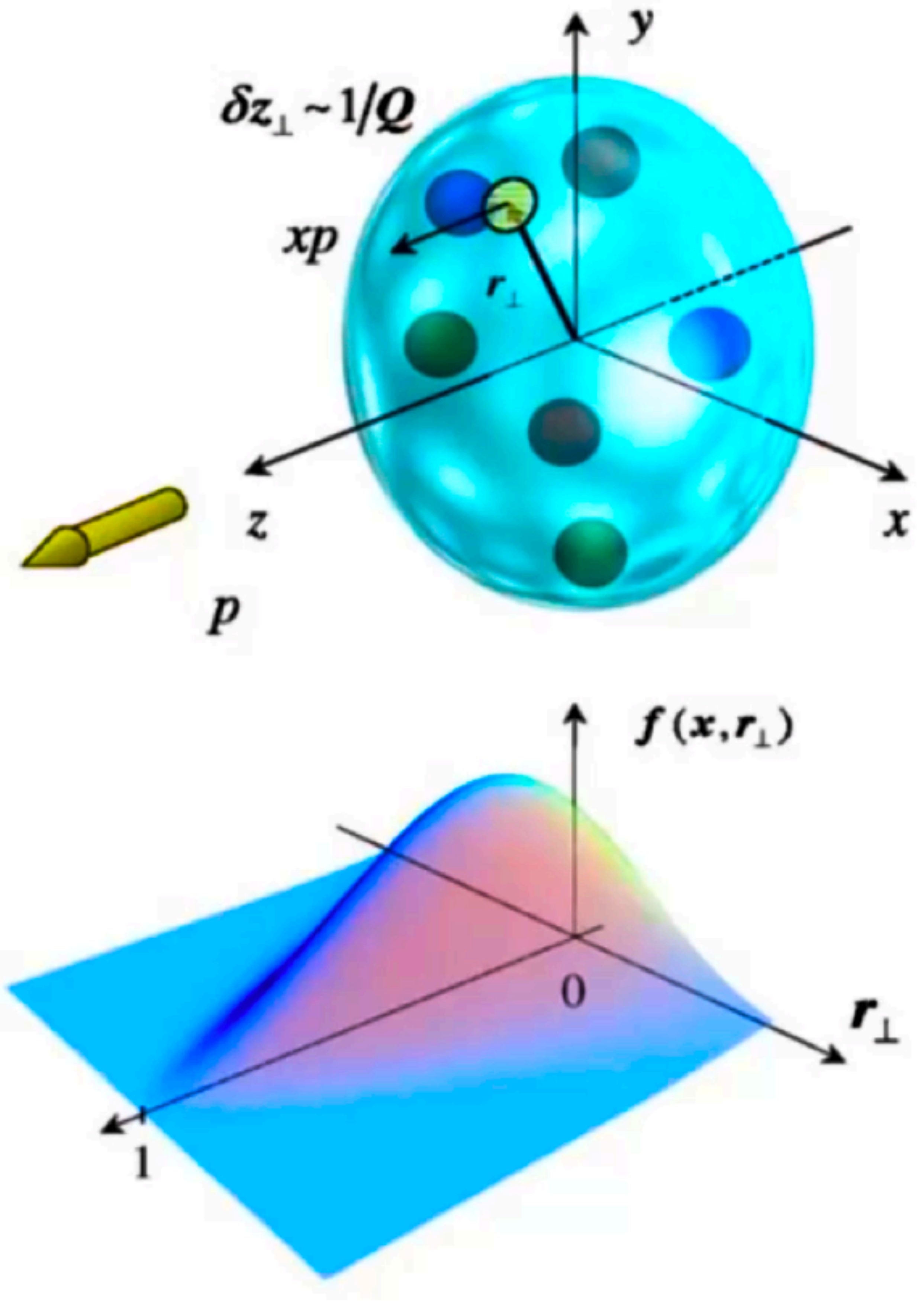
Generalized parton distributions

GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors.



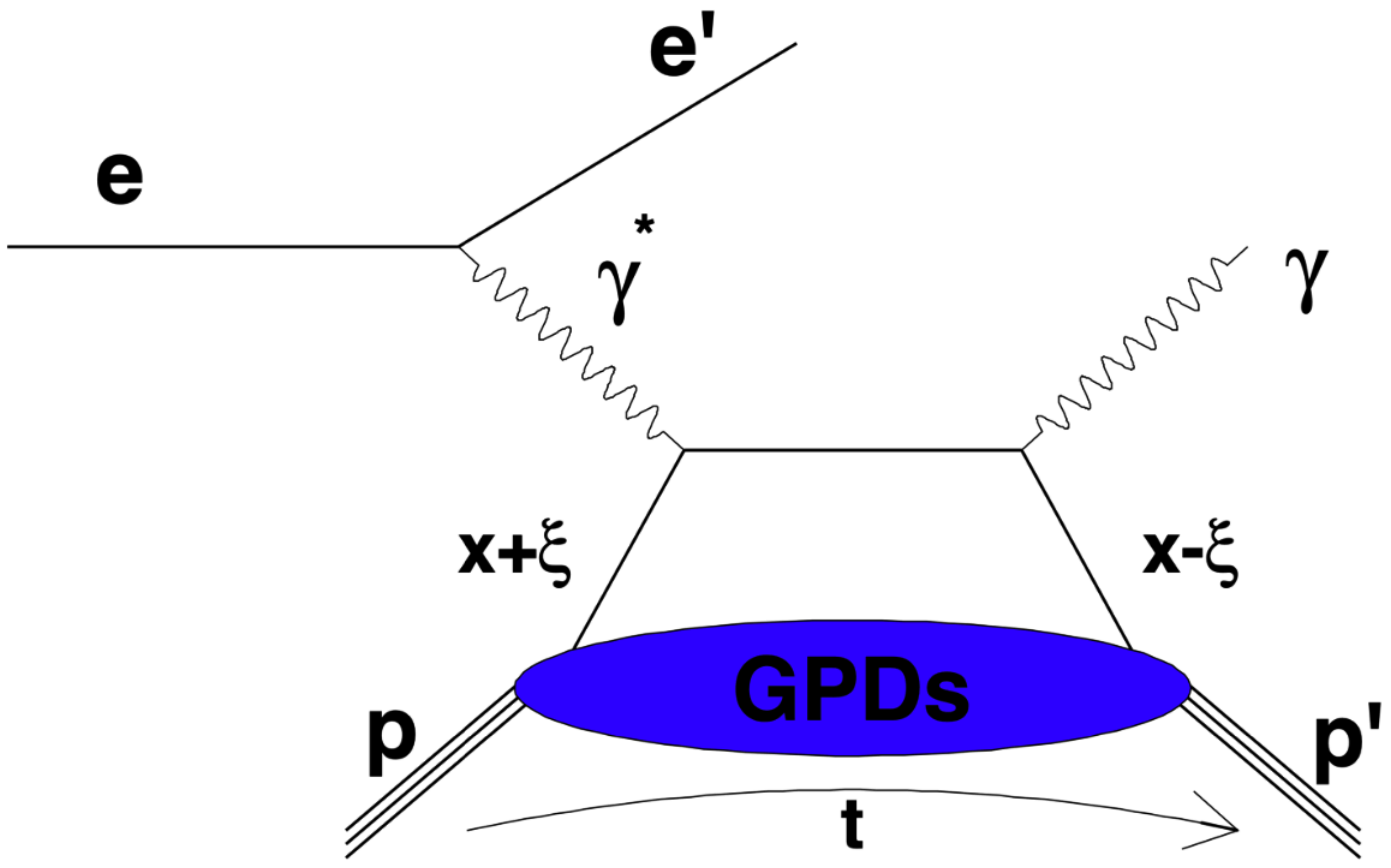
$$F_q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p_f | \bar{q}(-\tfrac{z}{2}) \gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

- Ji, PRL 78 (1997)
- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

Generalized parton distributions

DVCS



The golden process to study the quark GPDs is DVCS

Challenging:

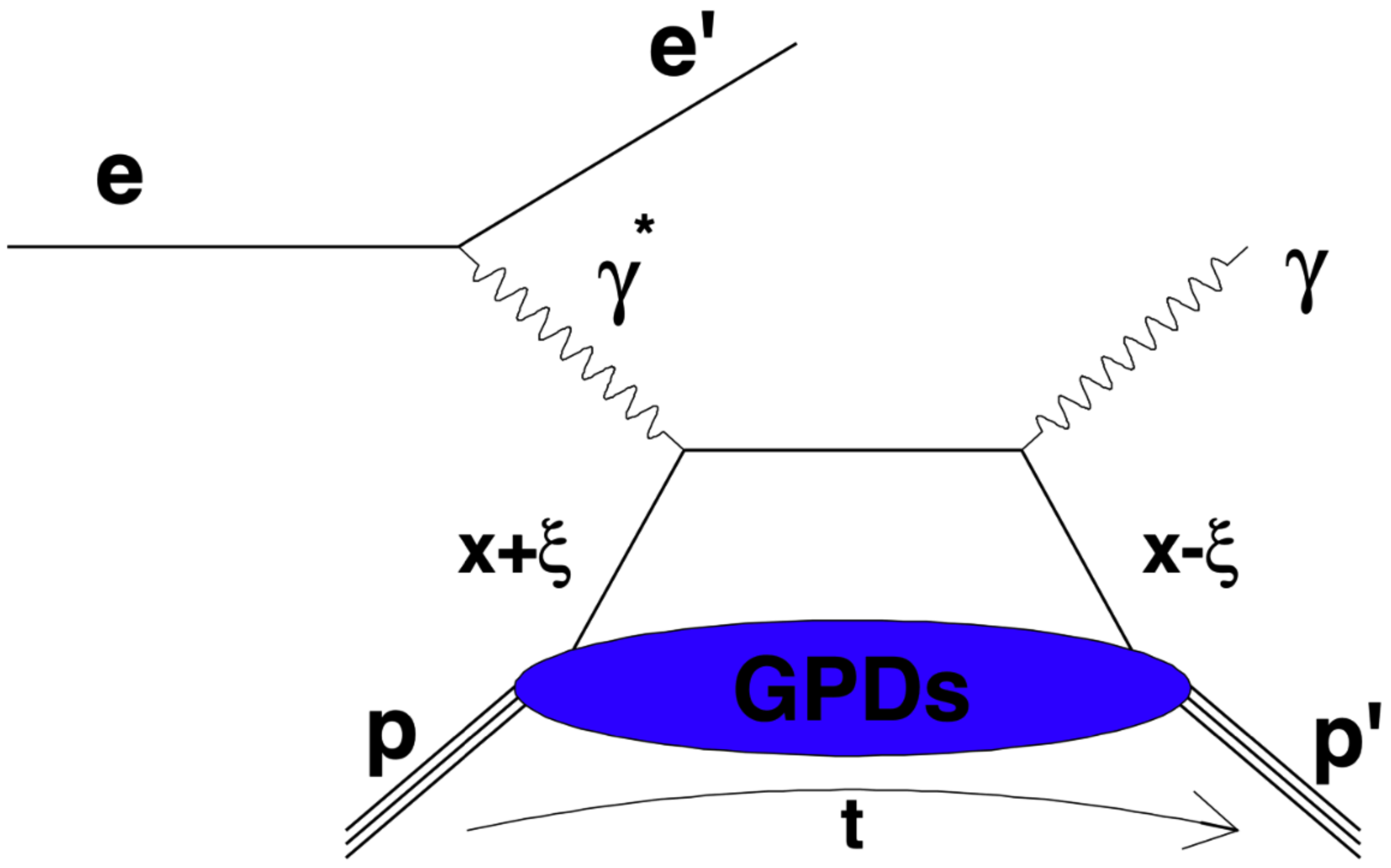
- Observables appear at the amplitude level.
- Multi-dimensionality (x, ξ, t).
- The momentum fraction x is integrated over (Compton Form Factors).

$$\mathcal{F}(\xi, t; Q^2) = \int_{-1}^{1} \mathrm{d}x \left[\frac{1}{\xi - x - i\epsilon} \pm \frac{1}{\xi + x - i\epsilon}\right] F(x, \xi, t; Q^2)$$

- Ji, PRL 78 (1997)

Generalized parton distributions

DVCS



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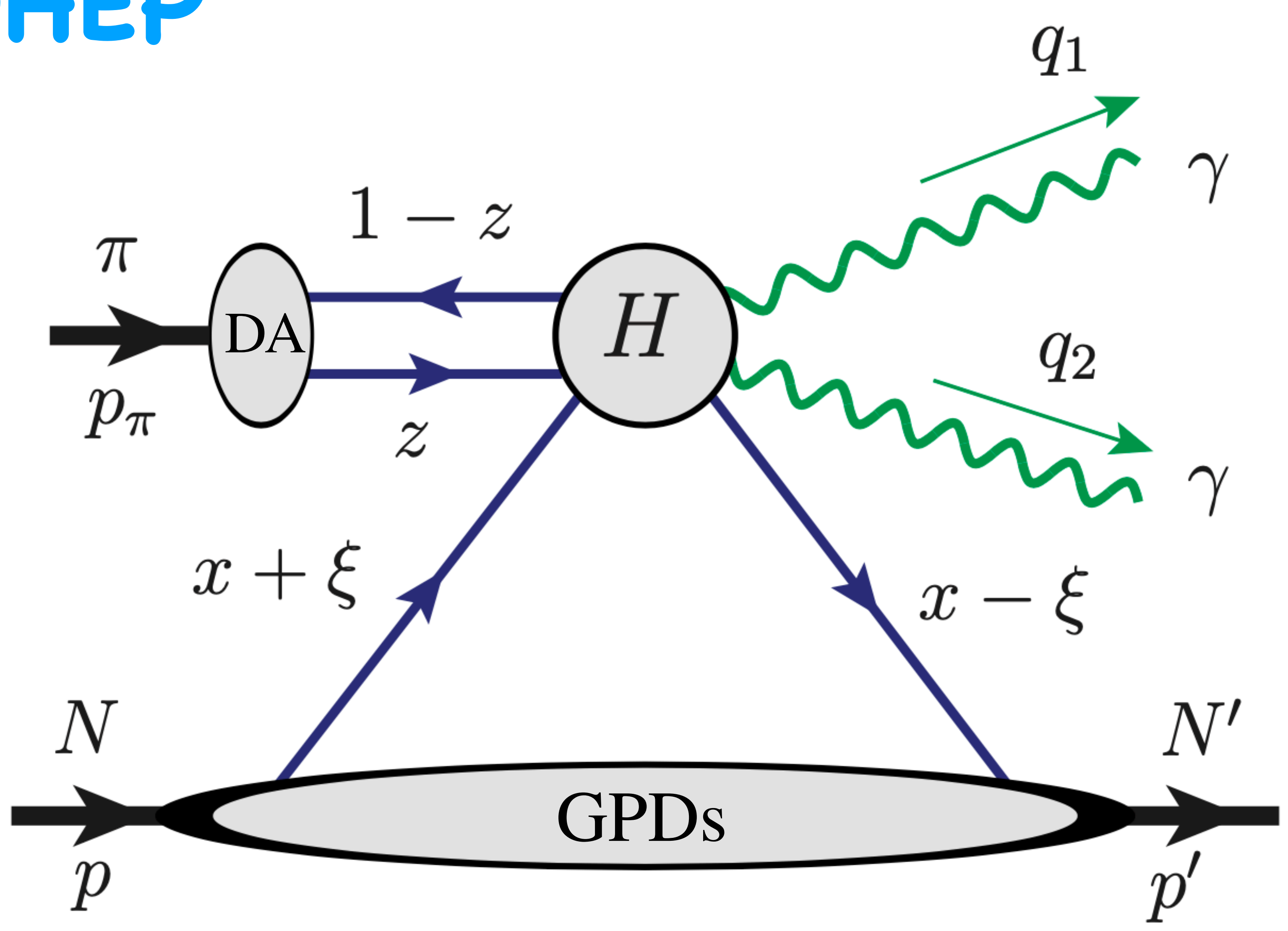
SDHEP

