

**Discussion session: the disconnected contribution to  $a_{\mu}^{\text{hvp}}$**

Harvey Meyer  
Johannes Gutenberg University Mainz

“The hadronic vacuum polarization from lattice QCD at high precision”  
Online workshop, 16-20 November 2020

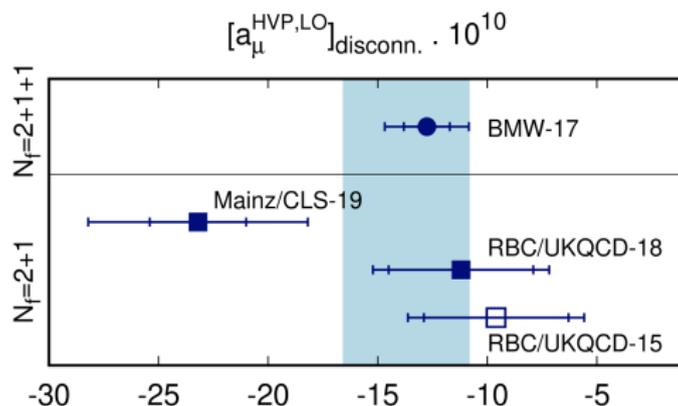
## The disconnected contribution $a_\mu^{\text{disc}}$ : developments up to 2016

- ▶ disconnected/connected  $\approx -1/10$  in partially quenched perturbation theory. [Jüttner, Della Morte 1009.3783 (JHEP)]
- ▶ disconnected/connected  $\approx -1/10$  is due to isospin symmetry and holds only asymptotically at long distances  
[Francis, Jäger, HM, Wittig 1306.2532 (PRD); Chakraborty et al, 1512.03270 (PRD)]
- ▶ “it appears very unlikely that the disconnected diagram contribution to  $a_\mu^{\text{hvp}}$  is more than 3%.”  
[HM, “g-2” workshop at Schloss Waldthausen, Mainz, April 2014; 1407.4021]
- ▶ “expected ratio disconnected/connected of -3.5%” (and in a slightly different estimate -3%) [Bijnens, Relefors 1609.01573 (JHEP)]

### First lattice calculations with a signal appeared in 2015:

- ▶ “the statistical error can be significantly reduced when using the same stochastic sources for light and strange loops.”  
[Gülpers et al. 1411.7592 (PoS)].
- ▶ First published result at the physical point:  
 $a_\mu^{\text{disc}} = -9.6(3.3)(2.3) \times 10^{-10}$  ( $\approx -1.4\% a_\mu^{\text{hvp}}$ ) .  
[RBC/UKQCD, 1512.09054 (PRL); see also B. Toth at LAT 15].

## 2020 White Paper overview of results

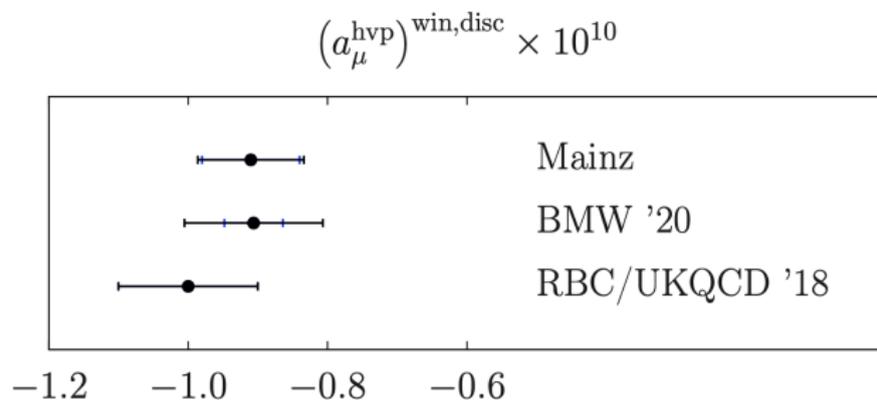


RBC/UKQCD-18: finite-size correction improved over RBC/UKQCD-15.

