RBC+UKQCD Hadronic Vacuum Polarization Update

Aaron S. Meyer (asmeyer.physics@gmail.com)

UC Berkeley/LBL

November 16, 2020

The Hadronic Vacuum Polarization from Lattice QCD at High Precision

Collaboration

PHYSICAL REVIEW LETTERS **121,** 022003 (2018)

Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

T. Blum,[1] P. A. Boyle,[2] V. Gülpers,[3] T. Izubuchi,[4,5] L. Jin,[1,5] C. Jung,[4] A. Jüttner,[3] C. Lehner,[4,*] A. Portelli,[2] and J. T. Tsang[2]

(RBC and UKQCD Collaborations)+ Mattia Bruno +Aaron S. Meyer

[1]*Physics Department, University of Connecticut, Storrs, Connecticut 06269-3046, USA*
[2]*School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3FD, United Kingdom*
[3]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[4]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[5]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 25 January 2018; published 12 July 2018)

We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\mathrm{HVP\ LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\mathrm{HVP\ LO}}$.

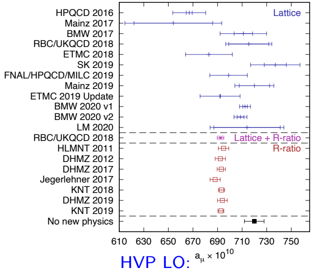
DOI: 10.1103/PhysRevLett.121.022003

Introduction

$g-2$ Theory Breakdown

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.893	0.010
EW	15.36	0.10
HVP LO	693.1	4.0
HVP NLO	-9.83	0.07
HVP NNLO	1.24	0.01
Hadronic light-by-light	9.2	1.8
Total SM prediction	11 659 181.0	4.3
BNL E821 result	11 659 208.9	6.3
Fermilab E989 target		≈ 1.6

[Phys.Rep. 887 (2020)]



HVP LO:

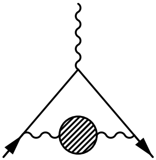
[PRD 101, 074515 (2020)]

[expt]$-$[theory] $= 27.9(7.6) \times 10^{-10} \implies 3.7\sigma$ tension

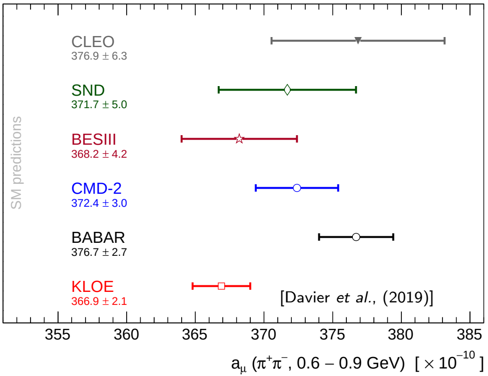
Hadronic parts dominate uncertainty

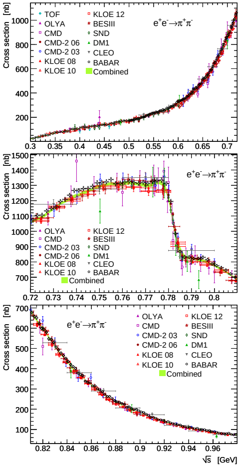
R-ratio precise, slowly improving
LQCD imprecise, rapidly improving

