Finite-volume corrections to the vector correlator from space-like observables

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November 19th, 2020

MTH and A. Patella, *PRL* 123 (2019) 172001
MTH and A. Patella, *JHEP* 10 (2020) 029

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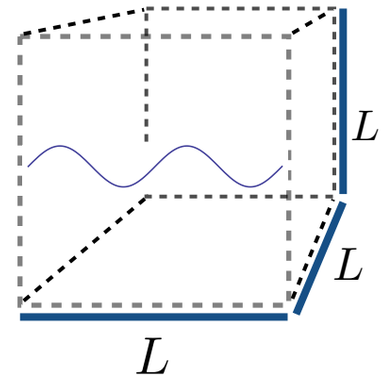
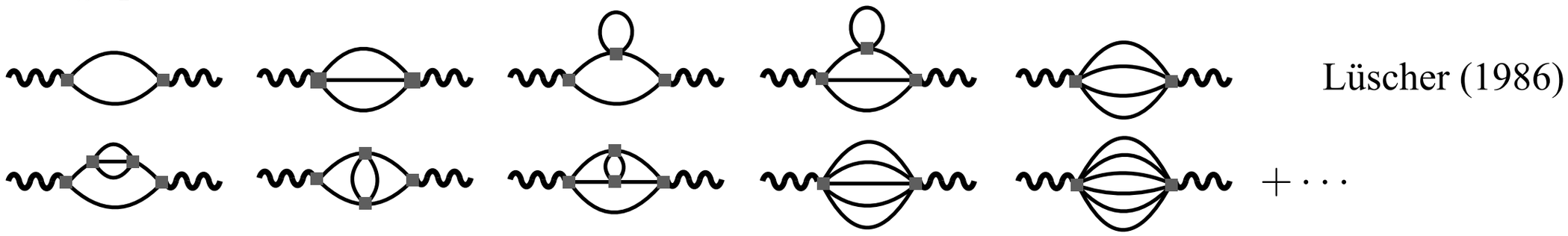
Set-up

- $T \times L^3$ periodic, Euclidean, all-orders representation



$$G(x_0|T,L) = -\frac{1}{3}\sum_{k=1}^{3}\int_0^L d^3x\,\langle j_k(x) j_k(0)\rangle_{T,L}$$



Lüscher (1986)

- Expand $\Delta G(x_0|T,L) = G(x_0|T,L) - G(x_0|\infty)$ …

$$\begin{aligned}\Delta G(x_0|T,L) = &\mathcal{O}(e^{-M_\pi L}) + \mathcal{O}(e^{-\sqrt{2}M_\pi L}) + \mathcal{O}(e^{-\sqrt{3}M_\pi L}) + \mathcal{O}(e^{-1.9M_\pi L}) + \cdots \\ &+ \mathcal{O}(e^{-M_\pi T}) + \mathcal{O}(e^{-2M_\pi T}) + \cdots \\ &+ \mathcal{O}(e^{-M_\pi\sqrt{T^2+L^2}}) + \cdots \\ &+ \mathcal{O}(e^{-M_K L}) + \cdots\end{aligned}$$

$\sqrt{2+\sqrt{3}} \approx 1.92$

short paper (*1904.10010*) long paper *(2004.03935)* this talk

- Similar expansion for $\Delta a_\mu(x_0|T,L)$

Result

Leading *L*-dependence

$$\Delta G(x_0|T,L) = -\sum_{\boldsymbol{n}\neq\boldsymbol{0}} \int \frac{dp_3}{2\pi} \frac{e^{-|\boldsymbol{n}|L\sqrt{M_\pi^2+p_3^2}}}{24\pi L|\boldsymbol{n}|} \int \frac{dk_3}{2\pi} \cos(x_0 k_3)\, \mathrm{Re}\, T(-k_3^2, -k_3 p_3) + \mathcal{O}(e^{-M_\pi T}, e^{-1.9 M_\pi L})$$