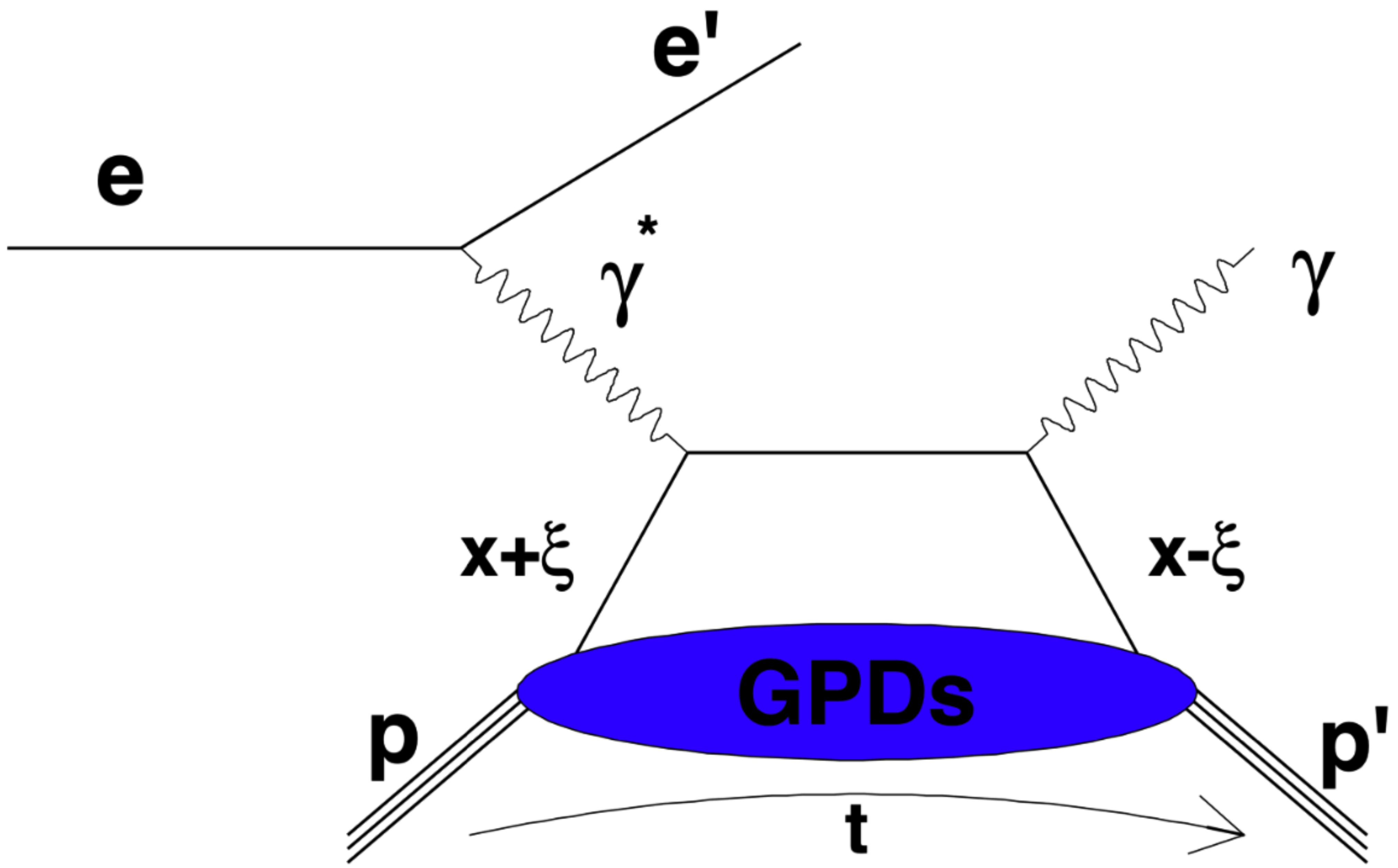
- J. Qiu and Z. Yu, JHEP 08 (2022)
- J. Qiu and Z. Yu, PRD 107 (2023)



- Single diffractive hard exclusive processes have enhanced sensitivity to the x-dependence.
- Require high transverse momentum final-state particles.

Generalized parton distributions

DVCS



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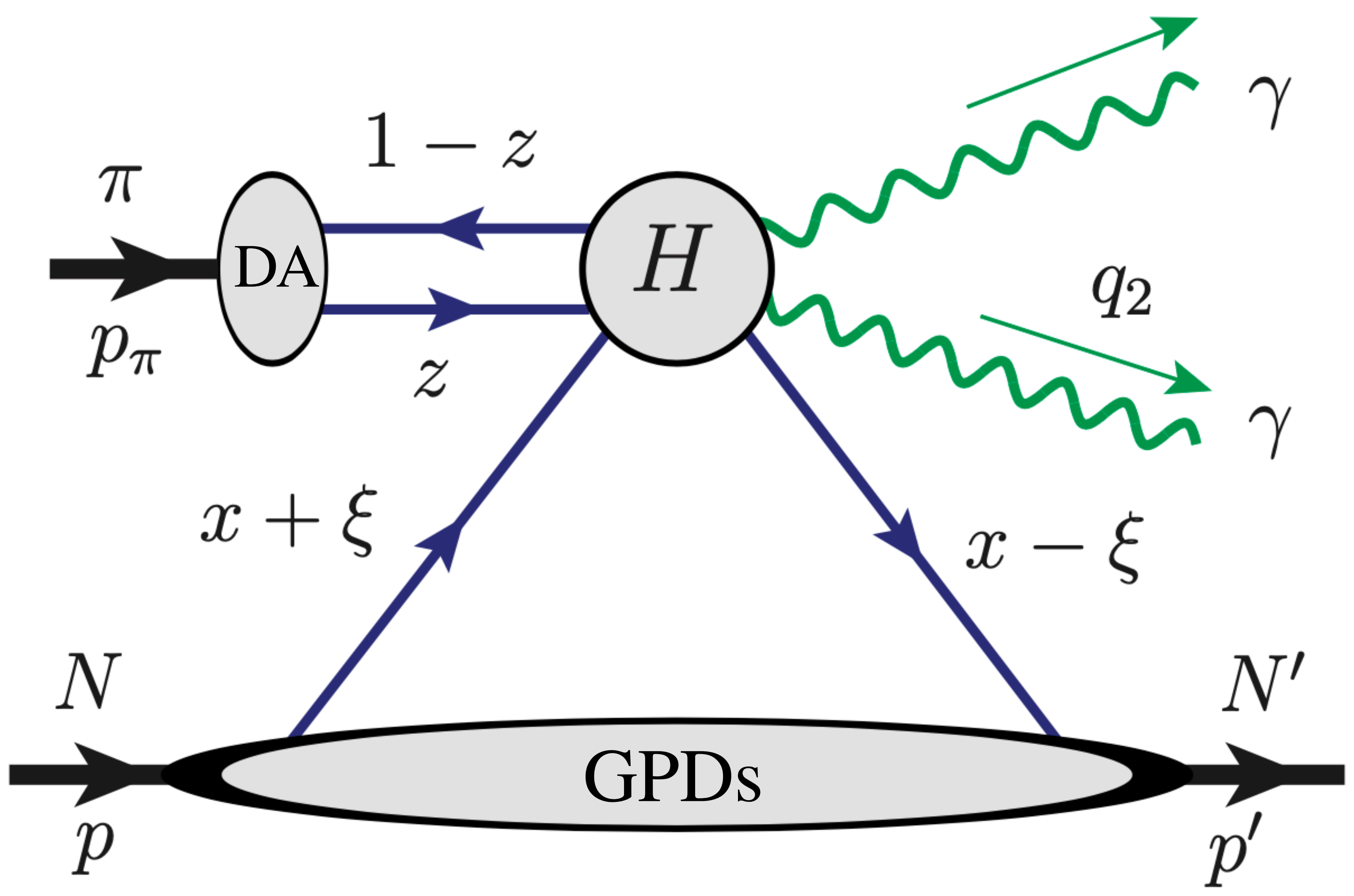
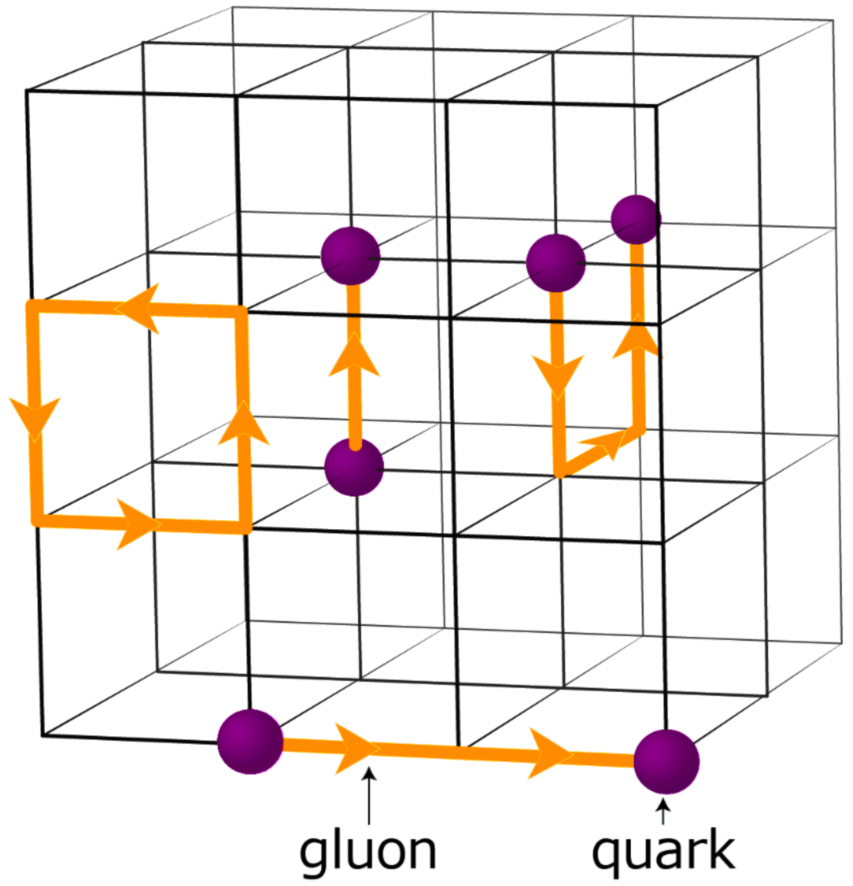


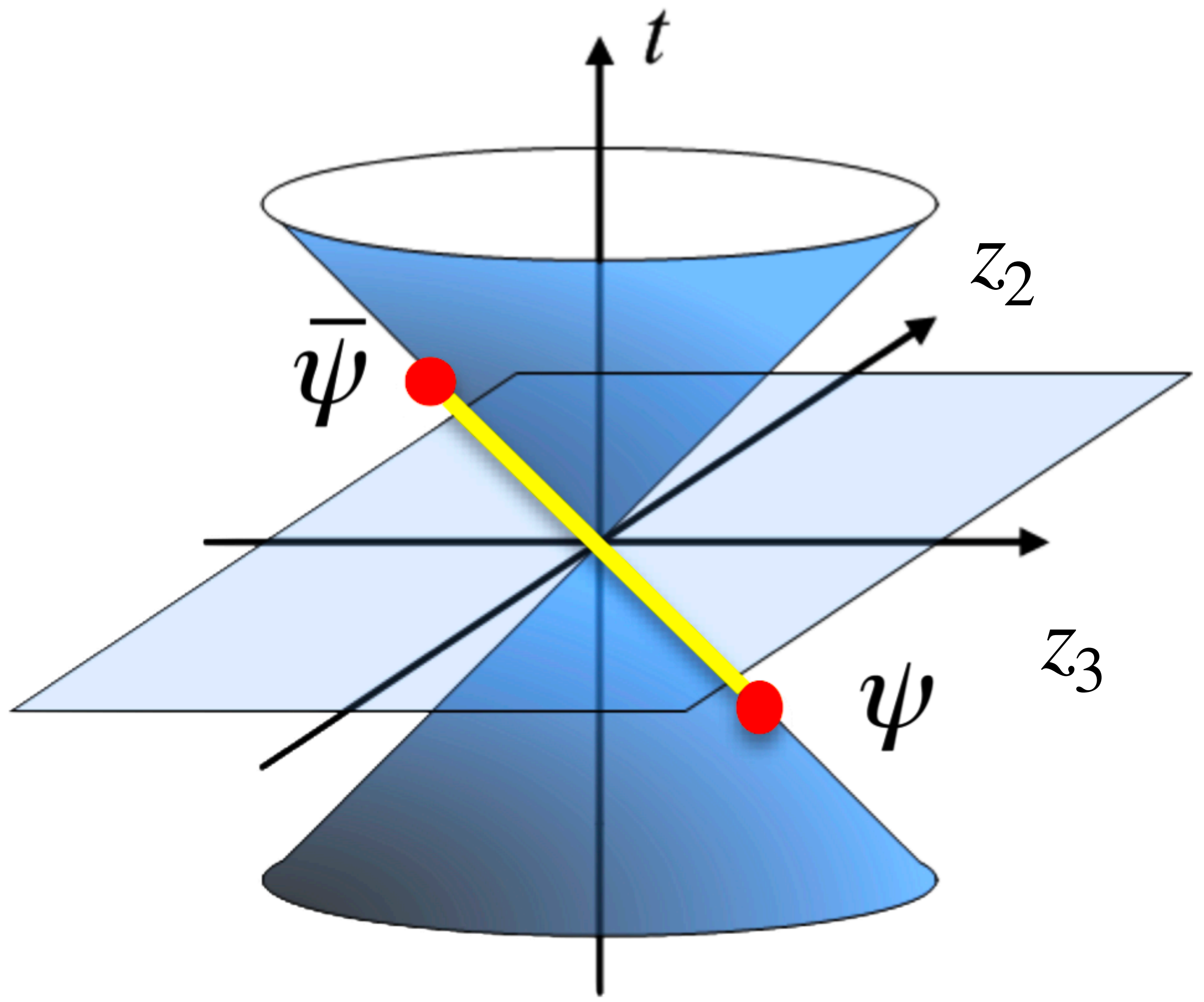
- Single diffractive hard
- Require high transvers
final-state particles.