Soon test LQCD against R-ratio/No NP



Tensions in R-ratio data





KLOE−BABAR $\Delta a_\mu^{\pi\pi,\mathrm{exc.}} \sim 10 \times 10^{-10}$

R-ratio $\delta a_\mu^{\mathrm{HVP}} \sim 3 \times 10^{-10}$

Tension $>$ quoted uncertainty

Avoid tension w/ lattice-only estimate of a_μ^{HVP}

Use LQCD to inform experiment, resolve discrepancy

RBC/UKQCD Error Budget

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.1)_\mathrm{V}(0.2)_\mathrm{A}(0.2)_\mathrm{Z}$	$649.7(14.2)_\mathrm{S}(2.8)_\mathrm{C}(3.7)_\mathrm{V}(1.5)_\mathrm{A}(0.4)_\mathrm{Z}(0.1)_\mathrm{E48}(0.1)_\mathrm{E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}$	$53.2(0.4)_\mathrm{S}(0.0)_\mathrm{C}(0.3)_\mathrm{A}(0.0)_\mathrm{Z}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_\mathrm{S}(0.1)_\mathrm{C}(0.0)_\mathrm{Z}(0.0)_\mathrm{M}$	$14.3(0.0)_\mathrm{S}(0.7)_\mathrm{C}(0.1)_\mathrm{Z}(0.0)_\mathrm{M}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}$	$-11.2(3.3)_\mathrm{S}(0.4)_\mathrm{V}(2.3)_\mathrm{L}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}$	$5.9(5.7)_\mathrm{S}(0.3)_\mathrm{C}(1.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(1.1)_\mathrm{E}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_\mathrm{S}(0.0)_\mathrm{C}(0.0)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}$	$-6.9(2.1)_\mathrm{S}(0.4)_\mathrm{C}(1.4)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(1.3)_\mathrm{E}$
a_μ^{SIB}	$0.1(0.2)_\mathrm{S}(0.0)_\mathrm{C}(0.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E48}$	$10.6(4.3)_\mathrm{S}(0.6)_\mathrm{C}(6.6)_\mathrm{V}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}(1.3)_\mathrm{E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.1)_\mathrm{V}(0.3)_\mathrm{A}(0.2)_\mathrm{Z}(0.0)_\mathrm{M}$	$705.9(14.6)_\mathrm{S}(2.9)_\mathrm{C}(3.7)_\mathrm{V}(1.8)_\mathrm{A}(0.4)_\mathrm{Z}(2.3)_\mathrm{L}(0.1)_\mathrm{E48}$ $(0.1)_\mathrm{E64}(0.0)_\mathrm{M}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_\mathrm{S}(0.0)_\mathrm{C}(0.2)_\mathrm{V}(0.0)_\mathrm{A}(0.0)_\mathrm{Z}(0.0)_\mathrm{E}(0.0)_\mathrm{E48}$	$9.5(7.4)_\mathrm{S}(0.7)_\mathrm{C}(6.9)_\mathrm{V}(0.1)_\mathrm{A}(0.0)_\mathrm{Z}(1.7)_\mathrm{E}(1.3)_\mathrm{E48}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_\mathrm{RST}(2.1)_\mathrm{RSY}$	
a_μ	$692.5(1.4)_\mathrm{S}(0.2)_\mathrm{C}(0.2)_\mathrm{V}(0.3)_\mathrm{A}(0.2)_\mathrm{Z}(0.0)_\mathrm{E}(0.0)_\mathrm{E48}$ $(0.0)_\mathrm{b}(0.1)_\mathrm{c}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_\mathrm{M}(0.7)_\mathrm{RST}(2.1)_\mathrm{RSY}$	$715.4(16.3)_\mathrm{S}(3.0)_\mathrm{C}(7.8)_\mathrm{V}(1.9)_\mathrm{A}(0.4)_\mathrm{Z}(1.7)_\mathrm{E}(2.3)_\mathrm{L}$ $(1.5)_\mathrm{E48}(0.1)_\mathrm{E64}(0.3)_\mathrm{b}(0.2)_\mathrm{c}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_\mathrm{M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

[PRL 121, 022003 (2018)]

Full program of computations to improve total uncertainties:

- Reduce statistical uncertainties on light connected contribution
- Improve QED+SIB contributions
- Improve lattice spacing determination
- Finite volume and continuum extrapolation study

RBC/UKQCD Error Budget

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.2)_{\rm A}(0.2)_{\rm Z}$	$649.7(14.2)_{\rm S}(2.8)_{\rm C}(3.7)_{\rm V}(1.5)_{\rm A}(0.4)_{\rm Z}(0.1)_{\rm E48}(0.1)_{\rm E64}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_{\rm S}(0.0)_{\rm C}(0.1)_{\rm A}(0.0)_{\rm Z}$	$53.2(0.4)_{\rm S}(0.0)_{\rm C}(0.3)_{\rm A}(0.0)_{\rm Z}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_{\rm S}(0.1)_{\rm C}(0.0)_{\rm Z}(0.0)_{\rm M}$	$14.3(0.0)_{\rm S}(0.7)_{\rm C}(0.1)_{\rm Z}(0.0)_{\rm M}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}$	$-11.2(3.3)_{\rm S}(0.4)_{\rm V}(2.3)_{\rm L}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$5.9(5.7)_{\rm S}(0.3)_{\rm C}(1.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.1)_{\rm E}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$-6.9(2.1)_{\rm S}(0.4)_{\rm C}(1.4)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E}$
a_μ^{SIB}	$0.1(0.2)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E48}$	$10.6(4.3)_{\rm S}(0.6)_{\rm C}(6.6)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E48}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm M}$	$705.9(14.6)_{\rm S}(2.9)_{\rm C}(3.7)_{\rm V}(1.8)_{\rm A}(0.4)_{\rm Z}(2.3)_{\rm L}(0.1)_{\rm E48}(0.1)_{\rm E64}(0.0)_{\rm M}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$	$9.5(7.4)_{\rm S}(0.7)_{\rm C}(6.9)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.7)_{\rm E}(1.3)_{\rm E48}$
$a_\mu^{\text{R}-\text{ratio}}$	$460.4(0.7)_{\rm RST}(2.1)_{\rm RSY}$	
a_μ	$692.5(1.4)_{\rm S}(0.2)_{\rm C}(0.2)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}(0.0)_{\rm b}(0.1)_{\rm c}(0.0)_{\overline{\rm S}}(0.0)_{\overline{\rm Q}}(0.0)_{\rm M}(0.7)_{\rm RST}(2.1)_{\rm RSY}$	$715.4(16.3)_{\rm S}(3.0)_{\rm C}(7.8)_{\rm V}(1.9)_{\rm A}(0.4)_{\rm Z}(1.7)_{\rm E}(2.3)_{\rm L}(1.5)_{\rm E48}(0.1)_{\rm E64}(0.3)_{\rm b}(0.2)_{\rm c}(1.1)_{\overline{\rm S}}(0.3)_{\overline{\rm Q}}(0.0)_{\rm M}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

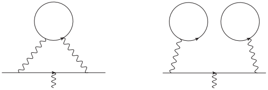
[PRL 121, 022003 (2018)]

Full program of computations to improve total uncertainties:

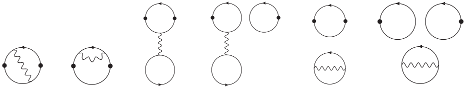
- Reduce statistical uncertainties on light connected contribution
- Improve QED+SIB contributions
- Improve lattice spacing determination
- Finite volume and continuum extrapolation study

