FNAL/MILC/HPQCD results

C.T.H. Davies, C. DeTar, A.X. El-Khadra, E. Gámiz,
S. Gottlieb, D. Hatton, A.S. Kronfeld, S. Lahert, J. Laiho,
G.P. Lepage, Yuzhi Liu, P.B. Mackenzie, C. McNeile, E.T. Neil,
T. Primer, J.N. Simone, D. Toussaint, R.S. Van de Water, and
A. Vaquero

November 17, 2020

Introduction

In Phys.Rev.D 100 (2019) 3, 034506 we published

$$10^{10} a_\mu^{\mathrm{HVP,LO}} = 699(15)_{u,d}(1)_{s,c,b}$$

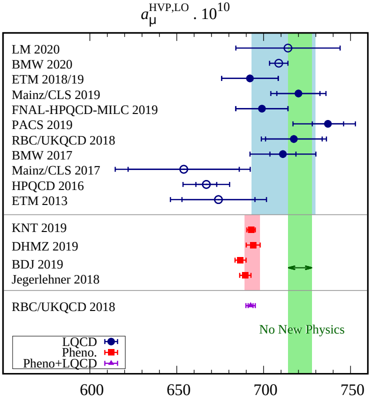
- We have calculated the LO-HVP contribution with an error of 2.2%.
- Our current target is for an error of 0.5%
- As usual in physics the errors are a mixture of systematic and statistical errors.

This talk will describe the various calculations in progress to reduce the overall error.

Why a target error of 0.5 % ?

- An error of 0.5% will give an accuracy similar to the phenomenological estimates.

Plot from arXiv:2006.04822



Current status

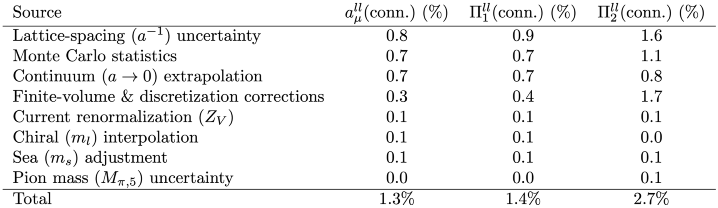
- Light quarks make by far the largest contribution compared with s, c, and b quarks.

Quark	$10^{10} a_\mu^{\mathrm{HVP,LO}}$
Light	630.8(8.8)(13)
Strange	53.40(60)
Charm	14.40(40)
Bottom	0.270(40)
Total	699(15)

- Second error of first line (13) is from estimate of strong iso-spin breaking, electromagnetism, and quark-disconnected diagrams. (See 1902.04223 for details.)

Connected light-quark error budget

- Error budget in percent for anomalous magnetic moment and two Taylor coefficients
- Top three contributions are dominant, but finite volume effects become important for larger moments



Source	a_μ^{ll}(conn.) (%)	Π_1^{ll}(conn.) (%)	Π_2^{ll}(conn.) (%)
Lattice-spacing (a^{-1}) uncertainty	0.8	0.9	1.6
Monte Carlo statistics	0.7	0.7	1.1
Continuum ($a \to 0$) extrapolation	0.7	0.7	0.8
Finite-volume & discretization corrections	0.3	0.4	1.7
Current renormalization (Z_V)	0.1	0.1	0.1
Chiral (m_l) interpolation	0.1	0.1	0.0
Sea (m_s) adjustment	0.1	0.1	0.1
Pion mass ($M_{\pi,5}$) uncertainty	0.0	0.0	0.1
Total	1.3%	1.4%	2.7%

What needs to be done

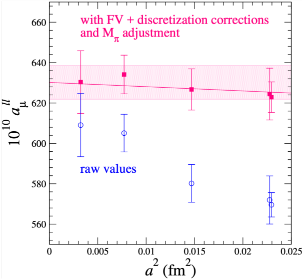
To reduce the errors on the light connected contribution

- Reduce the statistical errors
- Reduce errors on the continuum extrapolation
- Improve isospin breaking estimate
- Scale setting (determine the lattice spacing)

We also need to explicitly calculate

- Disconnected contribution
- QCD+QED contributions

Continnum limit of the connected contribution



- Must fit results as a function of a^2, taking quark-mass mistuning into account.

$$a_\mu^{ll} = a_\mu^{ll}(cont)\left(1 + c_s \sum_{f=l,l,s,c} \frac{\delta m_f}{\Lambda} + c_{a^2}\frac{(a\Lambda)^2}{\pi^2}\right)$$

- This is only light-quark connected contribution
- $\Lambda = 500$ MeV, and priors, $c_s = 0.0(3)$, $c_{a^2} = 0(1)$.