Complementary knowledge from lattice QCD is essential.

Generalized parton distributions

$$z_3 + ct = 0, \quad z_3 - ct \neq 0$$



Light-cone correlation: Cannot be calculated on the lattice

$$\langle p_f | \bar{q}(-\frac{z^-}{2})\gamma^\mu \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) | p_i \rangle$$

$$z^2 = 0$$

- Moments from leading-twist **local operators**.

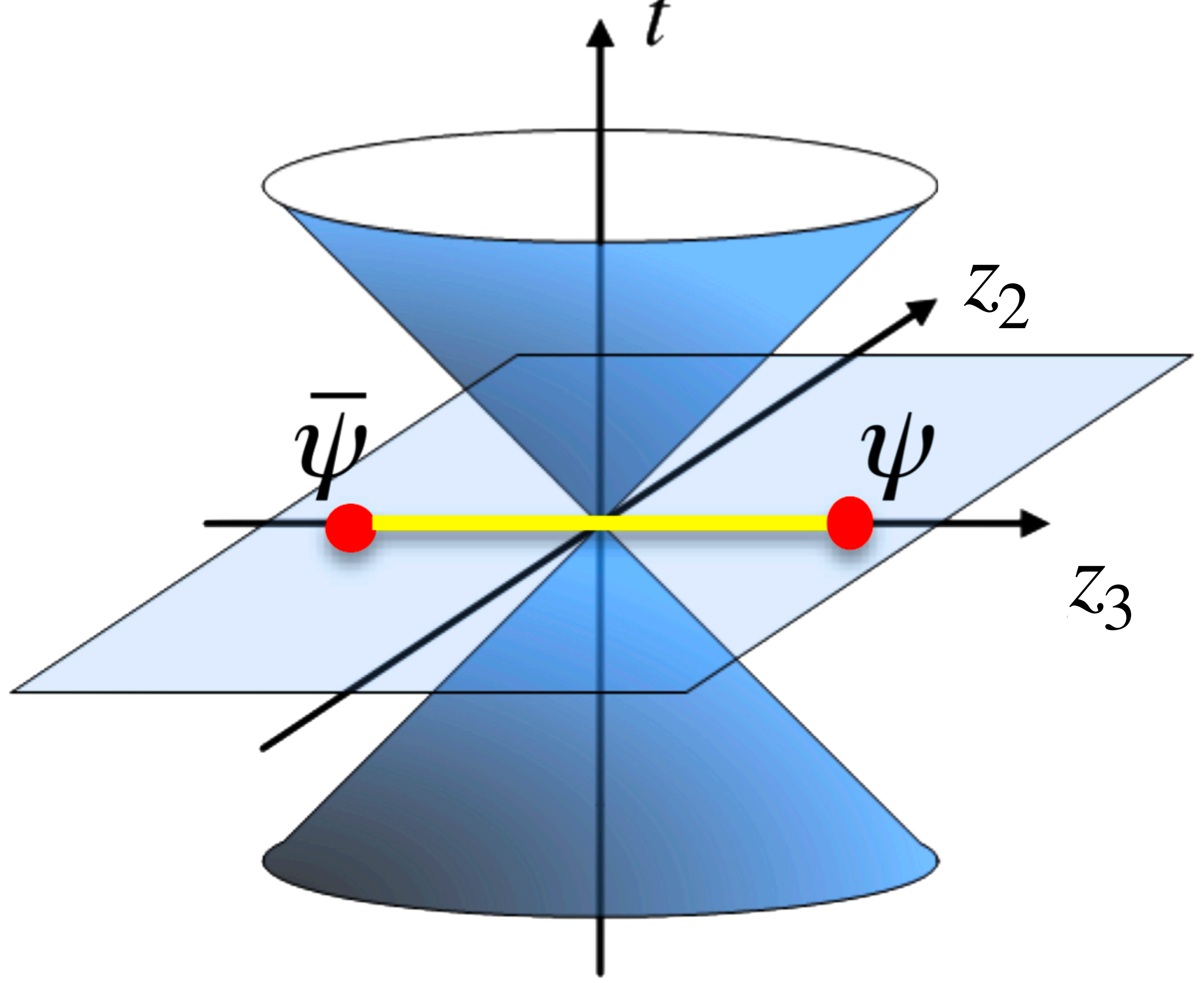
$$\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \dots \overleftrightarrow{D}^{\alpha_n} q$$

High dimensional

Limited up to $\langle x^3 \rangle$ due to signal decay and power-divergent mixing with lower moments under renormalization.

Generalized parton distributions

$t = 0, \quad z_3 \neq 0$



$$F^\mu(z, P, \Delta)$$
$$= \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$z = (0,0,0,z_3), \ z^2 = z_3^2$

- Moments from leading-twist **local operators**.

$$\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q$$

High dimensional

- **Large-momentum effective theory**: x-space matching of quasi-PDF.

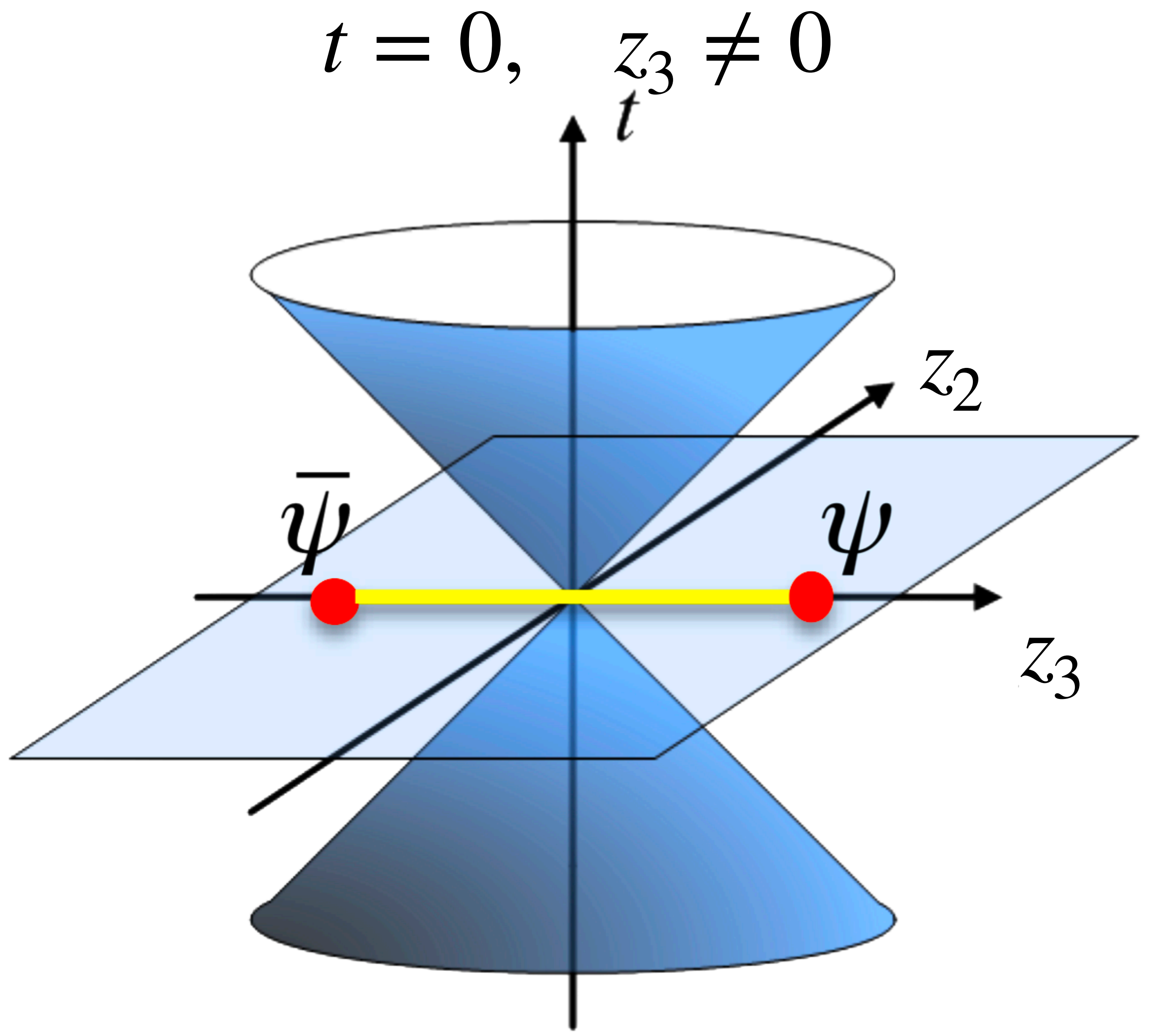
X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

- **Short distance factorization** of the quasi-PDF matrix elements in position space or the pseudo-PDF approach.

A. Radyushkin, PRD 100 (2019)
A. Radyushkin, Int.J.Mod.Phys.A 2020

- …

Short distance factorization



$$F^\mu(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$z = (0,0,0,z_3),\ z^2 = z_3^2$

SDF of the zero skewness GPD matrix elements:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

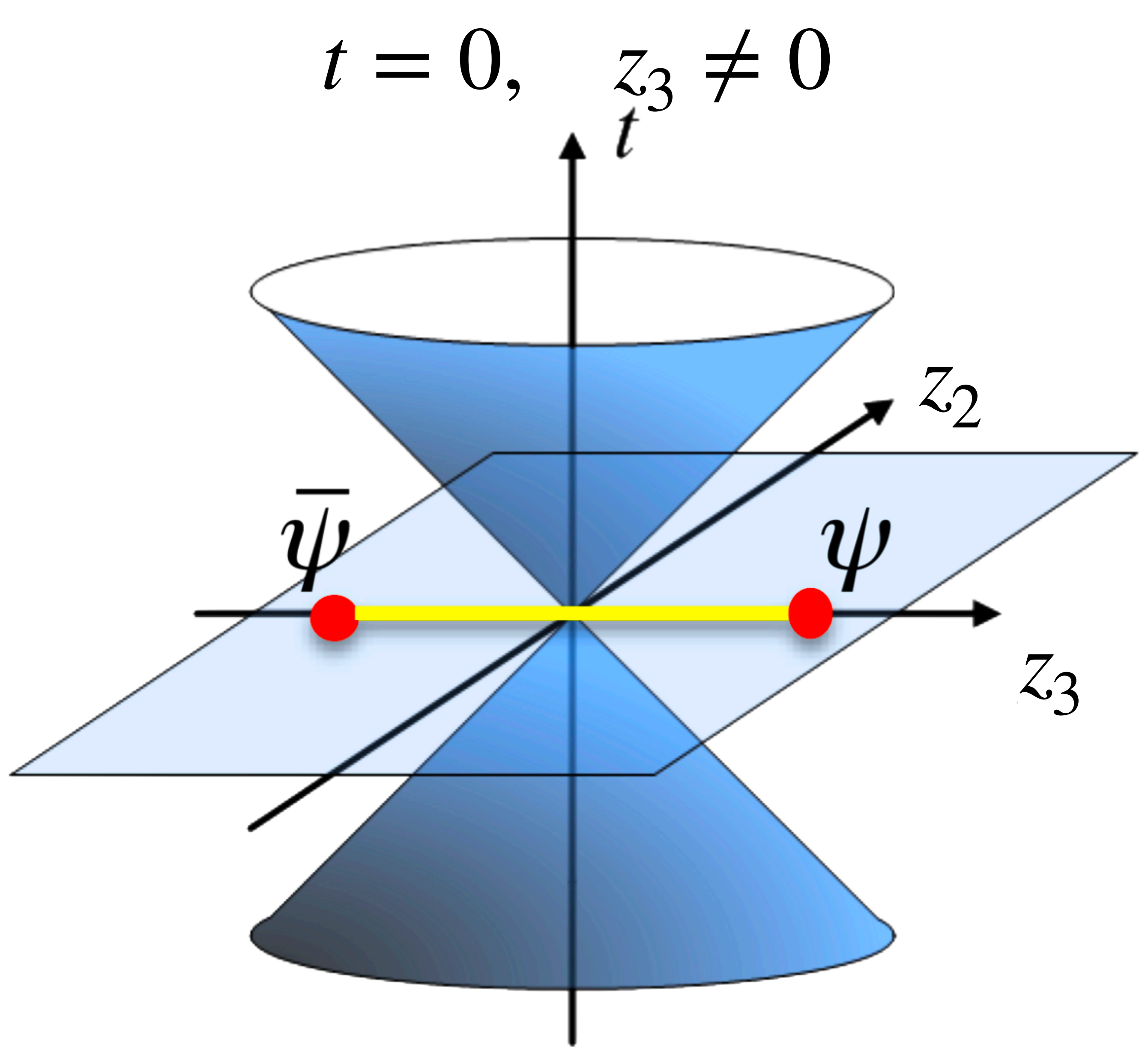
$$F^R(z, P, \Delta) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (t; \mu) + \mathcal{O}(z^2 \Lambda_{\rm QCD}^2)$$

Perturbative coefficients

- The perturbative matching is valid in short range of z^2.
- The information that lattice data contains is limited by the range of finite $\lambda = zP$.