Diagrams

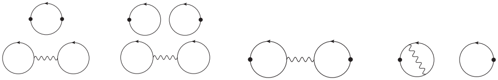
Isospin limit



QED corrections

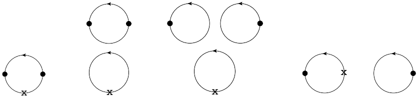


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking

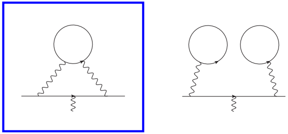


(a) M (b) R (c) R_d (d) O

Diagrams

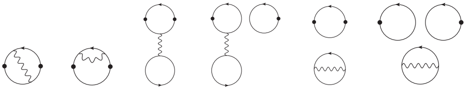
Isospin limit

isospin-symmetric light connected

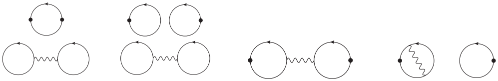


up,down qk loop in $I = 1$

QED corrections

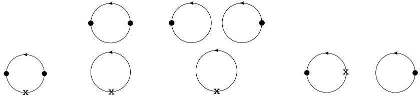


(a) V (b) S (c) T (d) T_d (e) D1 (f) $\mathrm{D1}_d$



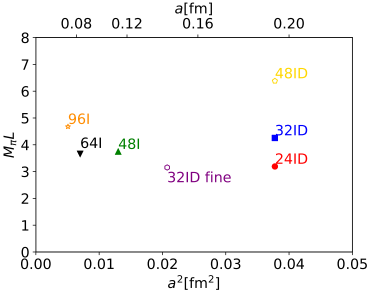
(g) D2 (h) $\mathrm{D2}_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

Lattice Ensembles



Many ensembles all at physical pion mass

$2+1$ flavor Möbius Domain Wall Fermions for valance & sea

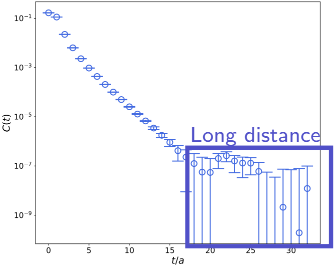
Connected light-quark distillation w/ 4 ensembles: 24ID, 32ID, 48I, 64I

High precision distillation study:

- significant improvement on statistics (improved bounding)
- measure FV directly, test model

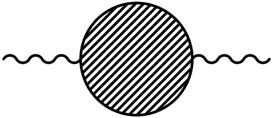
Light Connected HVP

Precision in LQCD HVP



$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle\Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$

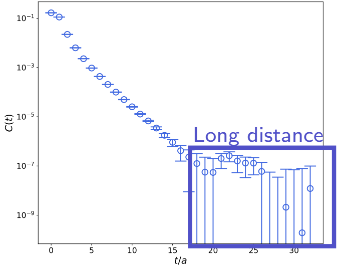
$$a_\mu = \sum_t w_t C(t)$$



LQCD $a_\mu^{\rm HVP}$ precision dominated by stat.uncertainty from long distance

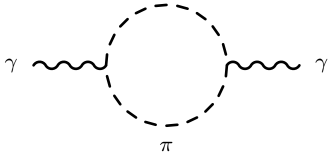
$C(t)$ small at large t, $\approx$ sum of few exponential

Precision in LQCD HVP



$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle\Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$

$$a_\mu = \sum_t w_t C(t)$$



LQCD a_μ^{HVP} precision dominated by stat.uncertainty from long distance

$C(t)$ small at large t, $\approx$ sum of few exponential

Long distance mostly $\pi\pi$, ρ

Calculate exclusive $\pi\pi$ in LQCD, replace data $\rightarrow$ fit (stat $\rightarrow$ syst)

Improved Operator Basis

Distillation smearing kernel $\mathcal{K} \implies$ access many operators

Use operator improvement for 2π [Phys.Rev.D 86 (2012) 034031]

Unimproved π, 2π operators:

$$O_\pi(\mathbf{p}) \sim \bar{Q}\mathcal{K}^\dagger \gamma_5 e^{-i\mathbf{p}\cdot\mathbf{x}} \mathcal{K} Q$$
$$O_{\pi\pi,j} \sim O_\pi(\mathbf{p}) O_\pi(-\mathbf{p})$$

Optimized π, Improved 2π:

$$O_\pi^{\text{opt.}}(\mathbf{p}) \sim \sum_k b_k \left(\bar{Q}\mathcal{K}^\dagger \Gamma_k e^{-i\mathbf{p}\cdot\mathbf{x}} \mathcal{K} Q \right), \quad \Gamma_k \in \{\gamma_5, \gamma_5\gamma_t, \gamma_5\gamma_i\}$$
$$O_{\pi\pi,j}^{\text{imp.}} \sim \left[O_\pi^{\text{opt.}}(\mathbf{p}) \right]^\dagger O_\pi^{\text{opt.}}(\mathbf{p})$$

Linear combo of π operators to remove excited state contamination from 1π

Expectation of reduced excited states in 2π operators

Operator Basis

$I = 1$, P-wave channel $\sim a_\mu^{\mathrm{HVP}}$

Vector current operators (2 total):

- Local $\mathcal{O}_{J_\mu} = \bar{Q}\gamma_\mu Q, \quad \mu \in \{1,2,3\}$
- Smeared $\mathcal{O}_{j_\mu} = \bar{Q}\mathcal{K}^\dagger \gamma_\mu \mathcal{K} Q$

2π operators (8 total) $\quad \mathbf{p} \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1),(2,0,0)\}$