### Updates since White Paper:

- ▶ BMW-20:  $-13.15(1.28)_{\text{stat}}(1.29)_{\text{syst}}$  (minor change from BMW-17).
- ▶ Mainz/CLS-20<sub>prelim</sub>:  $-14.8(2.8)_{\text{tot}}$  (difference btw 2019 & 2020  $1.5\sigma$ ).
- ▶ preliminary results by Fermilab/HPQCD/MILC (cf. 1912.04382 & talk by C. McNeile).

## Diagnostic quantity: the window $[0.4, 1.0]$ fm



- ▶ Very good agreement between three collaborations.

## Disconnected contribution: aspects to be scrutinized

1. signal-to-noise ratio: are we able to follow the signal to long enough distances?

- ▶ BMW-20:  $t_c = 2.5$  fm, bounding method

$$0 \leq -G_{\text{disc}}(t) \leq \frac{1}{10} G_l(t_c) e^{-E_{2\pi}^{\text{free}}(t-t_c)} + G_s(t) + G_c(t).$$

- ▶ RBC/UKQCD-15: summand summed up to  $t_c = 2.3$  fm.

- ▶ Mainz/CLS: bounding method on the isoscalar correlator,

$$0 \leq G^{I=0}(t) \leq G^{I=0}(t_c) e^{-m_\rho(t-t_c)}. \text{ Noise still main limitation?}$$

2. infrared effects: finite-size correction, chiral dependence, behaviour of the tail.

All understood to be  $-1/9$  of the isovector contribution.

3. cutoff effects: relevant for all calculations.

Main limitation of the BMW-20 disconnected calculation?

4. Reliable statistical errors?

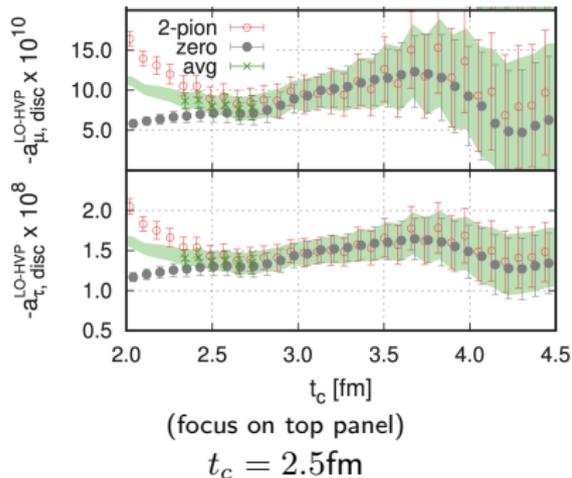
RBC/UKQCD-18 result based on 21 configurations ( $48^3 \times 96$ ,  $L = 5.5$  fm).

To put it provocatively: Is it now really excluded that

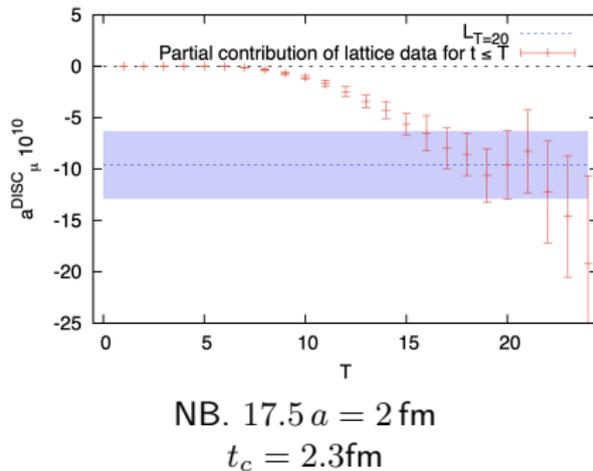
$$a_\mu^{\text{disc}} = -21 \times 10^{-10} \text{ (i.e. 3\% of total } a_\mu^{\text{hvp}} \text{)?}$$

# 1. Control over long-distance tail

BMW 1711.04980 (PRL)