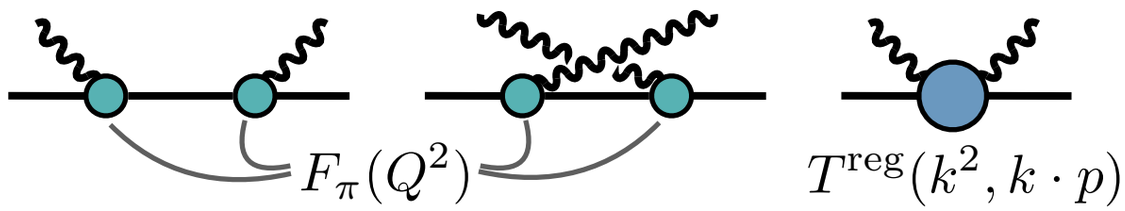
volume dependence — Compton amplitude

$$T(k^2, k\cdot p) \equiv i \lim_{\boldsymbol{p}'\to\boldsymbol{p}} \sum_{q=0,\pm1} \int d^4x\, e^{ikx}\, \langle \boldsymbol{p}', q|\mathrm{T}\mathcal{J}_\rho(x)\mathcal{J}^\rho(0)|\boldsymbol{p}, q\rangle$$

spacelike photons only

Modeling the amplitude

$$T(k^2, k\cdot p) =$$



Monopole form

$$F_\pi(Q^2) = \frac{1}{1+Q^2/M^2}$$

One-loop ChPT

$$T^{\mathrm{reg}} = c_0 + c_1\boldsymbol{k}^2 + \frac{7M_\pi^2 + 4\boldsymbol{k}^2}{6\pi^2 f_\pi^2}\, \sigma \cot^{-1}\sigma \Big|_{\sigma=\sqrt{1+4M_\pi^2/\boldsymbol{k}^2}}$$