- Unimproved: $O_{\pi\pi}(\mathbf{p}) \sim O_\pi^\dagger(\mathbf{p}) O_\pi(\mathbf{p})$
- Improved: $O_{\pi\pi}^{\text{imp.}}(\mathbf{p}) \sim \left[O_\pi^{\text{opt.}}(\mathbf{p})\right]^\dagger O_\pi^{\text{opt.}}(\mathbf{p})$

4π operators (2 total) $\quad \mathbf{p} = \frac{2\pi}{L} \times (1,0,0)$:

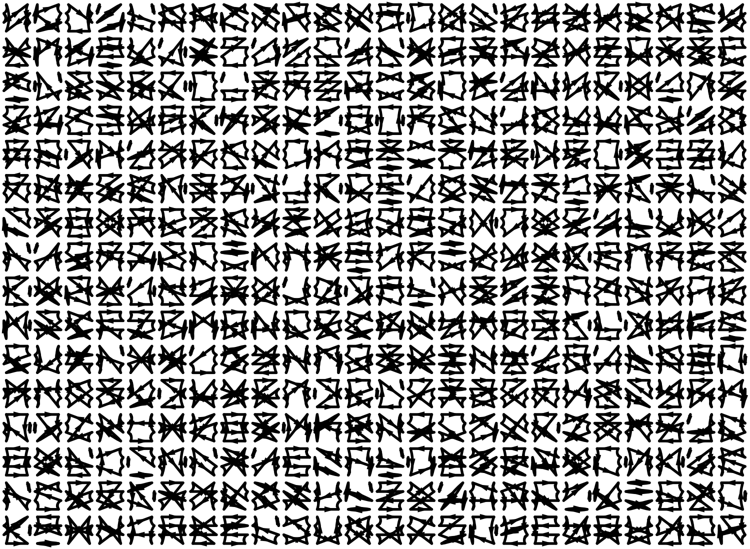
- $\mathcal{O}_{4\pi A}(\mathbf{p}) \sim O_{\pi\pi, I=1}(\mathbf{p}) O_{\pi\pi, I=2}(\mathbf{0})$
- $\mathcal{O}_{4\pi B}(\mathbf{p}) \sim O_{\pi\pi, I=1}(\mathbf{p}) O_{\pi\pi, I=0}(\mathbf{0})$

Other 4π isospin combinations vanish due to symmetrization

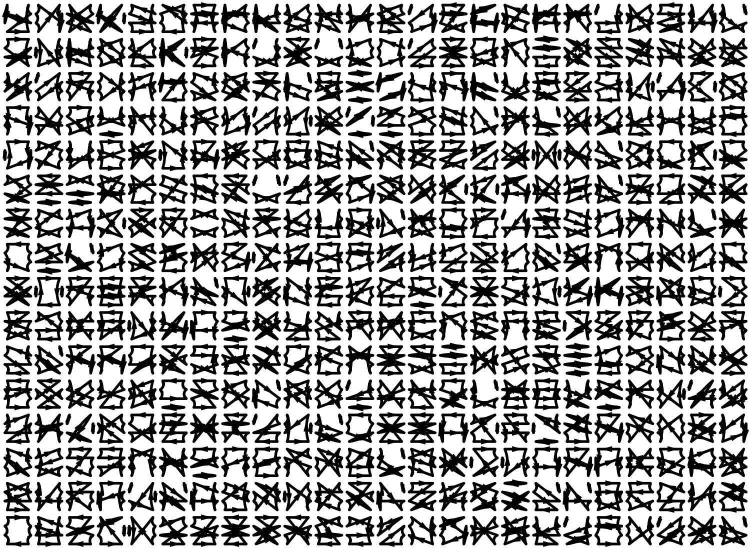
Nominal choice: 8 operators –

- 2 vector current
- 3 improved $\pi\pi$, $\quad \mathbf{p} \in \{(1,0,0),(1,1,0),(1,1,1)\}$
- 1 unimproved $\pi\pi$, $\quad \mathbf{p} \in \{(2,0,0)\}$
- 2 isospin combos 4π

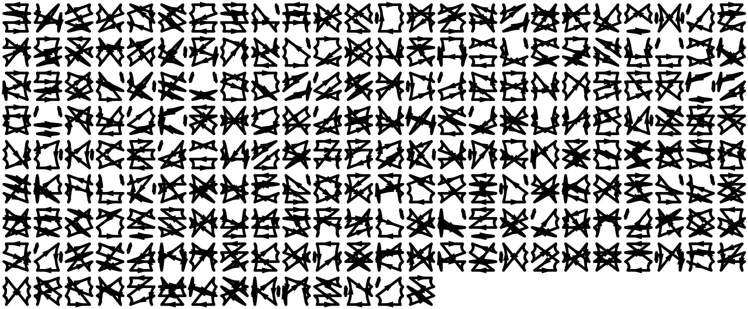
4π Contractions



4π Contractions cont...

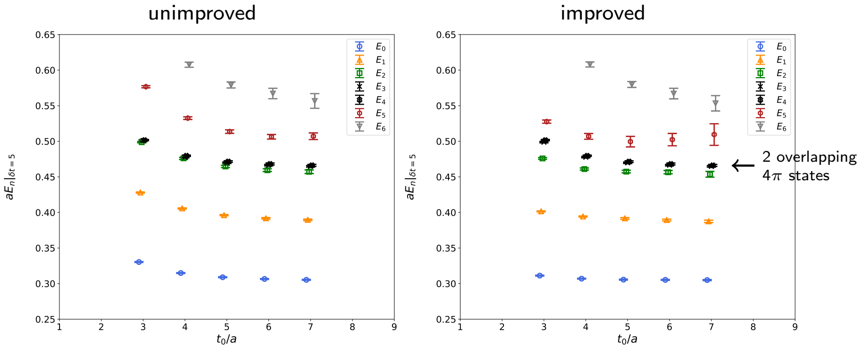


4π Contractions cont... cont...