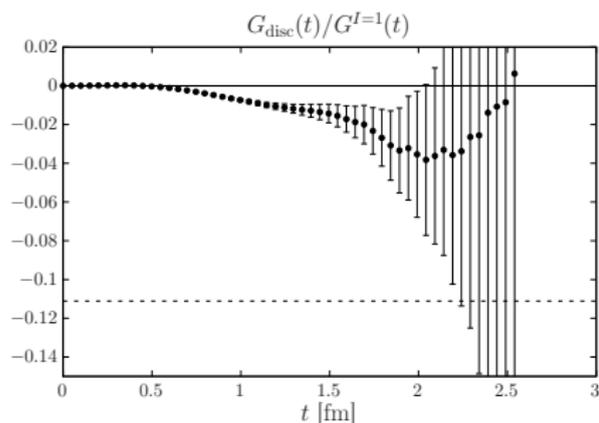
RBC/UKQCD 1512.09054 (PRL)



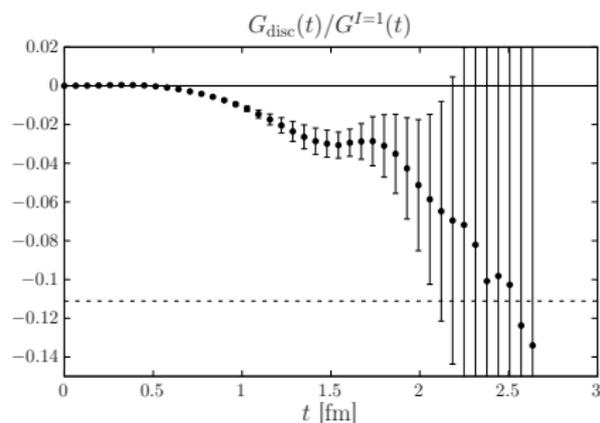
- ▶ the choices made look reasonable...

# 1. Ratio disconnected/isovector contribution (Mainz/CLS)

$$m_\pi = 170 \text{ MeV}, a = 0.04981 \text{ fm}$$



$$m_\pi = 130 \text{ MeV}, a = 0.06426 \text{ fm}$$



- ▶ asymptotic ratio of  $-1/9$  reached only at very long distances.

### 3. Cutoff effects

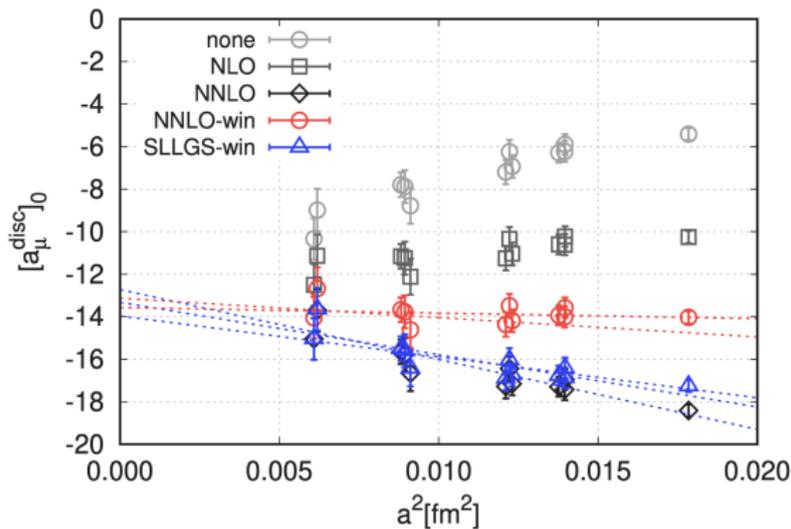
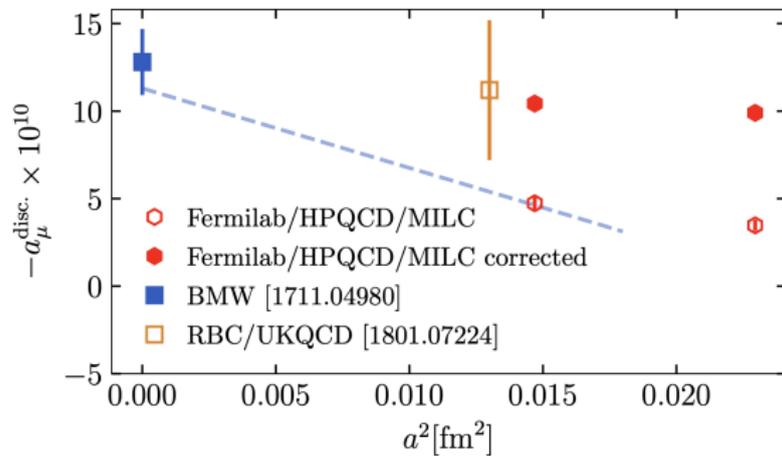