Lattice Ensembles

The starting point of the calculation are the gauge configurations.

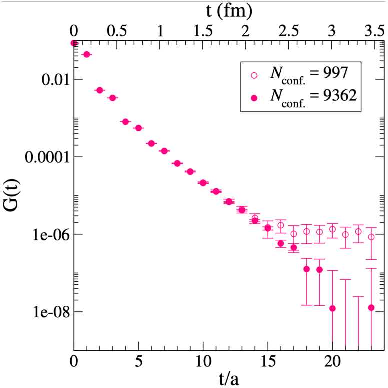
- We use Nf=2+1+1 HISQ ensembles from the MILC collaboration with physical light quark masses
- Results from above ensembles appeared in arXiv:1902.04223
- These use physical light quark masses.

$\approx a$ (fm)	$am_l^{\text{sea}}/am_s^{\text{sea}}/am_c^{\text{sea}}$	w_0/a	M_{π_5} (MeV)	$(L/a)^3 \times (T/a)$	$N_{\text{conf.}}$
0.15	0.00235/0.0647/0.831	1.13670(50)	133.04(70)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	1.13215(35)	134.73(71)	$32^3 \times 48$	9362
0.12	0.00184/0.0507/0.628	1.41490(60)	132.73(70)	$48^3 \times 64$	998
0.09	0.00120/0.0363/0.432	1.95180(70)	128.34(68)	$64^3 \times 96$	1557
0.06	0.0008/0.022/0.260	3.0170(23)	134.95(72)	$96^3 \times 192$	1230

- One ensemble with 1+1+1+1 sea quarks at 0.15 fm with 5000 configurations (1710.11212).
- We can reduce the statistical errors and error from the continuum extrapolation by generating more configurations.

Vector correlator at $a \approx 0.15$ fm

- Beyond about 2 fm the vector correlator gets very noisy. Higher statistics help. Improved measurement techniques can help as well.
- We fit the propagator at shorter distance and use fit to extend to $t > t^\star$.
- We also investigated the bounding box method.

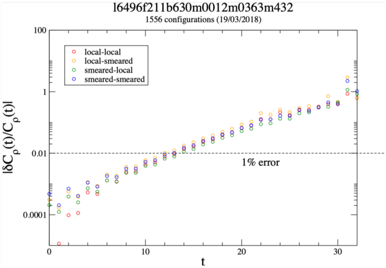


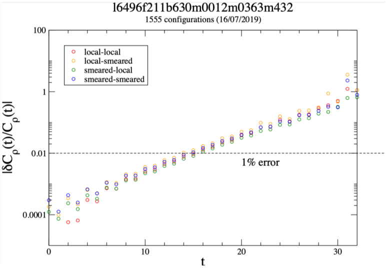
Current status of lattice generation

Ensemble	No. PRD paper	No. now	No. measurements
a=0.15 fm	9400	9400	9400
a=0.12 fm	1000	10000	5700
a=0.09 fm	1600	5400	5400
a=0.06 fm	1230	2000	1360

- The computational cost of the generation of the lattices increases as the lattice spacing decreases.
- The generation of the lattices with lattice spacings 0.12 and 0.06 fm are ongoing.
- Use the (TSM) truncated solver method for measurements.
- Two-pion contributions to the hadronic vacuum polarization with staggered quarks, Shaun Lahert, Thursday.

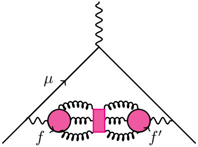
Reducing the errors on the connected correlator





- Connected vector correlator on the 0.09 fm ensemble.
- On the left we have only 16 sloppy solves per configuration, and on the right we have 48 sloppy solves
- Note that 1% error goes from t=12 to t=14.

Disconnected analysis



- Disconnected contribution is challenging to calculate.

a=0.15 fm	(1690 lattices measured) Yamamoto et al., 1811.06058 Lattice 2018
a=0.12 fm	(780 lattices measured) DeTar et al., 1912.04382 Lattice 2019
a=0.09 fm	(111 lattices measured) Ongoing

The runs on the 0.09 fm ensemble use deflation with 1000 eigenvectors. Low modes exactly and high modes via stochastic method with TSM.