Large P_z still essential!

Short distance factorization



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Perturbative coefficients

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$

quasi-GPD matrix elements

The unpolarized qGPD matrix elements in γ_0 definition:

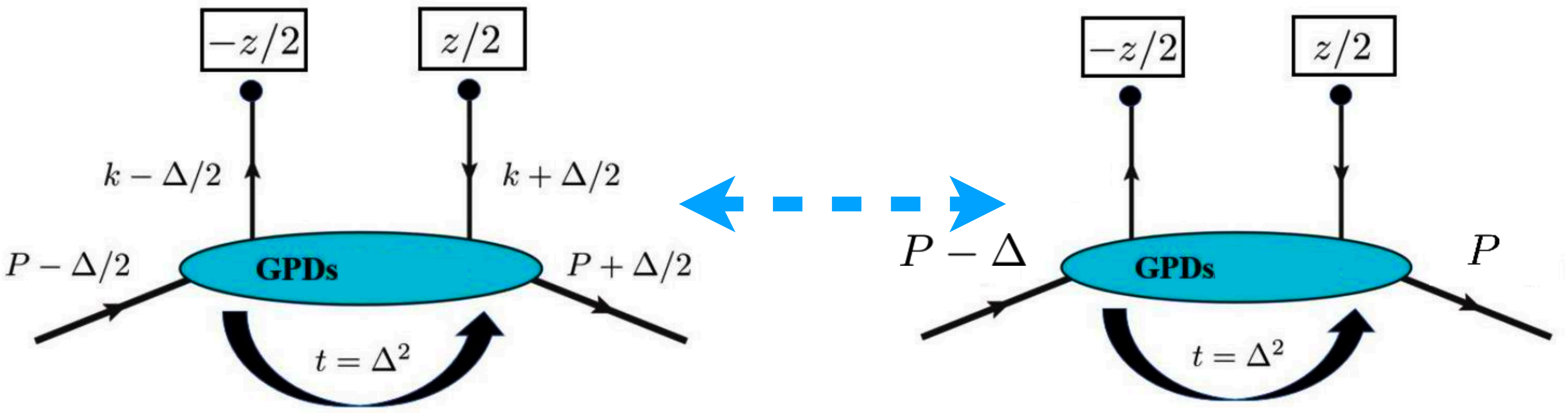
$$F^0(z,P,\Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^0 \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2}) | p_i \rangle$$

$$= \bar{u}(p_f,\lambda')\left[\gamma^0 \mathscr{H}_0(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathscr{E}_0(z,P,\Delta)\right]u(p_i,\lambda)$$

- Depend on Lorentz scalars:

  $t = \Delta^2,\ \xi = -(\Delta\cdot\hat{n}_3)/(2P\cdot\hat{n}_3)$

- And Lorentz vectors, e.g. $\mathcal{O}(\Delta/P)$: frame dependent at finte momentum.



$$F_0^s \longleftrightarrow \gamma F_0^a - \gamma\beta F_\perp^a$$

S. Bhattacharya, et al., PRD 106, 114512 (2022)

quasi-GPD matrix elements

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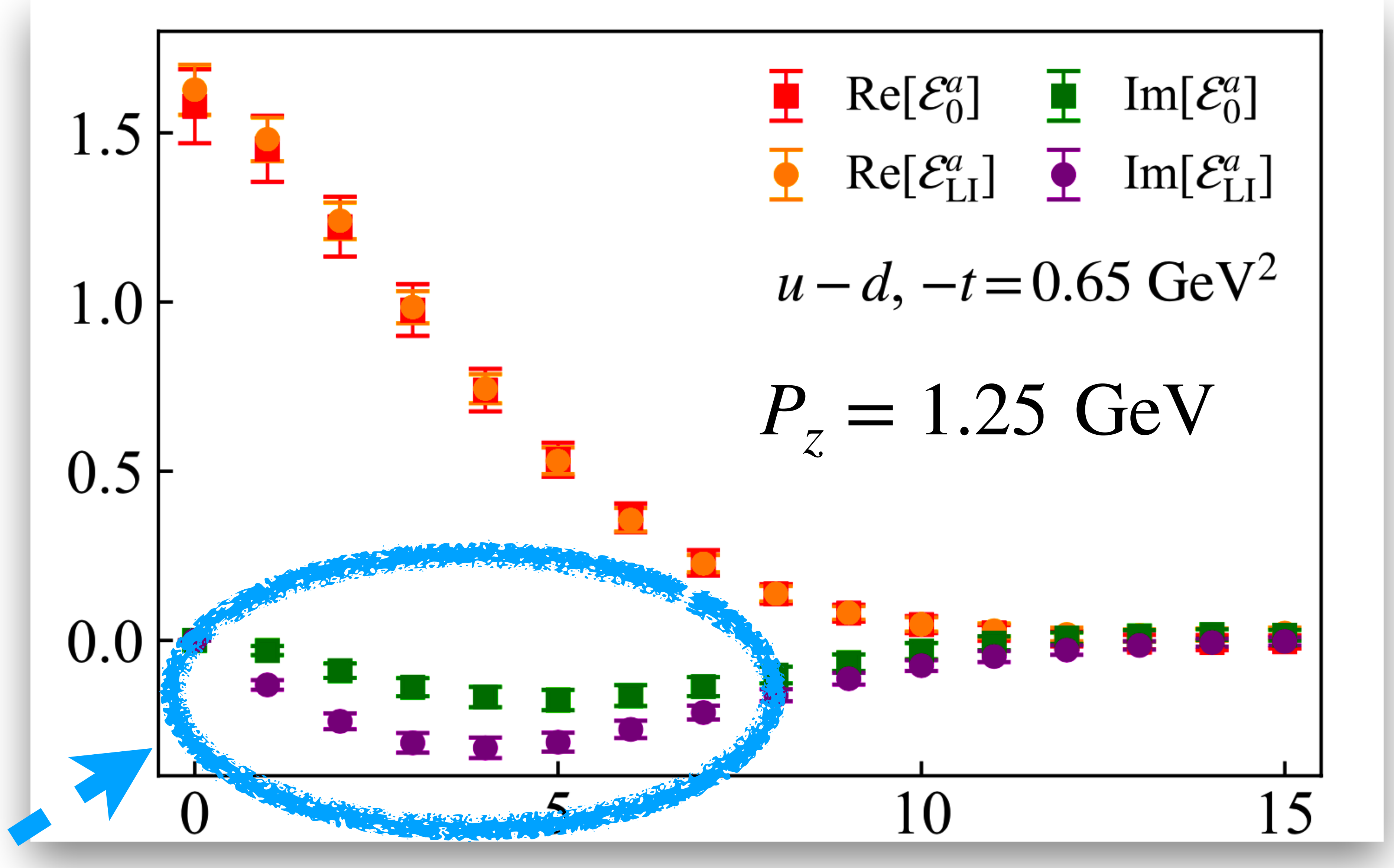


frame-dependent power corrections $\sim \Delta/P$ at the tree level

S. Bhattacharya, et al., PRD 106, 114512 (2022)

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of 8 **Lorentz invariant amplitudes**: $A_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$:

more details in Shohini Bhattacharya's talk

- S. Bhattacharya, et al., PRD 106 (2022)

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

- Construct qGPD using the A_i.
- Frame independent, computational cheaper for multiple t.
- Potentially reduce the power corrections with proper construction.

A Lorentz invariant (LI) choice analogous to the light-cone GPD:

$$\mathscr{H}(z,P,\Delta) = A_1 + \frac{\Delta\cdot z}{P\cdot z}A_3$$

$$\mathscr{E}(z,P,\Delta)$$

$$= -A_1 - \frac{\Delta\cdot z}{P\cdot z}A_3 + 2A_5 + 2P\cdot zA_6 + 2\Delta\cdot zA_8$$

- Differ from Light-cone GPD only by $z^2 \neq 0$

Renormalization

- The operator can be **multiplicatively renormalized**

- X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
- J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m(a)|z|}Z(a)[\bar{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- Short distance factorization with **ratio scheme renormalization**

- A. V. Radyushkin et al., PRD 96 (2017)
- BNL, PRD 102 (2020)

$$\mathscr{M}(z^2, zP, \Delta^2) = \frac{\mathscr{H}^R(z, P, \Delta; \mu)}{\mathscr{H}^R(z, P=0, \Delta=0; \mu)} = \frac{\mathscr{H}^B(z, P, \Delta; a)}{\mathscr{H}^B(z, P=0, \Delta=0; a)}$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle (\Delta^2; \mu) + \mathscr{O}(z^2\Lambda^2_{\rm QCD})$$

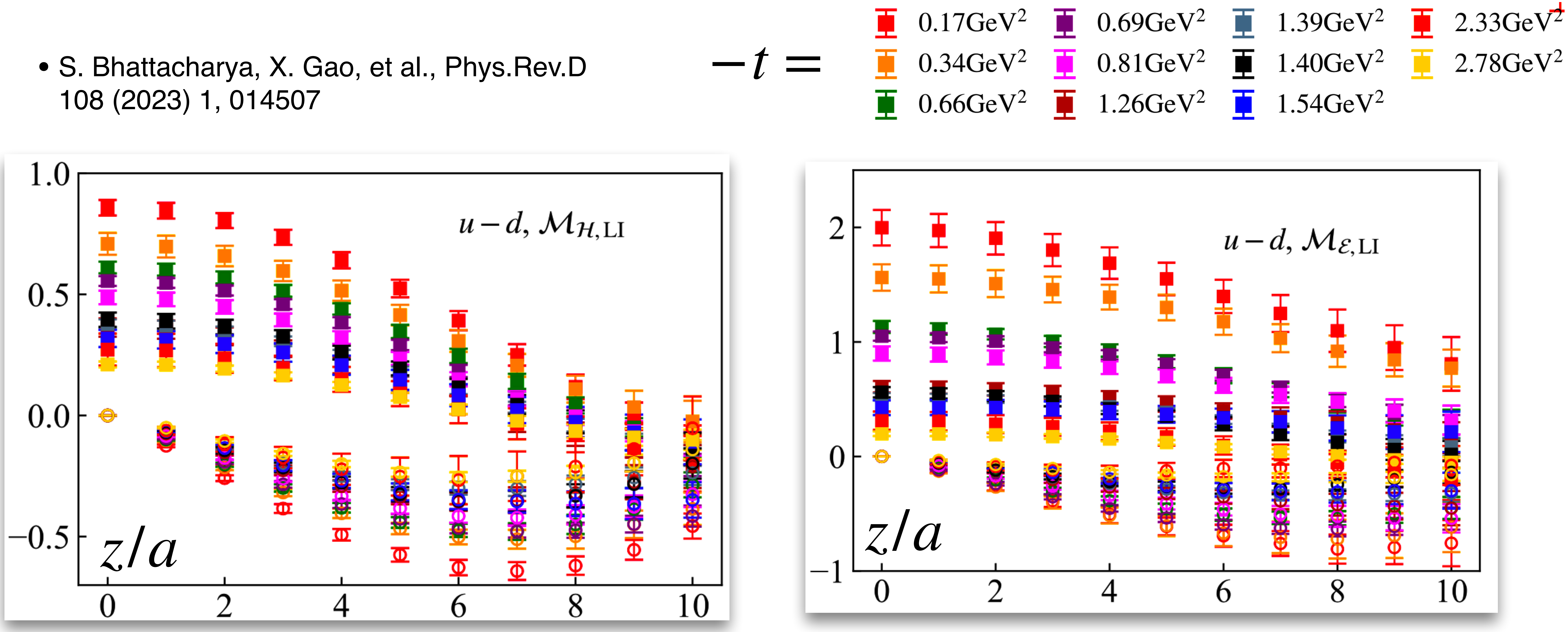
$C_n^{\overline{MS}}(\mu^2 z^2) = 1 + \alpha_s C^{(1)}(\mu^2 z^2) + \ldots$ up to NNLO

- **Lattice setup**

$m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$, $N_f = 2+1+1$ twisted mass fermions

Renormalized matrix elements

- S. Bhattacharya, X. Gao, et al., Phys.Rev.D 108 (2023) 1, 014507



- filled symbols: real part, sensitive to even moments
- unfilled symbols: imaginary part, sensitive to odd moments

$P_z = 1.25\ \text{GeV},\ a = 0.093\ \text{fm}$

SDF of qGPDs: γ_0 definition

$$\int_{-1}^{1} dx\, x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

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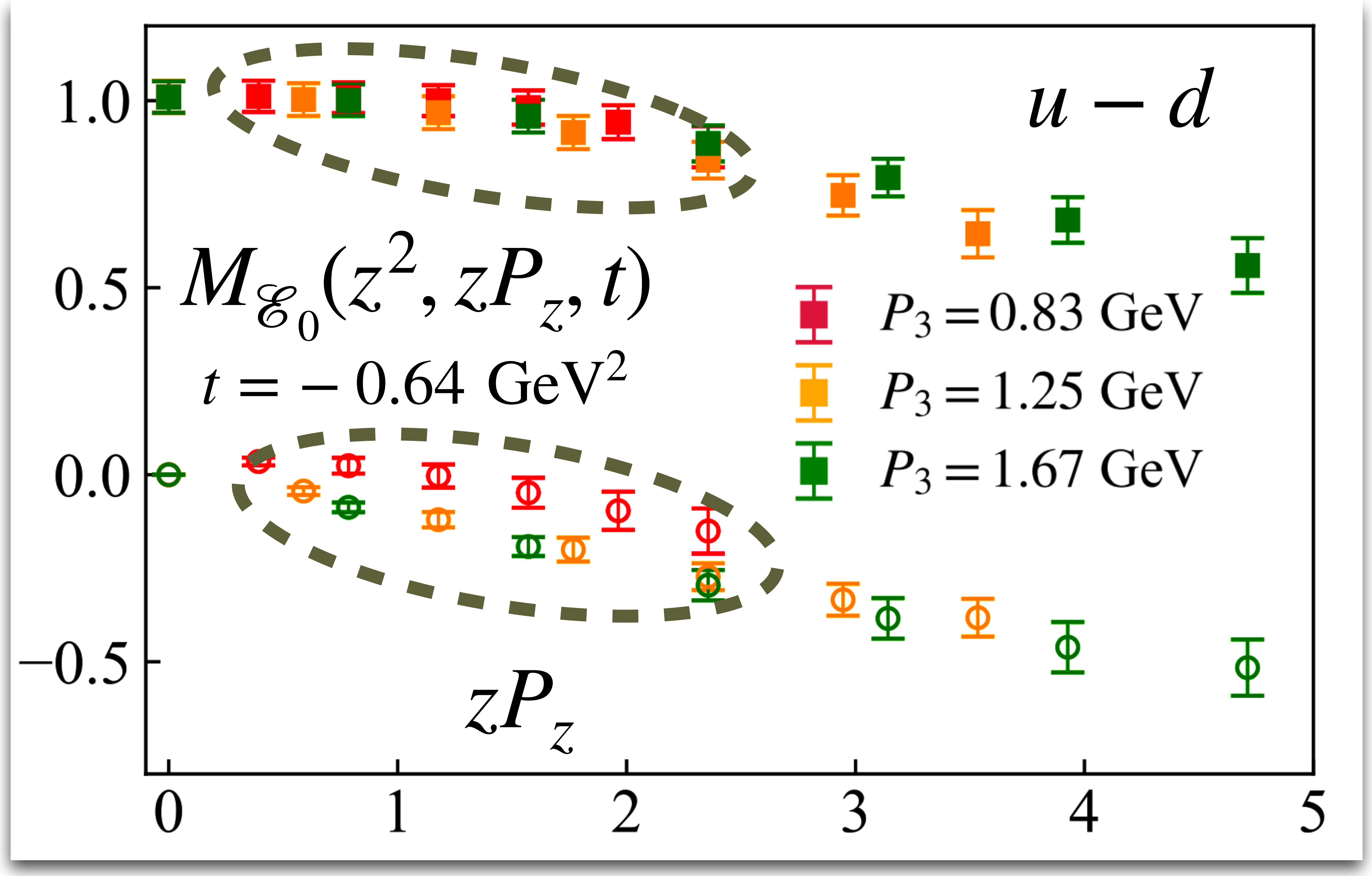