D. Brommel et al. (QCDSF/UKQCD) (2007)

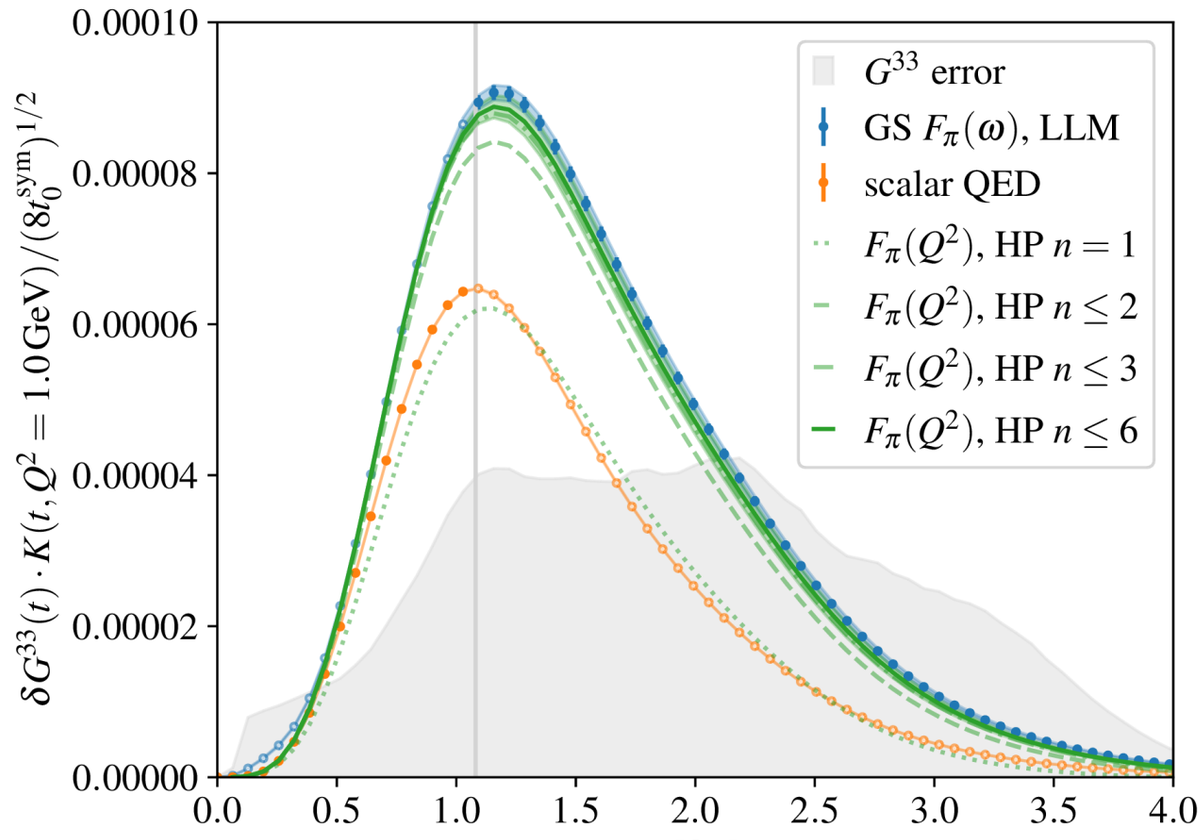
Numerical estimates

Non g-2 implications

$\Delta G(x_0 \,|\, \infty, L) \times$ kernel for vector polarization function at momentum transfer of $1\ \mathrm{GeV}^2$

CLS D200, $M_\pi \approx 200$ MeV, $M_\pi L \approx 4.2$

from M. Cè, K. Miura *et al.*, *in progress*



Implications for g-2

$$\Delta a_\mu(L) = \int_0^\infty dx_0\, \mathcal{K}(x_0) \Delta G(x_0|\infty, L)$$

$M_\pi = 137$ MeV $\qquad$ $m_\mu = 106$ MeV $\qquad$ $M = 727$ MeV

		$F_\pi(Q^2) = [1 + Q^2/M^2]^{-1}$					$T^{\mathrm{reg}}_{\mathrm{ChPT}}$
$M_\pi L$	$-10^{10} \times \Delta a_\mu(L)$	$e^{-M_\pi L}$	$e^{-\sqrt{2} M_\pi L}$	$e^{-\sqrt{3} M_\pi L}$	$e^{-2 M_\pi L}$	$e^{-\sqrt{5} M_\pi L}$	$e^{-M_\pi L}$
4	22.1	8.82	8.12	2.24	0.70	1.33	0.13
5	9.94	5.96	3.00	0.57	0.14	0.20	0.04
6	4.40	3.23	0.99	0.13	0.02	0.03	0.01

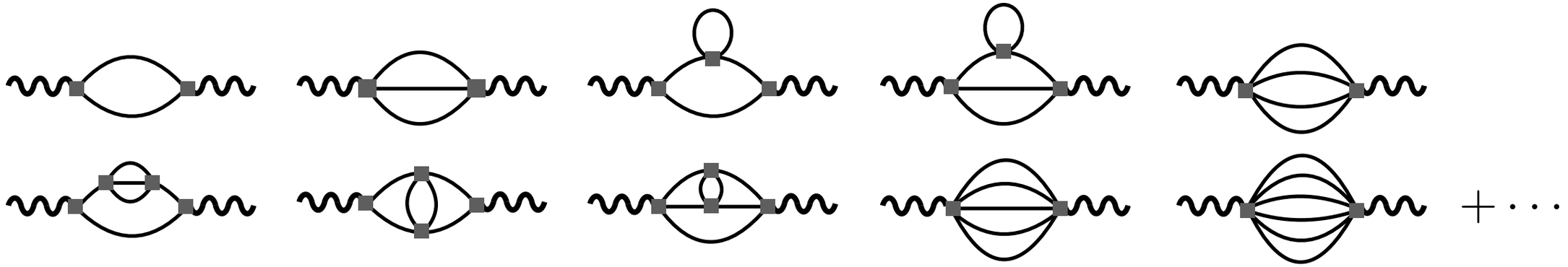
Also good agreement with BMW and RBC estimates

Thanks for listening!