Fig. from 2002.12347 (BMW).

- ▶ RBC/UKQCD: only one lattice spacing so far.
- ▶ Mainz/CLS: four lattice spacings, no discernable cutoff effect by eye; but, still to be investigated.

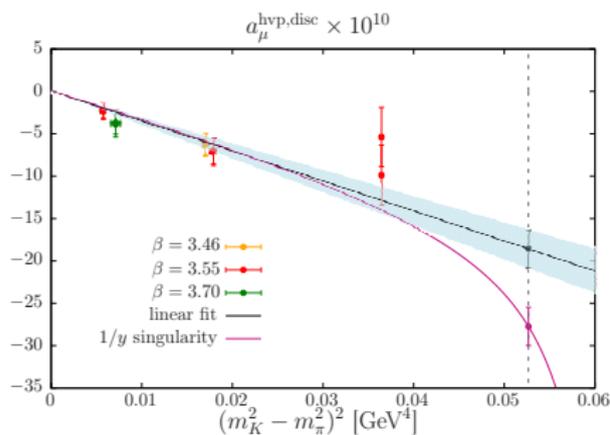
### 3. Cutoff effects



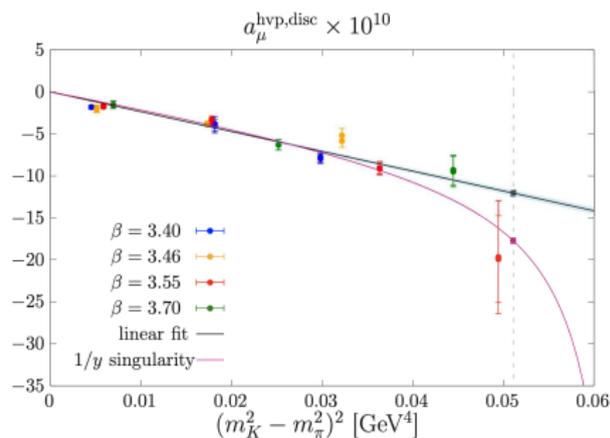
Fermilab/HPQCD/MILC 1912.04382

## 2. Chiral extrapolation

1904.03120 (PRD)



2020



- ▶ chiral extrapolations: Taylor expansion around  $SU(3)_f$  point including or not the singular  $a_\mu^{\text{disc}} \sim -\frac{\alpha^2}{810\pi^2} \frac{m_\mu^2}{4m_\pi^2}$  behaviour.
- ▶ Reduced statistical error: many more ensembles, higher statistics
- ▶ new stochastic estimator  $\rightsquigarrow$  next slide

## Mainz/CLS: algorithmic intermezzo

Using a set of random sources,  $\langle \eta(x) \bar{\eta}(y) \rangle_\eta = \delta_{xy}$ :

$$\text{Tr}\{\gamma_\mu D_f^{-1}(x, x)\} = \sum_y \langle \text{Tr}\{\gamma_\mu D_f^{-1}(y, x) \eta(x) \bar{\eta}(y)\} \rangle_\eta = \sum_y \langle \bar{\eta}(y) \gamma_\mu D_f^{-1}(y, x) \eta(x) \rangle_\eta.$$