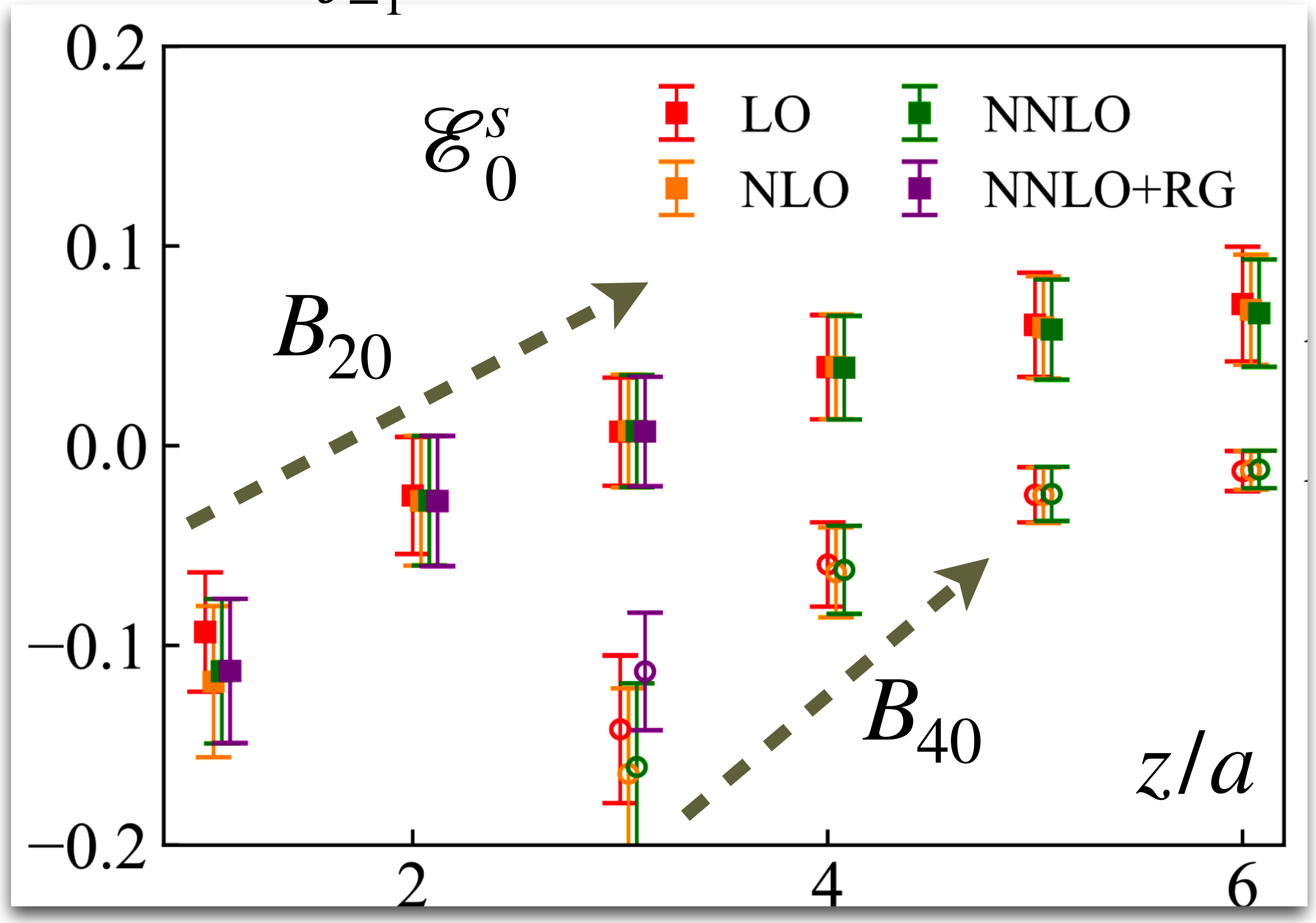

- no scaling with zP_z
- not constant in z

$$F^0(z, P, \Delta) = \bar{u}(p_f, \lambda')\left[\gamma^0 \mathscr{H}_0(z, P, \Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathscr{E}_0(z, P, \Delta)\right] u(p_i, \lambda)$$

$$\mathscr{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathscr{O}(z^2\Lambda^2_{QCD})$$

$\mu = 2$ GeV

SDF of qGPDs: LI definition

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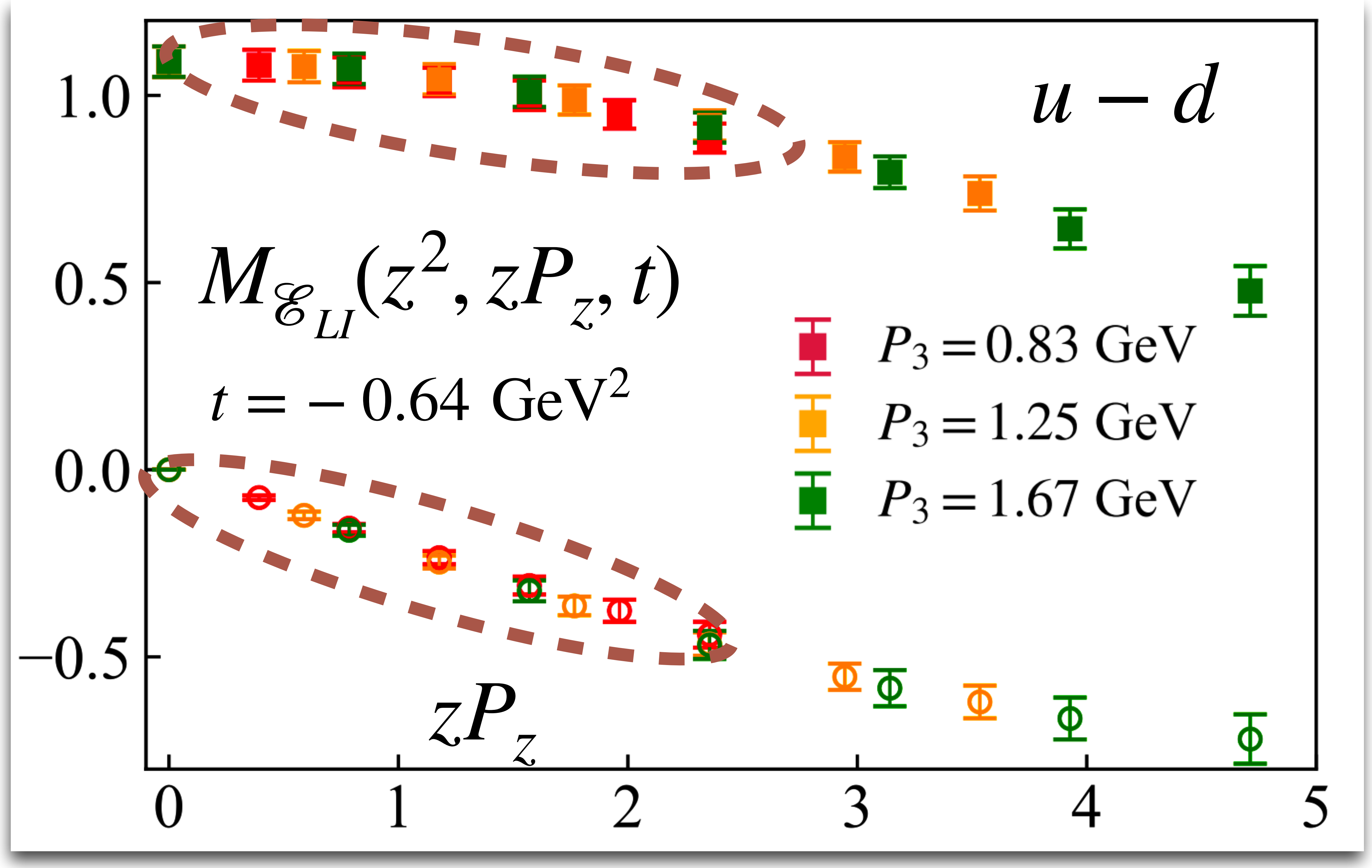
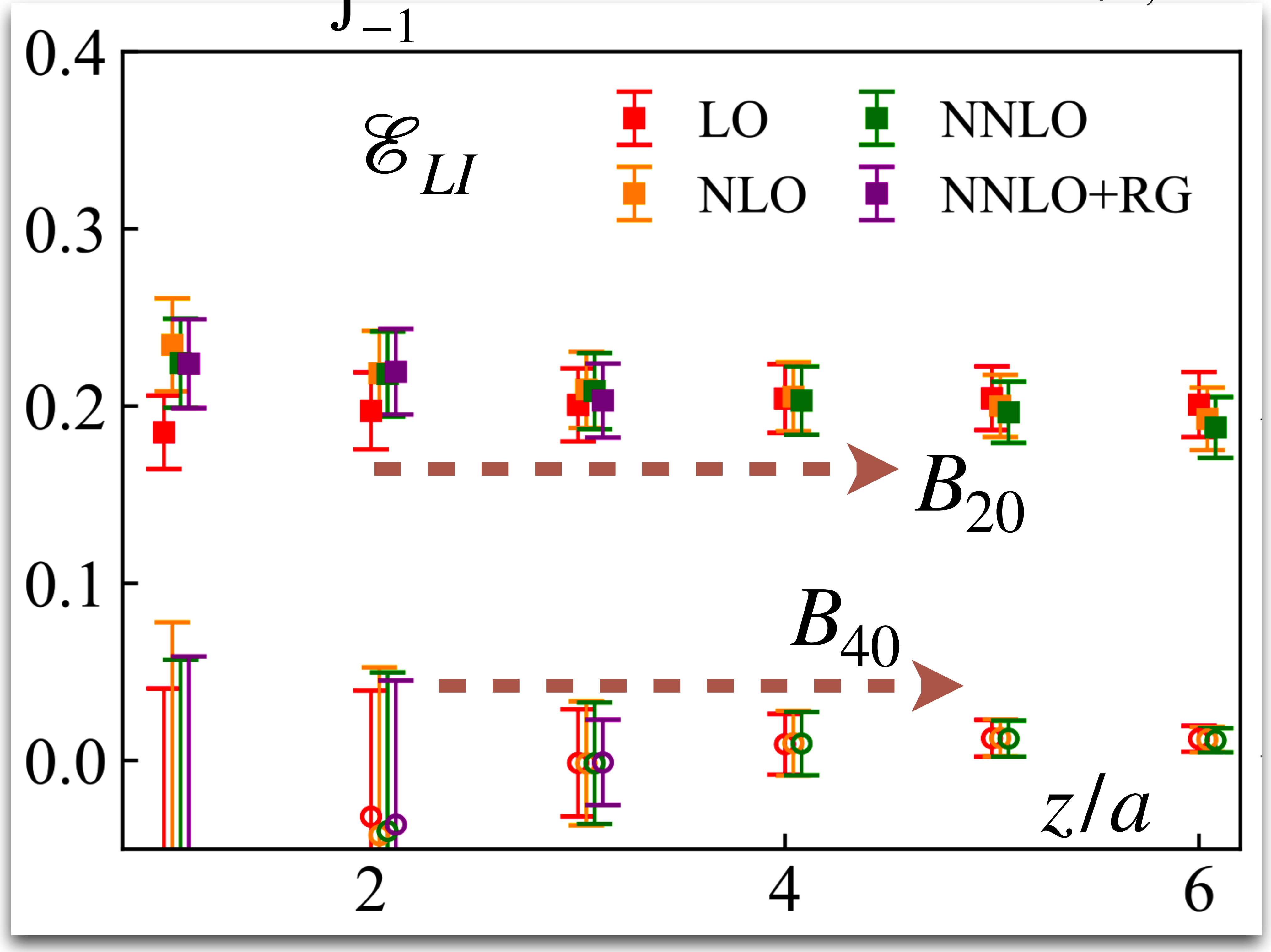