- automated operator generator [1912.04917[hep-lat]]
- automated Wick contractor https://github.com/lehner/Wick
- automated perambulator contractor https://github.com/lehner/Contractor

Spectrum - 48I



Generalized EigenValue Problem (GEVP):

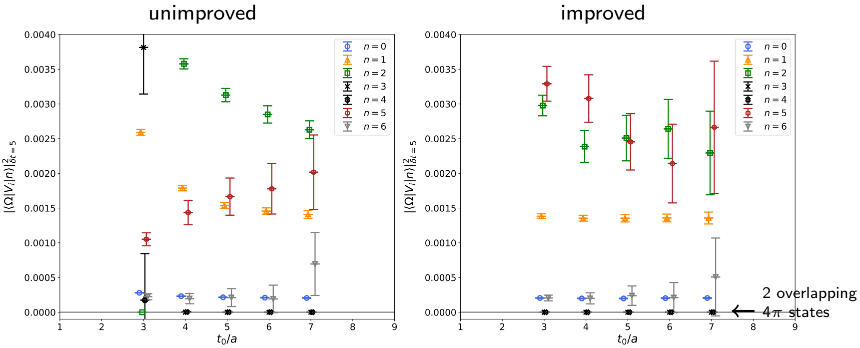
$$C(t_0)\,V = C(t_0+\delta t)\,V\,\Lambda(\delta t) \qquad \text{spectrum}: \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Fixed δt; increase $t_0 \implies$ less excited state contamination

Improved operators:

Faster convergence to plateau, slightly worse statistics

Overlaps - 48I



Generalized EigenValue Problem (GEVP):

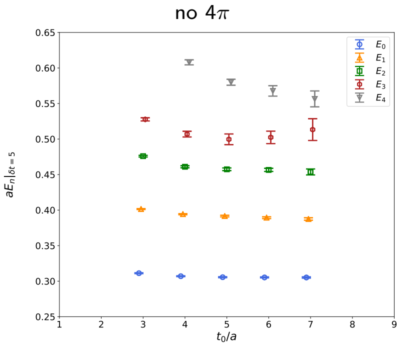
$$C(t_0)\,V = C(t_0+\delta t)\,V\,\Lambda(\delta t) \qquad \text{overlaps}: V_{im} \propto \langle\Omega|\,\mathcal{O}_i\,|m\rangle$$

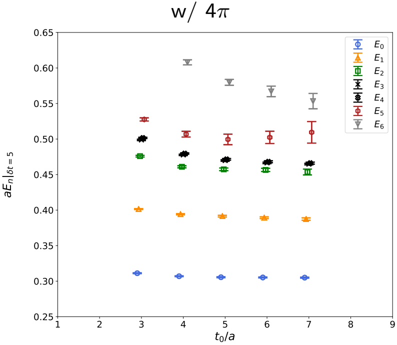
Fixed δt; increase t_0 $\implies$ less excited state contamination

Improved operators:

Faster convergence to plateau, slightly worse statistics

Spectrum - 48I

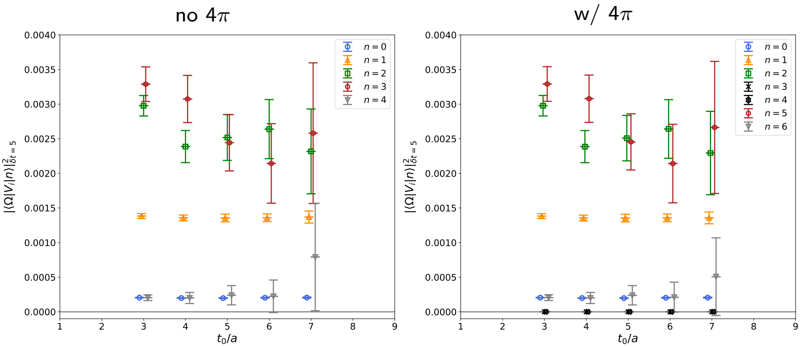




4π operators:

- No change to spectrum of 2π states

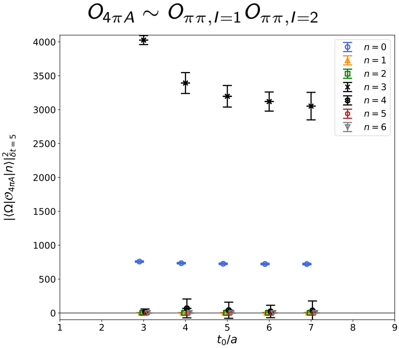
Overlaps - 48I

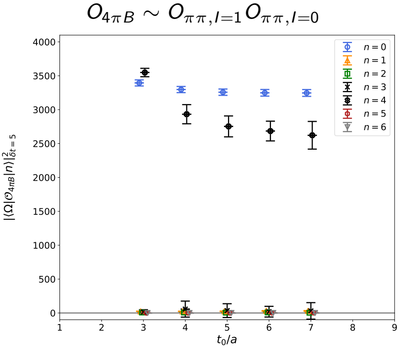


4π operators:

- No change to spectrum of 2π states
- No overlap with local V_μ operator

Overlaps - 48I

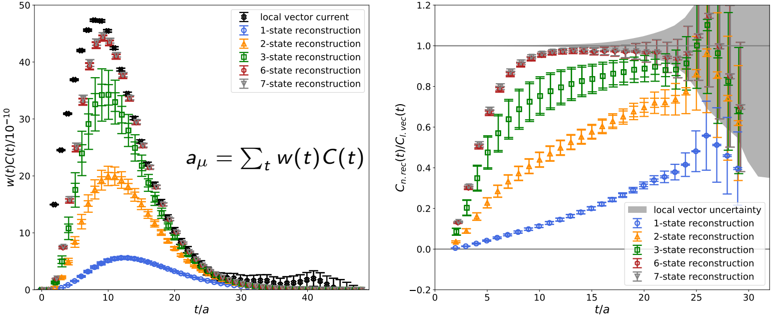




4π operators:

- No change to spectrum of 2π states
- No overlap with local V_μ operator
- Large overlap with 4π states

Correlation Function Reconstruction - 48I



Left: a_μ integrand, Right: ratio reconstruction/local vector

- More states $\implies$ better reconstruction
- 6 state $\implies$ 1σ consistent at $t \geq 16a \sim 1.7\ \mathrm{fm}$

(Improved) Bounding Method

Bounding method [BMW/RBC/UKQCD 2016]:
Use known results in spectrum to make a precise estimate of upper & lower bound on a_μ^{HVP}

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max}) e^{-E(t-t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E \leq E_0$, lowest state in spectrum

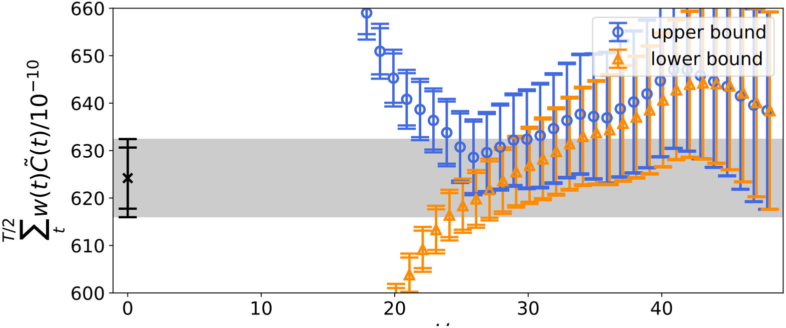
Lower bound: $E \geq \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

Improved bnd. mtd. [RBC/UKQCD 2018 (KEK workshop)]:
Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1} t}$
$\implies$ Smaller overall contribution from neglected states

Add $\sum_t \sum_n^N w(t) |c_n|^2 e^{-E_n t}$ after bounding correlator

Bounding Method Results - 48I



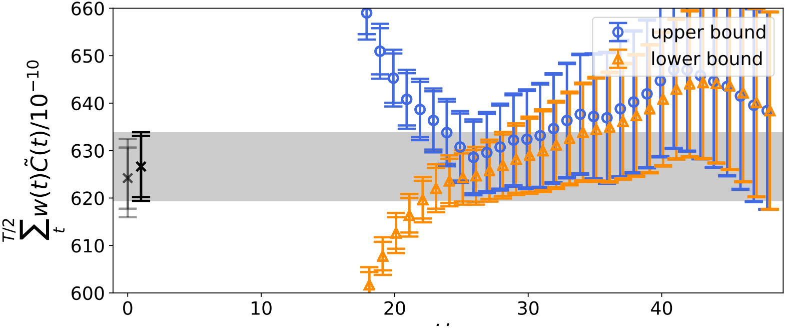
No bounding method: $a_\mu^{HVP} = 638(21)$

Bounding method $t_{\text{max}} = 3.0\ \text{fm}$, no reconstruction: $a_\mu^{HVP} = 624.2(8.2)$

Bounding method gives factor of 2.5 improvement over no bounding method

Improving the bounding method increases gain to factor of 5, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 638(21)$

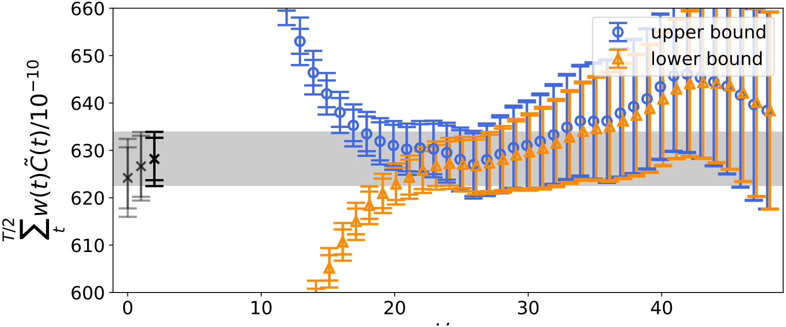
Bounding method $t_{\max} = 3.0$ fm, no reconstruction: $a_\mu^{HVP} = 624.2(8.2)$

Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 626.6(7.2)$

Bounding method gives factor of 2.5 improvement over no bounding method

Improving the bounding method increases gain to factor of 5, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 638(21)$

Bounding method $t_{\text{max}} = 3.0$ fm, no reconstruction: $a_\mu^{HVP} = 624.2(8.2)$