Backup

Diagrammatic expansion

- All orders diagrammatic expansion

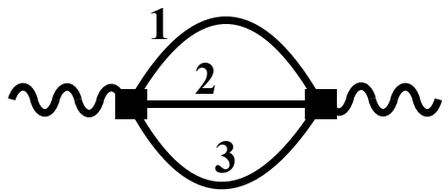
$a_\mu(L) =$  $+\cdots$

- Loop momenta are summed… rewrite using ***Poisson summation***

$$\frac{1}{L^3}\sum_{\boldsymbol{k}} \longrightarrow \sum_{\boldsymbol{n}} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e^{iL\boldsymbol{n}\cdot\boldsymbol{k}}$$

$\boldsymbol{n} = (n_x, n_y, n_z)$ can be interpreted as loop pion wrapping the torus

- Some subtleties in assigning ***Poisson modes***



$$\begin{matrix} \boldsymbol{n}_1 = (0,0,1) \\ \boldsymbol{n}_2 = (0,0,1) \end{matrix} \equiv \begin{matrix} \boldsymbol{n}_2 = (0,0,0) \\ \boldsymbol{n}_3 = (0,0,1) \end{matrix}$$

understood as a gauge redundancy → gauge fixing to catalogue effects

Lüscher (1986)

Leading terms

- Leading contributions: *only one pion wraps the torus*

$$\Delta a_\mu(L) = \int_0^\infty dx_0\, \mathcal{K}(x_0) \left[\; \boxed{L} \; + \frac{1}{2} \; \boxed{L} \; \right] + \mathcal{O}(e^{-1.9 M_\pi L})$$

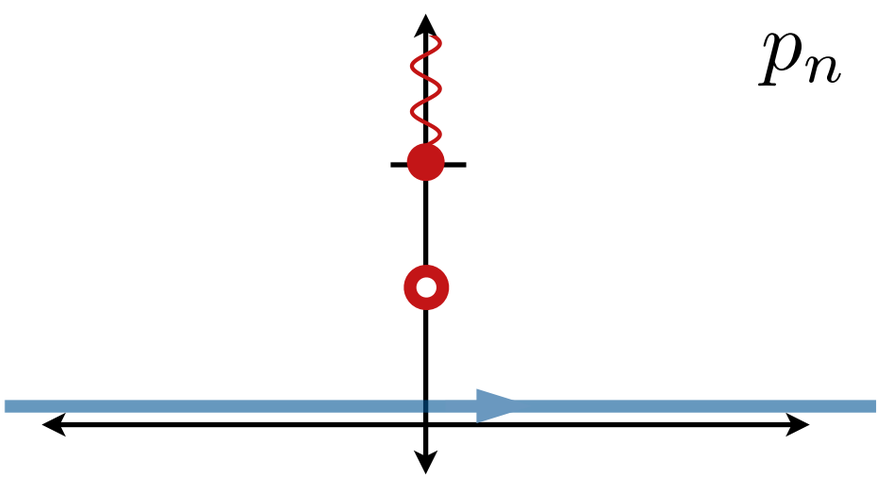
one-particle irreducible vertices

dressed propagators

leading 2-wrap contribution

- All L dependence inside… $\quad -\!\!\boxed{L}\!\!- \equiv \sum_{\boldsymbol{n}} \dfrac{e^{iL\boldsymbol{n}\cdot\mathbf{p}}}{p^2 + M_\pi^2 + \Sigma(p^2)}$

- Exponential decay? Decompose: $\mathbf{p} = p_n \hat{\boldsymbol{n}} + \mathbf{p}_\perp$



pole contributions → on-shell pions → ***physical Compton amplitude***