Disconnected HVP

- Beyond 2 fm, use fit to replace data to reduce noise
- Fit includes omega-like and rho-like states

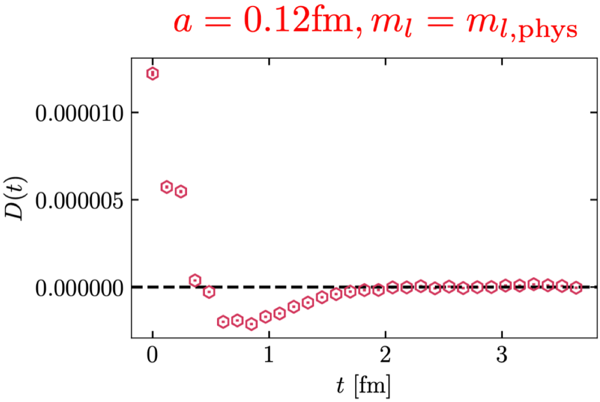
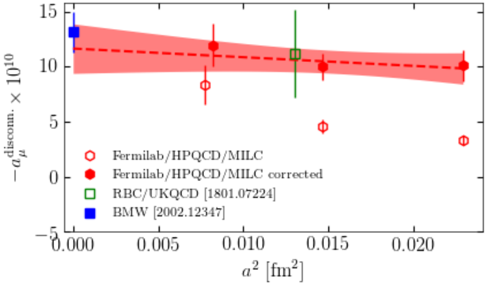


Figure: Disconnected correlator

Disconnected HVP Summary

Preliminary summary of the disconnected results.



- Corrections for taste-effects and finite volume between open and closed red circles.
- We also have results for isospin breaking effects in the disconnected correlator, but the analysis is still in progress.

Including QED with QCD

We are using two approaches:

1. Quenched QED with QCD. (Sea quarks are neutral).
2. Producing a dynamical QED+QCD ensemble.

Including quenched QED with QCD

Non-compact $A_\mu(k)$ generated in Feynman gauge for each QCD gluon field configuration. Use the QED_L formulation to deal with zero modes.

$$U_\mu^{QCD+QED} = \exp(ieQA_\mu)U_\mu^{QCD}$$

- We are currently computing connected QED+QCD corrections.
- The value of Z_V has been computed including quenched QED using the RI-SMOM scheme. (Hatton et al., HPQCD, PhysRevD.100.114513)

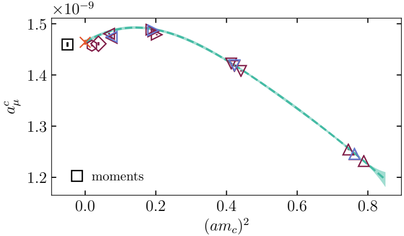
We are developing techniques to estimate some of the QED effects in the sea.

QCD+QED for charmonium properties

D. Hatton, C. T. H. Davies, B. Galloway, J. Koponen, G. P. Lepage, A. T. Lytle, HPQCD, arXiv:2005.01845.

- Precision study of quenched QED effects on properties of η_c and J/ψ
- HISQ action with lattice spacings: 0.15, 0.12, 0.09, 0.06, 0.045, 0.03 fm.
- Hyperfine mass splitting $M_{J/\psi}$ - M_{η_c}
- QED contribution to decay constants of J/ψ.
- QED contributions to mass of the charm quark m_c.
- Time-moments of the vector charmonium current-current correlators.

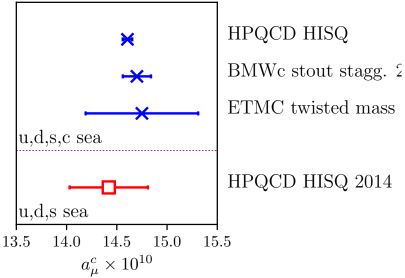
Continuum extrapolation of a_μ^c (2005.01845)

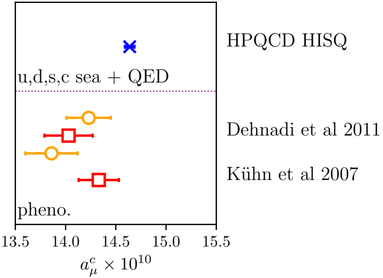


- m_c is tuned from the J/ψ mass both in pure QCD and in QCD+QED.
- $a_\mu^c = 14.638(47) \times 10^{-10}$, which is 2.5σ higher than the value derived for the the c quark contribution using experimental data on $R(e^+e^- \to \text{hadrons})$.
- The effect of QED $\delta a_\mu^c = 0.0313(28) \times 10^{-10}$.