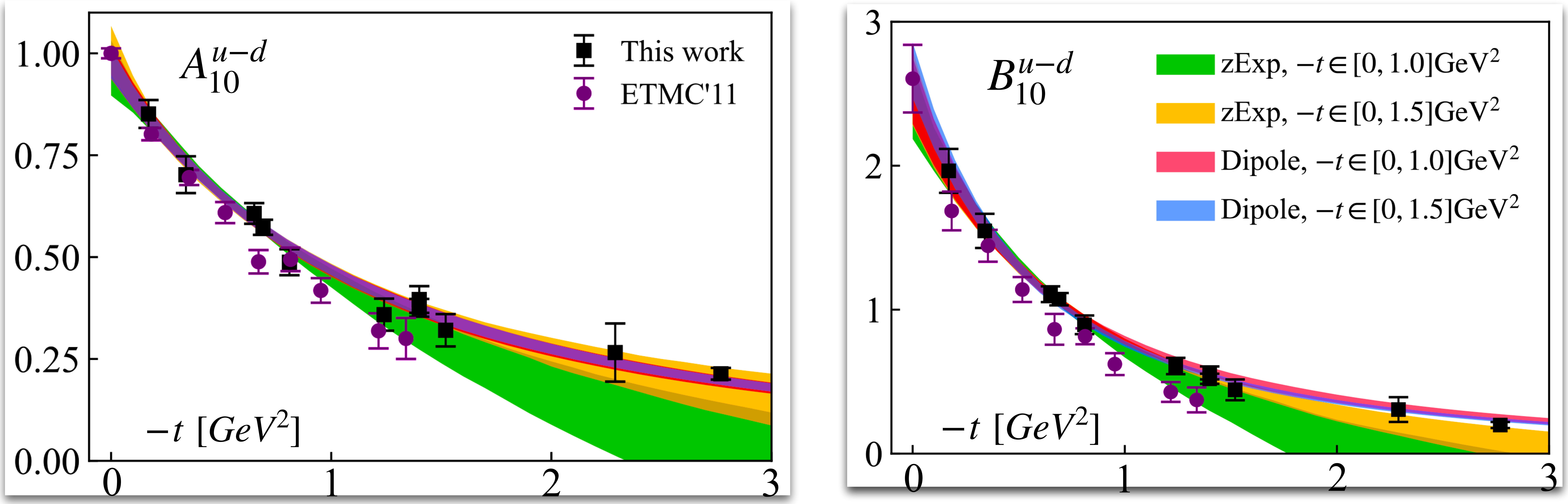
- Perturbative corrections $C_n(z^2\mu^2) = 1 + \mathcal{O}(\alpha_s)$
- Stable moments $\langle x^n \rangle(\mu)$

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

$\mu = 2$ GeV

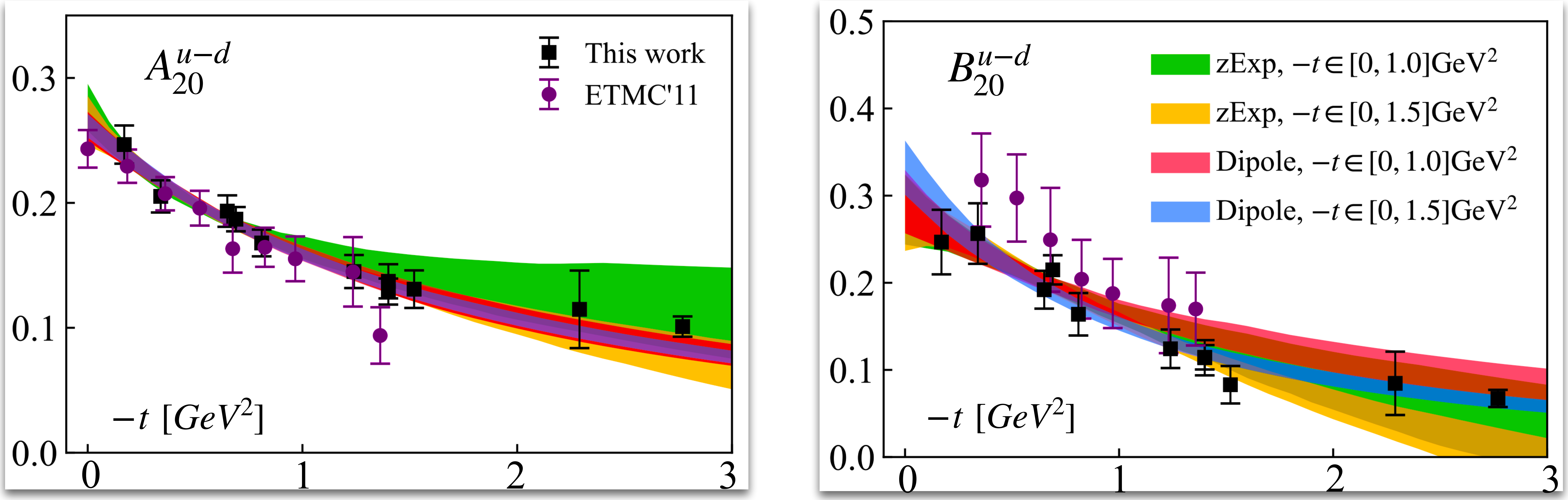
$$\mathcal{E}(z, P, \Delta) = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8$$

Mellin moments of GPDs



- Good agreement with available traditional lattice QCD calculations of GPD moments using local operators (ETMC’11)

Mellin moments of GPDs



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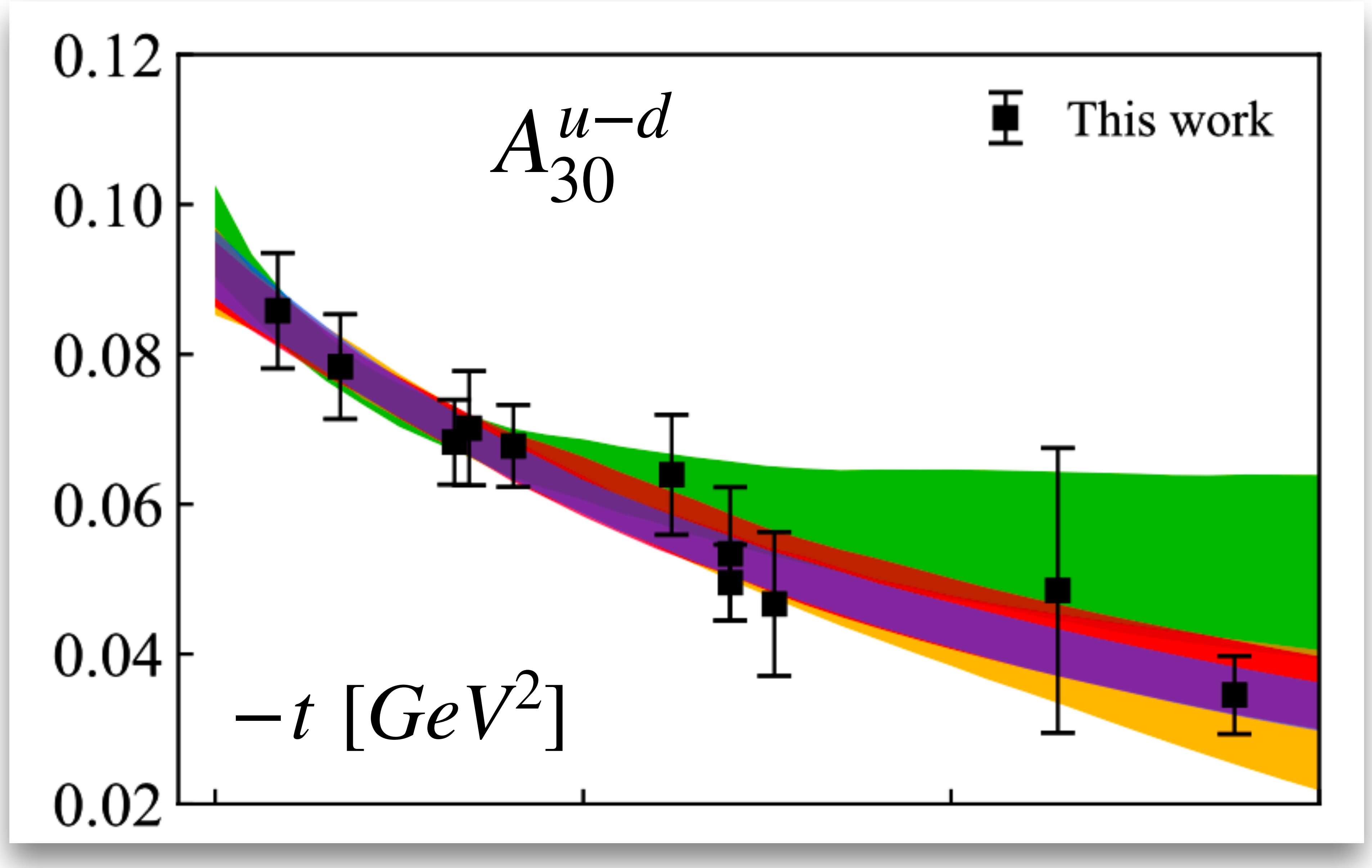
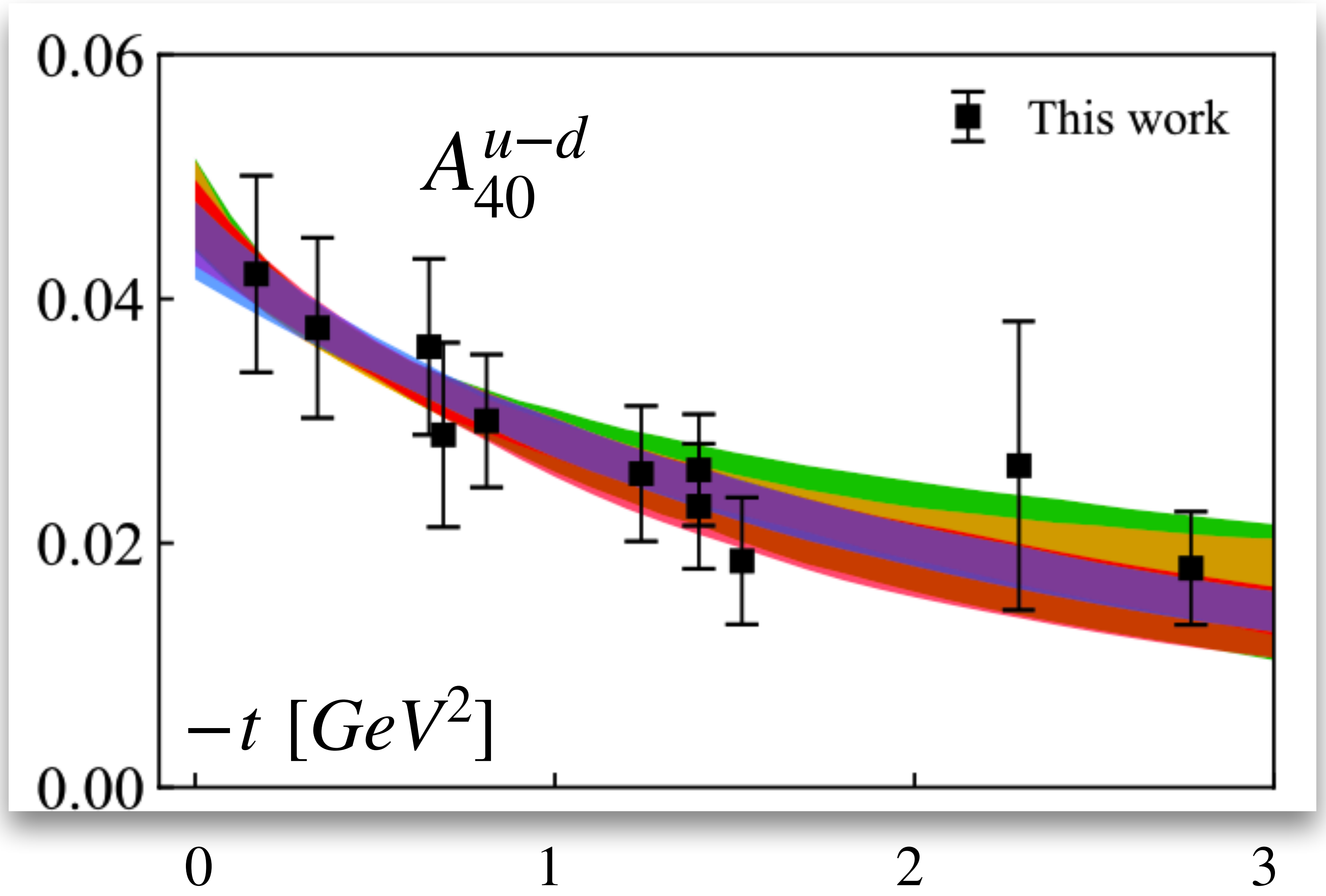
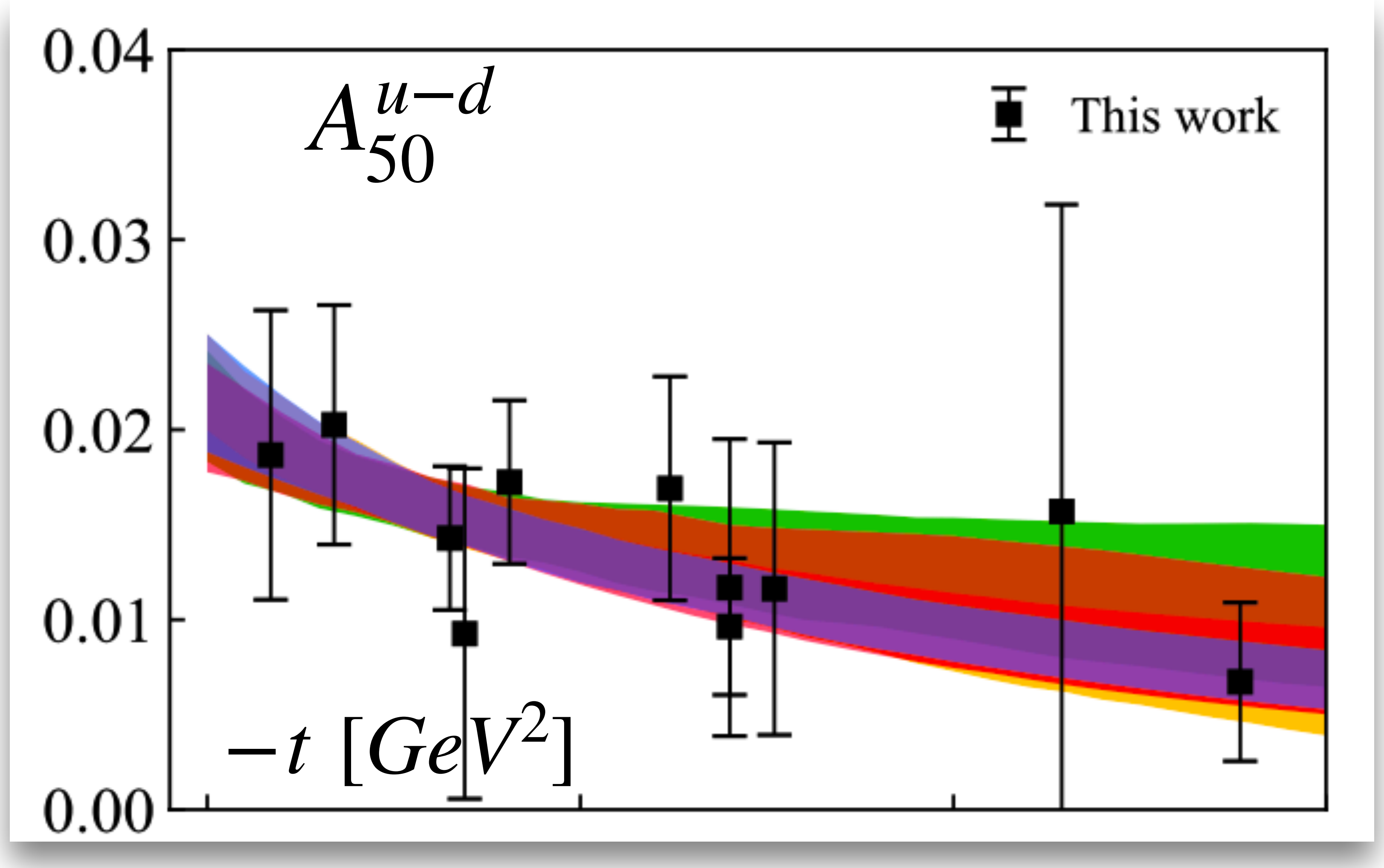
Mellin moments of GPDs