Exploit  $SU(3)_f$  symmetry, use same source for light and strange [\[1411.7592\]](#):

$$\text{Tr}\{\gamma_\mu (D_\ell^{-1} - D_s^{-1})(x, x)\} = \sum_y \left\langle \bar{\eta}(y) \gamma_\mu (D_\ell^{-1} - D_s^{-1})(y, x) \eta(x) \right\rangle_\eta.$$

However, from  $D_f = D + m_f$  note the identity

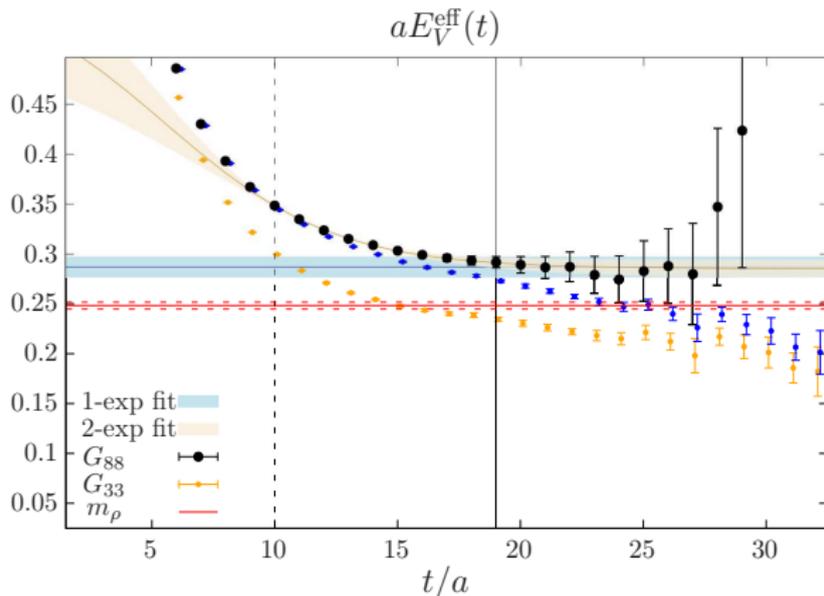
$$D_\ell^{-1} - D_s^{-1} = (m_s - m_l) D_\ell^{-1} D_s^{-1}.$$

This suggests inserting the random source  $\eta \bar{\eta}$  here  $\uparrow$  (rather than outside the product)

$$\begin{aligned} \Rightarrow & \text{Tr}\{\gamma_\mu (D_\ell^{-1} - D_s^{-1})(x, x)\} \\ &= (m_s - m_l) \left\langle \left( \sum_y \bar{\eta}(y) D_s^{-1}(y, x) \right) \gamma_\mu \left( \sum_z D_\ell^{-1}(x, z) \eta(z) \right) \right\rangle. \end{aligned}$$

For Gaussian sources, the variance of this estimator is demonstrably reduced.

## Effective masses in the isovector and isoscalar channel



- ▶ coupling to light  $\pi\pi$  states clearly seen in the isovector channel.
- ▶ in the isoscalar channel (black), the  $\pi\pi$  contribution is absent.
- ▶ blue points: leaving out the disconnected diagram from the isoscalar channel.

## Disconnected: from challenge to opportunity?

- ▶ the disconnected contribution is small ( $O(2\%)$ ), but is directly computable on the lattice (not through a large cancellation).
- ▶ phenomenological  $R$ -ratio: ignoring isospin breaking effects,

$$R_{\text{disc}} = R_{\text{isoscalar}} - \frac{1}{9}R_{\text{isovector}} - R_{\text{strange,conn}}.$$

Below  $\sqrt{s} = 980 \text{ MeV}$ ,  $R_{\text{strange,conn}} = 0$ ,

$$R_{\text{disc}} \approx R_{\pi^0\pi^+\pi^-} - \frac{1}{9}R_{\pi^+\pi^-}, \quad \sqrt{s} < 900 \text{ MeV}.$$

Due to the large cancellation, this is very sensitive to  $R_{\pi^+\pi^-}$ .

If the IB-breaking effects can be controlled, this could be used to confront different measurements of  $R_{\pi^+\pi^-}$  to lattice data.