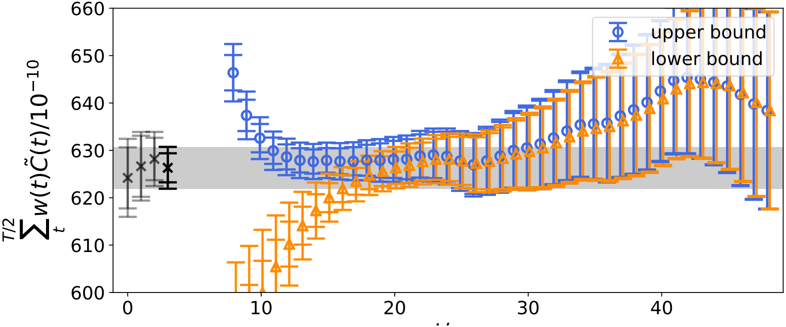
Bounding method $t_{\text{max}} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 626.6(7.2)$

Bounding method $t_{\text{max}} = 2.5$ fm, 2 state reconstruction: $a_\mu^{HVP} = 628.2(5.7)$

Bounding method gives factor of 2.5 improvement over no bounding method

Improving the bounding method increases gain to factor of 5, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 638(21)$

Bounding method $t_{\text{max}} = 3.0$ fm, no reconstruction: $a_\mu^{HVP} = 624.2(8.2)$

Bounding method $t_{\text{max}} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 626.6(7.2)$

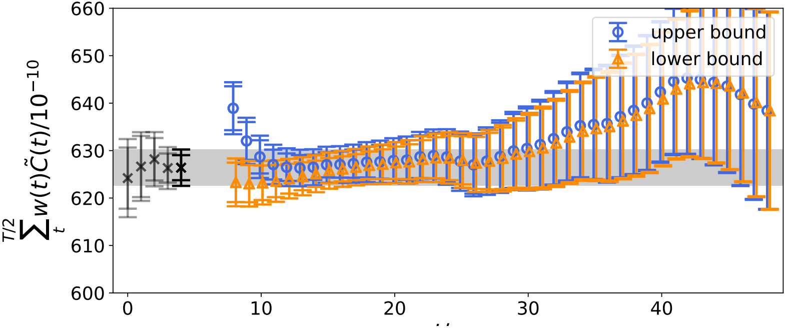
Bounding method $t_{\text{max}} = 2.5$ fm, 2 state reconstruction: $a_\mu^{HVP} = 628.2(5.7)$

Bounding method $t_{\text{max}} = 2.1$ fm, 5 state reconstruction: $a_\mu^{HVP} = 626.3(4.4)$

Bounding method gives factor of 2.5 improvement over no bounding method

Improving the bounding method increases gain to factor of 5, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 638(21)$

Bounding method $t_{\max} = 3.0$ fm, no reconstruction: $a_\mu^{HVP} = 624.2(8.2)$

Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 626.6(7.2)$

Bounding method $t_{\max} = 2.5$ fm, 2 state reconstruction: $a_\mu^{HVP} = 628.2(5.7)$

Bounding method $t_{\max} = 2.1$ fm, 5 state reconstruction: $a_\mu^{HVP} = 626.3(4.4)$

Bounding method $t_{\max} = 1.7$ fm, 6 state reconstruction:

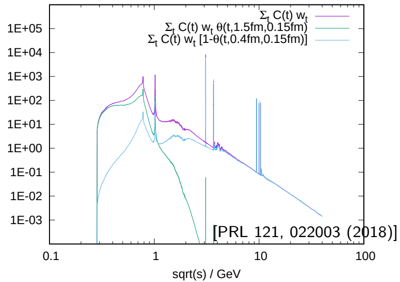
$$a_\mu^{HVP\text{,conn, iso, 48I}} = 626.6(2.7)_{\text{stat}}(0.4)_{Z_V\text{.48I}}(2.6)_{a^{-1}\text{.48I}}(0.5)_{\text{bound}}(0.5)_{\text{exc}}$$

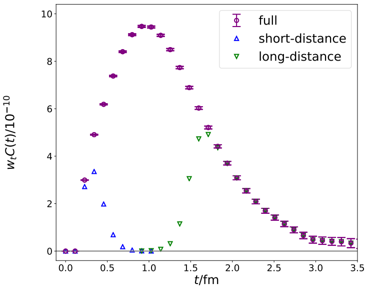
Bounding method gives factor of 2.5 improvement over no bounding method

Improving the bounding method increases gain to factor of 5, including systematics

Window Method/Cross Checks

Window Method





Build smooth step function:

$$\Theta(t, \mu, \Delta) = [1 + \tanh[(t - \mu)/\Delta]]/2$$

$$\implies a_\mu^{\text{conn.}}[t_0, t_1, \Delta] = \sum_t w_t C(t)[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)].$$

Window range $\implies$ $\sqrt{s}$ scale
Use window to isolate ρ, $\pi\pi$ physics

Cross Check Values

$$\Theta(t,\mu,\Delta) = [1+\tanh[(t-\mu)/\Delta]]/2$$

$$\{t_0, t_1, \Delta\} = \{0.4, 1.0, 0.15\}\ [\mathrm{fm}]$$

$$a_{\mu\ \text{short}}^{\text{ud, conn., isospin}} = \sum_t w_t C(t)[1-\Theta(t,t_0,\Delta)]$$

$$a_{\mu\ \text{win.}}^{\text{ud, conn., isospin}} = \sum_t w_t C(t)[\Theta(t,t_0,\Delta)-\Theta(t,t_1,\Delta)]$$

$$a_{\mu}^{\text{ud, conn, isospin}}/10^{-10} = 649.7(14.2)_S(2.8)_C(3.7)_V(1.5)_A(0.4)_Z(0.1)_{\text{E48}}(0.1)_{\text{E64}}$$

$$a_{\mu\ \text{win.}}^{\text{ud, conn, isospin}}/10^{-10} = 202.9(1.4)_S(0.2)_C(0.1)_V(0.2)_A(0.2)_Z$$

