

# Introduction

- scale-setting is as old as lattice QCD
- "The only constant in life is change."
  - 80's: string tension
  - 90's: rho mass, Sommer-scale
  - 00's: decay constants
  - 10's: gradient-flow, omega/xi mass
  - 20's: ??
- changing is not always simple, eg. interpolation to the physical point might turn into an extrapolation
- experimental scales (omega, decay constant, ...) vs. theory scales (gradient flow, ...)
- comprehensive review [[Sommer,Lattice13](#)]

# Scale precision for HVP

- first moment of vacuum polarization  $\Pi_1 = d\Pi/dQ^2$

$$\delta a/a = 1\% \rightarrow \delta\Pi_1/\Pi_1 = 2\%$$

(there might be corrections due to change in physical point)

- magnetic moment  $a_\mu$  [Mainz'17]

$$\delta a/a = 1\% \rightarrow \delta a_\mu/a_\mu = 1.8\%$$

sub-percent needs scale determination with few per-mill

- window observable

$$\delta a/a = 1\% \rightarrow \delta a_{\mu,\text{win}}/a_{\mu,\text{win}} = 0.5\%$$

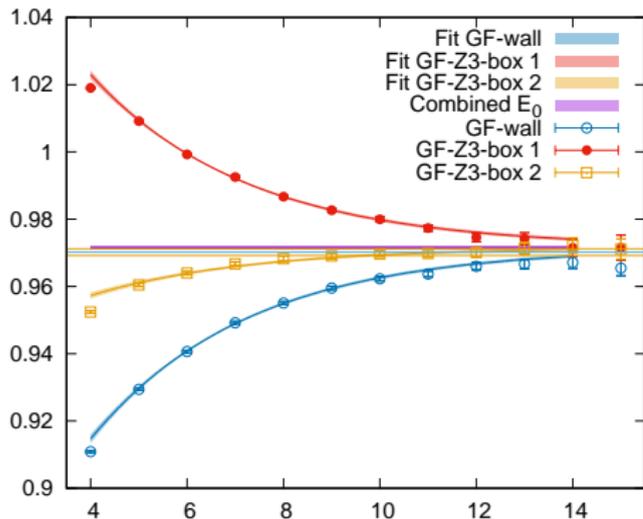
# Experimental scales

- 1 decay constant “Turning away from  $f_\pi$  ...”
  - EM correction [Carrasco et al '15, Giusti et al '18]
  - available in ETMC/Rome setup
- 2 omega:
  - already in use RBC →Cristoph Lehner, BMWc
  - Lukas Varnhorst, PACS (xi)
  - working on it (FNAL/HPQCD/MILC, ETMC/Rome, Mainz)
- 3 charmonium splitting  $\frac{1}{2}M_{J/\psi} - M_D$ 
  - FNAL/HPQCD/MILC achieve 0.3% precision on  $w_0$
  - Dan Hatton

## RBC/UKQCD $m_{\Omega^-}$

Example: 48l ensemble

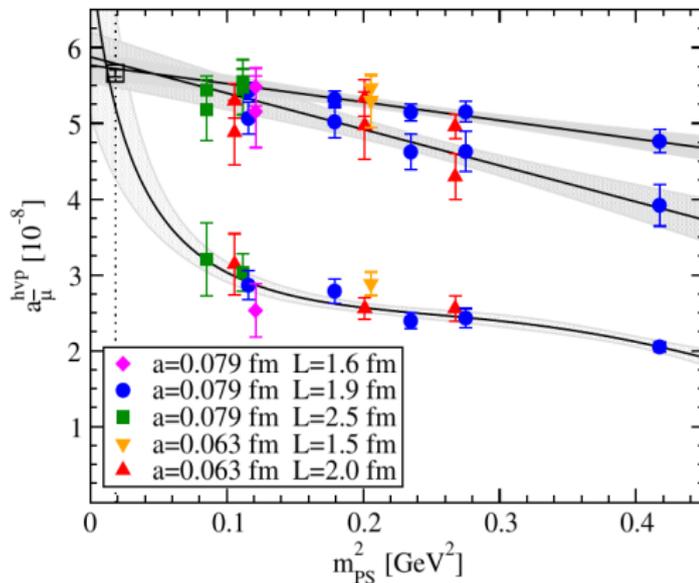
( $m_{\pi} = 135$  MeV,  $a^{-1} \approx 1.73$  GeV)



- ▶ Phys.Rev.D 93 (2016) 7, 074505: Using  $\Omega$  correlators of Coulomb GF wall – point and  $Z_3$  GF box ( $\frac{1}{3}$  lattice extent) – point
- ▶ RBC/UKQCD 2018: add quenched QED corrections to  $m_{\Omega^-}$ ,  $a^{-1}$  determined at 0.17%
- ▶ In progress: add additional box size, see left figure for  $\frac{1}{2}$  lattice extent on 48l
- ▶ In progress: continuum limit and beyond quenched QED of RBC/UKQCD 2018  $m_{\Omega^-}$  determination