Comparison of a_μ^c to other calculations (2005.01845)

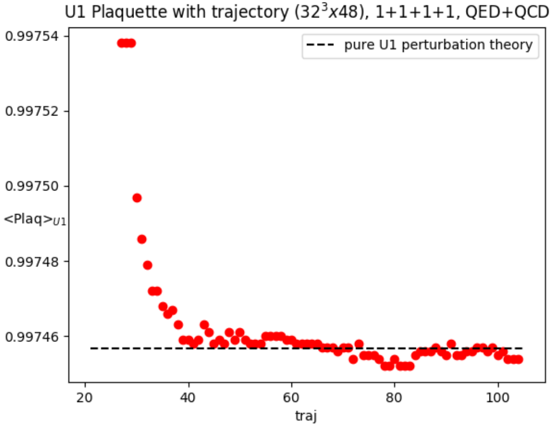
- Left plot is the HPQCD result for a_μ^c compared with other lattice results (QCD only).
- Right plot is the HPQCD result for a_μ^c compared with estimates from phenomenology.





Including QED with the QCD sea quarks

- Just starting (noncompact) QED + QCD HMC simulations.
- 1+1+1+1 sea quarks with QED, lattice spacing a =0.15 fm
- We are testing measurement strategies.



Scale setting

Scale setting accounts for the largest error to the light connected contribution.

- To convert the results from lattice units to physical units, such as GeV, requires the calculation of a “physical number”.
- High precision results requires a careful choice of the number to set the scale. For example, f_π (pion decay constant) has been used in the past, but the extraction of the experimental number depends on V_{ud}, which has recently changed by 2σ.

Unphysical numbers such as the Wilson flow parameter w_0 can be used to determine the physical scale.

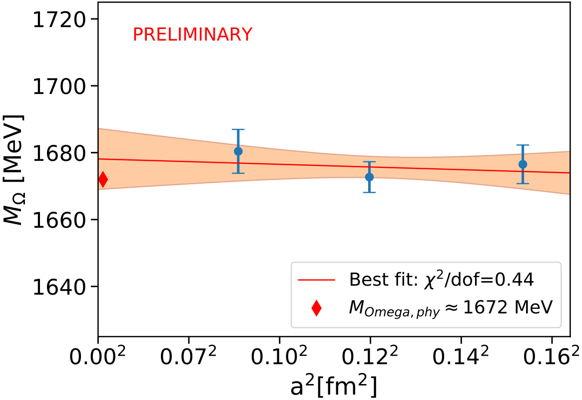
- It has excellent statistical precision, and it doesn’t require a sophisticated analysis to compute it from a simulation.
- It does not depend on valence quark masses.

However, we need to compute the physical value of w_0.

Computing the mass of the Ω baryon

.

HISQ (QCD only) simulations at lattice spacing = 0.15, 0.12, and 0.09 fm Ciaran Hughes, Yin Lin, Aaron S. Meyer, 1912.00028

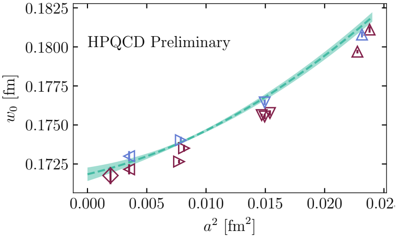


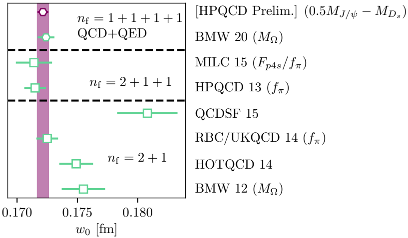
$M_\Omega = 1678(9)$ MeV (0.5 % error)

Calibration of the w_0 scale

.

- D. Hatton et al., Calculates w_0 by fixing the value of the mass difference $0.5M_{J/\psi} - M_{D_s}$ to the experimental average.
- Finite volume effects included.
- Using QCD only (red) and QCD+quenched QED (blue).
- Goal of 0.3% uncertainty in w_0. The analysis is still in progress.





Conclusions

I have given an overview of the calculations in progress to reduce the errors:

- Additional running and measurements on the connected correlators to reduce the statistics and continuum extrapolation error.
- Include two pion operators for connected contribution.
- Improved determination of lattice spacing (used to set scale).
- Computation of disconnected correlators.
- Inclusion of electromagnetic effects
- Improved estimate of isospin breaking