$a_{\mu\ \text{short}}^{\text{ud, conn, isospin}}/10^{-10} = 51.6(0.5)$ *[UNPUBLISHED]*

$$\Pi_n = (-1)^{n+1}\sum_t \frac{t^{2n+2}}{(2n+2)!}C(t)$$

$$\Pi_1^{\text{ud, conn, isospin}}/10^{-10} = +\,(Q_{\text{up}}^2 + Q_{\text{down}}^2)\ \mathrm{GeV}^{-2} \times 0.1713(46)_S(12)_V(8)_C$$

$$\Pi_2^{\text{ud, conn, isospin}}/10^{-10} = -\,(Q_{\text{up}}^2 + Q_{\text{down}}^2)\ \mathrm{GeV}^{-4} \times 0.352(37)_S(9)_V(4)_C$$

Published values in Ref. [PRL 121, 022003 (2018)]

Window Comparison

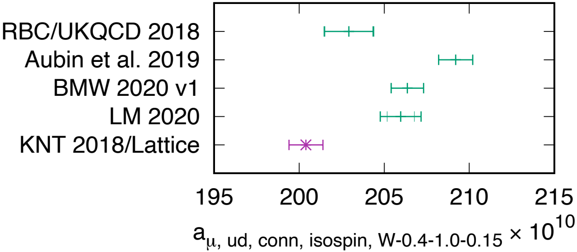
PHYSICAL REVIEW D **101**, 074515 (2020)

Consistency of hadronic vacuum polarization between lattice QCD and the R ratio

Christoph Lehner*
Universität Regensburg, Fakultät fr Physik, 93040, Regensburg, Germany and Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

Aaron S. Meyer†
Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA

(Received 15 March 2020; accepted 30 March 2020; published 23 April 2020)



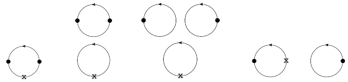
$a_\mu^{\text{ud, conn, isospin}}[0.4, 1.0, 0.15 \text{ fm}]/10^{-10} =$

RBC/UKQCD	$202.9(1.4)_S(0.2)_C(0.1)_V(0.2)_A(0.2)_Z$
Aubin *et al.*	209.2(1.0)
BMW	206.7(0.2)(1.4)
LM	205.97(0.79)(0.90)

Outlook

Other Improvements

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

- Strong Isospin Breaking

  Two computations of derivative:

  Fit valence pion mass dependence of connected

  Explicit calc. of diagram M

- Disconnected Contribution

  Study V dependence

  $a \to 0$ limit

  ++stats

- QED continuum limit & scale

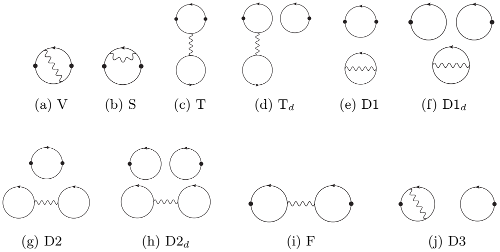
  QED from 2018 PRL now w/ $a \to 0$ limit

  new Z_V from $\langle \pi | V_4 | \pi \rangle$

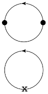
  improved M_Ω analysis

QED w/ HLbL data

QED corrections



- point-src data reused from HLbL calc
- reduced stat error $\approx \times 5 - \times 10$ on HVP QED
  (Mattia Bruno, 1812.09562[hep-lat])
- $V \to \infty$, $a \to 0$ limits for diagrams V,S,F
- first results for T,D1,R; other subleading in preparation
- target QED error $O(1 \times 10^{-10})$



(b) R

RBC/UKQCD 2020 Data on Ensembles at Physical M_π

- new A2A for conn. isospin symm:

24ID	0conf	$\rightarrow$	130conf (multi mass)
32ID	0conf	$\rightarrow$	88conf (multi mass)
48I	127conf	$\rightarrow$	400conf
64I	160conf	$\rightarrow$	250conf
96I:	ongoing data generation		

- new A2A for disc.:

24ID	0conf	$\rightarrow$	260conf (multi mass)
32IDf	0conf	$\rightarrow$	103conf
48I	0conf	$\rightarrow$	33conf

- QED & SIB corrections to meson/Ω masses, Z_V:

48I	0conf	$\rightarrow$	30conf
64I	0conf	$\rightarrow$	30conf

- QED & SIB from HLbL pt-src data:
  24ID, 32ID, 32IDf, 48I : $\sim$20conf each, 2000pt/conf
- Distillation:

24ID	0conf	$\rightarrow$	33conf
32ID	0conf	$\rightarrow$	11conf (multi mass)
48I	0conf	$\rightarrow$	33conf
64I	0conf	$\rightarrow$	(in prog.)

- New operators for M_Ω (control exc. state):
  48I: 130 conf

Conclusions

- Data for next paper almost ready, analysis progressing
- Full control of all systematics for Lattice QCD+QED calc
- Have all ingredients for HVP uncertainty $O(5 \times 10^{-10})$
- With lattice precision improvements, weigh in on BaBar/KLOE tension
- Ongoing analysis of τ decay for independent extraction (M.Bruno)
- On target to reach precision of Fermilab expt by 2022

Thanks for listening!