# Lepton mass rescaling [Feng et al '11]

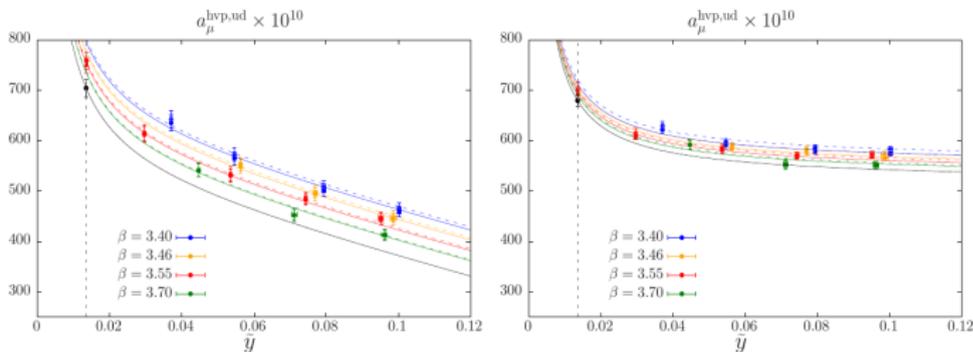
- make extrapolations (chiral or continuum) easier
- rescale lepton mass  $m_\mu \rightarrow \frac{M_\rho(a,m)}{M_\rho(\text{phys})} \cdot m_\mu$
- for  $\Pi(Q^2) = \frac{AQ^2}{Q^2 + M_\rho^2(a,m)}$  cancels all  $(a, m)$  dependence



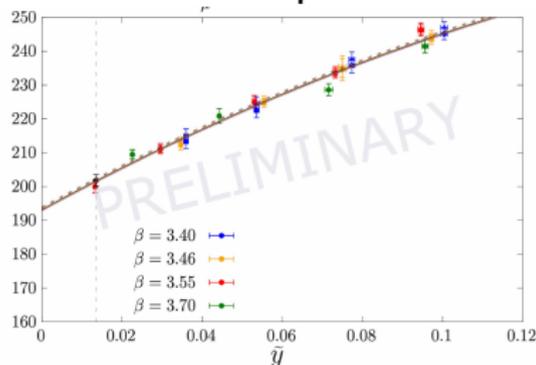
- first Wilson-prize award

# Lepton mass rescaling

- used by [Mainz, ETMC] with  $f_\pi(a, m)$  instead of  $M_\rho(a, m)$ . Is there a physics motivation?



- how does it perform in window?



no-rescaling

$$a_{\mu, \text{win}} = 207.2(4.5)(0.4)(1.2)$$

$f_\pi$ -rescaling

$$a_{\mu, \text{win}} = 201.8(2.0)(0.2)(0.8)$$

# Scale setting and isospin breaking

- scale setting → setting the physical point
- experimental observables

$$M_{\Omega}, \quad M_{\pi_0}^2, \quad M_{K_x}^2 = \frac{1}{2} (M_{K_0}^2 + M_{K_+}^2 - M_{\pi_+}^2), \quad \Delta M_K^2$$

- choose theory observables for isospin separation  
[BMWc'13'20] [QCDSF'16]

$$w_0, \quad M_{\pi_x}^2 = \frac{1}{2} (M_{uu}^2 + M_{dd}^2) \quad M_{ss}^2, \quad \Delta M^2 = M_{dd}^2 - M_{uu}^2$$

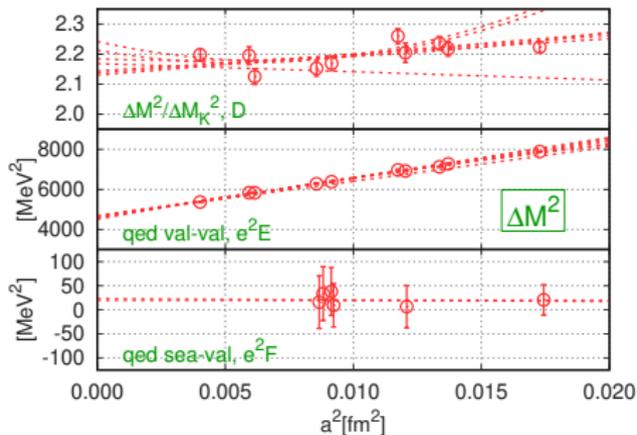
EM neutral; first three are symmetric under  $u \leftrightarrow d$  change

need physical values of  $w_0$ ,  $M_{ss}$  and  $\Delta M^2$  [BMWc'20]

$$w_0 = 0.17236[70] \text{ fm}, \quad M_{ss} = 689.89[49] \text{ MeV}, \quad \Delta M^2 = 13170[420] \text{ MeV}^2$$

can be simpler to handle than exp. observables and used by other groups

# Scale setting and isospin breaking [BMWc'20]



$$\frac{\Delta M^2}{M_\Omega^2} = D \left( \frac{\Delta M_K^2}{M_\Omega^2} \right) + E e^2 + F e^2 e_s$$