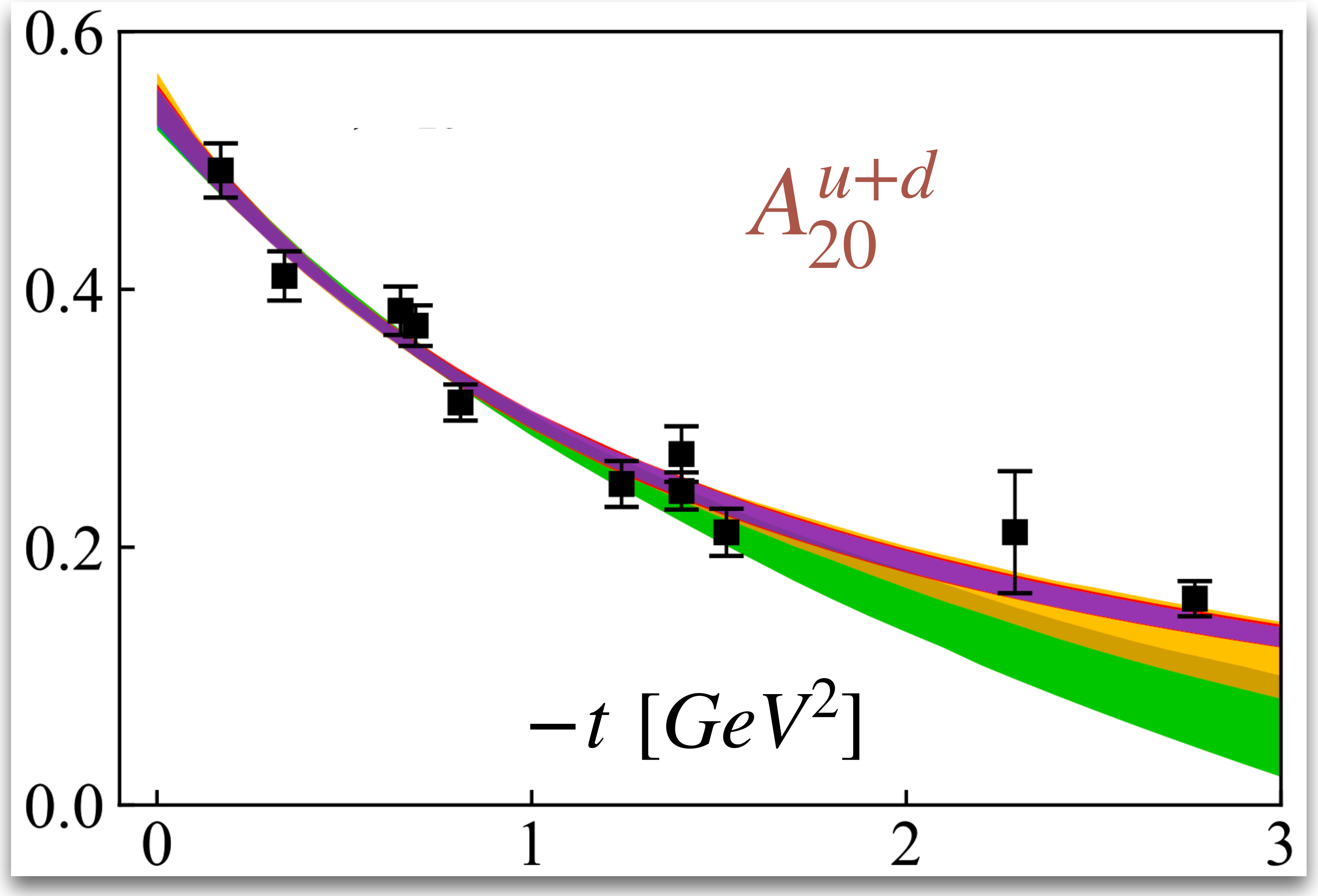


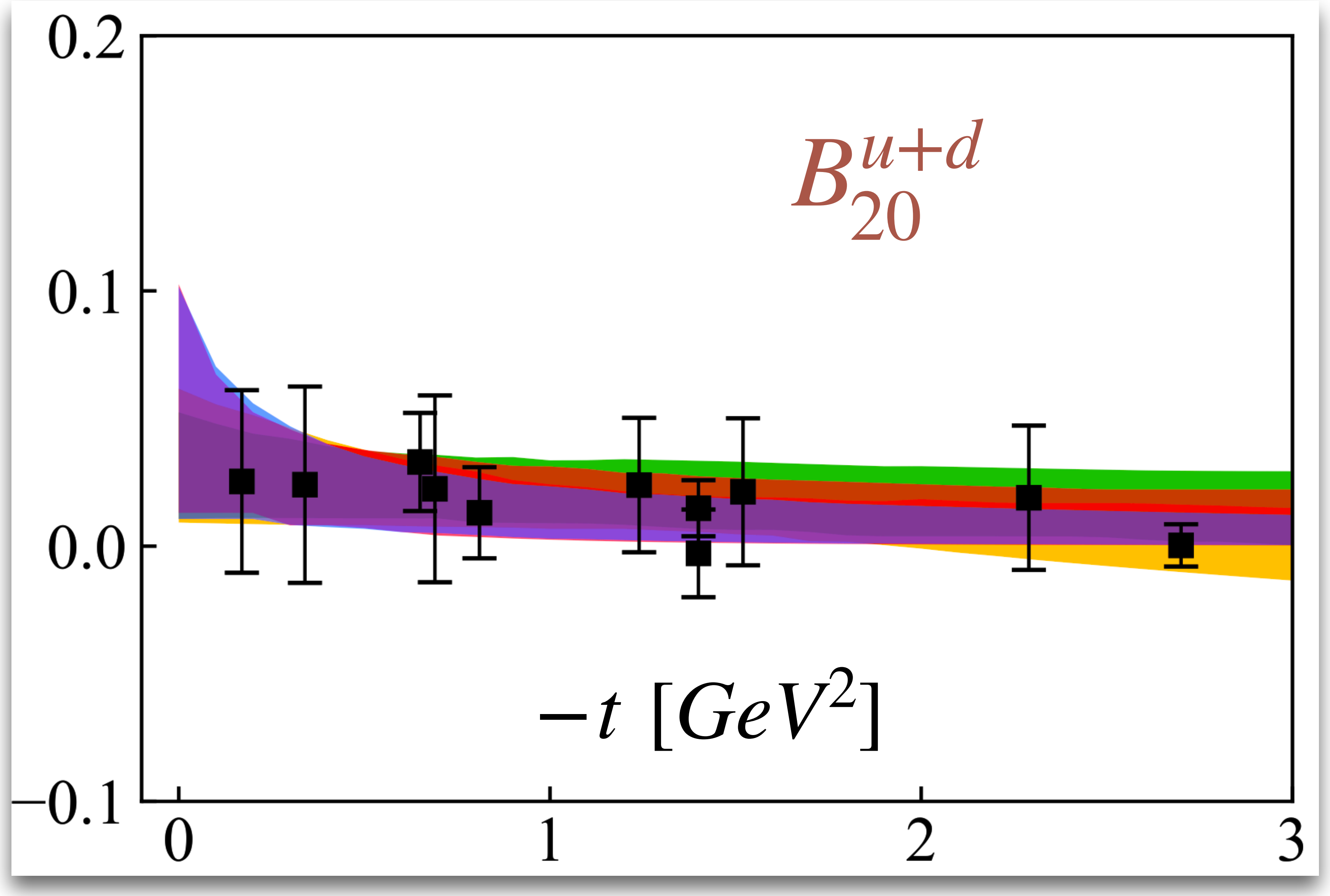




- Up to 5th moments of GPDs show reasonable signals and smooth $-t$ dependence.
- Higher moments can be constrained by increasing the hadron momentum.

Mellin moments of GPDs





- 2nd moments: Gravitational form factors

Ji sum rule: $J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$

$$J^{u-d} = 0.281(21)(11)$$

$$J^{u+d} = 0.296(22)(33)$$

- $m_\pi = 260$ MeV, $a = 0.093$ fm
- Disconnected diagrams neglected

Impact parameter space interpretation

- Unpolarized quark inside unpolarized nucleon

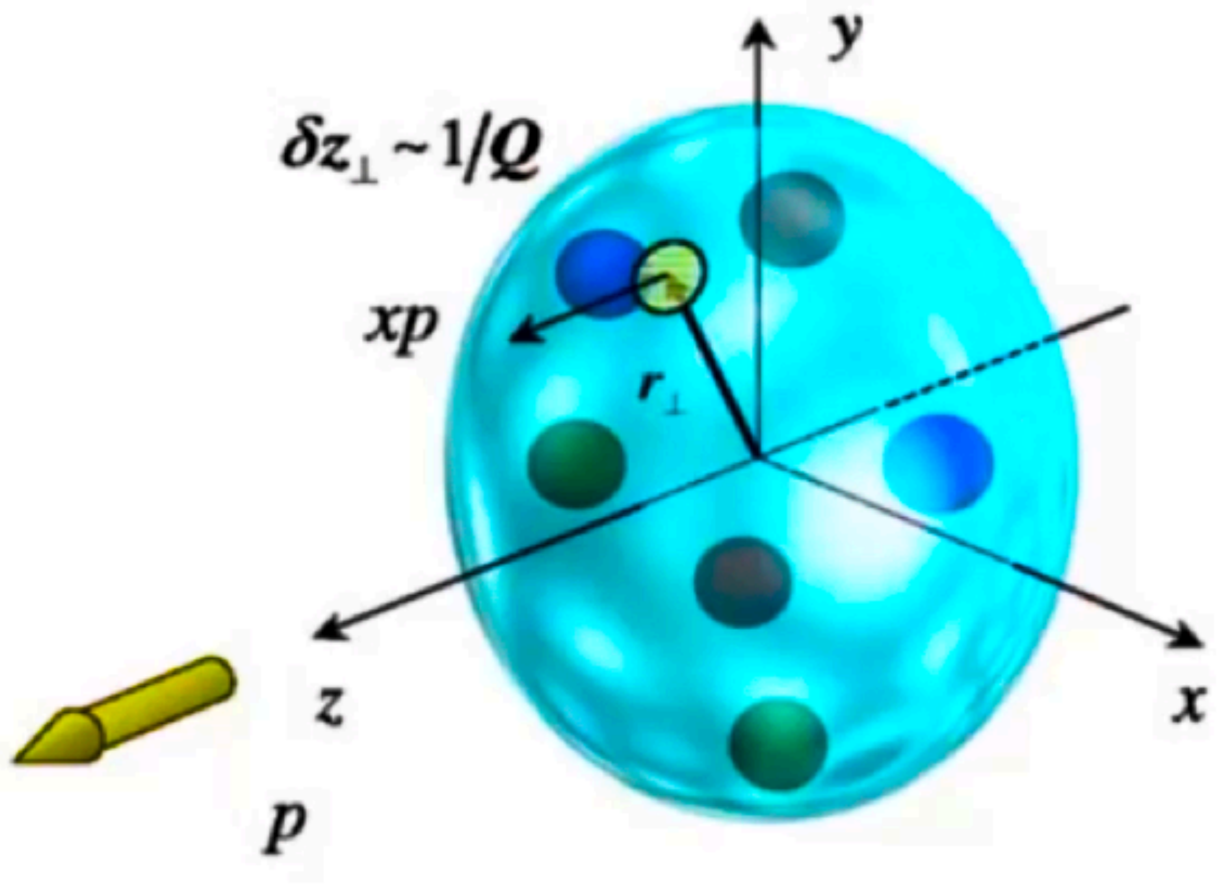
$$q(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} H(x,-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

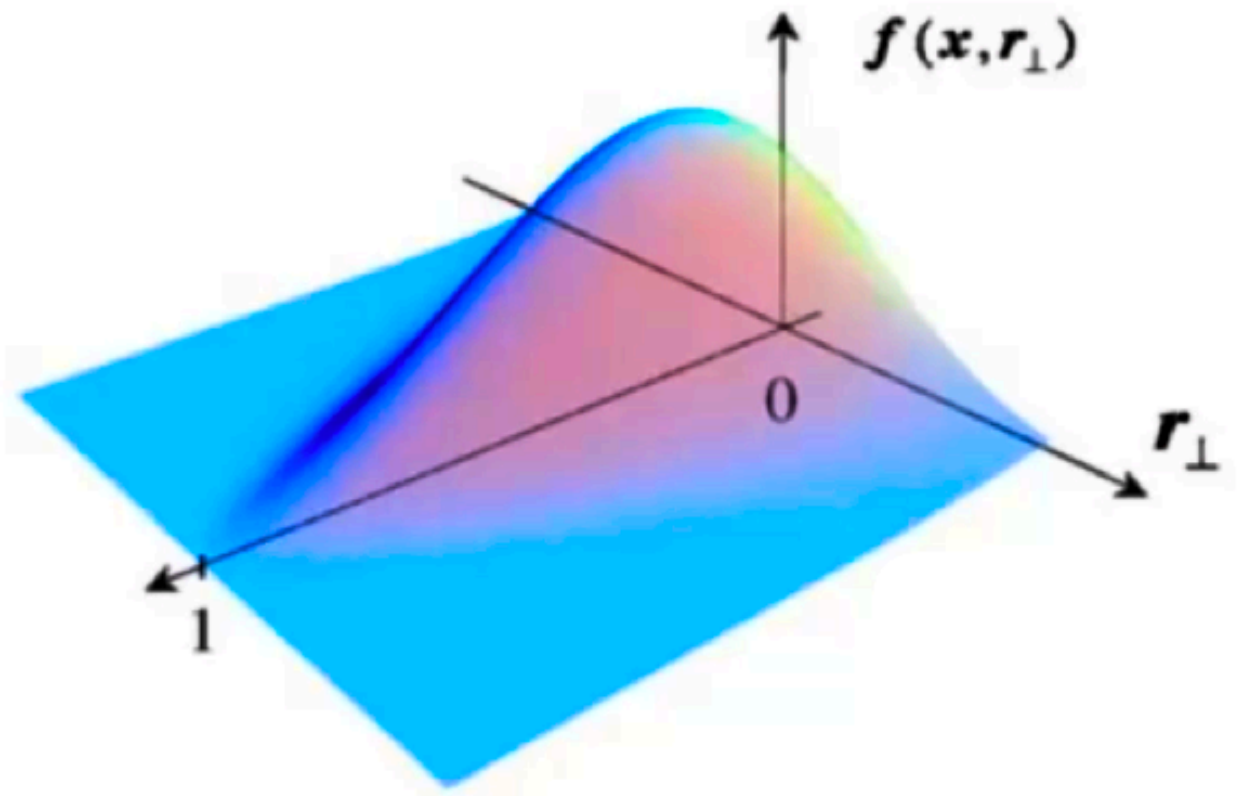
$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

- Unpolarized quark inside transversely polarized nucleon

$$q^T(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2}\left[H(x,-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} E(x,-\overrightarrow{\Delta}_\perp^2)\right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2}\left[A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} B_{n+1,0}(-\overrightarrow{\Delta}_\perp^2)\right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

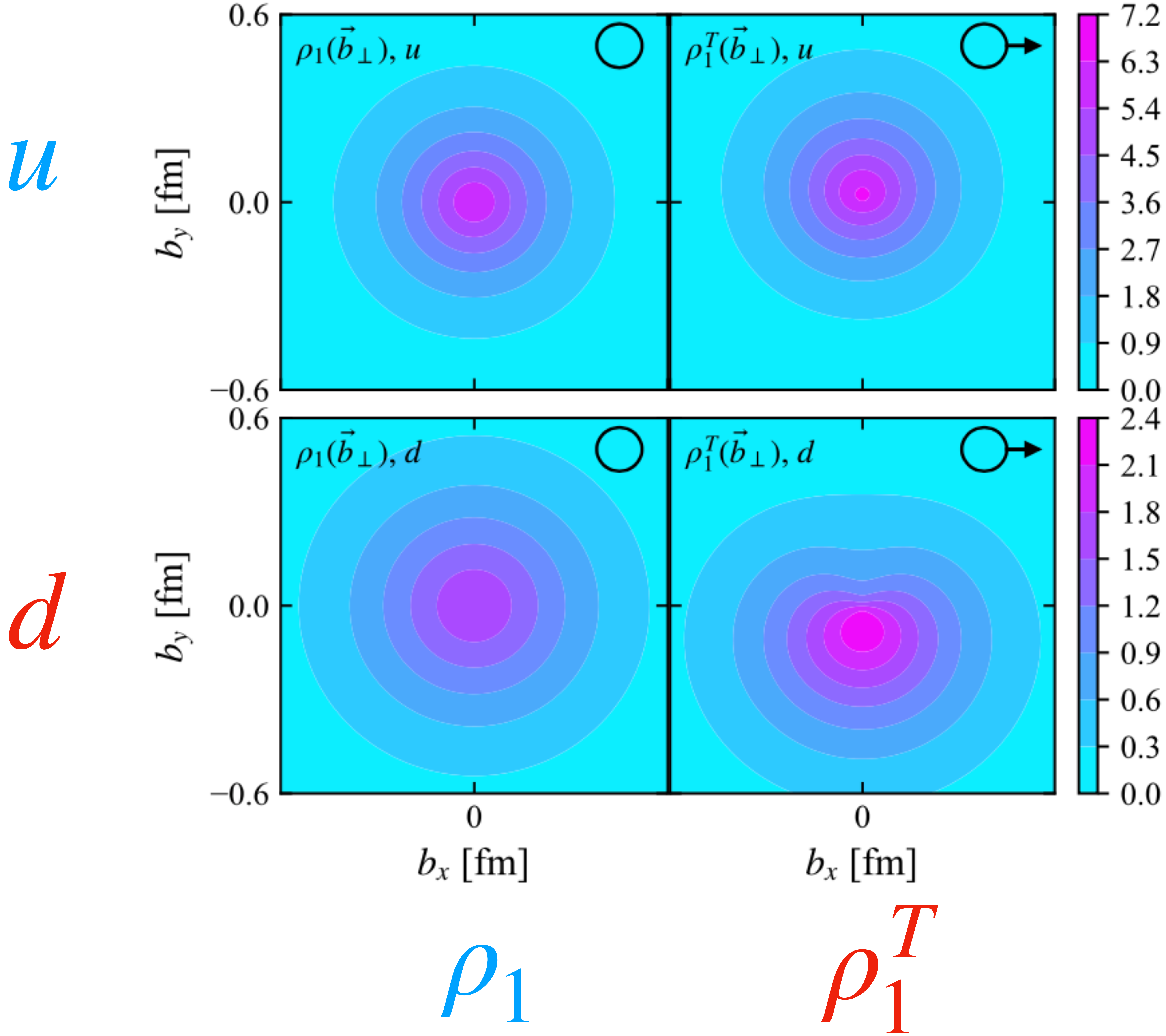




Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

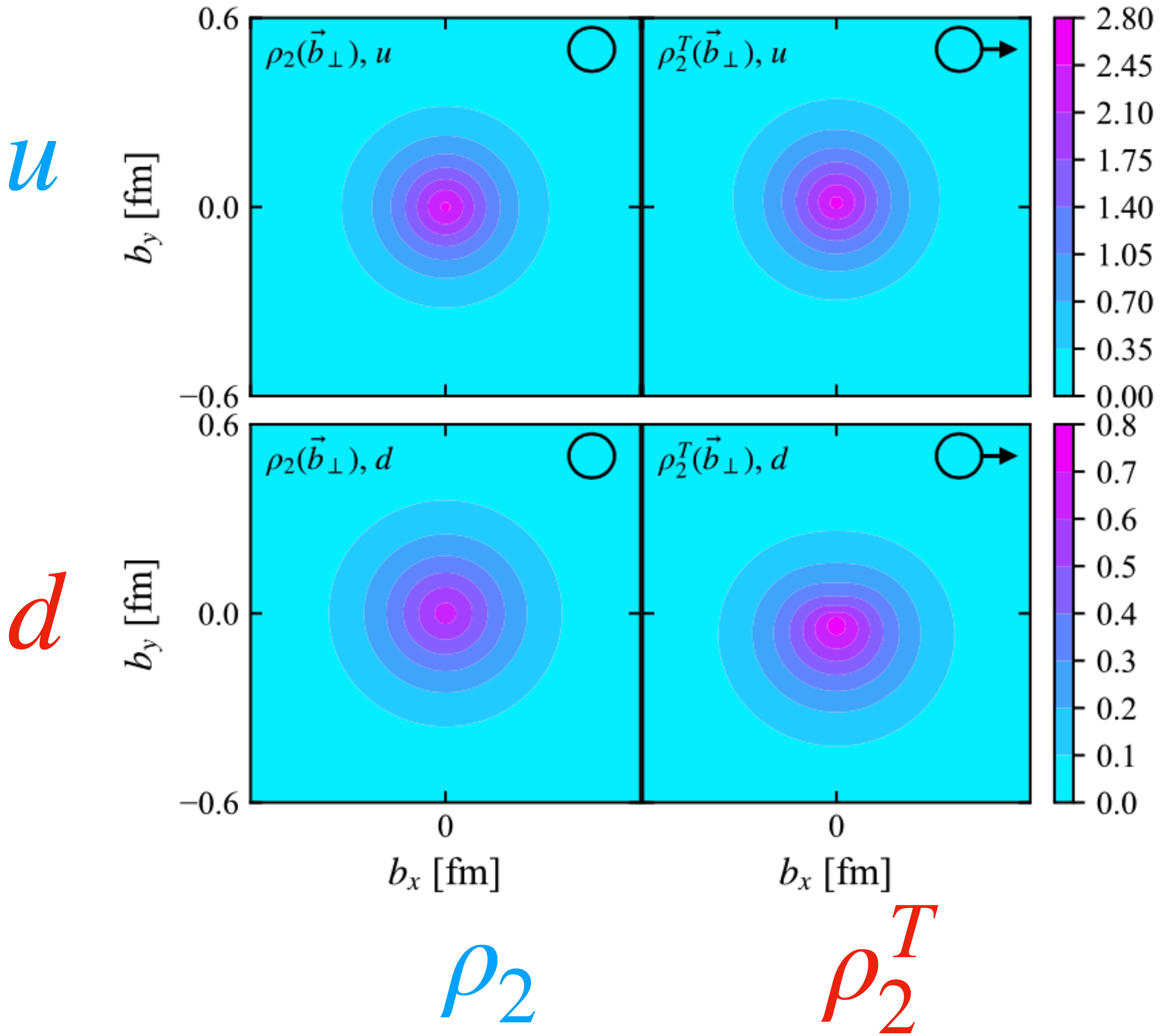
- Matthias Burkardt, Int.J.Mod.Phys.A 18 (2003) 173-208

Impact parameter space interpretation



- The **1st** moment: charge distribution
- **d** quark exhibits a broader distribution and smaller amplitudes
- When transversely polarized, the **u** and **d** quarks shift in different directions, with the **d** quarks showing larger distortion.

Impact parameter space interpretation

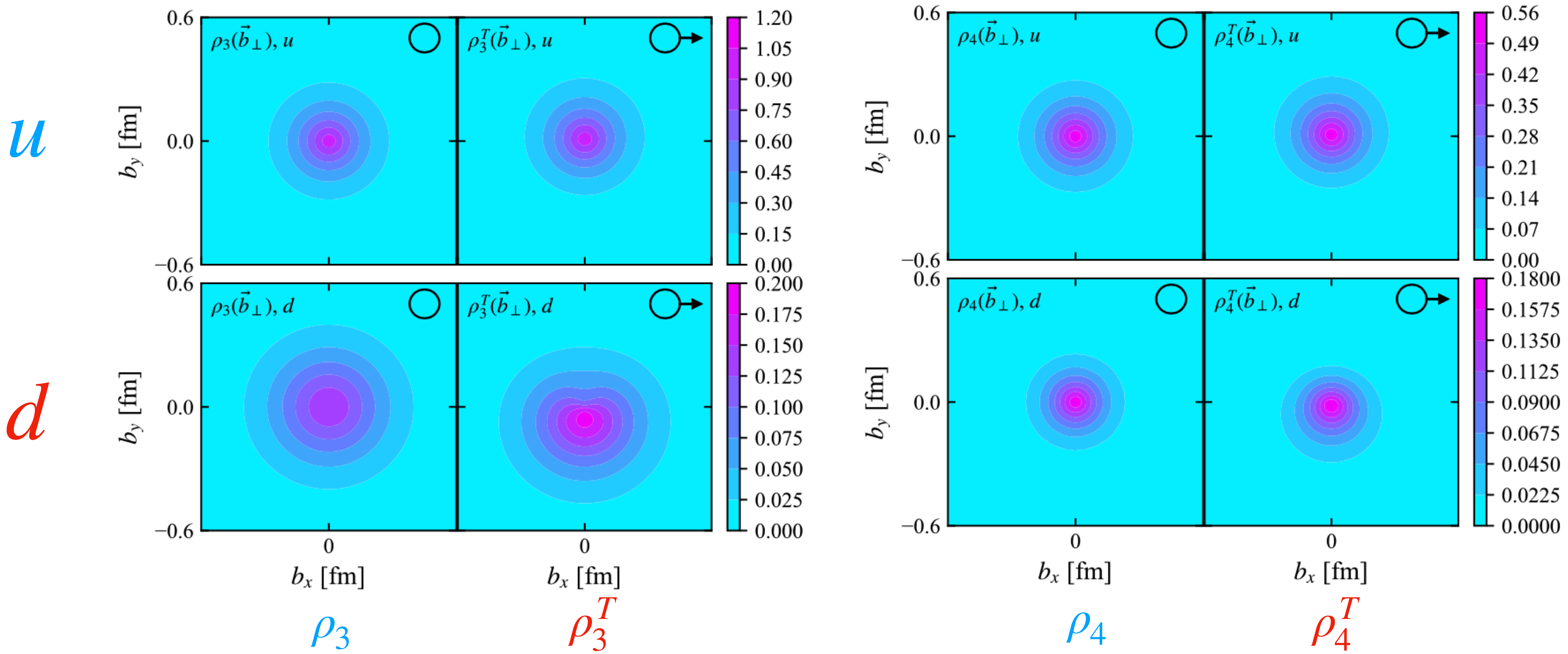


- The **2nd** moment: momentum distribution.
- The total contribution of **u** and **d** quarks to the transverse center of energy is small.

$$\sum_{u,d} \int d^2\vec{b}_\perp \, \vec{b}_\perp \rho_2^T(\vec{b}_\perp) = 1/(2M) B_{20}^{u+d}(0)$$

$$B_{20}^{u+d}(0) = 0.047(33)(65)$$

Impact parameter space interpretation



- Higher moments are weighted by x^n, exhibiting a sharper drop in the transverse distance.

Summary and outlook

- We carried out lattice calculation of the quasi-GPD matrix elements of proton using the Lorentz invariant amplitudes.
- The matrix elements are renormalized in ratio scheme and the Mellin moments up to the 5th ones were extracted using the leading-twist short distance factorization.
  - The methods can be extended to other kind of GPDs and non-zero skewness.
  - Hybrid renormalization and LaMET matching for x dependence ongoing.

Thanks for your attention!